Coupled Multi-Proca Vector Dark Energy

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CoCo 2020: Cosmología en Colombia

Preprint: arXiv:2004.06466 [gr-qc]

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According to GR both experience the same metric.

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Jordan frame: fields and curvature couple directly, but matter only involves the metric.

Einstein frame: the gravitational Lagrangian has the Einstein-Hilbert form but the matter sector is affected by fields, which mediates an additional force.

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Einstein frame: the gravitational Lagrangian has the Einstein-Hilbert form but the matter sector is affected by fields, which mediates an additional force.

Both representations are equivalent at the classical level.

Relation between gravity and matter

What happens if both sector have non-trivial relation?

$$\mathcal{S} = \int d^4x \, \sqrt{-g}(\mathcal{L}_{EH} + \mathcal{L}_V) + \int d^4x \, \sqrt{-\tilde{g}}\tilde{\mathcal{L}}_m),$$

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Disformal metric tranformation:

$$\tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)A_\mu A_\nu,$$
$$\tilde{g}_{\mu\nu} = C(Y)g_{\mu\nu} + D(Y)F_{\mu\rho}g^{\rho\sigma}F_{\sigma\nu},$$

$$X \equiv -\frac{1}{2}A_\mu A^\mu, \; Y \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

Coupled multi-Proca

A simple illustration: $\tilde{g}_{\mu\nu} = f(X) g_{\mu\nu}$.

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$$\mathcal{S} = \int d^4x \, \sqrt{-g} \left(\frac{M_p^2}{2} R - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - V(\tilde{X}) + f(X) \tilde{\mathcal{L}}_m \right),$$

$$F^a_{\mu\nu} \equiv \nabla_\mu A^a_\nu - \nabla_\nu A^a_\nu, \; \tilde{X} \equiv A^a_\mu A^\mu_a.$$

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$$\frac{M_p^2}{2} G_{\mu\nu} = T^A_{\mu\nu} + f \tilde{T}^m_{\mu\nu} + f_{,X} \tilde{\mathcal{L}}_m A^a_\mu A_{\nu a},$$

$$T^A_{\mu\nu} \equiv -2 \frac{\delta \mathcal{L}_A}{\delta g^{\mu\nu}} + \mathcal{L}_A g_{\mu\nu}$$

$$= F^a_{\mu\sigma} F^\sigma_{a\nu} + 2 V_{,\tilde{X}} A^a_\mu A_{\nu a} - \left(\frac{1}{4} F^a_{\rho\sigma} F_a^{\rho\sigma} + V(\tilde{X}) \right) g_{\mu\nu},$$

$$\tilde{T}^m_{\mu\nu} \equiv -2 \frac{\delta \tilde{\mathcal{L}}_m}{\delta g^{\mu\nu}} + \tilde{\mathcal{L}}_m g_{\mu\nu}. \quad \tilde{T}^m_{\mu\nu} = \sqrt{\frac{g}{\tilde{g}}} \frac{\partial g^{\alpha\beta}}{\partial \tilde{g}^{\mu\nu}} T^m_{\alpha\beta},$$

Coupled multi-Proca

$$\partial_\mu(\sqrt{-g}F^{a\mu\nu}) = 2\sqrt{-g}A^{\nu a}\left(V_{,\tilde{X}} + \frac{1}{2}\frac{f_{,X}}{f}\mathcal{L}_m\right).$$

$$\begin{aligned}\nabla_\mu T^{\mu m}_\nu = &- A^c_\beta \nabla_\mu A^\beta_c [\frac{f_{,X}}{f}(T^{\mu m}_\nu - f_{,X}A^{\mu a}A_{\nu a}\mathcal{L}_m) \\ &+ \frac{f_{,XX}}{f}\mathcal{L}_m A^\mu_a A^a_\nu] + \frac{f_{,X}}{f}A^\mu_c A^c_\nu[\nabla_\mu \mathcal{L}_m \\ &+ \frac{f_{,X}}{f}\mathcal{L}_m A^a_\beta \nabla_\mu A^\beta_a] + \frac{f_{,X}}{f}\mathcal{L}_m[A^c_\nu \nabla_\mu A^\mu_c \\ &+ A^\mu_c \nabla_\mu A^c_\nu],\end{aligned}$$

