Ultraviolet completion and predictivity from minimal parameterizations of Beyond-Standard-Model physics

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Asymptotic Safety - The concept

Definition:

$$\mathcal{L} = \sum_i g_i(k)\mathcal{O}_i(\varphi) \Longrightarrow \beta_i(g_j^*) = k\frac{dg_i}{dk} = 0\,.$$

Linearized analysis for $y_i = g_i - g_i^*$

$$\frac{dy_i}{dt} = M_{ij}y_j\,, \quad M_{ij} = \frac{\partial\beta_i}{\partial g_j} \tag{1}$$

$$(S^{-1})_{ij}M_{jl}S_{ln} = \delta_{in}\lambda_n\,, \quad z_i = S^{-1}_{ij}y_j \tag{2}$$

$$\frac{dz_i}{dt} = \lambda_i z_i \quad \text{and} \quad z_i(t) = c_i\,e^{\lambda_i t} = c_i\left(\frac{k}{k_0}\right)^{\lambda_i}. \tag{3}$$

- If $\Re(\lambda_i) > 0$, *irrelevant* direction.
- If $\Re(\lambda_i) < 0$, *relevant* direction.
- If $\Re(\lambda_i) = 0$, *marginal* direction.

Theory space

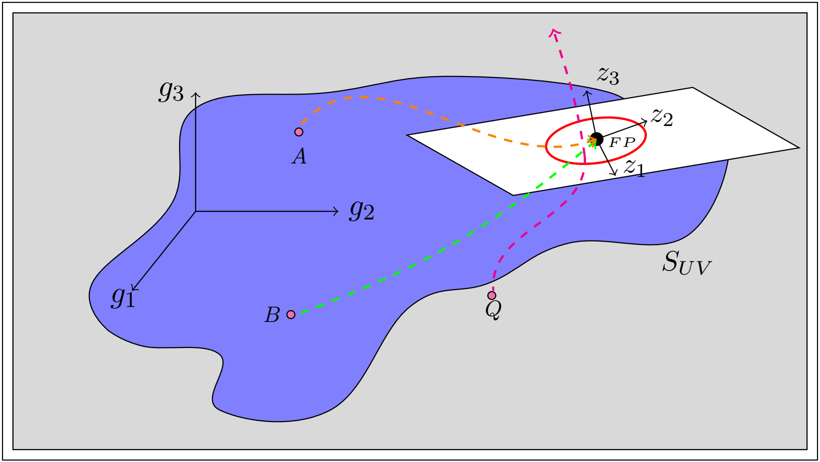


Figure : Theory space and fixed-point properties.

Running of the SM gauge couplings

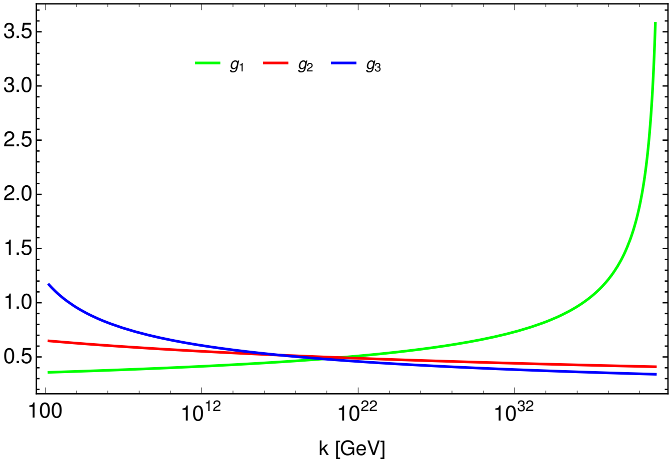


Figure : Running of the gauge couplings g_1, g_2 and g_3. At large values of k, g_1 begins its ascent towards the Landau pole.

Gauge theories in $d = 4$: one loop

- $SU(N_c)$ theory with N_f fermions in the fundamental representation

$$\mathcal{L} = -\frac{1}{4}\text{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi \tag{4}$$

The beta function of $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ is

$$\beta_g = -B\alpha_g^2 \tag{5}$$

$$B = -\frac{4}{3}\epsilon \; ; \qquad \epsilon = \frac{N_F}{N_c} - \frac{11}{2} \tag{6}$$

$$N_F < \frac{11}{2}N_c \Longrightarrow \epsilon < 0 \Longrightarrow B > 0 \Longrightarrow \quad \text{AF}$$

