RELATIVISTIC STARS IN THE GENERALIZED PROCA THEORY

CoCo 2020

William Jaimes Espíndola[1]
Yeinzon Rodríguez García[1,2,3]
Gabriel Gómez Díaz[4]

1. School of Physics, Universidad Industrial de Santander.
2. Research Centre on Basic and Applied Sciences, Universidad Antonio Nariño.
3. Simons Associate at The Abdus Salam International Centre for Theoretical Physics.
4. Physics Department, Universidad Santiago de Chile.

Some Interesting Features:

Generalized Proca Theory

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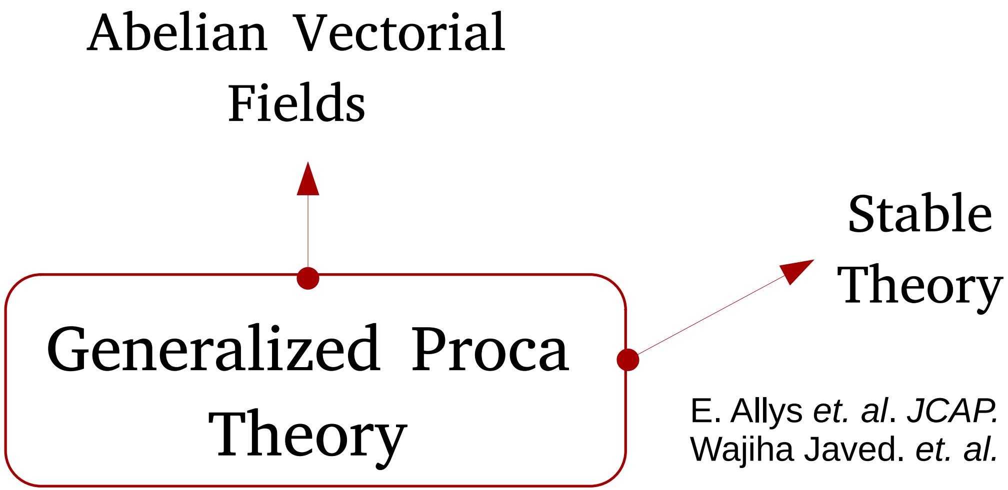

E. Allys *et. al*. *JCAP.* 2016.
Wajiha Javed. *et. al. Chin. Phys. C*. 2020

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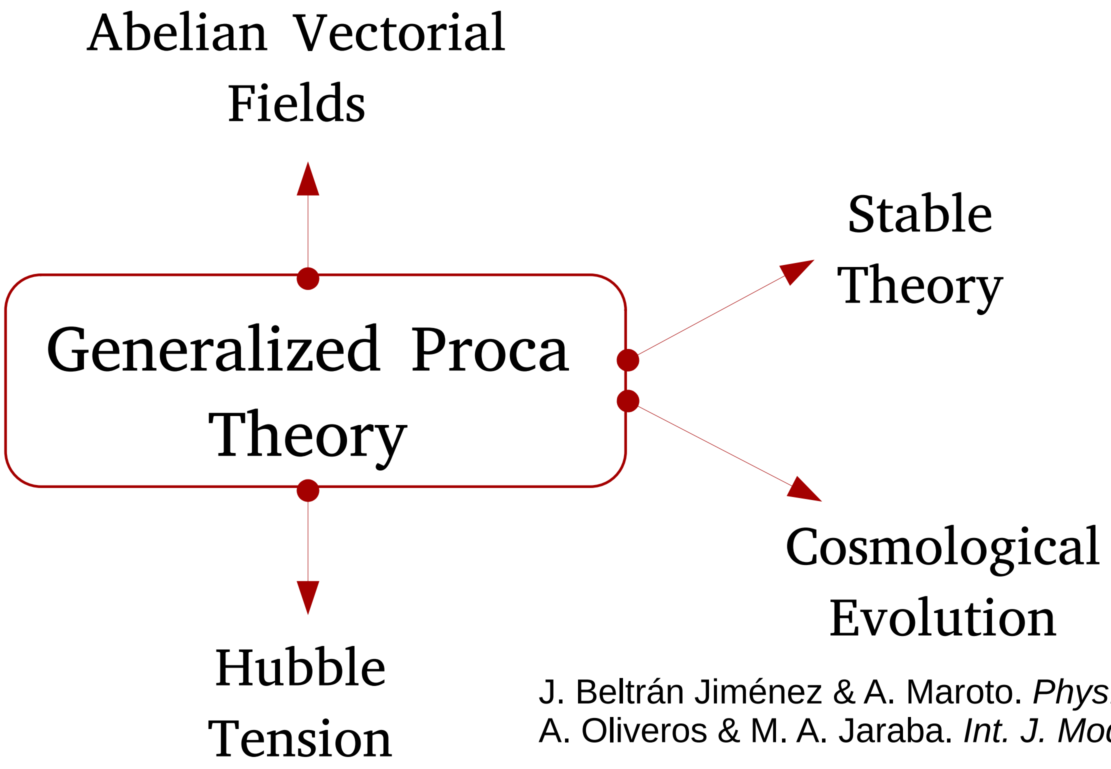