$$\nabla_\mu T^{\mu m}_\nu = -\nabla_\mu T^{\mu A}_\nu.$$

Particular setup

FRW background: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j.$

Cosmic triad: $A^a_\mu \equiv a(t)A(t)\delta^a_\mu.$

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$$3M_p^2H^2 = f\tilde{\rho}_m + \frac{3}{2}(\dot{A} + HA)^2 + V,$$

$$M_p^2(3H^2 + 2\dot{H}) = -\frac{1}{2}(\dot{A} + HA)^2 + V - 2V_{,\tilde{X}}A^2 + f_{,X}A^2\tilde{\rho}_m$$

$$\ddot{A} + \left(\frac{\ddot{a}}{a} + H^2\right)A + 3H\dot{A} + 2V_{,\tilde{X}}A - f_{,X}A\tilde{\rho}_m = 0.$$

$$\dot{\rho}_A + 3H(\rho_A + p_A) = 3A\dot{A}\frac{f_{,X}}{f}\rho_m,$$

$$\dot{\rho}_m + 3H\rho_m = -3A\dot{A}\frac{f_{,X}}{f}\rho_m.$$

Dynamical system analysis

Dimensionless variables:

$$x \equiv \sqrt{\frac{(\dot{A} + HA)^2}{2M_p^2 H^2}}; \quad y \equiv \sqrt{\frac{V}{3M_p^2 H^2}};$$

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$$x' = -x\left(2 + \frac{H'}{H}\right) + \frac{1}{x}\left[3\lambda y^2 u^2 - \sqrt{2}\frac{x}{u} q z^2 - y\left(y' + y\frac{H'}{H}\right)\right],$$

$$y' = -y\left[\frac{H'}{H} + 3u^2\lambda\left(\sqrt{2}\frac{x}{u} - 1\right)\right],$$

$$z' = -z\left[\frac{H'}{H} + \frac{3}{2} - q\left(\sqrt{2}\frac{x}{u} - 1\right)\right],$$

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$$\frac{H'}{H} = -\frac{3}{2}(1+w_{eff}),$$

$$w_{eff} = \frac{x^2}{3} - y^2 - 2\lambda y^2 u^2 + \frac{2}{3}qz^2,$$

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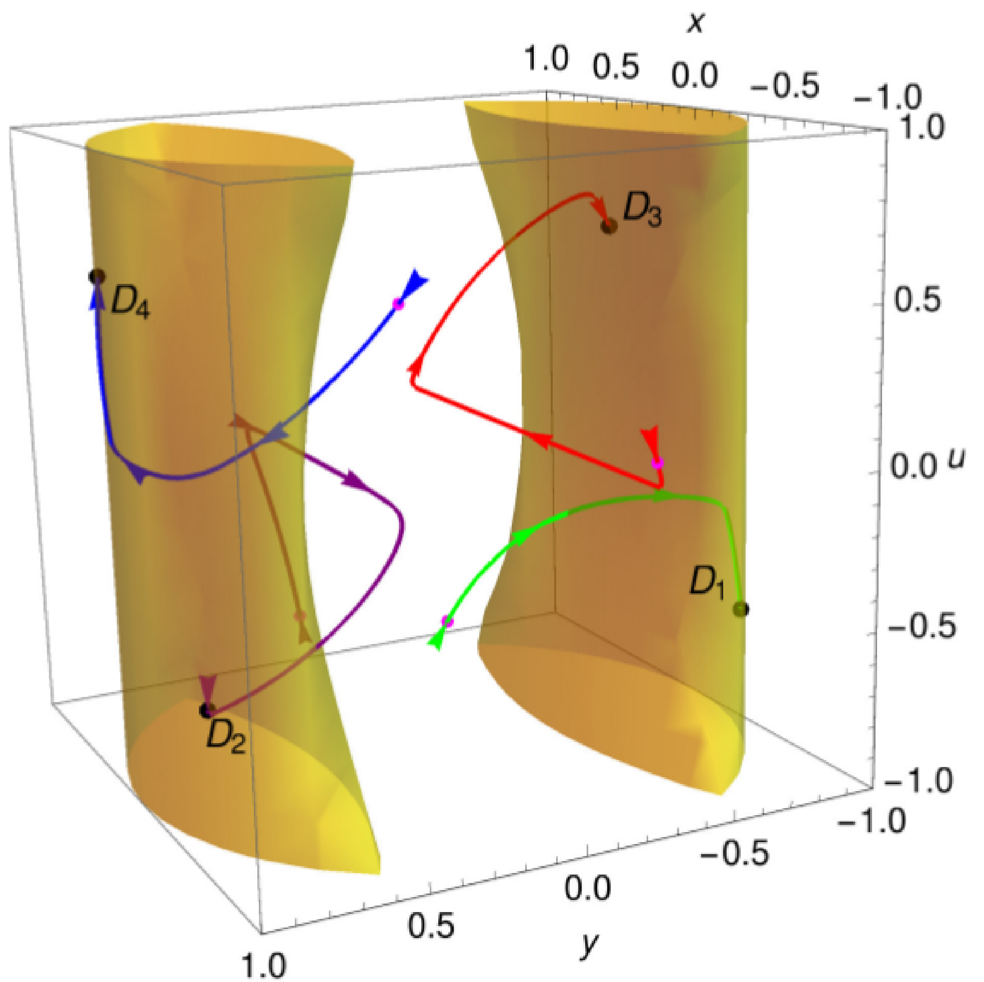
$$V(\tilde{X}) = V_0 e^{-\lambda\tilde{X}/M_p^2}, \; f(X) = \left(\frac{X}{M_p^2}\right)^q,$$

