Anisotropic Einstein Yang-Mills Higgs Dark Energy

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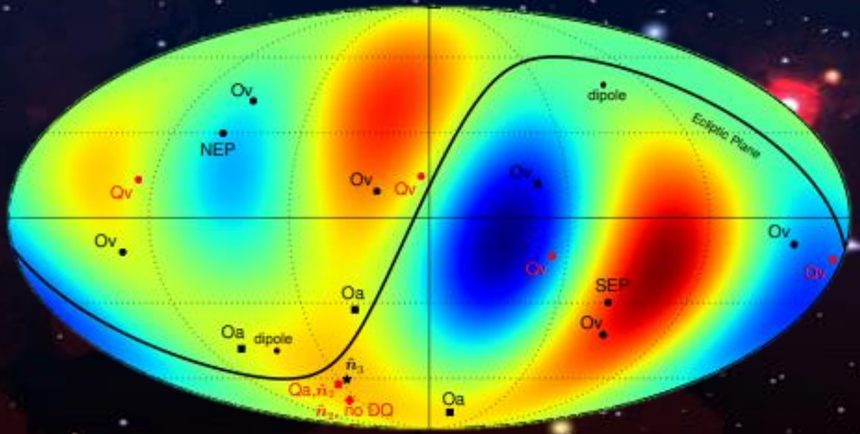
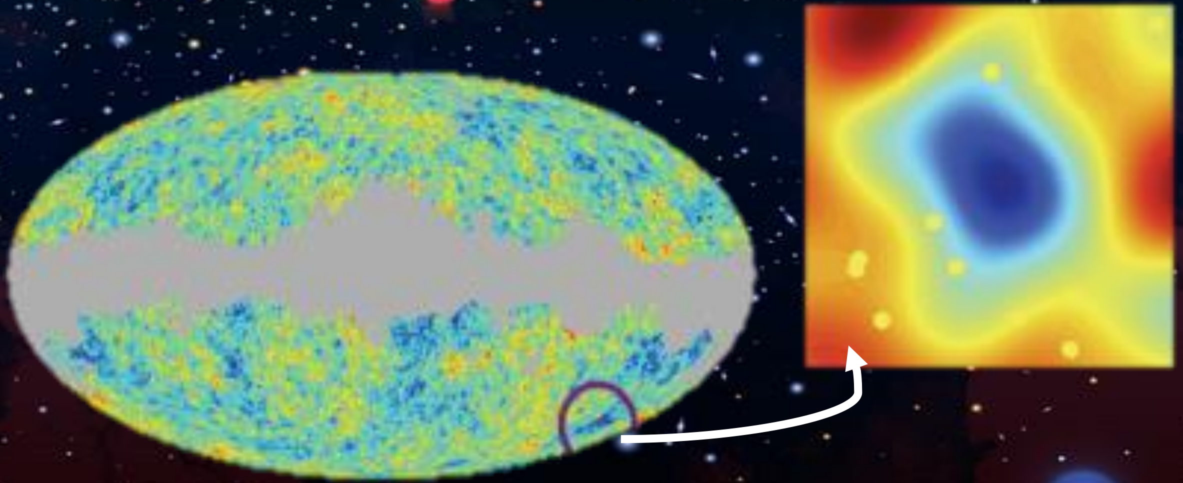
The Problems

$$|\Sigma_0| \leq \mathcal{O}(0.001)$$





ArXiv: 1510.07929 (Schwarz et al), 1012.5596 (Campanelli et al), 1802.04251 (Amirhashchi et al)

The Antecedents

Action →

$$S = \int \mathrm{d}^4x \sqrt{-\mathrm{det} g_{\mu\nu}} \left[\frac{m_{\mathrm{P}}^2}{2} R - \frac{1}{4} F^a{}_{\mu\nu} F_a{}^{\mu\nu} - \frac{1}{2} D_\mu \mathcal{H}^a D^\mu \mathcal{H}_a - V\left(\mathcal{H}^2\right) + \mathcal{L}_m + \mathcal{L}_r \right]$$