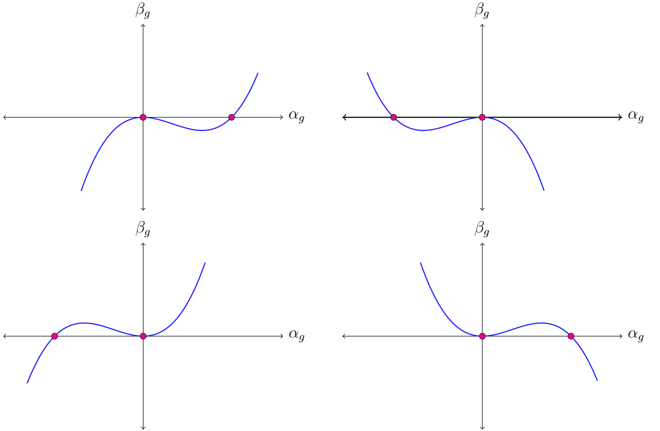
Gauge theories in $d = 4$: two loops

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$C > 0$ $C < 0$

$B > 0$

$B < 0$



Yukawa couplings

- Adding scalar fields H

$$\Delta\mathcal{L} = \mathrm{tr}(\partial^\mu H)^\dagger(\partial_\mu H) + y\,\mathrm{tr}(\bar{\psi}_L H \psi_R + \bar{\psi}_R H \psi_L) \tag{7}$$

$$\beta_g = \frac{d\alpha_g}{dt} = (-B + C\alpha_g - D\alpha_y)\alpha_g^2$$

$$\beta_y = \frac{d\alpha_y}{dt} = (E\alpha_y - F\alpha_g)\alpha_y$$

Beta functions for α_g and $\alpha_y = \frac{y^2 N_c}{(4\pi)^2}$

$$\begin{aligned} \beta_g &= \alpha_g^2\left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{3} + \epsilon\right)\alpha_y\right] \\ \beta_y &= \alpha_y\left[(13 + 2\epsilon)\alpha_y - 6\alpha_g\right] \end{aligned} \tag{8}$$

Fixed-point solutions

- For $\epsilon < 0$, Banks-Zaks

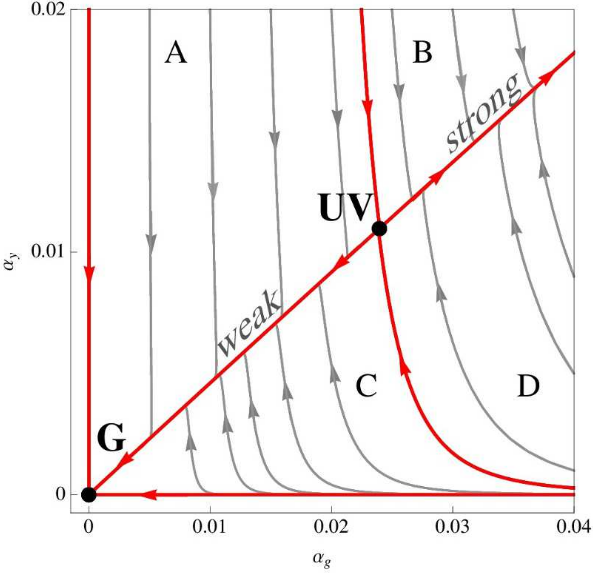
$$(\alpha_{g*}, \alpha_{y*}) = \left(-\frac{4\epsilon}{75 + 26\epsilon}, 0\right) \tag{9}$$

- For $\epsilon > 0$, Litim-Sannino

$$\begin{aligned}(\alpha_{g*}, \alpha_{y*}) &= \left(\frac{2\left(13\epsilon + 2\epsilon^2\right)}{57 - 46\epsilon - 8\epsilon^2}, \frac{12\epsilon}{57 - 46\epsilon - 8\epsilon^2}\right) \\ &\approx (0.456\epsilon + O(\epsilon^2), 0.211\epsilon + O(\epsilon^2))\end{aligned} \tag{10}$$

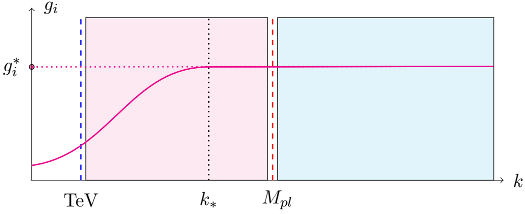
[D.F. Litim and F. Sannino, JHEP 1412 (2014) 178]

Phase diagram

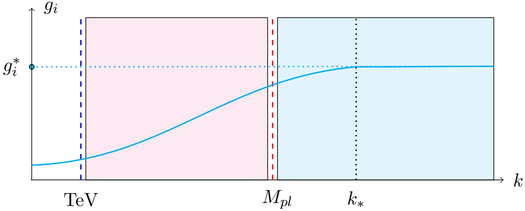


Fixed-point regimes

- $k_* < M_{pl}$



- $k_* > M_{pl}$



A class of BSM models

- Are there phenomenologically viable AS gauge-Yukawa theories?

