Towards the observed galaxy bispectrum

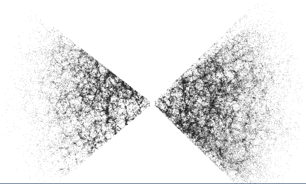
in the weak field approximation

Lina Julieth Castiblanco Tolosa

Based on : L.C, R.Gannouji, J.Noreña, C.Stahl (2018),
J.Calles, L.C, J.Noreña, C.Stahl (2019)

PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO

CoCo 2020



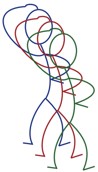
Inflation, Initial conditions

Inflation provides a mechanism to generate primordial perturbations which are the seed to structure formation.

Single field inflation?

Initial conditions: Gaussian, adiabatic and almost scale invariant.

Multi-field inflation? Exotic Inflation?



Predict large non-Gaussianity.

Consistency relations

One way to test the field content during inflation in a model independent way is through the consistency relations.

$$\lim_{q\to 0} \langle \zeta(\boldsymbol{q})\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle' \propto (n_s - 1) P_\zeta(q) \langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle'$$

$$\Downarrow$$

$$f_{NL}$$

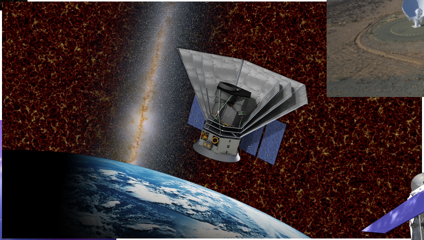
- Inflationary three point function does not have terms diverging like q^{-3} . Maldacena (2002), Creminelli.et.al (2004)
- Valid for single field models. Bravo.et.al. (2017)
- Diverging terms like q^{-2} are also absent. Creminelli.et.al.(2011)
- The consistency relation for the large scale structure is a continuation of the single field consistency relation. Creminelli.et.al (2013)

Violations to the consistency relations will rule out all the single field inflationary models.

We are in the era of precision cosmology.

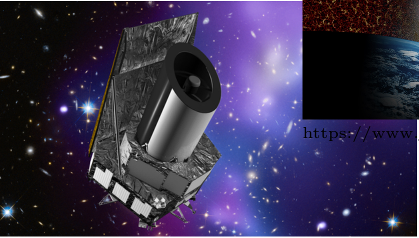
The Large Synoptic Survey Telescope
https://www.lsst.org/lsst

Spherex

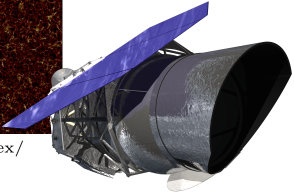


https://www.jpl.nasa.gov/missions/spherex/

Square Kilometre Array
https://www.skatelescope.org/



Euclid http://sci.esa.int/euclid/



Wide Field Infrared Survey Telescope
https://wfirst.gsfc.nasa.gov/index.html

Dark matter perturbations

Dark matter is considered as a barotropic irrotational perfect fluid

$$T_{\mu\nu} = \bar{\rho}\,(1+\delta)\,u_\mu u_\nu$$

Perturbed FLRW metric

$$ds^2 = -\,(1+2\phi)\,dt^2 + 2\omega_i dt dx^i + a(t)^2\left[(1-2\psi)\,\delta_{ij} + \gamma_{ij}\right]dx^i dx^j$$

$$\omega_i = \partial_i \omega + w_i \Longrightarrow \partial_i w_i = 0, \qquad \gamma_{ii} = \partial_i \gamma_{ij} = 0$$

- Weak field approximation:

$$\phi \sim \psi \sim \omega \sim \left(\frac{H^2}{\nabla^2}\right) = \mathcal{O}(\epsilon) \ll 1, \qquad w_i \sim \mathcal{O}(\epsilon^{3/2}), \quad \gamma_{ij} \sim \mathcal{O}(\epsilon^2)$$

- Gravitational field and velocity are small at small scales

$$\psi \sim \frac{H^2}{\nabla^2} \sim 10^{-5}, \qquad u^i \sim \frac{H}{\nabla} \sim 10^{-3}$$

Dark matter perturbations

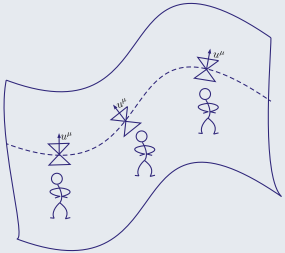
Comoving gauge

- Proper time coincides with time coordinate along the fluid.

