Generalized SU(2) Proca theory reloaded and beyond

Based on arXiv:2009.03241 [hep-th]

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Classical Gravity

Newton (Universal gravity law)

Einstein (General relativity)

Fundamental or effective?:
we don't know

Singularities → Effective

Breakdown of GR:

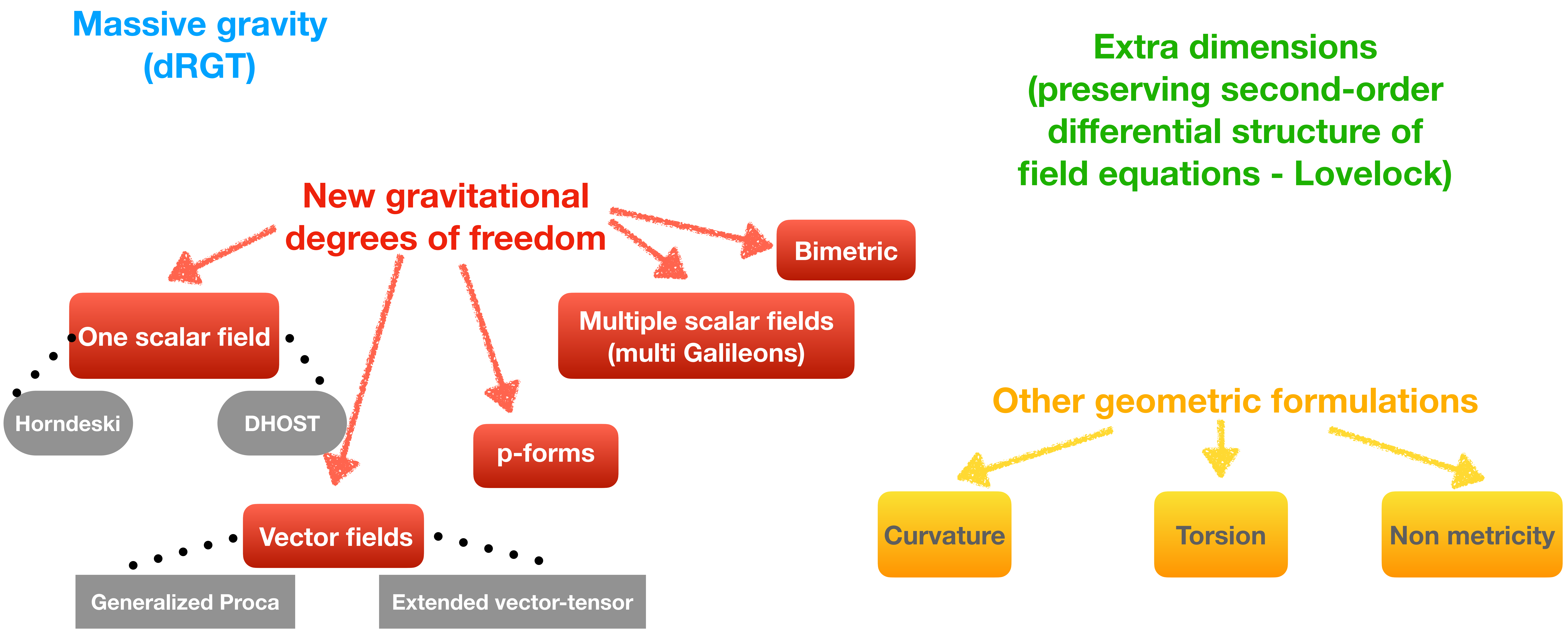
Infrared → True nature of accelerated expansion

Ultraviolet → Renormalizability

Intermediate scale in the strong gravity regime → Departures from GR could be seen in compact objects (multi-messenger astronomy)

Are we in the verge of a scientific crisis?

Different possibilities to extend GR:



L. Heisenberg, Phys. Rep. 2019

Vector fields in gravity and/or cosmology

Why to introduce them?

Why not?

Let's be pragmatical:
there are much more vector fields in nature
than fundamental scalar fields

Despite of these problems:

Ghosts, anisotropies
in cosmology, etc.

