No slow-roll inflation à la Generalized Chaplygin Gas in General Relativity

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The parameter space of the generalized Chaplygin gas (GCG)

The GCG model is characterized by $P = -A\rho^{-\alpha}$, $A > 0$

1. For $-1 < \alpha$, no expansion but accelerated contraction. Contrary to previous works.
2. For $\alpha < -5/3 \approx -1.7$, no sustained inflationary period, since the slow-roll parms are large.
3. Only for $\alpha \to -1$ the model produces enough e-folds.
4. We constrain the parameters from Planck results → The GCG model is still alive!!!.

The GCG model $P = -A\rho^{-\alpha}$, $A > 0$

Extensively studied due to its interesting properties and applicability in several contexts such as late-time acceleration and cosmic inflation.

- The model corresponds to a generalized Nambu-Goto action [Bento et al., 2002].
- $\alpha = 1$, pure Chaplygin gas model [Kamenshchik et al., 2001], originally introduced to explain late-time acceleration without DE.
- $\alpha = -1$ corresponds to a cosmological constant.
- Different modifications: modified Chaplygin gas in brane-world [Herrera, 2008] and GCG in modified gravity [Barreiro and Sen, 2004].

The GCG model applied to inflation

- First approach with $0 < \alpha < 1$ [del Campo, 2013]. From the Planck 2013 data $\alpha = 0.2578 \pm 0.0009$.
- Later, shown not a suitable for inflation in GR for $\alpha < -1$ since $N_* \approx 217$, in order to obtain the BICEP2 constraint $r = 0.2^{+0.07}_{-0.05}$ [Dinda et al., 2014].
- Finally, the generalized Chaplygin-Jacobi gas (GCJG) [Villanueva, 2015, Villanueva and Gallo, 2015]

$$P = -\frac{A\,\kappa}{\rho^{\alpha}} - 2\,(1-\kappa)\,\rho + \frac{(1-\kappa)}{A}\,\rho^{2+\alpha}, \tag{1}$$

where $0 \leq \alpha \leq 1$ and $0 \leq \kappa \leq 1$ is the modulus of the elliptic function.

The Hubble parameter H as the fundamental quantity

The Hamilton-Jacobi equation

$$\dot{\phi} = -\frac{m_{\rm Pl}^2}{4\pi} H'(\phi)\,, \tag{2}$$

$$[H'(\phi)]^2 - \frac{12\pi}{m_{\rm Pl}^2} H^2(\phi) = -\frac{32\pi^2}{m_{\rm Pl}^4} V(\phi)\,, \tag{3}$$

where $m_{\rm Pl}$ is the Planck mass and dots and primes denote derivatives with respect to the cosmic time and the scalar field ϕ, respectively.

The generating function $H(\phi)$ is given by [Dinda et al., 2014]

$$H(\phi) = \sqrt{\frac{8\pi}{3\, m_{\rm Pl}^2}}\, A^{\frac{1}{2(1+\alpha)}} \cosh^{\frac{1}{1+\alpha}}\left[\sqrt{\frac{6\pi}{m_{\rm Pl}^2}}(1+\alpha)(\phi-\phi_0)\right], \quad (4)$$

where ϕ_0 is an integration constant given by

$$\phi_0 = \phi_i - \frac{1}{(1+\alpha)}\sqrt{\frac{m_{\rm Pl}^2}{6\pi}}\,\text{arcsinh}\left[\sqrt{\frac{\rho_i^{1+\alpha}}{A} - 1}\right], \quad (5)$$

where $\rho_i = \rho(\phi_i)$ is the energy density at the beginning of inflation and $\rho_i^{1+\alpha}/A > 1$.

The scale factor [Dinda et al., 2014]

$$a(\phi) = a_i \left(\frac{\sinh[(1+\alpha)\Phi]}{\sinh[(1+\alpha)\Phi_i]} \right)^{-\frac{2}{3(1+\alpha)}}, \tag{6}$$

where for simplicity we use the dimensionless variable

$$\Phi(\phi) \equiv \sqrt{\frac{6\pi}{m_{\rm Pl}^2}}(\phi - \phi_0). \tag{7}$$

The slow-roll parameters [del Campo, 2013]

$$\epsilon_H(\phi) = \frac{3}{2}\tanh^2\left[(1+\alpha)\Phi\right], \quad (8)$$

$$\eta_H(\phi) = \frac{3}{2}\left(1+\alpha\,\mathrm{sech}^2\left[(1+\alpha)\Phi\right]\right), \quad (9)$$

$$\xi_H^2(\phi) = \frac{9}{8}\Big(1-2\alpha(1+2\alpha)+\cosh\left[2(1+\alpha)\Phi\right]\Big)\mathrm{sech}^2\left[(1+\alpha)\Phi\right]\tanh^2\left[(1+\alpha)\Phi\right]. \quad (10)$$

The condition for the end of inflation is $\epsilon_H(\Phi_e) = 1$.