Phase space trajectories



Point	x_c	y_c	z_c	u_c	Ω_A	w_A	$w_{\rm eff}$	Existence	Acceleration
$A_\pm$	± 1	0	0	$\pm\sqrt{2}$	1	$1/3$	$1/3$	$\forall q, \lambda$	No
$B_\pm$	$\pm\frac{\sqrt{2q}}{\sqrt{-1+2q}}$	0	$\pm\frac{1}{\sqrt{1-2q}}$	$\pm\sqrt{2}x_c$	$\frac{2q}{-1+2q}$	0	0	$\forall q, \lambda < 0$	No
$D_{1,2}$	$-\frac{\sqrt{-1+3\lambda}}{\sqrt{3\lambda}}$	$\mp\frac{1}{\sqrt{3\lambda}}$	0	$-\sqrt{2}x_c$	1	-1	-1	$\forall q, \lambda > 1/3$	$\forall q, \lambda$
$D_{3,4}$	$\frac{\sqrt{-1+3\lambda}}{\sqrt{3\lambda}}$	$\mp\frac{1}{\sqrt{3\lambda}}$	0	$\sqrt{2}x_c$	1	-1	-1	$\forall q, \lambda > 1/3$	$\forall q, \lambda$

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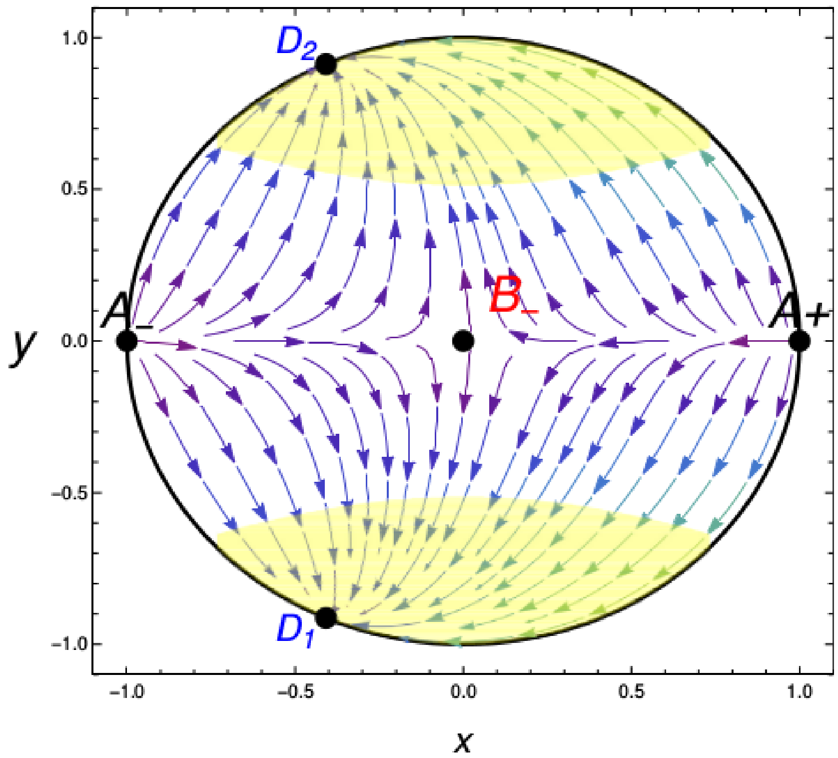

