Dark Energy in Five Dimensional Spherically Symmetric Universe with Future Singularity

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CoCo 2o2o: Cosmology in Colombia

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Overview

1. Abstract
2. Introduction
3. Formulation of problem with solutions
4. Discussion
5. Conclusions

Abstract

Within the limits of the present cosmological observations, an interacting model of holographic dark energy and matter in a five-dimensional spherically symmetric space-time setting has been analyzed within the framework of Brans-Dicke Theory. We obtain a model universe that undergoes super-exponential expansion. It is predicted that the universe is isotropic and will be continuously dark energy dominated. The universe doesn't evolve from an initial singularity but, ultimately ends at the big crunch singularity. The values of the Hubble's parameter, dark energy and matter density parameters are obtained as $H = 68.027, \Omega_{de} = 0.741$ and $\Omega_m = 0.203$ respectively which are very close to the values estimated by the latest Planck 2018 results.

Introduction

- Dark energy (DE) [1,2] with its huge negative pressure with gravity defying effect causes the accelerated expansion of the universe.
- DE is considered to be uniformly distributed and vary slowly or unchanged with time [3-6] .

1. A. G. Riess *et al.*, Astron. J. **116**, (1998) 1009.
2. S. Perlmutter *et al.*, Astrophys. J. **517**, (1999) 565.
3. M. H. Chan, J. Gravity **2015**, (2015) 384673.
4. S. M. Carroll, Living Rev. Rel. **4**, (2001) 1.
5. S. M. Carroll, arXiv:astro-ph/0107571.
6. P. J. Peebles, B. Ratra, Rev. Mod. Phys. **75**, (2003) 559.

Introduction

- Equation of state paramater (EoS) ω is studied to understand the nature and properties of DE. It is the ratio its pressure to density.
- $\omega = -1.03 \pm 0.03$ [7]
- To precisely understand the accelerating phenomenon, two well appreciated methods have been adopted. Firstly, different possible forms of DE are developed. Secondly, modifying the Einstein's theory of gravitation (ETG).

7. Planck Collaboration *et al.*, arXiv:1807.06209.

Introduction

- The natural candidate for DE is the cosmological constant with $\omega = -1$. But, it fails to illustrate many riddles of physics, e.g. the coincidence problem [8]. Therefore, many other viable candidates of dark energy have been introduced [9].
- Holographic dark energy (HDE), introduced by Gerardt Hooft [10], is obtained by the application of holographic principle [11] to DE. Accordingly, all the physical quantities inside the universe including the energy density of DE can be illustrated by some quantities on the boundary of the universe [12].

8. I. Zlatev, Phys. Rev. Lett. **82**, (1999) 896.
9. E.J. Copeland, Int. J. Mod. Phys. D **15**, (2006) 1753.
10. G.t. Hooft, arXiv:/gr-qc/9310026.
11. R. Bousso, Rev. Mod. Phys. **74**, 825 (2002).
12. S. Wang *et al.*, Phys Rep **696**, 1 (2017).

Introduction

- Brans-Dicke Theory (BDT) is an optimized modifications of the ETG equipped with a scalar field φ representing the space-time varying gravitational constant with the physical effect of interfering gravity, besides the metric.
- BDT is a good option to study DE and the accelerating universe.
  - φ and the theory itself can be regarded as possible contributor in the late time evolution [13]
  - BDT contributes to the present cosmic acceleration [14].
  - DBT itself is equivalent to a DE model [15].

13. H. Kim, Mon. Not. R. Astron. Soc. **364**, (2005) 813.
14. N. Banerjee, D. Pavon, Phys. Rev. D **63**, (2001) 043504.
15. A. Joyce *et al.*, Annu. Rev. Nucl. Part. Sci. **66**, (2016) 95.

Formulation of problem with solutions

The spherically symmetric metric of the following form is considered

$$ds^2 = dt^2 - e^{\mu}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right) - e^{\delta} dy^2 \tag{1}$$

where $\mu = \mu(t)$ and $\delta = \delta(t)$ are cosmic scale factors.
The Brans-Dicke field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R + \omega_{BD} \varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2} g_{ij}\varphi_{,k}\varphi^{,k}\right) + \varphi^{-1}\left(\varphi_{i;j} - g_{ij}\varphi^{,k}_{;k}\right) = -8\pi\varphi^{-1}\left(T_{ij} + S_{ij}\right) \tag{2}$$

where φ, T_{ij}, S_{ij} ,R and R_{ij} denote the Brans-Dicke scalar field, energy momentum tensors of matter, energy momentum tensors of HDE, Ricci scalar and Ricci tensor respectively.