Finite N_f and N_c

[A. Bond, D. Litim, Eur.Phys.J. C77 (2017) no.6, 429, arXiv:1608.00519 [hep-th]]
[A. Bond, G. Hiller, K. Kowalska, D. Litim, JHEP 1708 (2017) 004, arXiv:1702.01727 [hep-ph]]
[G.M. Pelaggi, A.D. Plascencia, A. Salvio, F. Sannino, Y. Smirnov, A. Strumia, Phys.Rev. D97 (2018) no.9, 095013 arXiv:1708.00437 [hep-th]]
[R.B. Mann, J.R. Meffe, F. Sannino, T.G. Steele, Z.W. Wang and C. Zhang, Phys. Rev. Lett. 119, 261802 (2017), arXiv:1707.02942 [hep-th]]

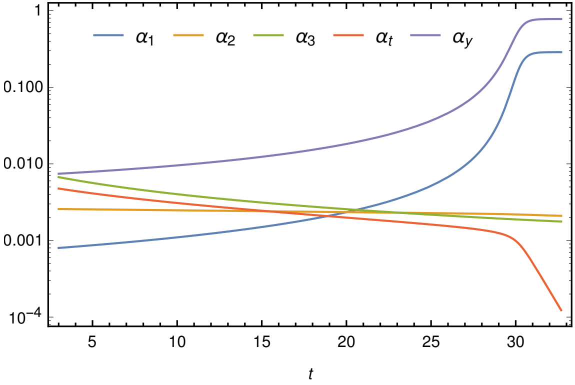
Simplest models use additional vector-like fermions.

A class of BSM models

- Group: $SU_c(3) \times SU_L(2) \times U_Y(1)$.
- Add N_f families of vector-like fermions ψ_i minimally coupled to SM and Yukawa interactions new scalars S.
- $\mathcal{L} = \mathcal{L}_{SM} + \mathrm{Tr}(\bar{\psi} i \not{D} \psi) + \mathrm{Tr}(\partial_\mu S^\dagger \partial_\mu S) - y\, \mathrm{Tr}(\bar{\psi}_L S \psi_R + \bar{\psi}_R S^\dagger \psi_L)$.
- Representation labels (p, q), ℓ, Y and N_f.
- Couplings: $(\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_y, \alpha_\lambda)$, where $\alpha_i \equiv \left(\frac{g_i}{4\pi}\right)^2$ and $\alpha_\lambda \equiv \frac{\lambda}{(4\pi)^2}$.

A promising model

$$N_f = 3,\ \ell = 1/2,\ Y = 3/2,\ p = q = 0$$
$$\alpha_1^* = 0.188,\ \ \alpha_2^* = 0,\ \ \alpha_3^* = 0,\ \ \alpha_t^* = 0,\ \ \alpha_y^* = 0.778$$
$$\lambda_1 = 33.2,\ \ \lambda_2 = -3.36,\ \ \lambda_3 = -0.817,\ \ \lambda_4 = 0,\ \ \lambda_5 = 0.$$



Requirements

- Stability under loop expansions

- SM matching at Fermi scale

Criteria for stability

- Small couplings
  $\alpha_i^* \equiv \left(\frac{g_i^*}{4\pi}\right)^2 \lesssim O(1)$.
- Small critical exponents
  $\beta_i = -d_i g_i + \beta_i^q(g_j)$, $M_{ij} = -d_i \delta_{ij} + \frac{\partial \beta_i^q}{\partial g_j}$, demand $|\lambda_i| \lesssim O(1)$.
- Hierarchy in the loop contributions
  At the FP $0 = \beta_i = A_*^{(i)} + B_*^{(i)} + C_*^{(i)}$,
  demand $\rho_i < \sigma_i < 1$ where $\rho_i = |C_*^{(i)}/A_*^{(i)}|$ and $\sigma_i = |B_*^{(i)}/A_*^{(i)}|$.

The search

Parameter space

- $N_f = 1, 2, ..., 300$
- $\ell = 1/2, 1, ..., 10$
- $Y = 0, 1/2, 1, ..., 10$

Singlet $SU(3) \implies (N, \ell) : (1, 1),\ (2, 1/2),\ (3, 1/2),\ (4, 1/2)$

Fundamental $SU(3) \implies (N, \ell) : (1, 1/2),\ (1, 5/2),\ (1, 3),\ (1, 7/2),$
$(1, 4),\ (1, 9/2),\ (2, 1),\ (2, 3/2),\ (2, 2),\ (3, 1/2),\ (3, 1),\ (4, 1/2),\ (5, 1/2).$

Adjoint $SU(3) \implies (N, \ell)$: None

Stable models have the $U(1)$ triviality problem.
$\implies$ no satisfactory UV fixed points.