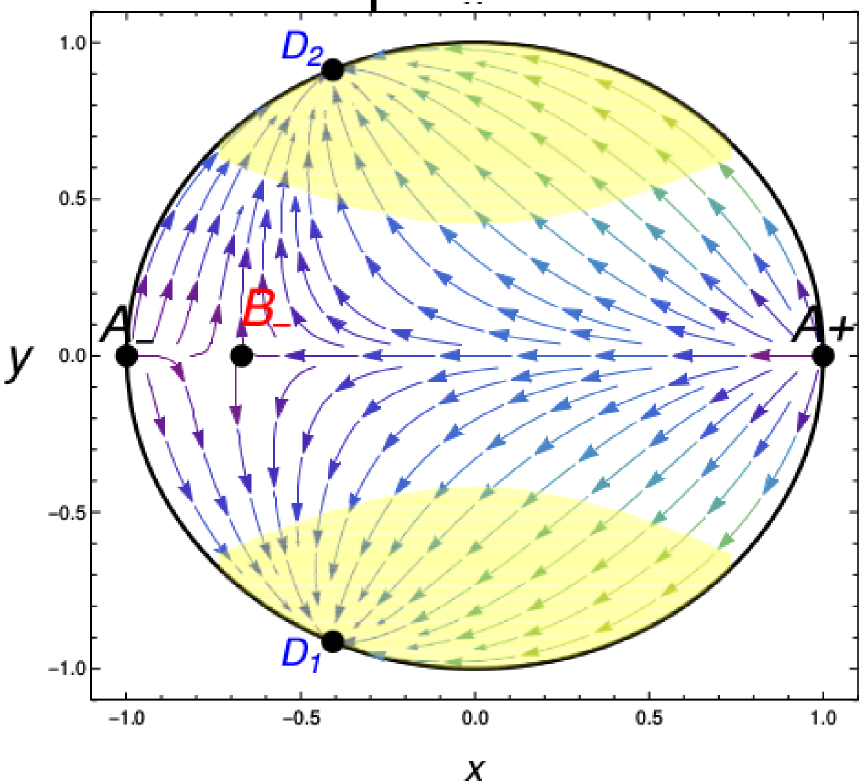

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