Covariant derivative

$$D_\mu \mathcal{H}^a \equiv \nabla_\mu \mathcal{H}^a + g \varepsilon^a{}_{bc} A^b{}_\mu \mathcal{H}^c$$

$$\mathcal{H} \equiv (\mathcal{H}_1 \quad \mathcal{H}_2 \quad \mathcal{H}_3)^{\mathrm{T}}$$

Mexican Hat Potential

$$V(\mathcal{H}^2) \equiv \frac{\lambda}{4} \left(\mathcal{H}^2 - \mathcal{H}_0^2\right)^2$$

FLRW Universe → Homogeneous and Isotropic

$$\mathrm{d}s^2 \equiv -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Cosmic Triad

$$A_0^a(t) \equiv 0\,, \quad A^a{}_i \equiv a(t)\psi(t)\delta^a{}_i$$

- 1508.04576 (Rinaldi).

The Inconsistencies

$$T_{\mu\nu} \equiv g_{\mu\nu}\mathcal{L}_{\rm mat} - 2\frac{\delta \mathcal{L}_{\rm mat}}{\delta g^{\mu\nu}}$$

$$2\frac{\delta \mathcal{L}_{\rm mat}}{\delta g^{\mu\nu}}\bigg|_{\mu\nu=i\neq j} = g^2\phi^2\mathcal{H}_i\mathcal{H}_j$$

Anisotropic Stress

$$2\frac{\delta \mathcal{L}_{\rm mat}}{\delta g^{\mu\nu}}\bigg|_{\mu\nu=0i} = g\,\phi\,\varepsilon_{aib}\dot{\mathcal{H}}^a\mathcal{H}^b$$

Momentum Density

Different Behaviours

$$\ddot{\phi} + H\dot{\phi} + \frac{2g^2\phi^3}{a^2} + g^2\phi\left(\mathcal{H}_2^2 + \mathcal{H}_3^2\right) = 0\,,\ a, i = 1$$

$$\ddot{\phi} + H\dot{\phi} + \frac{2g^2\phi^3}{a^2} + g^2\phi\left(\mathcal{H}_1^2 + \mathcal{H}_3^2\right) = 0\,,\ a, i = 2$$

$$\ddot{\phi} + H\dot{\phi} + \frac{2g^2\phi^3}{a^2} + g^2\phi\left(\mathcal{H}_1^2 + \mathcal{H}_2^2\right) = 0\,,\ a, i = 3$$

$$\mathcal{H}(t) \equiv (\Phi(t)\,,\ 0\,,\ 0)$$

$$T_{11} = a^2\left(\frac{1}{2}\frac{\dot{\phi}}{a^2} + \frac{1}{2}\frac{g^2\phi^4}{a^4} + \frac{1}{2}\dot{\Phi}^2 - \frac{g^2\phi^2\Phi^2}{a^2} - V\right)$$

$$T_{22} = T_{33} = a^2\left(\frac{1}{2}\frac{\dot{\phi}}{a^2} + \frac{1}{2}\frac{g^2\phi^4}{a^4} + \frac{1}{2}\dot{\Phi}^2 - V\right)\,;$$

The Proposal

Bianchi-I

$$ds^2 = -\mathrm{d}t^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}\mathrm{d}x^2 + e^{2\sigma(t)}\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)\right]$$

Dynamics

$$m_{\mathrm{P}}^2 G_{00} = T_{00} \longrightarrow 3m_{\mathrm{P}}^2 H^2 = \rho_r + \rho_m + \rho_{\mathrm{DE}}$$

$$m_{\mathrm{P}}^2\,\mathrm{Tr}\left(G_{ij}\right) = \mathrm{Tr}\left(T_{ij}\right) \longrightarrow -2m_{\mathrm{P}}^2\dot{H} = 3m_{\mathrm{P}}^2 H^2 + \frac{1}{3}\rho_r + p_{\mathrm{DE}}$$

$$G^2{}_2 - G^1{}_1 = (T^2_2 - T^1_1)/m_{\mathrm{P}}^2$$

$$3m_{\mathrm{P}}^2(\ddot{\sigma} + 3H\dot{\sigma}) = (G_1\dot{I})^2 - (G_2\dot{J})^2 + g^2(G_2J)^4 - g^2(G_1I)^2(G_2J)^2 + g^2\Phi^2(G_2J)^2$$

$$\rho_{\mathrm{DE}} \equiv \rho_{\mathrm{YM}} + \rho_{\mathcal{H}} + 3m_{\mathrm{P}}^2\dot{\sigma}^2$$

$$p_{\mathrm{DE}} \equiv \frac{1}{3}\rho_{\mathrm{YM}} + \frac{1}{2}\dot{\Phi}^2 - \frac{1}{3}g^2(G_2J)^2\Phi^2 - V + 3m_{\mathrm{P}}^2\dot{\sigma}^2$$