Results

- Instability of fixed points with large scaling exponents.
- Low-dimensional representations are preferable
- Stable solutions show that there are no UV perturbative FPs that can be connected to the TeV physics.
- Stability condition rules out other models appearing in the literature.

  [A. Bond, G. Hiller, K. Kowalska, D. Litim, Directions for model building from asymptotic safety, JHEP 1708 (2017) 004]
- Explore non-abelian embeddings.

  [B. Bajc, F. Sannino, Asymptotically safe grand unification, JHEP 1612 (2016) 141]
  [A. Eichhorn, A. Held, C. Wetterich, Quantum-gravity predictions for the fine-structure constant, Phys.Rev. D99 (2019) no.3, 035030]
- Consider gravity+matter systems.

Gravitational running

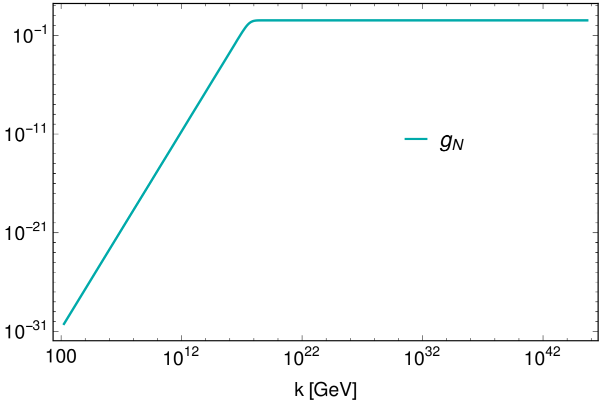


Figure : RG flow of the dimensionless Newton coupling g_N in the EH truncation.

Gravity plus matter

Gravity modifies the beta functions of matter couplings by universal (flavor-independent) terms.

$$\beta_{g_i} = \beta_{g_i}^{Matter} + f_g g_i \tag{11}$$

$$\beta_{Y_i} = \beta_{Y_i}^{Matter} + f_y Y_i \tag{12}$$

$$\beta_{\lambda_i} = \beta_{\lambda_i}^{Matter} + f_\lambda \lambda_i \tag{13}$$

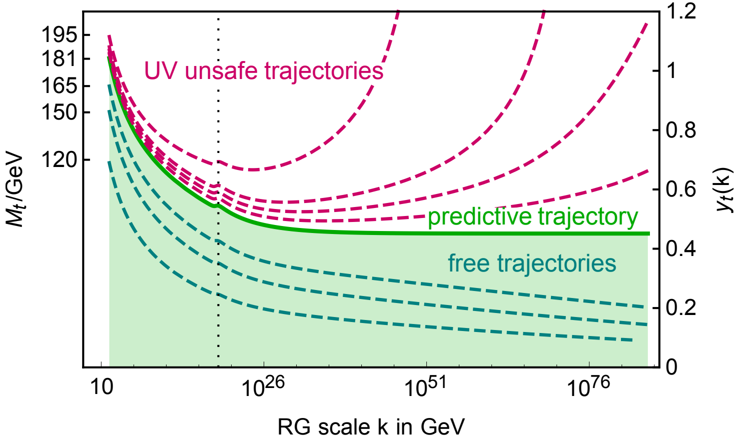
These can generate nontrivial fixed points for matter couplings

[S. P. Robinson, F. Wilczek, Complete asymptotically safe embedding of the standard model, Phys. Rev. Lett. 96 (2006) 231601]

[A. Salvio, A. Strumia, Agravity, JHEP 1406 (2014) 080]

[O. Zanusso, L. Zambelli, G. P. Vacca, R. Percacci, Gravitational corrections to Yukawa systems, Phys. Lett. B689 (2010) 90]

Calculation of top mass



$$f_g = G_N \frac{5(1-4\Lambda)}{18\pi(1-2\Lambda)^2}, \quad f_y = G_N \frac{\Lambda(235-\Lambda(103+56\Lambda))-96}{12\pi(3+2\Lambda(-5+4\Lambda))^2}$$

[A. Eichhorn and A. Held, Phys. Lett. B777, (2018) 217]