J. Beltrán Jiménez & A. Maroto. *Phys. Rev. D.* 2009.
A. Oliveros & M. A. Jaraba. *Int. J. Mod. Phys. D.* 2019.

A. De Felice. *et. al. JCAP.* 2020.

Some Interesting Features:

Abelian Vectorial Fields

Multi-Messenger Observations

R. Kase *et. al*. *Phys. Lett. B.* 2018.
R. Kase *et. al*. *Phys. Rev. D.* 2018
J. Chagoya *et. al*. *Clas. Quant. Grav.* 2017.

Stable Theory

Generalized Proca Theory

Weak Field Limit

A. De Felice. *et. al. Phys. Rev. D.* 2016.

Cosmological Evolution

Hubble Tension

The Generalized Proca Theory

$$S = \int d^4x\sqrt{-g}\left[F + \sum_{i=2}^{6}\mathcal{L}_i + \mathcal{L}_m\right],$$

$$\mathcal{L}_2 = G_2(X,F,Y),$$

$$\mathcal{L}_3 = G_3(X)\nabla_\mu A^\mu,$$

$$\mathcal{L}_4 = G_4(X)R + G_{4,X}(X)\left[(\nabla\cdot A)^2 - \nabla_\mu A_\nu \nabla^\mu A^\nu\right],$$

$$\mathcal{L}_5 = G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}\left[(\nabla\cdot A)^3 - 3(\nabla\cdot A)\nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\nu A^\rho \nabla^\sigma A_\nu\right]$$
$$-g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^{\beta}{}_{\mu}\nabla_\alpha A_\beta,$$

$$\mathcal{L}_6 = G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu,$$

$$X = -\frac{1}{2}A^\mu A_\mu,$$

$$F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha},$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta},$$

$$L^{\mu\nu\alpha\beta} = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}.$$

E: Allys *et. al*. *JCAP.* 2016.
E. Allys *et. al. JCAP.* 2016.
J. Beltrán Jiménez & L. Heisenberg. *Phys. Lett.* 2016.

Relativistic Stars

Line Element: Static and Spherically Symmetrical

$$ds^2 = -f(r)c^2dt^2 + h^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Vectorial Field Components

$$A_\mu = \left(cA_0(r), A_1(r), 0, 0\right).$$

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$$A_\mu = (cA_0(r), A_1(r), 0, 0)\,.$$

Material Content: Perfect Fluid

$$T_\mu{}^\nu = diag(-\rho c^2, P, P, P)$$

Equation of State

$$P = P(\rho)$$

Reference Case: General Relativity (GR)

$$G_4 = \frac{1}{16\pi G}, \quad G_2 = G_3 = G_5 = g_5 = G_6 = 0, \qquad A_\mu = 0.$$

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Field Equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \longrightarrow \begin{cases} M'(r) = 4\pi\rho(r) r^2 \\ f' = \dfrac{G}{c^2 r^2} \dfrac{\left(M + 4\pi P r^3/c^2\right)}{(1 - 2GM/c^2 r)} 2f \end{cases}$$

$$\nabla_\nu T^{\mu\nu} = 0 \longrightarrow P' = -\frac{G\left(\rho + P/c^2\right)\left(M + 4\pi P r^3\right)}{r^2 (1 - 2GM/r)}$$