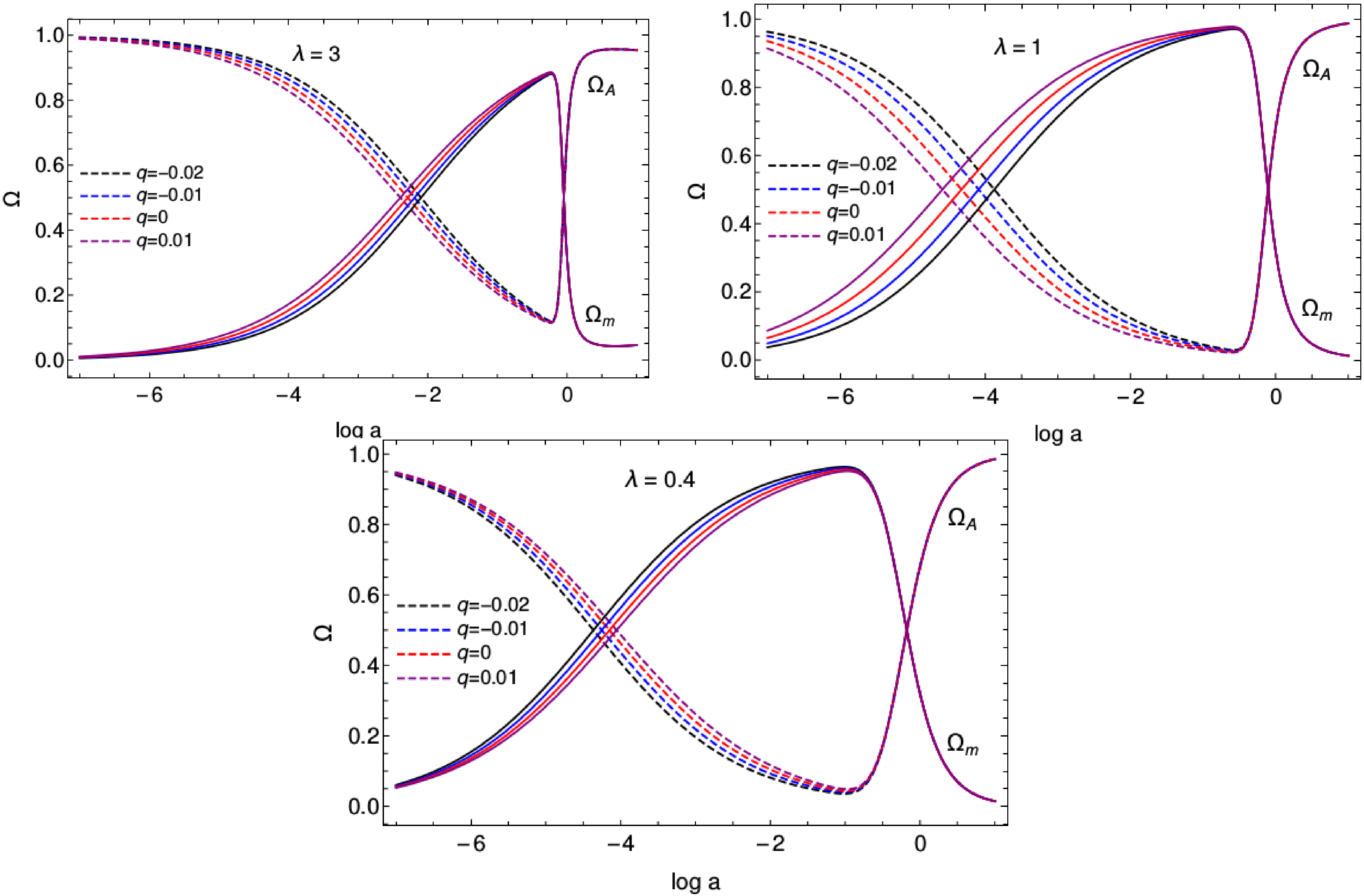
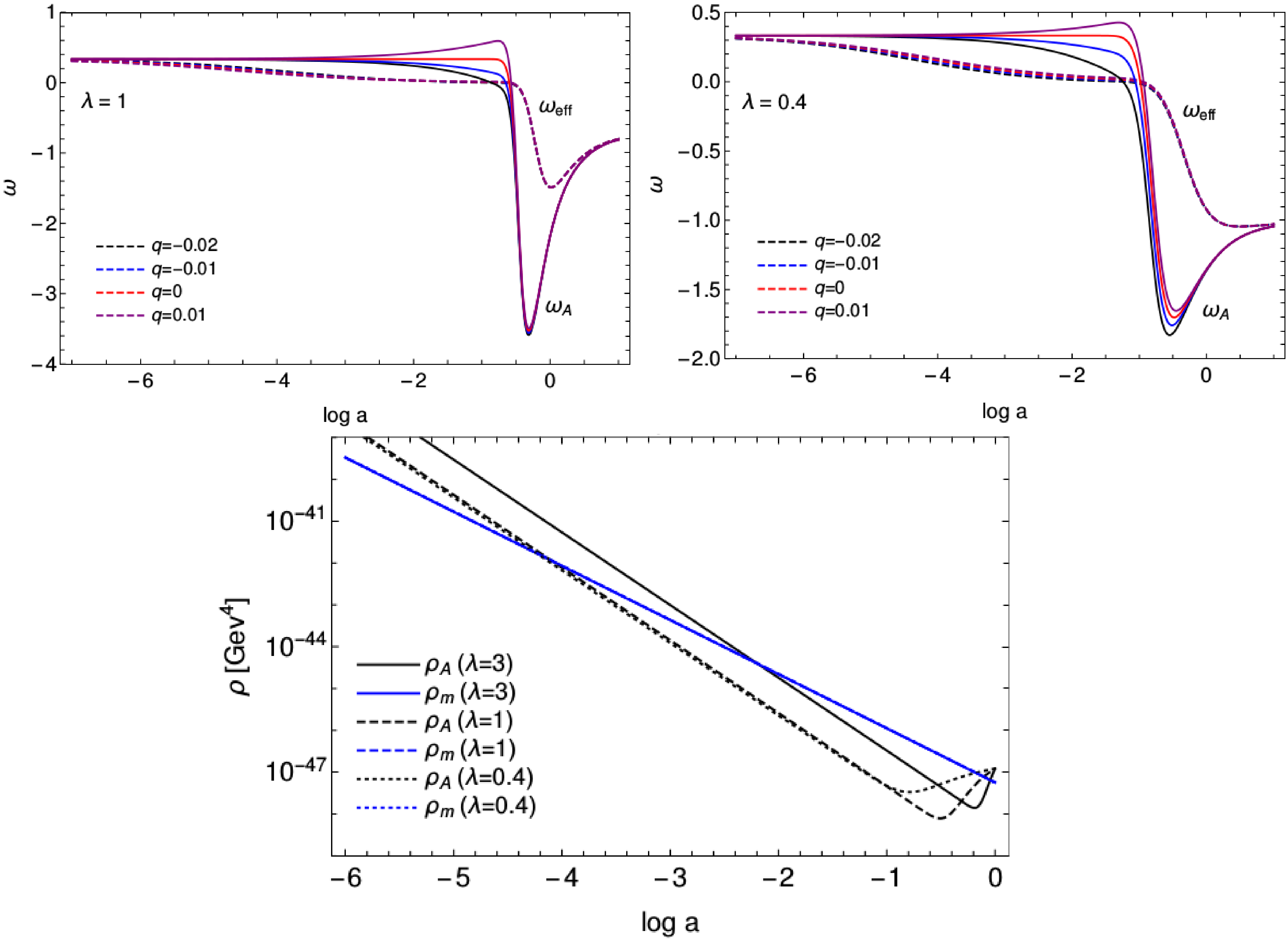