Gauge-Quark-Yukawa system

Gauge sector

$$\frac{dg_i}{dt} = \frac{1}{16\pi^2} b_i g_i^3 - f_g g_i.$$

Up-type Yukawas

$$\beta_{Y_U} = \frac{1}{16\pi^2}\left[\frac{3}{2} Y_U Y_U^\dagger Y_U - \frac{3}{2} Y_D Y_D^\dagger Y_U + 3\,\mathrm{Tr}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) Y_U - \left(\frac{17}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) Y_U\right] - f_y\, Y_U.$$

Down-type Yukawas

$$\beta_{Y_D} = \frac{1}{16\pi^2}\left[\frac{3}{2} Y_D Y_D^\dagger Y_D - \frac{3}{2} Y_U Y_U^\dagger Y_D + 3\,\mathrm{Tr}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) Y_D - \left(\frac{5}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) Y_D\right] - f_y\, Y_D.$$

Mass basis

- We introduce the unitary matrices V_L^U and V_L^D such that

$$V_L^U M_U V_L^{U\dagger} = D_U^2 = diag[y_u^2, y_c^2, y_t^2],$$
$$V_L^D M_D V_L^{D\dagger} = D_D^2 = diag[y_d^2, y_s^2, y_b^2].$$

- Thus, we get the CKM-matrix

$$V = V_L^U V_L^{D\dagger} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}.$$

- Set of variables: $y_i = (y_u, y_c, y_t)$, $y_\rho = (y_d, y_s, y_b)$ and 4 CKM elements $|V_{i\rho}|^2$

Diagonalized basis

Up-type Yukawas

$$\frac{dy_i}{dt} = \frac{y_i}{16\pi^2}\left(3\sum_j y_j^2 + 3\sum_\rho y_\rho^2 - \left(\tfrac{17}{12}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2\right) + \tfrac{3}{2}y_i^2 - \tfrac{3}{2}\sum_\rho y_\rho^2|V_{i\rho}|^2\right) - f_y\, y_i .$$

Down-type Yukawas

$$\frac{dy_\rho}{dt} = \frac{y_\rho}{16\pi^2}\left(3\sum_j y_j^2 + 3\sum_\alpha y_\alpha^2 - \left(\tfrac{5}{12}g_Y^2 + \tfrac{9}{4}g_2^2 + 8g_3^2\right) + \tfrac{3}{2}y_\rho^2 - \tfrac{3}{2}\sum_i y_i^2|V_{i\rho}|^2\right) - f_y\, y_\rho .$$

CKM elements

$$\beta_{|V_{i\rho}|^2} = -\frac{3}{2}\left(\sum_{\sigma,j\neq i}\frac{y_i^2+y_j^2}{y_i^2-y_j^2}y_\sigma^2\left(V_{i\sigma}V_{j\sigma}^*V_{j\rho}V_{i\rho}^* + V_{i\sigma}^*V_{j\sigma}V_{j\rho}^*V_{i\rho}\right)\right.$$
$$\left. + \sum_{j,\sigma\neq\rho}\frac{y_\rho^2+y_\sigma^2}{y_\rho^2-y_\sigma^2}y_j^2\left(V_{j\sigma}^*V_{j\rho}V_{i\sigma}V_{i\rho}^* + V_{j\sigma}V_{j\rho}^*V_{i\sigma}^*V_{i\rho}\right)\right) .$$

Two generations

- CKM matrix

$$V = \begin{bmatrix} V_{tb} & V_{ts} \\ V_{cb} & V_{cs} \end{bmatrix}.$$

- We choose $|V_{tb}|^2 = W$, such that $|V_{ts}|^2 = 1 - W$, $|V_{cb}|^2 = 1 - W$ and $|V_{cs}|^2 = W$.

$$V_2 = \begin{bmatrix} W & 1-W \\ 1-W & W \end{bmatrix},$$

$$\frac{dW}{dt} = -\frac{3}{16\pi^2}(1-W)W\left[\frac{y_t^2+y_c^2}{y_t^2-y_c^2}(y_b^2-y_s^2) + \frac{y_b^2+y_s^2}{y_b^2-y_s^2}(y_t^2-y_c^2)\right].$$

- Two important points: $W = 0$ and $W = 1$.