Gauge condition:

$$u^0 = 1$$

$$u^i = \partial_i \omega (1 + 2\psi) + w_i + \mathcal{O}(\epsilon^2)$$



- Comoving (synchronous) observer with the fluid.

Yoo.et.al. (2009)

Fluid Equations

Continuity equation

$$\dot{\delta} + \theta = -\partial_i \left(\delta u^i\right) + S_\delta \left[\psi, \delta, u^i\right]$$

Mass conservation

Euler equation

$$\dot{\theta} + 2H\theta + \frac{3}{2}H^2\delta = \partial_j \left(u^i \partial_j u^i\right) + S_\theta \left[\psi, \delta, u^i\right]$$

Momentum conservation

Einstein equations

$$\nabla^2\psi = \frac{5}{2}H^2\delta + S_\psi[\psi, \delta, \theta] \qquad \nabla^2 w_i = S_w[\psi, \delta, \theta]$$

- Separation between Newtonian result and relativistic (corrections suppressed by $\epsilon = \frac{H^2}{\nabla^2}$):

$$\delta = \delta_N + \delta_R, \qquad u^i = u^i_N + \left(u^i_R + u^i_T\right), \qquad \theta = \partial_i u^i$$

Matter perturbation dynamics

- Evolution equation for matter perturbations.

$$\ddot{\delta}(t,\boldsymbol{k}) + 2H\dot{\delta}(t,\boldsymbol{k}) - \frac{3}{2}H^2\delta(t,\boldsymbol{k}) = S(t,\boldsymbol{k})[\psi,\delta,\boldsymbol{u}^i]$$

- Solving with standard perturbation theory, $\delta \ll 1$ and $\theta \ll 1$

The solution to fluid equations is an expansion in powers of the linear density perturbation

$$\delta(t,\boldsymbol{k}) = \sum_{n=1}^{\infty} a^n(t) \int_{\boldsymbol{k}_1\cdots\boldsymbol{k}_n} (2\pi)^3 \delta_D(\boldsymbol{k}-\boldsymbol{k}_{1\ldots n}) \left[F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n) \right. \\ \left. + a^2(t)H^2(t)F_n^R(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\right] \delta_\ell(\boldsymbol{k}_1)\cdots\delta_\ell(\boldsymbol{k}_n)$$

- During matter domination $H^2 \sim \frac{1}{a^3}$

Initial conditions

- Initial conditions are taken to match the full GR calculation to leading order. Boubekeur.et.al (2008)
- Initial conditions need to be fixed up to second order.
- Higher order initial conditions are subdominant with respect to the source.

$$F_2^R(\boldsymbol{k}_1, \boldsymbol{k}_2) = \left(-\frac{5}{2} \frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2} + \frac{5}{4} \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2} \right)$$

Has the same behavior as:

Primordial non-Gaussianity of the local type

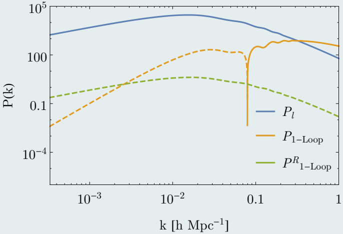
$$\psi_o = \psi_G + f_{NL} \psi_G^2 \Longrightarrow F_2^{NL}(\boldsymbol{k}_1, \boldsymbol{k}_2) = -\frac{3}{2} \frac{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2}{\boldsymbol{k}_1^2 \boldsymbol{k}_2^2}$$

Correlation functions for matter perturbations

Power spectrum up to one loop

$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_{12}) P(k_1)$$

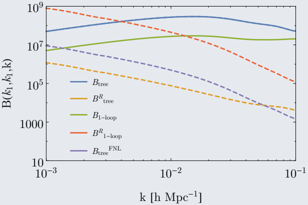
$$P(k) = P_{11}(k) + P_{22}(k) + P_{13}(k)$$



Bispectrum up to one loop

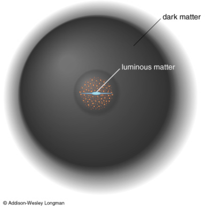
$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_{123}) B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = B_{211} + B_{321} + B_{222} + B_{411}$$



$k_1 = 0.1\text{h/Mpc}$

Galaxy Bias



- Galaxy formation is a local processes.
- Expansion in terms of second derivatives of the gravitational potential $\nabla^2\psi \sim \delta$.
- Galaxies moves with dark matter fluid

$$\Downarrow$$

Neglect bias velocity.