The total number of *e*-folds N from the beginning of inflation

The total number of *e*-folds at the end of inflation N_e is

$$N_e(\alpha, A) = \frac{1}{3(1+\alpha)} \ln\left\{\frac{1}{2}\left(\frac{\rho_i^{1+\alpha}}{A} - 1\right)\right\}. \qquad (11)$$

N_e can be arbitrarily large for either $\alpha \to -1$ or $\rho_i^{1+\alpha}/A \to 1$.

We can also see that $1 < \rho_i^{1+\alpha}/A < 3$.

The scale factor a and ϵ_H

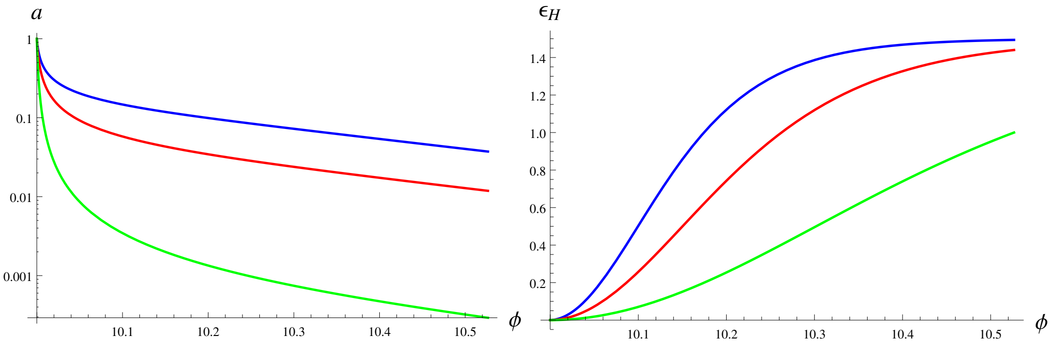


Figure: The scale factor a and ϵ_H are plotted for $\alpha = 0.5, 0, -0.5$ and $\rho_i^{1+\alpha}/A = 1.00001$ in the blue, red, and green lines, respectively. Notice that, although $\epsilon_H < 1$, the scale factor is in fact decreasing.

$$|\eta_H| < 1 \quad \text{only for} \quad -1.7 \approx -\frac{5}{3} < \alpha < -1. \tag{12}$$

$$|\xi_H| < 1 \quad \text{only for} \quad -1.5 = -\frac{3}{2} < \alpha < -1. \tag{13}$$

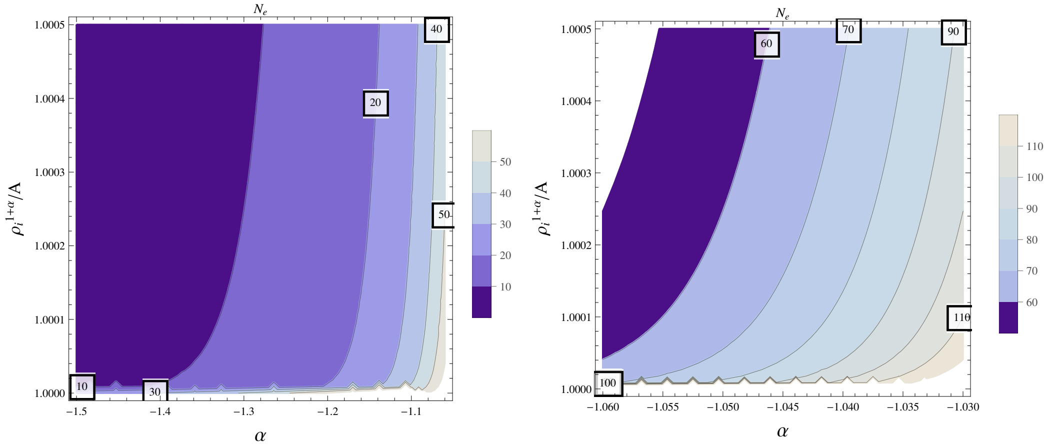


Figure: The contour values of N_e are plotted as functions of α and $\rho_i^{1+\alpha}/A$. It can be seen that even for $\alpha < -1.1$ and $\rho_i^{1+\alpha}/A > 1.0001$ we have $N < 60$. N_e increases as either $\alpha \to -1$ or $\rho_i^{1+\alpha}/A \to 1$, although the effect $\alpha \to -1$ is larger.