Formulation of problem with solutions

T_{ij} and S_{ij} are defined as

$$T_{ij} = \rho_m u_i u_j \tag{3}$$

$$S_{ij} = (\rho_{de} + p_{de})\, u_i u_j - g_{ij} p_{de}, \tag{4}$$

where ρ_m, ρ_{de} and p_{de} denote the energy density of matter, energy density of HDE and pressure of the HDE respectively.
For energy conservation, we have

$$T^{ij}_{;j} + S^{ij}_{;j} = 0 \tag{5}$$

Here, φ satisfy the following wave equation

$$\varphi^{k}_{;k} = 8\pi \left(3 + 2\omega_{_{BD}}\right)^{-1} (T + S) \tag{6}$$

Formulation of problem with solutions

The surviving field equations are obtained as follows

$$\ddot{\mu}+\frac{3}{4}\dot{\mu}^2+\frac{\ddot{\delta}}{2}+\frac{\dot{\delta}^2}{4}+\frac{\dot{\mu}\dot{\delta}}{2}+\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\ddot{\varphi}}{\varphi}+\frac{\dot{\varphi}}{\varphi}\left(\frac{2\dot{\mu}+\dot{\delta}}{2}\right)=-8\pi\varphi^{-1}p_{de} \quad (7)$$

$$\frac{3}{2}\left(\ddot{\mu}+\dot{\mu}^2\right)+\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\ddot{\varphi}}{\varphi}+\frac{3}{2}\frac{\dot{\varphi}}{\varphi}\dot{\mu}=-8\pi\varphi^{-1}p_{de} \quad (8)$$

$$\frac{3}{4}\left(\dot{\mu}^2+\dot{\mu}\dot{\delta}\right)-\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\dot{\varphi}}{\varphi}\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=8\pi\varphi^{-1}\left(\rho_m+\rho_{de}\right) \quad (9)$$

From eq. (6)[1], we have

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=8\pi\left(3+2\omega_{BD}\right)^{-1}\left(\rho_m-4p_{de}+\rho_{de}\right) \quad (10)$$

where an overhead dot denotes differentiation w.r.t time t.

[1] $\varphi^{k}_{;k}=8\pi\left(3+2\omega_{BD}\right)^{-1}\left(T+S\right)$

Formulation of problem with solutions

We assume ω as the EoS parameter of the HDE and hence, we have

$$p_{de} = \omega \rho_{de} \tag{11}$$

The conservation equation takes the obvious form as given by

$$\rho_m \left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \dot{\rho}_m + \dot{\rho}_{de} + \rho_{de}(1+\omega)\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) = 0 \tag{12}$$

Due to their minimal interaction, HDE and matter conserve separately so that by refs. [16,17], we obtain

$$\rho_m \left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \dot{\rho}_m = 0 \tag{13}$$

$$\rho_{de}(1+\omega)\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \dot{\rho}_{de} = 0 \tag{14}$$

16. S. Sarkar, Astrophys. Space Sci. **349**, (2014) 985.
17. S. Sarkar, Astrophys. Space Sci. **352**, (2014) 245.

Formulation of problem with solutions

Also,

$$(\rho + p)\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) + \dot{\rho} = 0 \tag{15}$$

From eqs. (7)[2] and (8)[3], the expression for the cosmic scale factors are obtained as

$$\mu = l_1 + \log(n - t)^{-\frac{2}{3}} \tag{16}$$

$$\delta = m_1 + \log(n - t)^{-\frac{2}{3}} \tag{17}$$

where l_1, m_1 and n are arbitrary constants.

[2] $\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} + \frac{\omega}{2}\frac{\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{2\dot{\mu}+\dot{\delta}}{2}\right) = -8\pi\varphi^{-1}p_{de}$

[3] $\frac{3}{2}\left(\ddot{\mu} + \dot{\mu}^2\right) + \frac{\omega}{2}\frac{\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{3}{2}\frac{\dot{\varphi}}{\varphi}\dot{\mu} = -8\pi\varphi^{-1}p_{de}$

Formulation of problem with solutions

Now, from eqs. (13)[4], (14)[5], (16)[6] and (17)[7], we have

$$\rho_m = l_0 e^{-\frac{1}{2}(3l_1+m_1)} (n-t)^{-\frac{4}{3}} \tag{18}$$

$$\rho_{de} = m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)} (n-t)^{\frac{4}{3}(1+\omega)} \tag{19}$$

so that the energy density of our universe is given by

$$\rho = \rho_m + \rho_{de} \tag{20}$$

where l_0 and m_0 are arbitrary constants.