λ = 3
1.0
0.8
0.6
0.4
0.2
0.0
Ω
q=−0.02
q=−0.01
q=0
q=0.01
Ω_A
Ω_m
−6
−4
−2
0
log a
λ = 1
1.0
0.8
0.6
0.4
0.2
0.0
Ω
q=−0.02
q=−0.01
q=0
q=0.01
Ω_A
Ω_m
−6
−4
−2
0
log a
λ = 0.4
1.0
0.8
0.6
0.4
0.2
0.0
Ω
q=−0.02
q=−0.01
q=0
q=0.01
Ω_A
Ω_m
−6
−4
−2
0
log a



λ = 1
ω
ω_eff
ω_A
q=−0.02
q=−0.01
q=0
q=0.01
log a
λ = 0.4
ω
ω_eff
ω_A
q=−0.02
q=−0.01
q=0
q=0.01
log a
ρ [Gev^4]
ρ_A (λ=3)
ρ_m (λ=3)
ρ_A (λ=1)
ρ_m (λ=1)
ρ_A (λ=0.4)
ρ_m (λ=0.4)
log a

Conclusions and perpectives

We proposed a novel coupling between multi-Proca vector fields and CDM at the level of the action through a mass-type term.

We studied the role of space-like vector fields (cosmic triad) in the cosmological background dynamics: Dark Radiation, Dark Energy, Scaling Solutions.

This result is quite general in the sense that encompasses Abelian and non-Abelian vector fields.

As future theoretical research, we propose building more general couplings involving, for instance, the field strength or its dual in order to make the structure of the group explicit.

Constrain the background dynamics, matter density perturbations: spherical collapse, number countst, the growth rate and the redshift-space distorsion.