Equations of Motion

Equations of Motion

$$0 = (G_1\ddot{I}) + (H + 4\dot{\sigma})(G_1\dot{I}) + 2g^2(G_2J)^2(G_1I)$$

$$0 = (G_2\ddot{J}) + (H - 2\dot{\sigma})(G_2\dot{J}) + g^2(G_2J)(G_1I)^2 + g^2(G_2J)^3 + g^2(G_2J)\Phi^2$$

$$\ddot{\Phi} + 3H\dot{\Phi} + 2g^2(G_2J)^2\Phi + \frac{dV}{d\Phi} = 0$$

Continuity Equations

$$\dot{\rho}_r + 4H\rho_r = 0$$

$$\dot{\rho}_m + 3H\rho_m = 0$$

Dynamical System

Normalized Expansion Variables:

$$x \equiv \frac{1}{\sqrt{3}m_P}\frac{(G_1 \dot{I})}{H} \quad y \equiv \frac{1}{\sqrt{3}m_P}\frac{(G_2 \dot{J})}{H} \quad w \equiv \frac{1}{\sqrt{3}m_P}\frac{g(G_2 J)\Phi}{H} \quad z \equiv \frac{1}{\sqrt{6}}\frac{\dot{\Phi}}{H} \quad v \equiv \frac{1}{m_P H}\sqrt{\frac{V}{3}} \quad \xi \equiv \frac{\sqrt{3}m_P}{(G_2 J)}$$

$$p \equiv \sqrt{\frac{g}{\sqrt{3}m_P H}}(G_1 I) \quad s \equiv \sqrt{\frac{g}{\sqrt{3}m_P H}}(G_2 J) \quad \Sigma \equiv \frac{\dot{\sigma}}{H} \quad \Omega_r \equiv \frac{\rho_r}{3m_P^2 H^2} \quad \Omega_m \equiv \frac{\rho_m}{3m_P^2 H^2}$$

Fixed Points

Radiation - Saddle →

$$z = 0\,,\ w = 0\,,\ v = 0\,,\ \Sigma = 0\,,\ \Omega_m = 0\,,\ \Omega_r = 1 - \Omega_{DE}\,,\ \Omega_{DE} = \frac{3}{2}\left(s^4 + x^2\right)$$

Matter - Saddle →

$$x = 0\,,\ y = 0\,,\ \xi = 0\,,\ p = 0\,,\ s = 0\,,\ z = 0\,,\ w = 0\,,\ v = 0\,,\ \Sigma = 0\,,\ \Omega_m = 1$$

DE Attractor →

$$x = 0\,,\ y = 0\,,\ \xi = 0\,,\ p = 0\,,\ s = 0\,,\ z = 0\,,\ w = 0\,,\ v = 1\,,\ \Sigma = 0\,,\ \Omega_m = 0$$

Numerical Analysis

$x_i = y_i = \xi_i = 10^{-8}$ $v_i = 2 \times 10^{-14}$ $z_i = w_i = 10^{-12}$ $p_i = s_i = 10^{-12}$