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[4] $\rho_m \left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right) + \dot{\rho}_m = 0$

[5] $\rho_{de}(1+\omega)\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right) + \dot{\rho}_{de} = 0$

[6] $\mu = l_1 + \log(n-t)^{-\frac{2}{3}}$

[7] $\delta = m_1 + \log(n-t)^{-\frac{2}{3}}$

Formulation of problem with solutions

Again, using eqs. (16)[8], (17)[9] and (20)[10] in equation (15)[11], the pressure of our universe is obtained as

$$p = \frac{1}{3} l_0 e^{-\frac{1}{2}(3l_1+m_1)} (n-t)^{-\frac{4}{3}} + m_0 \left(\frac{4\omega+1}{3} \right) e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)} (n-t)^{\frac{4}{3}(1+\omega)} \tag{21}$$

From eqs. (12) and (22), the pressure of DE is given by

$$p_{de} = \omega m_0 e^{-\frac{1}{2}(3l_1+m_1)(1+\omega)} (n-t)^{\frac{4}{3}(1+\omega)} \tag{22}$$

[8] $\mu = l_1 + \log (n-t)^{-\frac{2}{3}}$

[9] $\delta = m_1 + \log (n-t)^{-\frac{2}{3}}$

[10] $\rho = l_0 e^{-\frac{1}{2}(3l_1+m_1)} (n-t)^{-\frac{4}{3}} + m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)} (n-t)^{\frac{4}{3}(1+\omega)}$

[11] $(\rho + p) \left(\frac{3\dot{\mu}+\dot{\delta}}{2} \right) + \dot{\rho} = 0$

Formulation of problem with solutions

At any time $t = t_0$, we can assume that $p = p_{de}$ so that from eqs. (21)[12] and (22)[13], we have

$$l_0 e^x (n - t_0)^{-\frac{4}{3}} + m_0(1+\omega) e^{(1+\omega)x} (n - t_0)^{\frac{4}{3}(1+\omega)} = 0 \qquad (23)$$

where $x = -\frac{1}{2}(3l_1 + m_1)$

Eq.(23) will give us the expression for EoS parameter ω.

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[12] $p = \frac{1}{3} l_0 e^{-\frac{1}{2}(3l_1+m_1)} (n-t)^{-\frac{4}{3}} + m_0 \left(\frac{4\omega+1}{3}\right) e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)} (n-t)^{\frac{4}{3}(1+\omega)}$

[13] $p_{de} = \omega\, m_0 e^{-\frac{1}{2}(3l_1+m_1)(1+\omega)} (n-t)^{\frac{4}{3}(1+\omega)}$

Formulation of problem with solutions

Using eqs. (11)[14], (16)[15], (17)[16] , (18)[17] and (19)[18] in eq. (10)[19], the scalar field φ is obtained as below

$$\varphi = A_0 \left(n-t\right)^{\frac{2}{3}} + B_0 \left(n-t\right)^{\frac{10+4\omega}{3}} \tag{24}$$

where

$$A_0 = \frac{36}{35}\pi\, l_0 \left(3+2\omega\right)^{-1} e^{-\frac{1}{2}\left(3l_1+m_1\right)} \tag{25}$$

$$B_0 = 8\pi m_0 \left(1-4\omega\right)\left(3+2\omega\right)^{-1}\left(\frac{10+4\omega}{3}\right)^{-1}\left(\frac{11+4\omega}{3}\right)^{-1} e^{\frac{-1}{2}(1+\omega)(3l_1+m_1} \tag{26}$$

[14] $p_{de} = \omega\rho_{de}$

[15] $\rho_m \left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right) + \dot{\rho}_m = 0$

[16] $\delta = m_1 + \log\left(n-t\right)^{-\frac{2}{3}}$

[17] $\rho_m = l_0 e^{-\frac{1}{2}\left(3l_1+m_1\right)}\left(n-t\right)^{-\frac{4}{3}}$

[18] $\rho_{de} = m_0 e^{-\frac{1}{2}(1+\omega)\left(3l_1+m_1\right)}\left(n-t\right)^{\frac{4}{3}(1+\omega)}$

[19] $\ddot{\varphi} + \dot{\varphi}\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right) = 8\pi\left(3+2\omega\right)^{-1}\left(\rho_m - 4p_{de} + \rho_{de}\right)$

Formulation of problem with solutions

Finally, the expressions of the different cosmological parameters are obtained as follows.

