Dark Energy from Coupled p–forms

Alejandro Guarnizo
Universidad Antonio Nariño

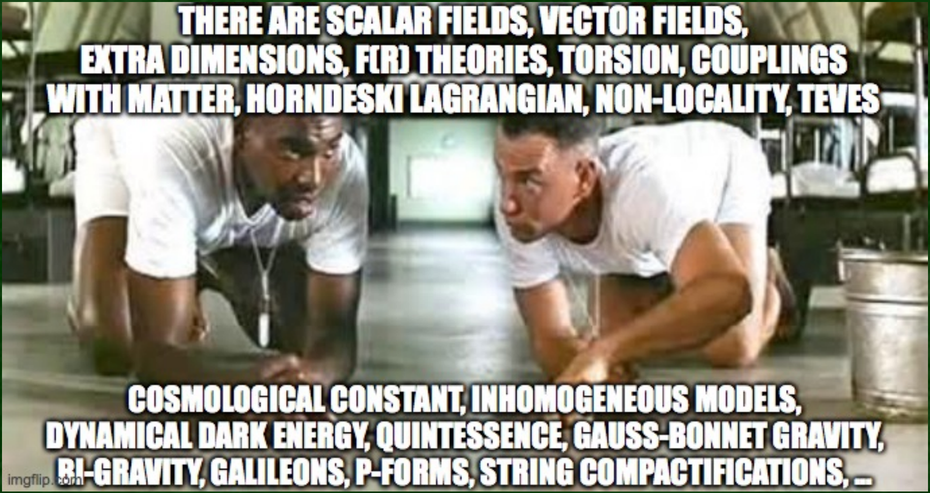
In collaboration with Juan P. Beltrán, Ryotaro Kase, Shinji Tsujikawa and César A. Valenzuela

Work in progress

CoCo 2020


DARK ENERGY
Made from...
well, we dont know!
EXPAND YOUR UNIVERSE!

General Relativity | Modified Gravity


THERE ARE SCALAR FIELDS, VECTOR FIELDS,
EXTRA DIMENSIONS, F(R) THEORIES, TORSION, COUPLINGS
WITH MATTER, HORNDESKI LAGRANGIAN, NON-LOCALITY, TEVES
COSMOLOGICAL CONSTANT, INHOMOGENEOUS MODELS,
DYNAMICAL DARK ENERGY, QUINTESSENCE, GAUSS-BONNET GRAVITY,
BI-GRAVITY, GALILEONS, P-FORMS, STRING COMPACTIFICATIONS, ...
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The idea

To built the most general Lagrangian allowing couplings between different p−forms, and also couplings with a scalar field (a 0−form)

- Juan P. Beltrán-Almeida, AG and César A. Valenzuela Toledo, Class. Quantum Grav. 37 (2020) 035001
  The construction and some applications at background level
- Juan P. Beltrán Almeida, AG, Ryotaro Kase, Shinji Tsujikawa, César A. Valenzuela-Toledo, JCAP 03 (2019) 025
  Anisotropic inflation: the coupled system of 1−and 2−forms sustaining inflation
- Juan P. Beltrán Almeida, AG, Ryotaro Kase, Shinji Tsujikawa, César A. Valenzuela-Toledo, Phys. Lett. B 793 (2019) 396-404
  Anisotropic dark energy from a 2−form field: cosmologically viable

We want to apply the whole coupled system as a source of dark energy

The model

We start from the action

$$\mathcal{S}=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R+\underbrace{P(\phi,X)}_{3-\text{form}}-\underbrace{\frac{f_1(\phi)F^2}{4}}_{1-\text{form}}-\underbrace{\frac{f_2(\phi)H^2}{12}}_{2-\text{form}}-\underbrace{\frac{m_v B\tilde{F}}{2}}_{\text{coupled}}\underbrace{+P_f(Z)}_{k-\text{essence}}\right]$$

with

$$F_{\alpha\beta}=\partial_{[\alpha}A_{\beta]},\quad H_{\alpha\beta\gamma}=\partial_{[\alpha}B_{\beta\gamma]},$$

$$F^2\equiv F_{\alpha\beta}F^{\alpha\beta}\,,\quad H^2\equiv H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}\,,\quad B\tilde{F}\equiv B_{\alpha\beta}\tilde{F}^{\alpha\beta}\,.$$

$$X=-\partial_\mu\phi\partial^\mu\phi/2,\quad Z=-\partial_\mu\chi\partial^\mu\chi/2$$

Background equations

A_μ in the x direction $A_\mu = (0, v_A(t), 0, 0)$.
$B_{\mu\nu}$ orthogonal to the 1-form field $B_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = 2v_B(t)\mathrm{d}y \wedge \mathrm{d}z$.