The Mukhanov-Sasaki equation [Sasaki, 1983]

The evolution of the comoving curvature perturbation $\mathcal{R}$ is governed by

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - 2a^2 H^2 \left[1 + f_H(\phi)\right]\right) u_k = 0, \tag{14}$$

where $d\tau \equiv dt/a$ is the conformal time, k the comoving wave number, $u \equiv z\mathcal{R}m_{\rm Pl}/\sqrt{8\pi}$ the Mukhanov-Sasaki variable, and $z = a\sqrt{2\epsilon_H}$.

f_H characterizes the validity of the slow-roll conditions

$$f_H(\phi) \equiv \epsilon_H - \frac{3}{2}\eta_H + \epsilon_H^2 - 2\epsilon_H\eta_H + \frac{1}{2}\eta_H^2 + \frac{1}{2}\xi_H^2 . \tag{15}$$

Despite its appearance as an expansion in the slow-roll parameters it is an exact expression.

We can estimate that

$$|f_H| < 1 \quad \text{only for} \quad -1.4 \approx -\frac{\sqrt{17}}{3} < \alpha < -1, \tag{16}$$

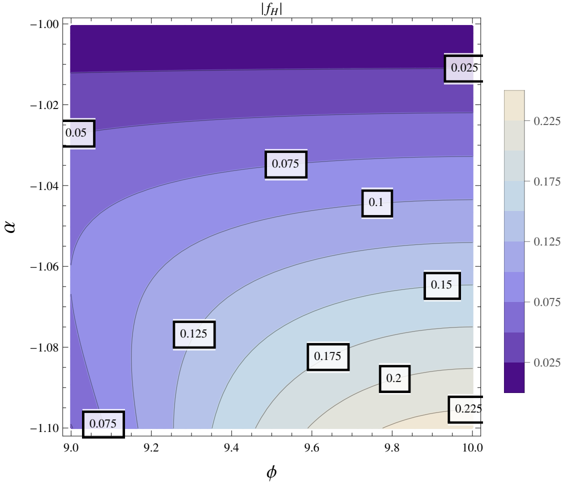


Figure: The contour values of $|f_H|$ are plotted as functions of $\phi \leq \phi_e$ and α. Only for $-1.04 < \alpha < -1$ we have $|f_H| \ll 1$.

Scalar and tensor spectra

Under the slow-roll approximation, $-1.04 < \alpha < -1$, the scalar spectrum $P_{\mathcal{R}}$ and tensor P_h spectrum are given by

$$P_{\mathcal{R}}(k_*) = \frac{1}{\pi m_{\rm Pl}^2} \frac{H^2}{\epsilon_H}\bigg|_{k_* = a_* H_*}, \quad (17)$$

$$P_h(k_*) = \frac{16}{\pi m_{\rm Pl}^2} H^2\bigg|_{k_* = a_* H_*}. \quad (18)$$

Scalar and tensor spectra

ϕ_* is the value when the mode $k_* = a_* H_*$ crosses the Hubble radius

$$\phi_* = \phi_0 + \sqrt{\frac{m_{\rm Pl}^2}{6\pi}} \frac{1}{1+\alpha} \text{arcsinh} \left[\sqrt{\left(\frac{\rho_i^{1+\alpha}}{A} - 1 \right) e^{-3(1+\alpha)N_*}} \right]. \tag{19}$$

The scalar spectral index n_s and the tensor-to-scalar ratio r

$$n_s - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k} = -4\epsilon_H + 2\eta_H, \tag{20}$$

$$r \equiv \frac{P_h}{P_{\mathcal{R}}} = 16\epsilon_H. \tag{21}$$

The Planck 2018 bounds are [Akrami et al., 2018]

$A_s = (2.0989 \pm 0.10141) \times 10^{-9}$, $n_s = 0.9649 \pm 0.0042$, and $r < 0.064$.

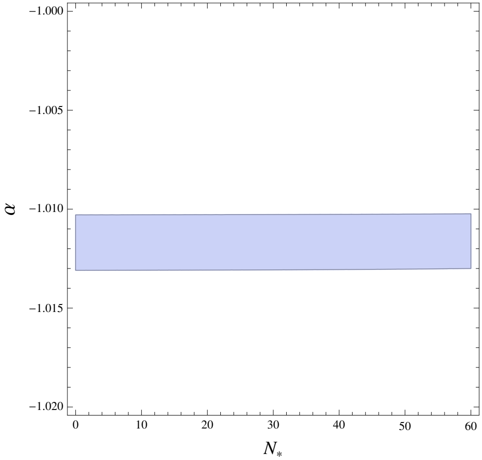


Figure: The allowed region in the (N_*,α) plane. The shaded region corresponds to the values of N_* and α for which the constraints from Planck 2018 on n_s and r are fulfilled. As it can be seen $-1.0131 < \alpha < -1.0103$.