Spatial volume:

$$v = e^{\frac{3l_1+m_1}{2}}(n-t)^{-\frac{4}{3}} \tag{27}$$

Scalar expansion:

$$\theta = \frac{4}{3}(n-t)^{-1} \tag{28}$$

Hubble's parameter:

$$H = \frac{1}{3}(n-t)^{-1} \tag{29}$$

Deceleration parameter:

$$q = -4 \tag{30}$$

Shear scalar:

$$\sigma^2 = \frac{2}{9}\left(\frac{1}{n-t} - 1\right)^2 \tag{31}$$

Formulation of problem with solutions

Anisotropic parameter:

$$A_h = 0 \tag{32}$$

Dark energy density parameter:

$$\Omega_{de} = \frac{\rho_{de}}{3H^2} = 3m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}(n-t)^{\frac{2}{3}(5+2\omega)} \tag{33}$$

Matter density parameter:

$$\Omega_m = \frac{\rho_m}{3H^2} = 3l_0 e^{-\frac{1}{2}(3l_1+m_1)}(n-t)^{\frac{2}{3}} \tag{34}$$

Overall density parameter:

$$\Omega = 3\left(l_0 e^{-\frac{1}{2}(3l_1+m_1)} + m_0 e^{-\frac{1}{2}(1+\omega)(3l_1+m_1)}(n-t)^{\frac{8+4\omega}{3}}\right)(n-t)^{\frac{2}{3}} \tag{35}$$

Jerk parameter:

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} = 28 \tag{36}$$

Discussion

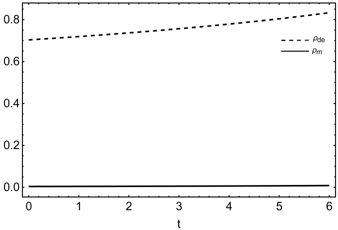


Figure: 1. DE density ρ_{de} and DM density ρ_m with t when $l_0 = l_1 = m_0 = m_1 = 1, n = 14.1245$.

- ρ_{de} increases slowly as DE varies slowly or unchanged with time [3-6]
- ρ_m increases negligibly.
- So, the model universe is increasingly DE dominated.

3. M. H. Chan, J. Gravity **2015**, (2015) 384673.
4. S. M. Carroll, Living Rev. Rel. **4**, (2001) 1.
5. S. M. Carroll, arXiv:astro-ph/0107571.
6. P. J. Peebles, B. Ratra, Rev. Mod. Phys. **75**, (2003) 559.

Discussion

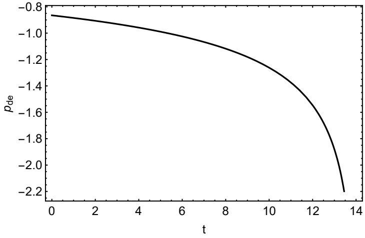
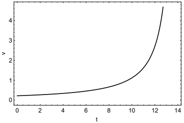


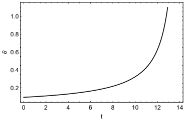
Figure: 2. DE pressure p_{de} with t when $l_1 = m_0 = m_1 = 1, n = 14.1245$.

- p_{de} ranges in the negative plane.
- Agrees with the property of DE which causes the accelerated expansion of the universe.

Discussion



(a) Volume v with t



(b) Scalar expansion θ with t

Figure: 3. v and θ with t when $l_1 = m_1 = 1, n = 14.1245$.

- v and θ are increasing, implying the accelerated expansion.
- At $t = 0$, $v =$ constant($\neq 0$) . Free from initial singularity.
- At $t \to \infty$, both v and $\theta \to 0$ as DE might start to diminish rapidly than DM resulting DE to vanish at $t \to \infty$ [6].
- The expansion will start to decelerate, there will be domination by gravity and ultimately ending at the big crunch singularity.

6. P. J. Peebles, B. Ratra, Rev. Mod. Phys. **75**, (2003) 559

Discussion

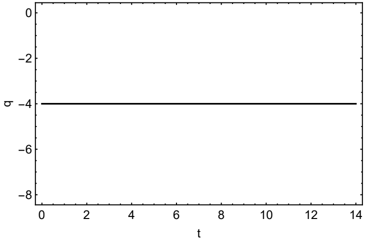


Figure: 4. Deceleration parameter q vs t

- $q = -4$ throughout evolution.
- As $q < -1$, the model universe experiences super-exponential expansion all through [18].