The line element corresponds to a Bianchi I type with lapse function N

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}\mathrm{d}x^2 + e^{2\sigma(t)}(\mathrm{d}y^2 + \mathrm{d}z^2)\right] ,$$

$a \equiv e^{\alpha(t)}$ is an isotropic scale factor, $\sigma(t)$ is a spatial shear. In the background

$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{3M_{\mathrm{pl}}^2 e^{3\alpha}}{N}\left(\dot{\sigma}^2 - \dot{\alpha}^2\right) + Ne^{3\alpha}P(\phi, X) + \frac{f_1(\phi)}{2N}e^{\alpha+4\sigma}\dot{v}_A^2 \right.$$
$$\left. + \frac{f_2(\phi)}{2N}e^{-\alpha-4\sigma}\dot{v}_B^2 + m_v\dot{v}_A v_B + Ne^{3\alpha}P_f(Z)\right]$$

varying w.r.t $N, \alpha, \sigma, \phi, Z$

$$3M_{\rm pl}^2 H^2\left(1-\Sigma^2\right) = \dot{\phi}^2 P_{,X} - P + \rho_A + \rho_B + \rho_f \,,$$

$$M_{\rm pl}^2\left(\dot{H} + 3H^2\Sigma^2\right) = -\frac{1}{2}\dot{\phi}^2 P_{,X} - \frac{2}{3}\rho_A - \frac{1}{3}\rho_B - \frac{1}{2}\left(\rho_f + P_f\right) \,,$$

$$M_{\rm pl}^2\left[H\dot{\Sigma} + \left(\dot{H} + 3H^3\right)\Sigma\right] = \frac{2}{3}(\rho_A - \rho_B)\,,$$

$$\left(P_{,X} + 2XP_{,XX}\right)\ddot{\phi} + 3P_{,X}H\dot{\phi} + P_{,X\phi}\dot{\phi}^2 - P_{,\phi} - \frac{f_{1,\phi}}{f_1}\rho_A - \frac{f_{2,\phi}}{f_2}\rho_B = 0\,,$$

$$\dot{\rho}_f + 3H\left(\rho_f + P_f\right) = 0\,,$$

where $P_{,X} \equiv \partial P/\partial X$, $P_{,XX} \equiv \partial^2 P/\partial X^2$, and

$$H \equiv \dot{\alpha}\,, \qquad \Sigma \equiv \dot{\sigma}/H\,.$$

and the energy densities

$$\rho_A = \frac{f_1(\phi)}{2} e^{-2\alpha+4\sigma}\dot{v}_A^2\,, \qquad \rho_B = \frac{f_2(\phi)}{2} e^{-4\alpha-4\sigma}\dot{v}_B^2\,, \qquad \rho_f = \dot{\chi}^2 P_{f,Z} - P_f\,.$$

The energy densities obey

$$\dot{\rho}_A = -4\rho_A H \left(1 + \Sigma + \frac{\dot{f}_1}{4f_1}\right) - 2m_v \sqrt{\frac{\rho_A \rho_B}{f_1 f_2}},$$

$$\dot{\rho}_B = -2\rho_B H \left(1 - 2\Sigma + \frac{\dot{f}_2}{2f_2}\right) + 2m_v \sqrt{\frac{\rho_A \rho_B}{f_1 f_2}}.$$

For the couplings $f_1(\phi)$ and $f_2(\phi)$, we consider the exponential functions:

$$f_1(\phi) = \bar{f}_1 e^{-\mu_1 \phi / M_{\rm pl}}, \qquad f_2(\phi) = \bar{f}_2 e^{-\mu_2 \phi / M_{\rm pl}},$$

where $\bar{f}_1, \bar{f}_2, \mu_1, \mu_2$ are constants.

We are extending the analysis to the dynamics of dark energy in the presence of coupled 1- and 2-forms with matter, i.e., $\bar{f}_1 \neq 0$, $\bar{f}_2 \neq 0$, $m_v \neq 0$, and $P_f \neq 0$.