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Analytical solutions around the center of the star:

$$f(r) = 1 + \sum_{i=2}^{\infty} f_i r^i,$$

$$h(r) = 1 + \sum_{i=2}^{\infty} h_i r^i,$$

$$P(r) = p_c + \sum_{i=2}^{\infty} P_i r^i,$$

$$\rho(r) = \rho_c + \sum_{i=2}^{\infty} \rho_i r^i.$$

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$$f'(0) = h'(0) = \rho'(0) = P'(0) = 0$$

$$f(r) = 1 + \frac{4\pi G\left(c^2\rho_c + 3p_c\right)}{3c^4} r^2 + \mathcal{O}\left(r^4\right),$$

$$h(r) = 1 - \frac{8\pi G\rho_c}{3c^2} r^2 + \mathcal{O}\left(r^4\right),$$

$$P(r) = p_c - \frac{4\pi G\left(c^2\rho_c + 3p_c\right)\left(c^2\rho_c + p_c\right)}{3c^4} r^2 + \mathcal{O}\left(r^4\right).$$

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$$M > 0, \quad p_2 < 0.$$

Case 1: Cubic Couplings $G_3(X) = \beta_3 X^n$

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Field Equations:

$$\frac{1-h}{8\pi G r^2} - \frac{4nX^{n-1}X_0\beta_3 A_1 h}{r} - \frac{hA_0^2 + 2nX^{n-1}\beta_3 h\left(A_0 A_1 A_0' + 2XfA_1'\right)}{2f} - \frac{1+8\pi G r n X^n \beta_3 A_1}{8\pi G r}h' = \frac{\rho}{c^2},$$

$$\frac{h-1}{8\pi G r^2} + \frac{4bX^{n-1}X_1\beta_3 A_1 h}{r} + \frac{hA_0^2 - 2nX^{n-1}\beta_3 A_0 A_1 A_0' h}{2f} + \frac{1+8\pi G r n X^n \beta_3 A_1}{8\pi G r f}hf' = \frac{P}{c^4},$$

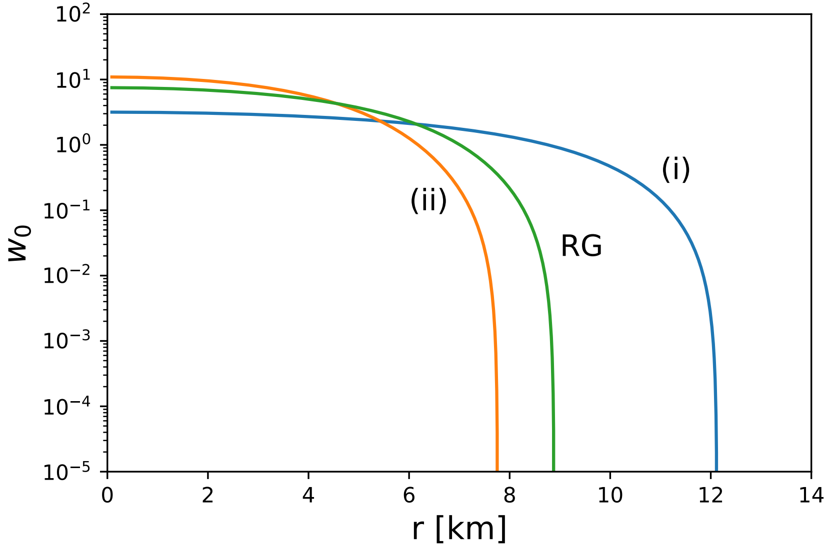
$$P' + \frac{f'}{2f}\left(\rho c^2 + P\right) = 0,$$

$$r\left[r\left(f'X - A_0 A_0'\right) + 4fX_1\right]\beta_3 n X^{n-1} = 0,$$

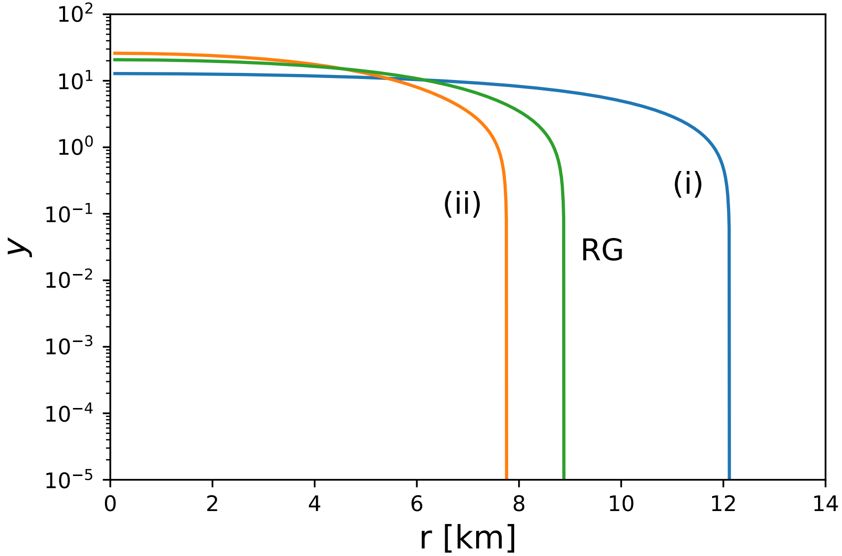
$$rf\left[2fh\left(rA_0'' + 2A_0'\right) + r\left(fh' - f'h\right)A_0'\right] - rfA_0\left[2rfhA_1' + \left(rf'h + rfh' + 4fh\right)A_1\right]\beta_3 n X^{n-1} = 0.$$