Two generations

- Yukawa beta-functions

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\Bigg(3(y_t^2+y_c^2+y_b^2+y_s^2)+3y_t^2-\frac{3}{2}y_b^2 W-\frac{3}{2}y_s^2\,(1-W) \\ -\frac{9}{4}g_2^2-8g_3^2-\frac{17}{12}g_1^2\Bigg)-f_y\,y_t. \tag{14}$$

$$\beta_{y_b} = \frac{y_b}{16\pi^2}\Bigg(3(y_t^2+y_c^2+y_b^2+y_s^2)+3y_b^2-\frac{3}{2}y_t^2 W-\frac{3}{2}y_c^2\,(1-W) \\ -\frac{9}{4}g_2^2-8g_3^2-\frac{5}{12}g_1^2\Bigg)-f_y\,y_b. \tag{15}$$

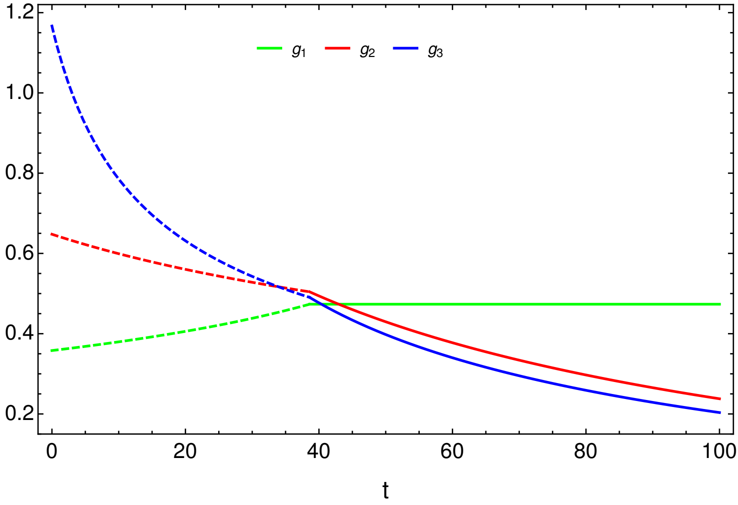
For y_c and y_b, we have $(t,b)\leftrightarrow(c,s)$.

- Using $g_{1\,*}=4\pi\sqrt{6f_g/41}$, demanding $y_{t\,*}>y_{c\,*}$ and $y_{b\,*}>y_{s\,*}$

$$W_*=0,\quad y_{t\,*}=\frac{4\pi}{\sqrt{15}}\sqrt{f_g+2f_y},\quad y_{b\,*}=\frac{4\pi}{\sqrt{615}}\sqrt{-19f_g+82f_y}. \tag{16}$$

$$y_{t\,*}^2-y_{b\,*}^2=\frac{2}{3}g_{1\,*}^2. \tag{17}$$

Two generations - Gauge running



Two generations - Yukawa running

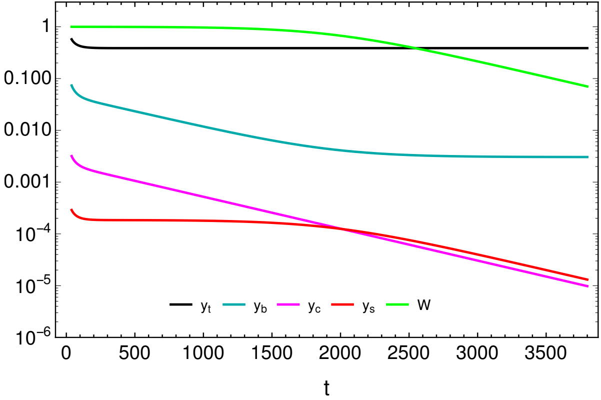


Figure : Trajectory emanating from the asymptotically safe fixed point. IR predictions: $M_t = 185$ GeV (173.21), $M_c = 1.27$ GeV, $M_b = 4.18$ GeV, $M_s = 0.096$ GeV, $W = 0.9984$(0.9980).

Three generations

- Yukawa couplings $y_i = (y_u, y_c, y_t)$, $y_\rho = (y_d, y_s, y_b)$.
- CKM matrix

$$V = V_L^U \, V_L^{D\dagger} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}.$$

- We choose $|V_{ud}|^2 = X$, $|V_{us}|^2 = Y$, $|V_{cd}|^2 = Z$, $|V_{cs}|^2 = W$

$$V_2 = \begin{bmatrix} X & Y & 1 - X - Y \\ Z & W & 1 - Z - W \\ 1 - X - Z & 1 - Y - W & X + Y + Z + W - 1 \end{bmatrix}.$$