Dynamical System

We introduce the following dimensionless quantities

$$x_1 = \frac{\dot{\phi}}{\sqrt{6} H M_{\rm pl}}\,, \qquad x_2 = \frac{M_{\rm pl} e^{-\lambda\phi/(2M_{\rm pl})}}{\sqrt{3}H}\,, \qquad \Sigma = \dot{\sigma}/H\,,$$

$$\Omega_A = \frac{\rho_A}{3H^2 M_{\rm pl}^2}\,, \qquad \Omega_B = \frac{\rho_B}{3H^2 M_{\rm pl}^2}\,, \qquad \Omega_r = \frac{\rho_r}{3H^2 M_{\rm pl}^2}\,, \qquad \Omega_m = \frac{\rho_m}{3H^2 M_{\rm pl}^2}\,.$$

Also

$$\frac{Y}{M_{\rm pl}^4} = \frac{x_1^2}{x_2^2}\,.$$

and the constraint

$$\Omega_m = 1 - (g + 2g_1)\, x_1^2 - \Sigma^2 - \Omega_A - \Omega_B - \Omega_r\,,$$

where we adopt the notation

$$g_n(Y) \equiv Y^n \frac{{\rm d}^n g(Y)}{{\rm d}Y^n}\,.$$

The dynamical system reads

$$
\begin{aligned}
x_1' =& -\frac{x_1}{2}\left(\sqrt{6}\lambda x_1 - 3gx_1^2 - 3\Sigma^2 - \Omega_A + \Omega_B - \Omega_r - 3\right) \\
& + A\left[\frac{\sqrt{6}}{2}\left\{(g+2g_1)\lambda x_1^2 - \mu_1\Omega_A - \mu_2\Omega_B\right\} - 3\left(g+g_1\right)x_1\right], \\
x_2' =& -\frac{x_2}{2}\left(\sqrt{6}\lambda x_1 - 3gx_1^2 - 3\Sigma^2 - \Omega_A + \Omega_B - \Omega_r - 3\right), \\
\Sigma' =& \frac{3}{2}\Sigma^3 + \frac{\Sigma}{2}\left(3gx_1^2 + \Omega_A - \Omega_B + \Omega_r - 3\right) + 2\Omega_A - 2\Omega_B, \\
\Omega_A' =& \Omega_A\left(3gx_1^2 + \sqrt{6}\mu_1 x_1 + 3\Sigma^2 - 4\Sigma + \Omega_A - \Omega_B + \Omega_r - 1\right) - 2\mathcal{M}\mathcal{F}\sqrt{\Omega_A\Omega_B}, \\
\Omega_B' =& \Omega_B\left(3gx_1^2 + \sqrt{6}\mu_2 x_1 + 3\Sigma^2 + 4\Sigma + \Omega_A - \Omega_B + \Omega_r + 1\right) + 2\mathcal{M}\mathcal{F}\sqrt{\Omega_A\Omega_B}, \\
\Omega_r' =& \Omega_r\left(3gx_1^2 + 3\Sigma^2 + \Omega_A - \Omega_B + \Omega_r - 1\right),
\end{aligned}
$$

with

$$
\mathcal{M} = \frac{m_v}{H}, \qquad \mathcal{F} = \frac{e^{(\mu_1+\mu_2)\phi/(2M_{\rm pl})}}{\sqrt{\bar{f}_1\bar{f}_2}}.
$$

The dimensional quantities $\mathcal{M}$ and $\mathcal{F}$ obey

$$
\mathcal{M}' = \frac{1}{2}\mathcal{M}\left(3 + 3gx_1^2 + 3\Sigma^2 + \Omega_A - \Omega_B + \Omega_r\right) \qquad \mathcal{F}' = \frac{\sqrt{6}}{2}\mathcal{F}(\mu_1+\mu_2)x_1 .
$$

Provided that the function $g(Y)$ is specified, the cosmological dynamics is known by solving the previous set of equations with given initial values of $x_1, x_2, \Sigma, \Omega_A, \Omega_B, \Omega_r, \mathcal{M}, \mathcal{F}$.

To characterize the evolution of the system, we define

$$\omega_{\rm eff} = g x_1^2 + \Sigma^2 + \frac{1}{3}\left(\Omega_A - \Omega_B + \Omega_r\right).$$

$$\Omega_{DE} = \frac{\rho_{DE}}{3H^2 M_{pl}^2} = (g + 2g_1)x_1^2 + \Sigma^2 + \Omega_A + \Omega_B = 1 - \Omega_m - \Omega_r$$

$$\omega_{DE} = \frac{P_{DE}}{\rho_{DE}} = \frac{3\left(g x_1^2 + \Sigma^2\right) + \Omega_A - \Omega_B}{3\left(x_1^2(g + 2g_1) + \Omega_A + \Omega_B + \Sigma^2\right)}.$$

with

$$\rho_{DE} = \dot{\phi}^2 P_{,X} - P + \rho_A + \rho_B + 3M_{\rm pl}^2 H^2 \Sigma^2\,,$$

$$P_{DE} = P + \frac{1}{3}\left(\rho_A - \rho_B\right) + 3M_{\rm pl}^2 H^2 \Sigma^2\,.$$

Fixed points (uncoupled case $m_v = 0$)