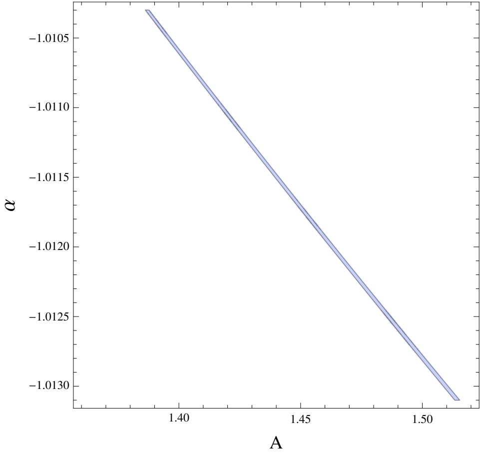


Figure: The allowed region in the (A,α) plane. The shaded region corresponds to the values of A and α for which the constraints from Planck 2018 on A_s are fulfilled. $1.391 < A < 1.522$.

Planck 2018 constraints

Constraints on α from Planck 2018 results

$$-1.0131 < \alpha < -1.0103. \tag{22}$$

Constraints on A from Planck 2018 results

$$1.391 < A < 1.522, \tag{23}$$

The generating function [Villanueva, 2015, Villanueva and Gallo, 2015]

$$H(\phi, \kappa) = H_0 \, \mathrm{nc}^{\frac{1}{1+\alpha}} \left[(1+\alpha)\, \Phi \right], \tag{24}$$

where $\mathrm{nc}(x) \equiv \mathrm{nc}(x|\,\kappa)$ is the Jacobi elliptic cosecant function, and κ is the modulus.

The generating function of the GCG is recovered when $\kappa = 1$.

The total number of e-folds at the end of inflation N_e is now given by

$$N_e(\alpha, A, \kappa) = \frac{1}{3(1+\alpha)} \ln \left\{ \frac{1}{\mathcal{G}(\kappa)} \left(\frac{\rho_i^{1+\alpha}}{A} - 1 \right) \right\}, \tag{25}$$

where $\mathcal{G} \equiv \mathrm{sd}^2 \left[F \left[\arcsin \left(\sqrt{y(\kappa)} \right), \kappa \right] \right]$.

$\mathcal{G}$ plays the role of the factor 2.

$\rho_i^{1+\alpha}/A$ is now reduced to the boundary $1 < \rho_i^{1+\alpha}/A < 1.4$.

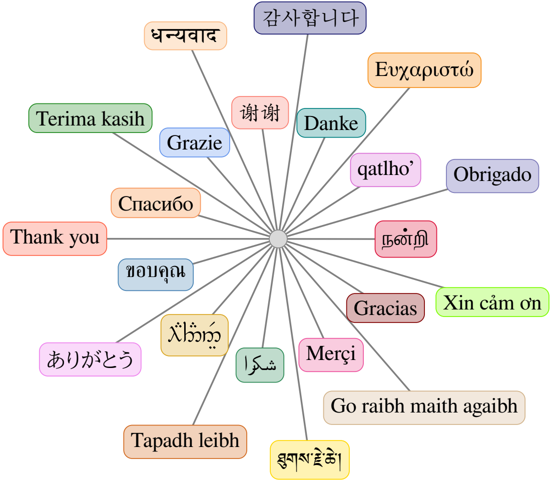
The introduction of the new parameter κ does not solve the previous results of $N \ll 60$ since N_e does not strongly depend on κ.

Conclusions I

1. For $-1 < \alpha$, no expansion but accelerated contraction. Contrary to previous works.
2. Only for $\alpha \to -1$ the model produces enough e-folds.
3. We constrain the parameters to

$$1.391 < A < 1.522 \quad \text{and} \quad -1.0131 < \alpha < -1.0103$$

4. There is no need to consider modified-gravity scenarios during inflation for the GC(J)G model.
5. The violation of the slow-roll conditions is a generic feature of the GC(J)G models.
6. The GC(J)G model is still alive!!!


감사합니다
धन्यवाद
Ευχαριστώ
谢谢
Terima kasih
Danke
Grazie
qatlho'
Obrigado
Спасибо
Thank you
நன்றி
ขอบคุณ
Xin cảm ơn
Gracias
ありがとう
شكرا
Merçi
Go raibh maith agaibh
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