Case 1: Cubic Couplings $G_3(X) = \beta_3 X^n$

Numerical Solutions



Dimensionless Pressure variation



Dimensionless Density variation

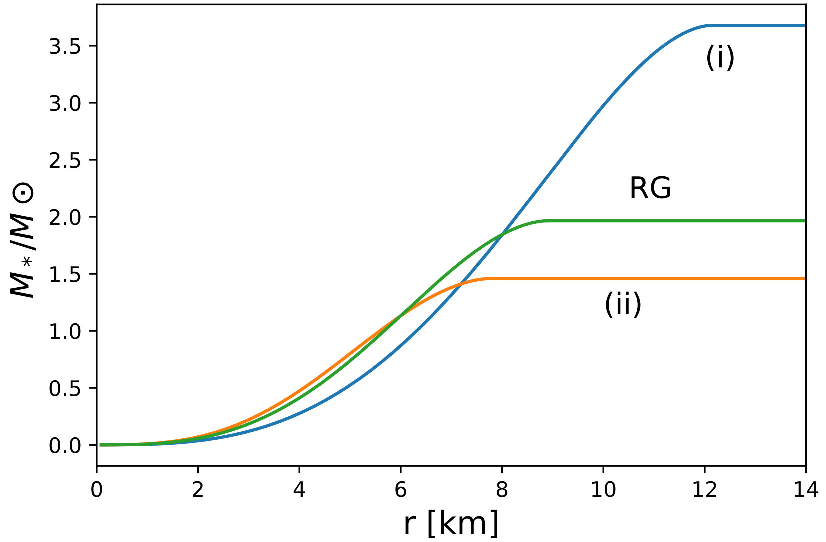
(i) $\tilde{\beta}_3 = -1,\ \bar{a}_0 = 2.2,\ w_{0c} = 3.168.$

(ii) $\tilde{\beta}_3 = 1,\ \bar{a}_0 = 2.0,\ w_{0c} = 10.939.$

GR $\tilde{\beta}_3 = 0,\ \bar{a}_0 = 0,\ w_{0c} = 7.505.$

Case 1: Cubic Couplings $G_3(X) = \beta_3 X^n$

Numerical Solutions



Mass distribution

(i) $\tilde{\beta}_3 = -1,\ \bar{a}_0 = 2.2,\ w_{0c} = 3.168.$

(ii) $\tilde{\beta}_3 = +1,\ \bar{a}_0 = 2.0,\ w_{0c} = 10.939.$

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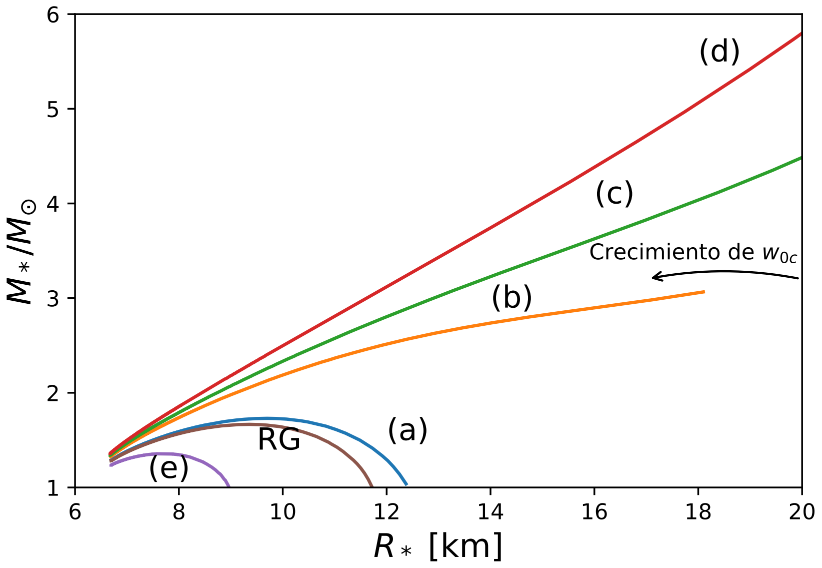
Numerical Solutions