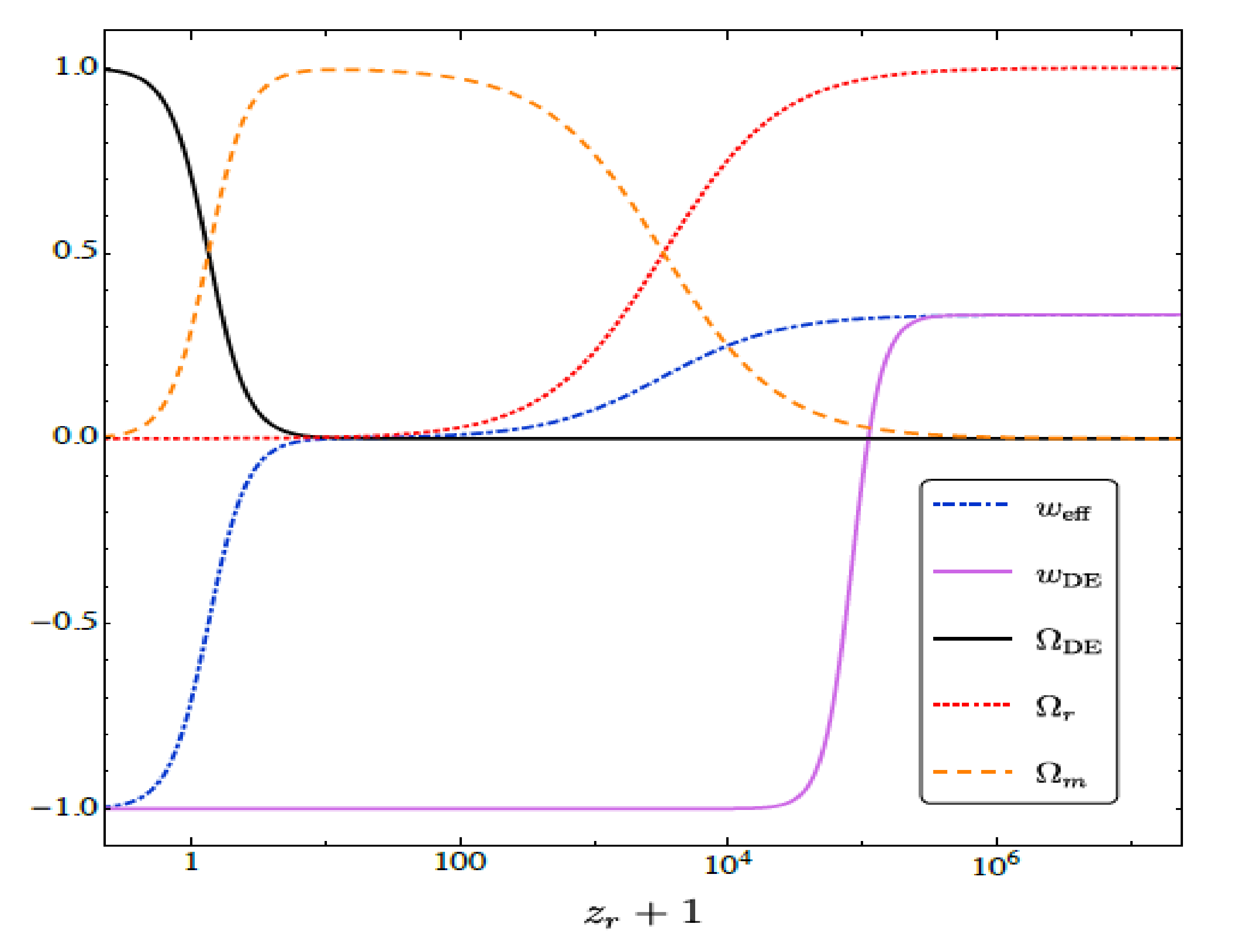
Initial Conditions

$z_r \cong 6.566 \times 10^7$

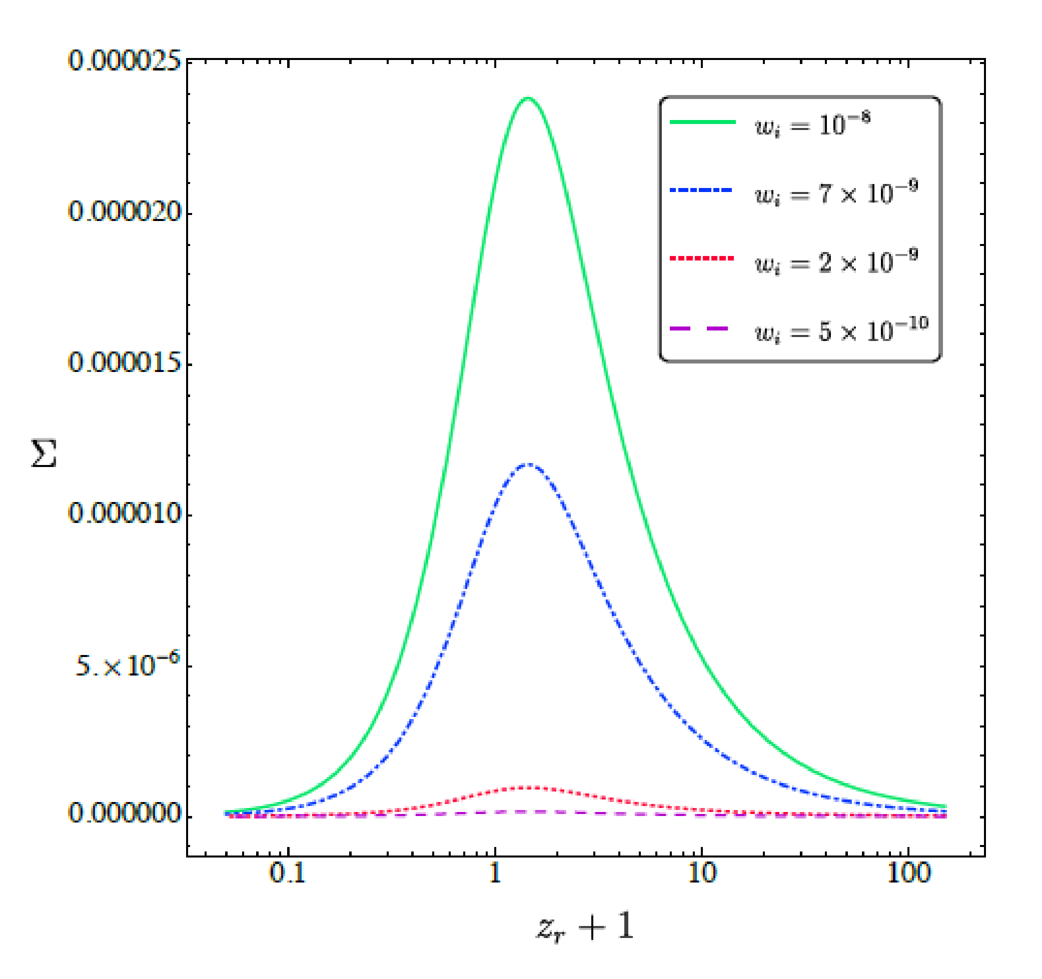
$\Sigma_i = 10^{-20}$

$\Omega_{r_i} = 0.99995$

