Kantowski-Sachs Cosmological Model with Chaplygin Gas

Marcela M Rodrigues, Gil de Oliveira Neto

Physics Department, Federal University of Juiz de Fora, Brazil

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Sumário

Introduction

- In search of a cosmological model that describes the early stages of the universe, since its properties might have been unusual;
- The cosmological principle is not satisfied, because the model is homogeneous and anisotropic;
- The matter content of the model is Chaplygin gas;
- Analysis of the evolution of the scale factors and anisotropy parameter of the model.

Kantowski-Sachs Universe

Kantowski-Sachs (KS) cosmological model is spatially homogeneous and anisotropic. The KS metric is given by

$$ds^2 = -dt^2 + a^2(t)dr^2 + b^2(t)[d\theta^2 + sen^2\theta d\phi^2]. \qquad (1)$$

Kantowski-Sachs Universe

Considering $c = 8\pi G = 1$, the Einstein's equations for the KS metric and the energy-momentum tensor for a perfect fluid yield

$$G_{00} = 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = \rho; \tag{2}$$

$$G_{11} = G_{22} = \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} + 2\frac{\ddot{b}}{b} = p; \tag{3}$$

$$G_{33} = \frac{\ddot{b}}{b} + \frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} = p. \tag{4}$$

Kantowski-Sachs Universe

The anisotropy parameter is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{5}$$

where H_i are the directional Hubble parameters and H is the average Hubble parameter.

Therefore,

$$\Delta = 2\frac{(b\dot{a} - \dot{b}a)^2}{(b\dot{a} + 2\dot{b}a)^2}. \tag{6}$$

When analysing (6), we observed that the maximum value allowed is 2. When the parameter approaches zero, we observe the "isotropization"of the universe.

Chaplygin Gas

Chaplygin gas is a perfect fluid with an exotic equation of state, given by

$$p = \frac{-A}{\rho}, \tag{7}$$

where A is a positive constant.

Chaplygin Gas

Replace equation (7) in the conservation of energy equation and we get

$$\rho_{gc} = \left(A + \frac{c_1}{a^2 b^4} \right)^{1/2}, \tag{8}$$

Hence,

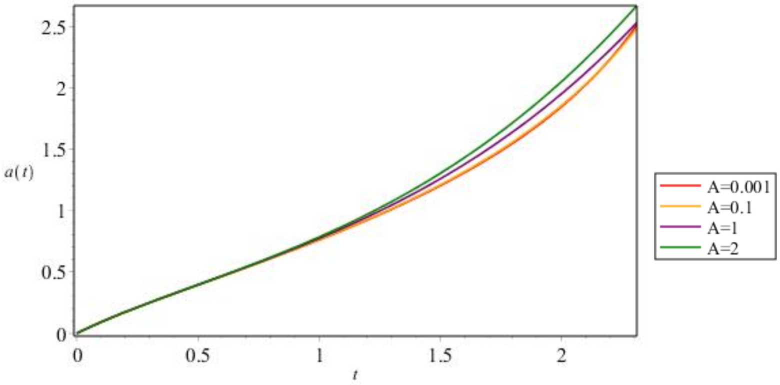
$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = \left(A + \frac{c_1}{a^2 b^4} \right)^{1/2} \tag{9}$$

$$\frac{\ddot{b}}{b} - \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = 0; \tag{10}$$

where c_1 is a positive integration constant.

Results

Initial conditions: $b(0) = 0.5$, $\dot{a}(0) = 1$, $\dot{b}(0) = 1$ e $c = 1$.



Evolution of the scale factor $a(t)$

Results

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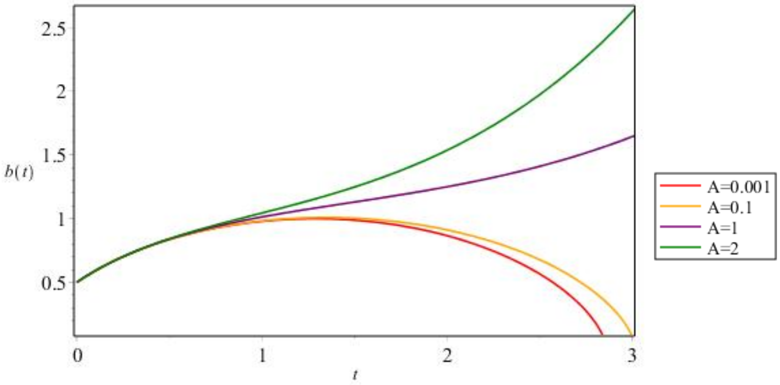

Evolution od the scale factor $b(t)$

Results

	$A = 0.001$	$A = 0.1$	$A = 1$	$A = 2$
t_s	2.857	3.015	206.667	173.488
Δ_i	2.00	2.00	2.00	2.00
Δ_f	2.00	2.00	3.058×10^{-20}	5.884×10^{-20}

Parameters obtained for the initial conditions

Conclusions

- Two types of solutions were found, one where $b(t)$ is bounded and $a(t)$ is expansive and another one where both scale factors are expansive;
- We observed that the velocity of the expansion is directly related to A;
- When $b(t)$ is bounded, Big Crunch singularity and constant anisotropy parameter;
- When both scale factors are expansive, Big Rip singularity and anisotropy parameter approaches zero.

References

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Acknowledgements

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