CKM beta-functions

$$\frac{dX}{dt} =$$
$$-\frac{3}{(4\pi)^2}\left[\frac{y_u^2+y_c^2}{y_u^2-y_c^2}\left\{(y_d^2-y_b^2)XZ+\frac{(y_b^2-y_s^2)}{2}(W(1-X)+X-(1-Y)(1-Z))\right\}\right.$$
$$+\frac{y_u^2+y_t^2}{y_u^2-y_t^2}\left\{(y_d^2-y_b^2)X(1-X-Z)+\frac{(y_b^2-y_s^2)}{2}((1-Y)(1-Z)-X(1-2Y)-W(1-X))\right\}$$
$$+\frac{y_d^2+y_s^2}{y_d^2-y_s^2}\left\{(y_u^2-y_t^2)XY+\frac{y_t^2-y_c^2}{2}(W(1-X)+X-(1-Y)(1-Z))\right\}$$
$$\left.+\frac{y_d^2+y_b^2}{y_d^2-y_b^2}\left\{(y_u^2-y_t^2)X(1-X-Y)+\frac{y_t^2-y_c^2}{2}((1-Y)(1-Z)-X(1-2Z)-W(1-X))\right\}\right]$$

$$\frac{dY}{dt} = -\frac{3}{(4\pi)^2}\left[\frac{y_u^2+y_c^2}{y_u^2-y_c^2}\left\{\frac{(y_b^2-y_d^2)}{2}(W(1-X)+X-(1-Y)(1-Z))+(y_s^2-y_b^2)YW\right\}\right.$$
$$+\frac{y_u^2+y_t^2}{y_u^2-y_t^2}\left\{\frac{(y_b^2-y_d^2)}{2}((1-Y)(1-Z)-W(1-X)-X(1-2Y))+(y_s^2-y_b^2)Y(1-Y-W)\right\}$$
$$+\frac{y_s^2+y_d^2}{y_s^2-y_d^2}\left\{(y_u^2-y_t^2)XY+\frac{y_t^2-y_c^2}{2}(W(1-X)+X-(1-Y)(1-Z))\right\}$$
$$\left.+\frac{y_s^2+y_b^2}{y_s^2-y_b^2}\left\{(y_u^2-y_t^2)Y(1-X-Y)+\frac{(y_c^2-y_t^2)}{2}(W(1-X-2Y)+X-(1-Z)(1-Y))\right\}\right]$$

CKM beta-functions

$$\frac{dZ}{dt} = -\frac{3}{(4\pi)^2}\left[\frac{y_c^2+y_u^2}{y_c^2-y_u^2}\left\{(y_d^2-y_b^2)XZ+\frac{(y_b^2-y_s^2)}{2}(W(1-X)+X-(1-Z)(1-Y))\right\}\right.$$
$$+\frac{y_c^2+y_t^2}{y_c^2-y_t^2}\left\{(y_d^2-y_b^2)Z(1-X-Z)+\frac{(y_s^2-y_b^2)}{2}(W(1-X-2Z)+X-(1-Y)(1-Z))\right\}$$
$$+\frac{y_d^2+y_s^2}{y_d^2-y_s^2}\left\{\frac{(y_u^2-y_t^2)}{2}((1-Y)(1-Z)-X-W(1-X))+(y_c^2-y_t^2)ZW\right\}$$
$$\left.+\frac{y_d^2+y_b^2}{y_d^2-y_b^2}\left\{\frac{(y_t^2-y_u^2)}{2}((1-Z)(1-Y)-W(1-X)-X(1-2Z))+(y_c^2-y_t^2)Z(1-Z-W)\right\}\right]$$

$$\frac{dW}{dt} = -\frac{3}{(4\pi)^2}\left[\frac{y_c^2+y_u^2}{y_c^2-y_u^2}\left\{(y_s^2-y_b^2)WY+\frac{(y_b^2-y_d^2)}{2}((1-X)W+X-(1-Y)(1-Z))\right\}\right.$$
$$+\frac{y_c^2+y_t^2}{y_c^2-y_t^2}\left\{(y_s^2-y_b^2)W(1-Y-W)+\frac{(y_b^2-y_d^2)}{2}((1-Y)(1-Z)-X-W(1-X-2Z))\right\}$$
$$+\frac{y_s^2+y_d^2}{y_s^2-y_d^2}\left\{(y_c^2-y_t^2)WZ+\frac{(y_t^2-y_u^2)}{2}Z((1-X)W+X-(1-Y)(1-Z))\right\}$$
$$\left.+\frac{y_s^2+y_b^2}{y_s^2-y_b^2}\left\{(y_c^2-y_t^2)W(1-Z-W)+\frac{(y_t^2-y_u^2)}{2}((1-Y)(1-Z)-X-W(1-X-2Y))\right\}\right]$$

Three generations

- CKM fixed points

$$M_{123} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_{132} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{321} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$M_{213} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_{312} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{231} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

- We seek for solutions of $\beta_{y_i} = 0 = \beta_{y_\rho}$ in each of the cases $|V|^2 = M_{abc}$.
- Solutions for $|V|^2$ are obtained from M_{123} by applying the corresponding M_{abc}^{-1} to $y_\rho = (y_d, y_s, y_b)$
- We select the positive solutions having at most two zero couplings.