Mass-radius profile

(a) $\tilde{\beta}_3 = -1 \;\; \bar{a}_0 = 1.0.$

(b) $\tilde{\beta}_3 = -1, \;\; \bar{a}_0 = 2.0.$

(c) $\tilde{\beta}_3 = -1, \;\; \bar{a}_0 = 2.2.$

(d) $\tilde{\beta}_3 = -1, \;\; \bar{a}_0 = 2.4.$

(e) $\tilde{\beta}_3 = +1, \;\; \bar{a}_0 = 2.0.$

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Case 2: Quartic Couplings $G_4(X) = \frac{1}{16\pi G} + \beta_4 X^n$

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Field Equations

$$\frac{2X^n\beta_4(1-h) + 8(n-1)nX^{n-2}X_0X_1\beta_4 h + 4nX^{n-1}\beta_4\left(Xh - X_0\right)}{r^2} + \frac{8nX^{n-2}\beta_4 h\left[(2n-1)XA_1fhA_1' - 2(n-1)X_1A_0A_0'\right] + rhA_0^2}{2fr} + \frac{(1-h)X - \left[X + 16\pi GX^{n-1} - 32Gn\pi X^n\left(X_0 + 2nX_1\right)\beta_4\right]rh'}{8\pi Gr^2X} = \frac{\rho}{c^2},$$

$$\frac{2X^n\beta_4(h-1) - 8(n-1)nX^{n-2}X_1^2\beta_4 h - 4nX^{n-1}X_1\beta_4(2h-1)}{r^2} + \frac{8nX^{n-2}\left[X + 2(n-1)X_1\right]\beta_4A_0hA_0' + rhA_0'^2}{2fr} + \frac{Xf(h-1) + \left[X + 16\pi GX^{n-1}\beta_4 - 32\pi GnX^n\left(X_0 + 2nX_1\right)\beta_4\right]rhf'}{8\pi Gr^2Xf} = \frac{P}{c^4},$$

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Field Equations

$$8(n-1)nX^{n-2}\beta_4 A_0\left[rX_1hf' + f\left(X_1h - rA_1h^2A_1' + rX_1h'\right)\right]$$
$$+4nX^{n-1}\beta_4 A_0 f\left(-1+h+rh'\right) + r\left[rA_0'\left(-hf'+fh'\right) + 2fh\left(2A_0' + rA_0''\right)\right] = 0,$$

$$\beta_4 A_1\left(A_0^2 - fhA_1^2\right)^{n-2}\left[A_1^2 fh\left\{(1+h-2nh)f + (1-2n)rf'h\right\}\right.$$
$$\left.+A_0^2\left\{f(h-1) + (2n-1)rf'h\right\} - 4r(n-1)A_0A_0'fh\right] = 0,$$

$$P' + \frac{f'}{2f}\left(\rho c^2 + P\right) = 0.$$

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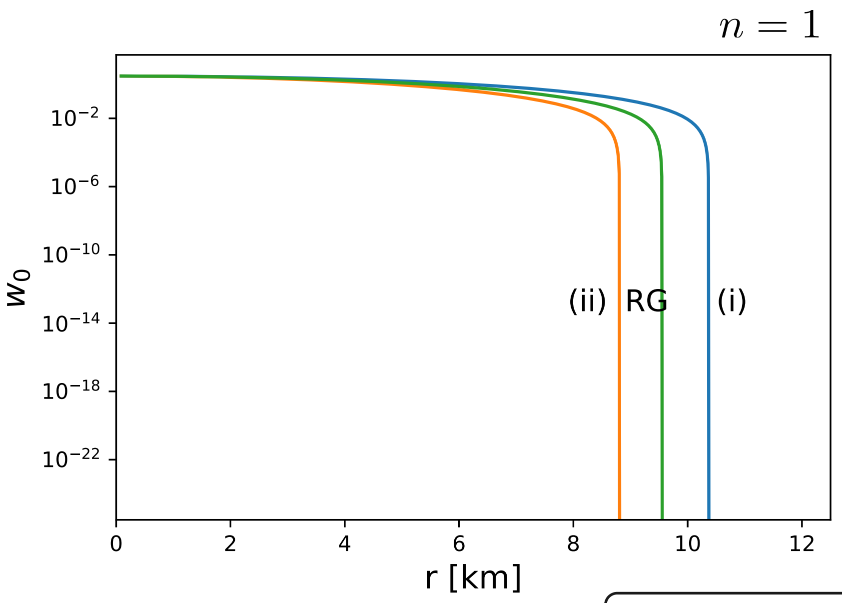