→

The role of vector fields in gravitation, astrophysics, and cosmology has attracted a lot of interest in recent years

↓

Generalized Proca theory

Generalized Proca theory

G. Tasinato, JHEP 2014
L. Heisenberg, JCAP 2014
E. Allys, P. Peter, and Y. Rodríguez, JCAP 2016
E. Allys, P. Peter, J. P. Beltrán Almeida, and Y. Rodríguez, JCAP 2016
J. Beltrán Jiménez and L. Heisenberg, Phys. Lett. B 2016

$$\mathcal{L} = \frac{m_p^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + \boxed{\cdots}$$

Proca theory in curved spacetime

Terms that break internal gauge symmetries

Principle of construction:

A_μ has four degrees of freedom but only three can propagate → According to the structure of the irreducible representations of the Poincaré group

Then, it must be degenerate by construction

Its decoupling limit reduces to the Horndeski theory, so it's healthy

Generalized Proca theory in astrophysics and cosmology
L. Heisenberg, Phys. Rep. 2019
Compact objects
Inflation
Dark energy
Too much anisotropy
Restoration of the isotropy
Rapid oscillations
Dilution by a companion scalar field
Suppression of the spatial components against the temporal one
Cosmic triad

Cosmic triad → Particular case of the most general spherically symmetric configuration:

$$A_0^a(r,t) = A_0(r,t)\hat{r}_a$$

$$A_i^a(r,t) = A_1(r,t)\hat{r}_i\hat{r}_a + \frac{a(r,t)}{r}\sin[\omega(r,t)][\delta_{ia} - \hat{r}_i\hat{r}_a] + \frac{a(r,t)\cos[\omega(r,t)] - 1}{r}\epsilon_{ial}\hat{r}_l$$

Except for the suppression of the spatial components against the temporal one, all the other options require global invariance of the action under SU(2)

This is the main motivation for the construction of the generalized SU(2) Proca theory (GSU2P)

E. Witten, Phys. Rev. Lett. 1977
D. Sivers, Phys. Rev. D 1986
P. Forgács and N. S. Manton, Commun. Math. Phys. 1980

GSU2P → Globally invariant under internal SU(2)

Spherically symmetric configuration

It spontaneously breaks:

1. the internal SU(2) symmetry
2. the Lorentz rotational symmetry
3. the Lorentz boosts

This is, anyway, extraordinarily reasonable: it seems to be nature's strategy to produce all patterns we see in condensed matter systems (fluids, superfluids, solids, supersolids)

A. Nicolis, R. Penco, F. Piazza, and R. Rattazzi, JHEP 2015

Construction of GSU2P:

To avoid the propagation of the fourth degree of freedom

- A primary constraint enforcing relation
- A secondary constraint enforcing relation

Already implemented in the first version of the GSU2P (the “old” GSU2P)

E. Allys, P. Peter, and Y. Rodríguez, Phys. Rev. D 2016
J. Beltrán Jiménez and L. Heisenberg, Phys. Lett. B 2017

Later discovered. Trivially satisfied for the generalized Proca theory but not for the GSU2P

V. Errasti Díez, B. Gording, J. A. Méndez-Zavaleta and A. Schmidt-May, Phys. Rev. D 2020

The old GSU2P

Implemented by (in strict order):

- The construction of Lorentz-invariant and group-invariant Lagrangian building blocks
- The primary constraint enforcing relation
- The remotion of redundant terms via total derivatives
- The covariantization
- The addition of healthy terms that only exist in curved spacetime
- The addition of counterterms to make the decoupling limit healthy

Healthiness: second-order field equations

E. Allys, P. Peter, and Y. Rodríguez, Phys. Rev. D 2016

The old GSU2P

Caveats:

- The secondary constraint enforcing relation
- The remotion of redundant terms before the covariantization

The remotion was performed via total derivatives that do not satisfy, in general, the secondary constraint enforcing relation

This prevented us from finding the beyond GSU2P

A. Gallego Cadavid and Y. Rodríguez, Phys. Lett. B 2019

Reconstructing the GSU2P

- Decomposition of $\partial_\mu A^a_\nu$ into

$$S^a_{\mu\nu} \equiv \partial_\mu A^a_\nu + \partial_\nu A^a_\mu \qquad \text{(its symmetric part)}$$

$$A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \qquad \text{(its antisymmetric part)}$$

(this allows us to deal with fewer Lagrangian building blocks)

- The primary constraint enforcing relation

$$\mathcal{H}^{0\nu}_{ab} = 0 \quad \text{with} \quad \mathcal{H}^{\mu\nu}_{ab} \equiv \frac{\partial^2 \mathcal{L}}{\partial \dot{A}^a_\mu \partial \dot{A}^b_\nu}$$

- The secondary constraint enforcing relation

$$\tilde{\mathcal{H}}^{00}_{ab} = 0 \quad \text{with} \quad \tilde{\mathcal{H}}^{\mu\nu}_{ab} \equiv \frac{\partial^2 \mathcal{L}}{\partial \dot{A}^{[a}_\mu \partial A^{b]}_\nu}$$