Three generations

- 392 solutions for each V_2
- We find 1 plane of fixed-points and 6 lines. We impose the conditions $y_{t*} > y_{c*} > y_{u*}$ and $y_{b*} > y_{s*} > y_{d*}$.
- Only the case remains

$$V_2^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Three solutions remain

$$y_{u*}^2 = \frac{4\pi^2}{123}(47 f_g + 82 f_y) - y_{c*}^2 - y_{t*}^2 \,, \quad y_{s*}^2 = -\frac{32 f_g \pi^2}{41} + y_{c*}^2,$$
$$y_{d*}^2 = \frac{4\pi^2}{123}(23 f_g + 82 f_y) - y_{c*}^2 - y_{t*}^2 \,, \quad y_{b*}^2 = -\frac{32 f_g \pi^2}{41} + y_{t*}^2 \,.$$

Three generations

$$y_{c\,*}^2 = \frac{4\pi^2}{123}(35 f_g + 82 f_y) - y_{t\,*}^2 \;, \quad y_{b\,*}^2 = -\frac{32 f_g \pi^2}{41} + y_{t\,*}^2,$$
$$y_{s\,*}^2 = \frac{4\pi^2}{123}(11 f_g + 82 f_y) - y_{t\,*}^2 \;, \quad y_{d\,*}^2 = 0 \;, \quad y_u^2 = 0 \;.$$

$$y_{c\,*}^2 = \frac{4\pi^2}{123}(23 f_g + 82 f_y) - y_{t\,*}^2 \;, \quad y_{b\,*}^2 = -\frac{32 f_g \pi^2}{41} + y_{t\,*}^2,$$
$$y_{s\,*}^2 = \frac{4\pi^2}{123}(-f_g + 82 f_y) - y_{t\,*}^2 \;, \quad y_{u\,*}^2 = \frac{32 f_g \pi^2}{41} \;, \quad y_d^2 = 0 \;.$$

- These fixed-points are UV repulsive.
- Other non-trivial CKM fixed-points generate negative Yukawa couplings.

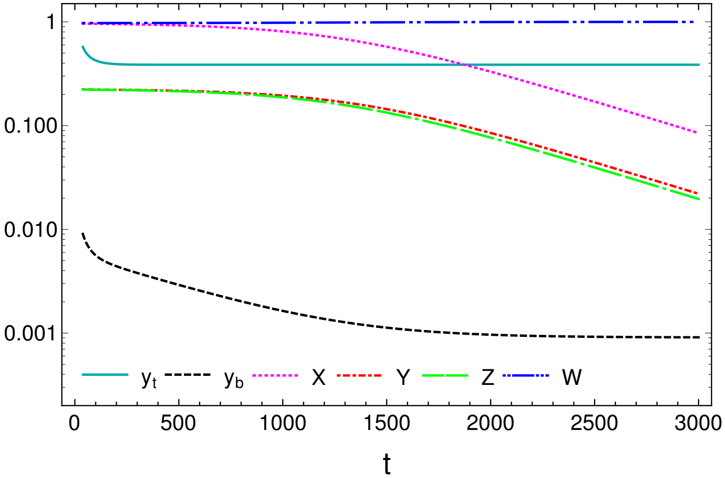
Limit

We approach $y_u \to 0$, $y_d \to 0$, $y_s \to 0$, $y_c \to 0$,
$\implies$ solvable system

$$y_{t\,*} = \frac{4\pi}{\sqrt{15}}\sqrt{f_g + 2f_y}, \quad y_{b\,*} = \frac{4\pi}{\sqrt{615}}\sqrt{-19f_g + 82f_y}.$$

$$V_2^* = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Running couplings



	Flow	Exp.
M_t	185 GeV	173 GeV
M_b	4.2 GeV	4.18 GeV
M_c	1.27 GeV	1.275 GeV
M_s	96 MeV	95 MeV
M_d	4.7 MeV	4.7 MeV
M_u	2.2 MeV	2.2 MeV
X	0.9326	0.9495
Y	0.05053	0.05040
Z	0.05035	0.05034
W	0.94961	0.94788

Conclusions

- It seems it is not possible to render the SM AS with a finite number of extra fields.
- A GUT extension can provide a solution to the triviality problem.
- The solution of the Landau pole in the $U(1)$ sector opens the possibility of exploring interesting UV properties of the SM.
- Using gravity effects, we can (partially) have an understanding of the mass hierarchy in the quark sector.
- A full understanding of the complete flavor structure of the SM remains an open question.