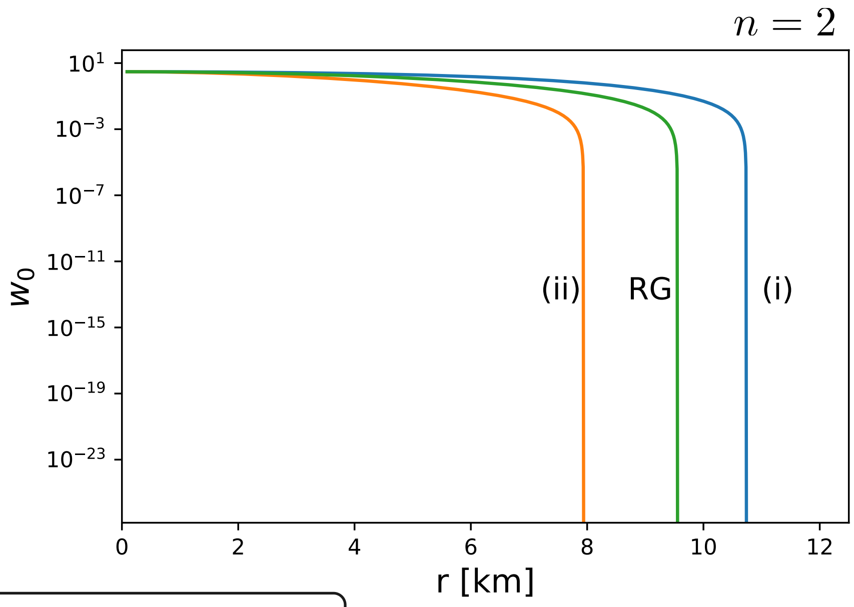

$K = 0.010 \;\; \Gamma = 2.34 \;\; w_{0c} = 3.020$

Dimensionless Pressure variation

(i) $\bar{\beta}_4 = -0.06,$ $\bar{a}_0 = 1.5.$

(ii) $\bar{\beta}_4 = +0.06,$ $\bar{a}_0 = 1.5.$

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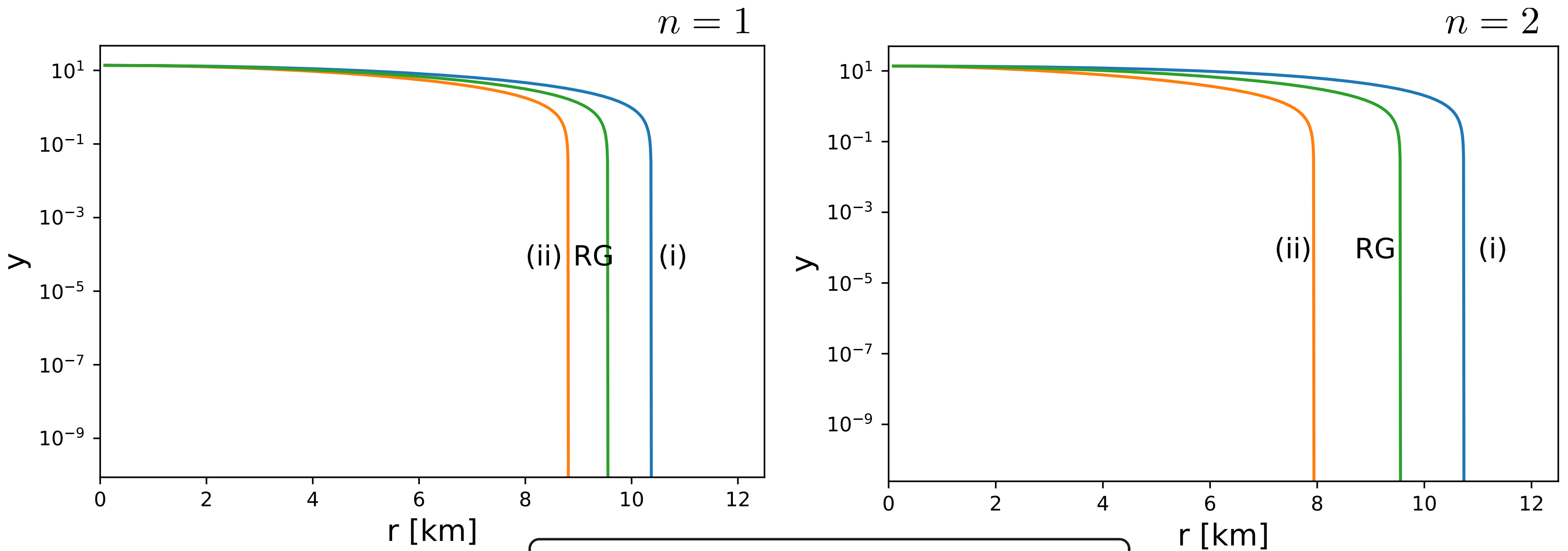
Numerical Solutions