(Thanks to the antisymmetry of $A^a_{\mu\nu}$, any Lagrangian $\mathcal{L}^A_i = \mathcal{L}^A_i(A^a_{\mu\nu}, A^a_\mu)$ satisfies both constraint enforcing relations)

The reloaded GSU2P and beyond

(up to six space-time indices in the Lagrangian building blocks before contractions)

$$\mathcal{L}_2 = \mathcal{L}_2(A^a_{\mu\nu}, A^a_\mu)$$

$$\mathcal{L}_{4,0} = G_{\mu\nu} A^{\mu a} A^\nu_a$$

$$\mathcal{L}_{4,2} = \sum_{i=1}^{6} \frac{\alpha_i}{m_P^2} \mathcal{L}^i_{4,2} + \sum_{i=1}^{3} \frac{\tilde{\alpha}_i}{m_P^2} \tilde{\mathcal{L}}^i_{4,2}$$

$$\tilde{\mathcal{L}}_{5,0} = A^{\nu a} R^\sigma{}_{\nu\rho\mu} A^b_\sigma \tilde{A}^{\mu\rho c} \epsilon_{abc}$$

$$\begin{aligned}
\mathcal{L}^1_{4,2} =& (A_b \cdot A^b)[S^{\mu a}_\mu S^\nu_{\nu a} - S^{\mu a}_\nu S^\nu_{\mu a}] \\
&+ 2(A_a \cdot A_b)[S^{\mu a}_\mu S^{\nu b}_\nu - S^{\mu a}_\nu S^{\nu b}_\mu] \\
\mathcal{L}^2_{4,2} =& A^a_{\mu\nu} S^{\mu b}_\sigma A^\nu_a A^\sigma_b - A^a_{\mu\nu} S^{\mu b}_\sigma A^\nu_b A^\sigma_a + A^a_{\mu\nu} S^{\rho b}_\rho A^\mu_a A^\nu_b \\
\mathcal{L}^3_{4,2} =& A^{\mu a} R^\alpha{}_{\sigma\rho\mu} A_{\alpha a} A^{\rho b} A^\sigma_b + \frac{3}{4}(A_b \cdot A^b)(A^a \cdot A_a) R \\
\mathcal{L}^4_{4,2} =& [(A_b \cdot A^b)(A^a \cdot A_a) + 2(A_a \cdot A_b)(A^a \cdot A^b)] R \\
\mathcal{L}^5_{4,2} =& G_{\mu\nu} A^{\mu a} A^\nu_a (A^b \cdot A_b) \\
\mathcal{L}^6_{4,2} =& G_{\mu\nu} A^{\mu a} A^{\nu b} (A_a \cdot A_b) \\
\tilde{\mathcal{L}}^1_{4,2} =& A^a_{\mu\nu} S^{\mu b}_\sigma A_{\alpha a} A_{\beta b} \epsilon^{\nu\sigma\alpha\beta} - \tilde{A}^{\alpha\beta}_a S^b_{\rho\alpha} A^{\rho a} A_{\beta b} \\
&+ \tilde{A}^{\alpha\beta}_a S^\rho_{\rho b} A^a_\alpha A^b_\beta \\
\tilde{\mathcal{L}}^2_{4,2} =& A^b_\beta R^\alpha{}_{\sigma\rho\mu} A^a_\alpha (A_a \cdot A_b) \epsilon^{\mu\rho\sigma\beta} \\
\tilde{\mathcal{L}}^3_{4,2} =& A_{\beta a} R^\alpha{}_{\sigma\rho\mu} A^a_\alpha (A^b \cdot A_b) \epsilon^{\mu\rho\sigma\beta}
\end{aligned}$$

Open questions

Technical questions:

- Does the theory propagate the right number of degrees of freedom in curved spacetime?
- Stability issues (partially studied in [1])
- The cutoff scale of the theory

Consistency with observations:

- The causal structure of the theory
- Does there exist a screening mechanism?
- Applications to inflation
- Applications to dark energy (partially explored in [2])
- Consistency with GW170817

[1] L. G. Gómez and Y. Rodríguez, Phys. Rev. D 2019

[2] Y. Rodríguez and A. A. Navarro, Phys. Dark Univ. 2018

Is the GSU2P a good candidate for an effective theory for the gravitational interaction?

“To be complete a theory of gravity must be capable of analyzing from “first principles” the outcome of every experiment of interest. It must therefore mesh with and incorporate a consistent set of laws for electromagnetism, quantum mechanics, and all other physics.”

C. W. Misner, K. S. Thorne, and J. A. Wheeler, “Gravitation”, 1973