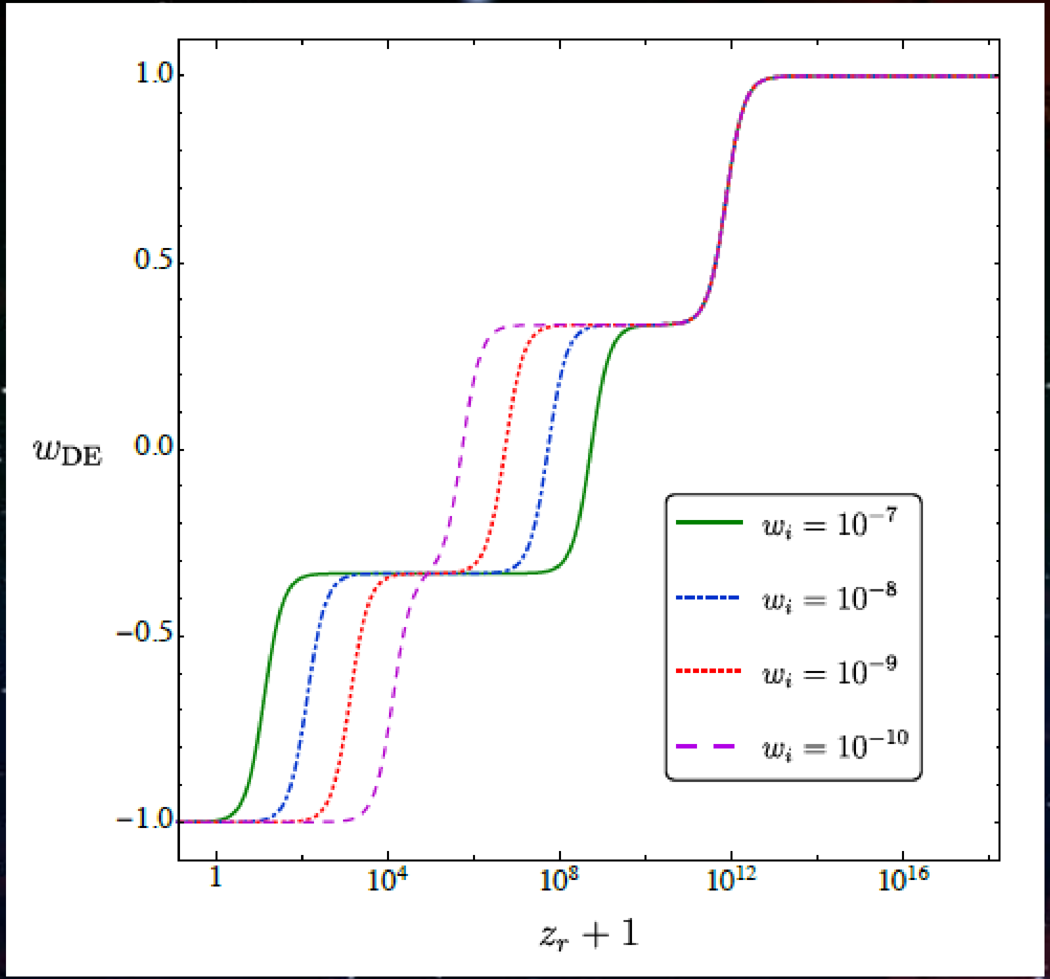
$\Omega_{m_i} = 4.99 \times 10^{-5}$



Anisotropy



Equation of State



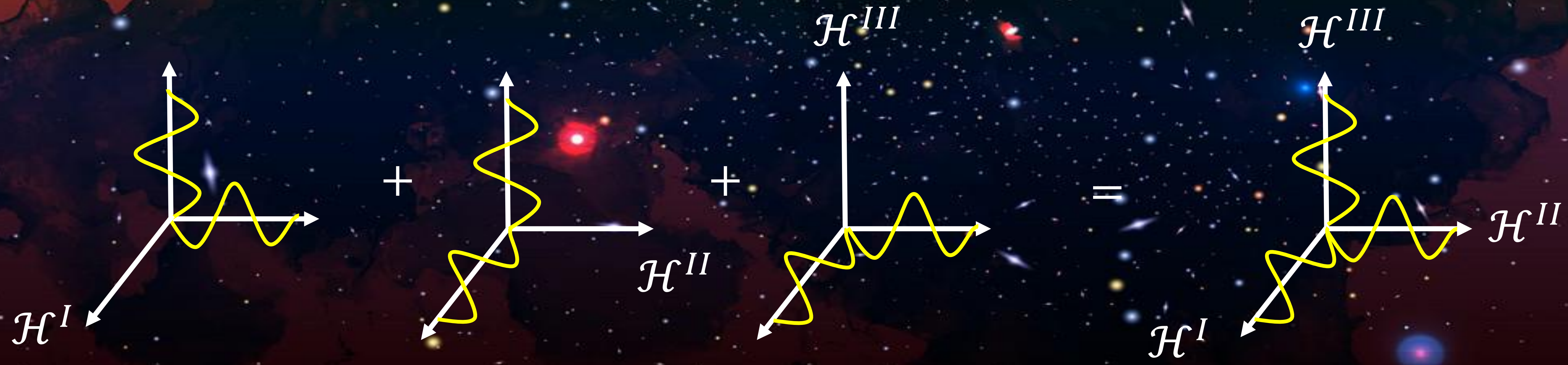
$$x = 0\,,\; y = 0\,,\; \xi = 0\,,\; p = 0\,,\; s = 0\,,\; z = 0\,,\; w = 0\,,\; v = 1\,,\; \Sigma = 0\,,\; \Omega_m = 0$$

Positive Eigenvalue → **Effective Attractor!!!**

The “Higgs Triad”