$K = 0.010, \Gamma = 2.34, w_{0c} = 3.020.$

Dimensionless density variation

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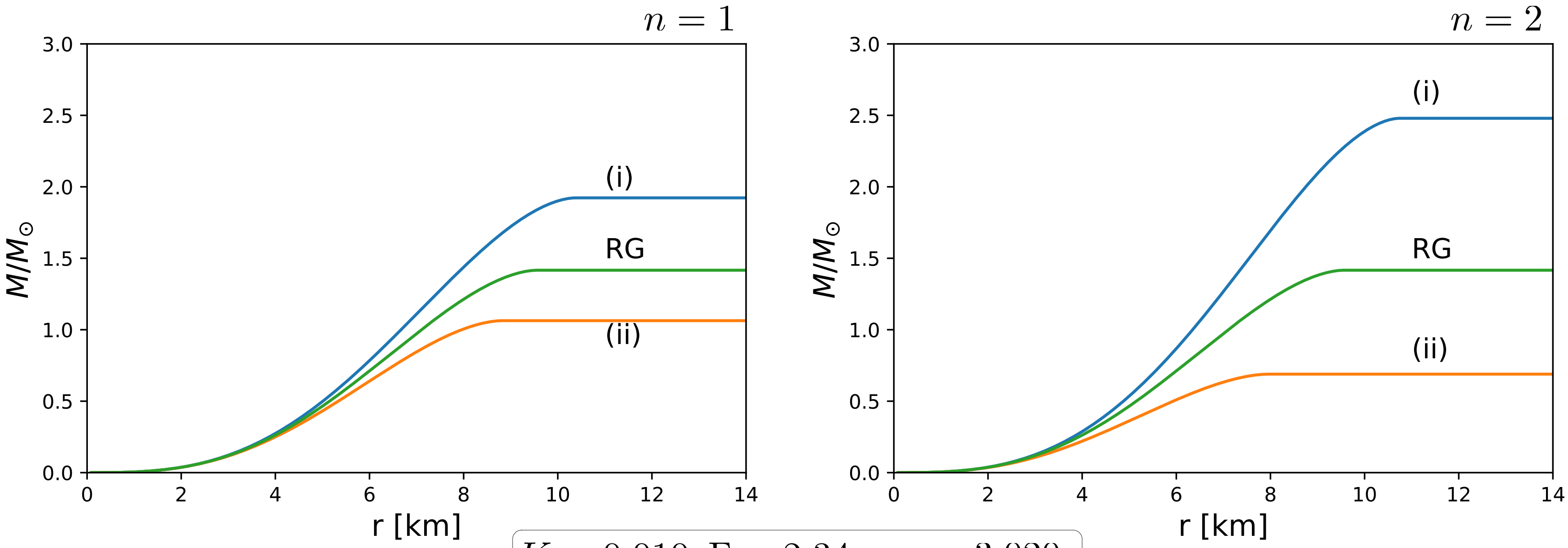
(ii) $\bar{\beta}_4 = +0.06,$ $\bar{a}_0 = 1.5.$