18. G.P. Singh, B.K. Bishi, Adv. High Energy Phys. **2017**, (2017) 1390572.

Discussion

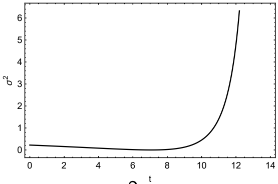


Figure: 5. Shear scalar σ^2 with t when $n = 14.1245$.

- σ^2 shows us the rate of deformation of the matter flow within the massive cosmos [19].
- σ^2 decreases very slowly at the early stage and then, diverges. From eq. (35), the anisotropic parameter $A_h = 0$.
- The isotropic universe expands with a slow and uniform change of shape in the early evolution whereas the change tends to become faster at late times.

19. G.F.R. Ellis, H.V. Elst, NATO Adv. Study Inst. Ser. C. Math. Phys. Sci. **541**, (1999) 1.

Discussion

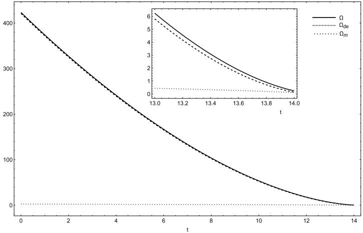


Figure: 6. Overall density parameter Ω, DE density parameter Ω_{de} and DM density parameter Ω_m with t when $l_0 = l_1 = m_0 = m_1 = 1, n = 14.1245$.

- Ω and Ω_{de} decrease with t and tend to become constant.
- Ω_m decreases with a greater extent as matter density decreases as galaxies move apart from each other due to expansion [5]
- Implying that that the universe is be continuously DE dominated.

5. S. M. Carroll, arXiv:astro-ph/0107571.

Discussion

- The present age of the universe by the latest Planck 2018 result [7] - 13.787 ± 0.020 Gyr.
- At $t_0 = 13.77$, eq. (23)[20] gives $\omega = -1.23$.
- With $\omega = -1.23$ at $t_0 = 13.77$, the present age of the universe, we obtain
  - $\Omega_{de} = 0.741$
  - $\Omega_m = 0.203$
  - $\Omega \approx 1$ (present cosmological belief)
  - These values are very close to $\Omega_m = 0.256^{+0.023}_{-0.031}$ and $\Omega_{de} = 0.6847 \pm 0.0073$ of the latest Planck 2018 result [7].

[20] $l_0 e^x (n - t_0)^{-\frac{4}{3}} + m_0(1+\omega) e^{(1+\omega)x} (n - t_0)^{\frac{4}{3}(1+\omega)} = 0$

Discussion

- At at $t = 13.77$, when $n = 13.7749$, from eq. (32)[21], Hubble's parameter $H = 68.027$, almost equal to the value $H_0 = 67.36 \pm 0.54 \text{km}^{-1}\text{Mpc}^{-1}$ of the latest Planck 2018 result [7].
- Transition from decelerating to accelerating phase occurs when jerk parameter is positive and deceleration parameter is negative [21]. From eqs. (30)[22] and (36)[23], this condition is satisfied in our model.

[21] $H = \frac{1}{3}(n-t)^{-1}$

[22] $q = -4$

[23] $j(t) = q + 2q^2 - \frac{\dot{q}}{H} = 28$

7. Planck Collaboration *et al.*, arXiv:1807.06209v2. (2019)

21. K.P. Singh, P.S. Singh, Chin. J. Phys., **60**, (2019) 239.

Conclusions

- The model universe is increasingly DE dominated.
- The pressure of DE is negative all through which agrees with the mystic property of DE causing the accelerated expansion of the universe.
- The universe doesn't evolve from an initial singularity but ends at the big crunch singularity.
- The universe is isotropic and experiences super-exponential expansion.
- The model universe changes its shape slowly and uniformly in the early evolution whereas the change tends to become faster at late times.

Conclusions

- At $t_0 = 13.77$ Gyr, the present age of the universe, we obtain $\Omega_{de} = 0.741$ and $\Omega_m = 0.203$, very close to the values of the latest Planck 2018 result [7].
- $\Omega \approx 1$ which is a present cosmological belief.
- At $t_0 = 13.77$ Gyr, Hubble's parameter $H = 68.027$, almost equal to the value of the latest Planck 2018 result [7].

7. Planck Collaboration *et al.*, arXiv:1807.06209.

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Thank You