- Isotropic point (A1): The isotropic point, which corresponds to the vanishing shear ($\Sigma = 0$), obeys

$$P_{,X} = \frac{\lambda}{\sqrt{6}x_1}\,, \quad g_1 = \frac{6-\sqrt{6}\lambda x_1}{6x_1^2} \quad \Sigma = 0\,, \qquad \Omega_A = 0\,, \quad \Omega_B = 0\,,$$

- 1−form dominated point (A2)

$$P_{,X} = \frac{(\lambda+\mu_1)[2\sqrt{6}-(\lambda+3\mu_1)x_1]}{8x_1}\,,$$

$$g_1 = \frac{[2\sqrt{6}-(\lambda+\mu_1)x_1](\sqrt{6}-\lambda x_1)}{8x_1^2}\,,$$

$$\Sigma = \frac{\sqrt{6}}{4}(\lambda+\mu_1)x_1 - 1\,,$$

$$\Omega_A = \frac{1}{8}\left[3(\lambda+\mu_1)x_1 - 2\sqrt{6}\right]\left(\sqrt{6}-\lambda x_1\right)\,, \qquad \Omega_B = 0\,.$$

Fixed points (uncoupled case $m_v = 0$)

- 2–form dominated point (A3)

$$P_{,x} = \frac{(2\lambda + \mu_2)\sqrt{6} - (2\lambda + 3\mu_2)(\lambda + \mu_2)x_1}{8x_1},$$

$$g_1 = \frac{[\sqrt{6} - (\lambda + \mu_2)x_1](\sqrt{6} - \lambda x_1)}{4x_1^2}, \qquad \Sigma = \frac{1}{2} - \frac{\sqrt{6}}{4}(\lambda + \mu_2)x_1,$$

$$\Omega_A = 0, \qquad \Omega_B = \frac{1}{8}\left[3(\lambda + \mu_2)x_1 - \sqrt{6}\right]\left(\sqrt{6} - \lambda x_1\right).$$

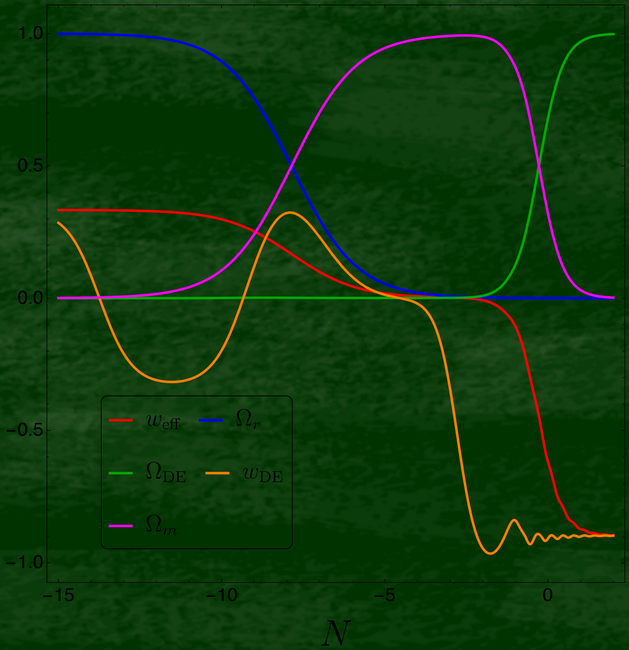
- Fixed point (A4) supported by 1– and 2–forms

$$x_1 = \frac{\sqrt{6}}{2\lambda + \mu_1 + \mu_2}, \qquad g = -\frac{1}{4}\left(\mu_1^2 + \mu_2^2\right) - \frac{\lambda}{8}\left(2\mu_1 + 3\mu_2\right) - \frac{1}{8}\mu_1\mu_2,$$

$$\Sigma = -\frac{\lambda - \mu_1 + 2\mu_2}{2(2\lambda + \mu_1 + \mu_2)}, \quad \Omega_A = \frac{3(\lambda + \mu_1 + \mu_2)(\lambda + \mu_1) - 12g_1}{2(2\lambda + \mu_1 + \mu_2)^2},$$

$$\Omega_B = \frac{3(\lambda + \mu_1 + \mu_2)(3\lambda + \mu_1 + 2\mu_2) - 24g_1}{4(2\lambda + \mu_1 + \mu_2)^2}.$$

Numerical solutions



Conclusions

1. p–forms could generate signals of anisotropic dark energy (in the decoupled case)
2. To do: if possible, a stability analysis of the coupled case. If not, a "rigorous numerical treatment"
3. Coupled p–forms joins the vast family of modified gravity theories
4. Dark energy problem is still unsolved