RG $\bar{\beta}_4 = 0,$ $\bar{a}_0 = 0.$

Case 2: Quartic Couplings $G_4(X) = \frac{1}{16\pi G} + \beta_4 X^n$

Numerical Solutions



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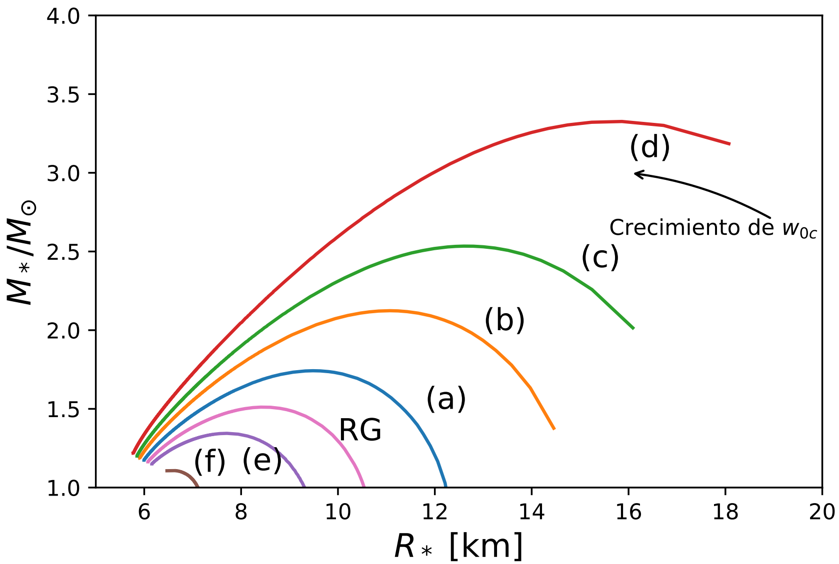

Mass-radius profile

$K = 0.010, \Gamma = 2.34.$

(a) $\bar{\beta}_4 = -0.1, \quad \bar{a}_0 = 1.0.$

(b) $\bar{\beta}_4 = -0.1, \quad \bar{a}_0 = 1.2.$

(c) $\bar{\beta}_4 = -0.1, \quad \bar{a}_0 = 1.3.$

(d) $\bar{\beta}_4 = -0.1, \quad \bar{a}_0 = 1.4.$

(e) $\bar{\beta}_4 = +0.1, \quad \bar{a}_0 = 1.0.$

(f) $\bar{\beta}_4 = +0.1, \quad \bar{a}_0 = 1.5.$

GR $\bar{\beta}_4 = 0, \quad \bar{a}_0 = 0.$

Why is this important?

NAME	MASS M_0	Reference
PSR J1748-2021B	2.74±0.21	P. Freire, *et. al. Astrophys. J*. 2008.
4U 1700-37	2.44±0.27	J. S. Clark, *et. al. Astron. Astrophys.* 2002.
PSR J1311-3430	2-15±2.7	R. Romani, *et. al. Astrophys. J. Lett.* 2012.
PSR B1957+20	2.4±0.12	M. H. Van Kerkwijk, *et. al. Astrophys. J*. 2011.
PSR J1600-3053	2.3±0.7	Z. Arzoumanian, *et. al. Astrophys. J. Suppl. Ser.* 2018.
PSR J2215+5135	2.27±0.17	M. Linares, *et. al. Astrophys. J.* 2018.
XMMU J013236.7+303228	2.2±0.8	B. Varun, *et. al. Astrophys. J.* 2012.
PSR J0740+6620	2.14±0.8	H. Cromartie, *et. al. Nature. Astron.* 2019.
PSR J0751+1807	2.10±0.2	D. Nice, *et. al. Astrophys. J*. 2005.
PSR J0348+0432	2.01±0.04	P. B. Demorest, *et. al. Nature*. 2010.
PSR B1516+02B	1.94±0.17	P. Freire, *et. al.* *AIP. Conf. Proc.* 2008.
PSR J1614-2230	1.908±0.016	F. Crawford, *et. al. Astrophys. J*. 2006.
Vela X-1	1.88±0.13	H. Quaintrell, *et. al. Astron. Astrophys.* 2003.

Some Conclusions

We found the role of cubic Galileon in the modifications of the internal structure of the star and its effects on mass and radius deviations.

Under the same EOS, the generalized Proca theory predicts objects of greater or less compactness than in GR (depending on the sign of the coupling). This is useful to make comparisons with observational data.

The solution at the exterior of the star matches the Reissner-Nordström vacuum solution (L. Heisenberg, *et. al. JCAP.* 2017).

Future work should consider more realistic situations that can explain the current observations.

THANK YOU!