Lagrangias bias expansion $\Longmapsto$ Adiabatic initial conditions

Building blocks in terms of the extrinsic curvature of the constant-time hypersurfaces, and the matter density contrast.

$$\delta_g(a_*) = \sum_{n=1}^{4} \frac{b_n^*}{n! a_*^n}\delta^n + \sum_{n=2}^{4} \frac{b_{s^n}^*}{a_*^n}(S^n) + \frac{b_{\delta s^2}^*}{a_*^3}(S^2)\delta + \frac{b_{\delta^2 s^2}^*}{a_*^4}(S^2)\delta^2 + \frac{b_{\delta s^3}^*}{a_*^4}(S^3)\delta + \frac{b_{(s^2)^2}^*}{a_*^4}(S^2)^2$$

$$S^i{}_j \equiv (K^\ell{}_\ell \delta^i_j/3 - K^i{}_j)/H^2$$

Galaxy bias evolution

Conserved number of galaxies

$$\dot{\delta}_g + \theta = -\partial_i \left(\delta_g u^i\right) + S_{\delta_g}\left[\psi, \delta_g, u^i\right]$$

At first order:

$$\delta_g(\eta_*) = b_1^* \delta_\ell \Longrightarrow \delta_g^{(1)} = a\delta_\ell \left(1 + \frac{b_1^*}{a}\right)$$

Eulerian Galaxy bias up to fourth order in perturbations

$$\delta_g(\boldsymbol{k}, a) = \delta(\boldsymbol{k}, a) + \sum_{n=1}^{\infty} a^n \int_{\boldsymbol{k}_1 \cdots \boldsymbol{k}_n} \delta_D(\boldsymbol{k} - \boldsymbol{k}_{1\cdots n}) \sum_{\mathcal{O}} b_{\mathcal{O}}^{\mathcal{L}} M_n^{\mathcal{O}}(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n, a) \delta_\ell(\boldsymbol{k}_1) \cdots \delta_\ell(\boldsymbol{k}_n)$$

$$M_n^{\mathcal{O}}(\boldsymbol{k}, \eta) = M_n^{\mathcal{O},N}(\boldsymbol{k}) + a^2 H^2 M_n^{\mathcal{O},R}(\boldsymbol{k})$$

Renormalization of the bias operators

The bias solution $\delta_g = \delta + \mathcal{O}$ generates correlations functions which have divergences at large scales coming from the composed operators $\mathcal{O}$.

Renormalization condition

$$\lim_{\boldsymbol{q}_i \to 0} \left\langle [\mathcal{O}_{\boldsymbol{k}}]_\Lambda \, \delta^{(1)}_{\boldsymbol{q}_1} \cdots \delta^{(1)}_{\boldsymbol{q}_n} \right\rangle = \left\langle [\mathcal{O}_{\boldsymbol{k}}] \, \delta^{(1)}_{\boldsymbol{q}_1} \cdots \delta^{(1)}_{\boldsymbol{q}_n} \right\rangle_{\text{tree}}$$

Assassi.et.al (2014)

Renormalization for the operator proportional to b_2^*

$$\frac{b_2^*}{2a_*^2} \left\langle \delta^2 \right\rangle = \frac{1}{2} b_2^* \int_{\boldsymbol{q}} P_L(q) = \frac{1}{2} b_2^* \sigma_\Lambda^2(q)$$

$$\frac{1}{2a^2} b_2^* \lim_{\boldsymbol{k}\to 0} \left\langle \delta_\ell(\boldsymbol{k}) \delta^2(-\boldsymbol{k}) \right\rangle' = \left(-\frac{5}{k^2} \sigma^2(\Lambda) - 5\sigma_{-2}^2(\Lambda) \right) b_2^* a^3 H_*^2 P(k)$$

$$\left[\frac{1}{2a_*^2} b_2^* \delta^2 \right]_\Lambda = \frac{1}{2a_*^2} b_2^* \delta^2 - \frac{1}{2} b_2^* \sigma^2 + \frac{5}{k^2} b_2^* \sigma^2 \delta_\ell + 5 b_2^* a_*^3 H_*^2 \sigma_{-2}^2 \delta_\ell$$

The comoving physical cutoff scale is modified by the presence of the long-wavelength perturbation $\sigma(\Lambda_{\text{phy}}) = \left(1 - \frac{10}{k^2}\delta_\ell\right)$. de Putter.et.al (2015)

Renormalization for the evolved bias expansion

Counter-terms for the evolved bias solution are obtained by evolving renormalized operators and subtracting extra cut-off dependence due to non-linear evolution.