Inspired in the “Cosmic Triad”

$$\mathcal{H}^{I} \equiv \begin{pmatrix} \Phi \\ 0 \\ 0 \end{pmatrix} \;,\; \mathcal{H}^{II} \equiv \begin{pmatrix} 0 \\ \Phi \\ 0 \end{pmatrix} \;,\; \mathcal{H}^{II} \equiv \begin{pmatrix} 0 \\ 0 \\ \Phi \end{pmatrix}$$

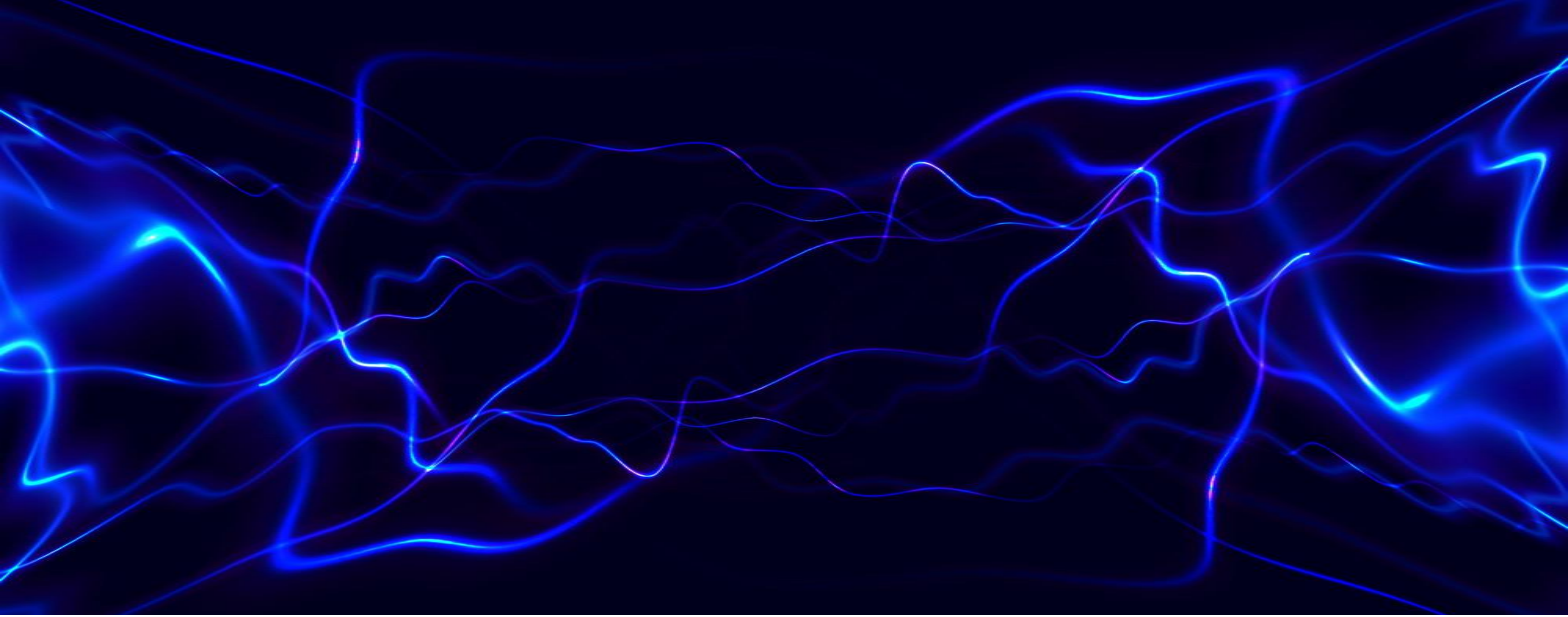


Conclusions

- The reduced theory worked out in 1508.04576 (Rinaldi) is inconsistent with FLRW.
- The anisotropic EYMH theory in $SO(3)$ reproduces the right expansión history.
- Isotropic dark energy domination is the only attractor point.
- The Universe can be anisotropic today.

The Universe can have hair although it will lose it in the future.

- The EYMH model can be compatible with FLRW $\rightarrow$ “Higgs Triad”
- Further details: 2006.14016 (Orjuela-Quintana et al)... Soonly available in JCAP.



Thank you!!!