$\langle \mathcal{O}(\boldsymbol{k}, a) \rangle = 0$

$M_0^{\mathcal{O},\mathrm{c.t}}(\boldsymbol{k}, a)$

$\langle \mathcal{O}(\boldsymbol{k}, a) \delta_\ell(-\boldsymbol{k}) \rangle$

$M_1^{\mathcal{O},\mathrm{c.t}}(\boldsymbol{k}, a)$

$\langle \mathcal{O}(\boldsymbol{k}, a) \delta_\ell(\boldsymbol{q}_1) \delta_\ell(\boldsymbol{q}_2) \rangle$

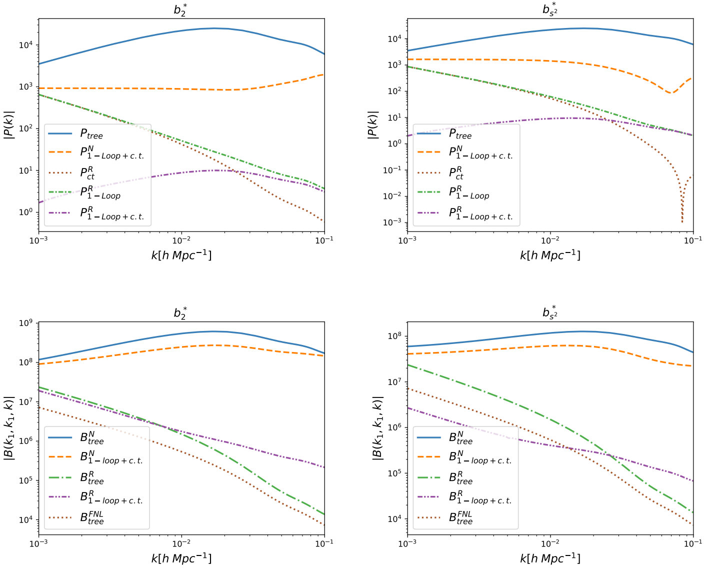
$M_2^{\mathcal{O},\mathrm{c.t}}(\boldsymbol{k}, a)$

Counter-terms contribute up to tree level to the galaxy correlation functions.

$$P_g^{c.t} = 2 \sum_{\mathcal{O}} b_{\mathcal{O}}^{\mathcal{L}} M_1^{\mathcal{O},c.t.}(\boldsymbol{k}) P_L(k)$$

$$\begin{aligned} B_g^{c.t.}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) =& 2 \sum_{\mathcal{O}} b_{\mathcal{O}}^{\mathcal{L}} M_2^{\mathcal{O},c.t.}(\boldsymbol{k}_2, \boldsymbol{k}_3) P_L(k_2) P_L(k_3) + 2\,\mathrm{perm} \\ &+ 2 \sum_{\mathcal{O}} b_{\mathcal{O}}^{\mathcal{L}} M_1^{\mathcal{O},c.t.}(\boldsymbol{k}_1) F_2(\boldsymbol{k}_2, \boldsymbol{k}_3) P_L(k_2) P_L(k_3) + 2\,\mathrm{perm} \end{aligned}$$

Galaxy correlation functions



Conclusions

Relativistic correction to the bispectrum is as large as the Newtonian result in the squeeze limit.

Also in this limit it is degenerated with the primordial non-Gaussianity signal of the local type $f_{NL} \sim \mathcal{O}(1)$.

Galaxy bispectrum is not gauge-independent. Still missing propagation effects.

Agencia Nacional de Investigación y Desarrollo

Ministerio de Ciencia, Tecnología, Conocimiento e Innovación

Gobierno de Chile

References

Rafael Bravo, Sander Mooij, Gonzalo A. Palma, and Bastian Pradenas.
Vanishing of local non-Gaussianity in canonical single field inflation.
JCAP, 05:025, 2018.

Paolo Creminelli, Guido D'Amico, Marcello Musso, and Jorge Norena.
The (not so) squeezed limit of the primordial 3-point function.
JCAP, 11:038, 2011.

Paolo Creminelli, Jorge Norena, Marko Simonovic, and Filippo Vernizzi.
Single-Field Consistency Relations of Large Scale Structure.
JCAP, 12:025, 2013.

Paolo Creminelli and Matias Zaldarriaga.
Single field consistency relation for the 3-point function.
JCAP, 10:006, 2004.

Roland de Putter, Olivier Doré, and Daniel Green.
Is There Scale-Dependent Bias in Single-Field Inflation?
JCAP, 10:024, 2015.

Juan Martin Maldacena.
Non-Gaussian features of primordial fluctuations in single field inflationary models.
JHEP, 05:013, 2003.

Jaiyul Yoo.
Proper-time hypersurface of nonrelativistic matter flows: Galaxy bias in general relativity.
Phys. Rev. D, 90(12):123507, 2014.