Topological mass generation and generalized 2-forms actions

Juan P. Beltrán Almeida

Departamento de Física

Facultad de Ciencias

UNIVERSIDAD NACIONAL DE COLOMBIA

CoCo 2020

September 23rd, 2020

Some motivations

1. Generalized "Galileon"-like models.
2. Nowadays we have high precision data from cosmological observations and we can test models.
3. Anisotropic and parity breaking signatures during the primordial inflationary expansion and in the large scale structure (LSS) formation process.
4. Testing non minimal couplings with gravity during inflation and LSS.

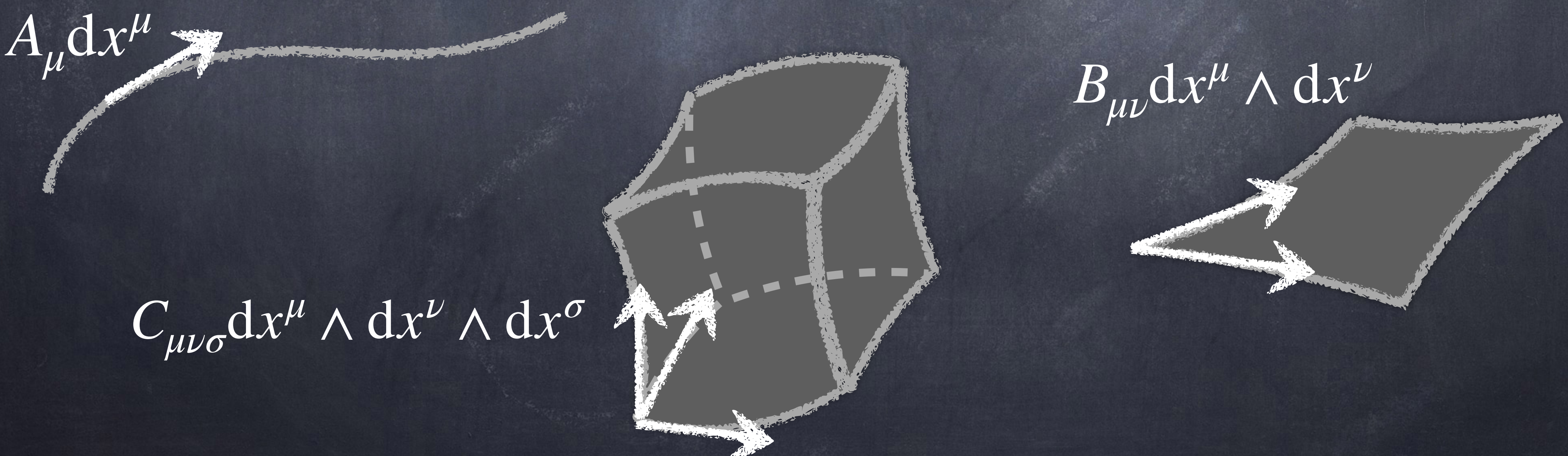
P-forms in D-dimensions

Based on:

JPBA, A. Guarnizo and C. Valenzuela-Toledo, Class.Quant.Grav. 37 (2020) 3, 035001, arXiv:1810.05301 [astro-ph.CO].

JPBA, A. Guarnizo, R. Kase, S. Tsujikawa and C. Valenzuela-Toledo: JCAP 1903, 025 (2019), arXiv:1901.06097 [gr-qc] and Phys.Lett. B793, 396 (2019), arXiv:1902.05846 [hep-th].

JPBA, A. Guarnizo, L. Heisenberg, C. Valenzuela-Toledo and J. Zosso: Phys. Rev. D **102**, 063521 (2020), arXiv:2003.11736 [hep-th].



P-forms in D-dimensions

Definitions, notation and formalities[1]

Totally antisymmetric rank p tensors in D-dim manifold M.

A p-form ⟶ $$A_{(p)} = \frac{1}{p!} A_{(p)\mu_1\mu_2\cdots\mu_p} \mathrm{d}x^{\mu_1} \wedge \cdots \wedge \mathrm{d}x^{\mu_p}$$

$$A_{(p)\mu_1\cdots\mu_p} = p! A_{(p)[\mu_1\cdots\mu_p]}$$

⟶ (p) labels the rank of the antisymmetric tensor

⟶ [...] = (1/p!)x(Antisymmetric Perms.)

Wedge product $\wedge$ ⟶ Totally antisymmetric tensor product.
Ex.: $\mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} = \mathrm{d}x^{\mu_1} \otimes \mathrm{d}x^{\mu_2} - \mathrm{d}x^{\mu_2} \otimes \mathrm{d}x^{\mu_1}$

Exterior product. In general, any two (r) and (q)-forms generate a (r+q)-form:

$$A_{(r)} \wedge B_{(q)} = (-1)^{qr} B_{(q)} \wedge A_{(r)} .$$

[1] See e.g. M. Nakahara, Geometry, Topology and Physics, (2003).

P-forms in D-dimensions

Definitions, notation and formalities[1]

Associated with $A_{(p)}$, two forms can be defined:

1. Field strength. Provides dynamics to $A_{(p)}$.

(p+1)-form $F_{(p)} = dA_{(p)} = \frac{1}{p!}\nabla_{[\mu_1}A_{(p)\mu_2\mu_3\cdots\mu_{p+1}]}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_{p+1}}$

2. Hodge dual. Establishes an isomorphism $\Omega^p(M)\rightarrow\Omega^{D-p}(M)$.

(D-p)-form $\star A_{(p)} = \frac{\sqrt{-g}}{p!(D-p)!}\epsilon_{\mu_1\cdots\mu_p\nu_1\cdots\nu_{D-p}}A_{(p)}^{\mu_1\cdots\mu_p}dx^{\nu_1}\wedge\cdots\wedge dx^{\nu_{D-p}}$

$\epsilon_{\mu_1\cdots\mu_D}$ → Totally antisymmetric Levi-Civita tensor

Volume element

$\epsilon = \sqrt{-g}\,\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^D$

[1] See e.g. M. Nakahara, Geometry, Topology and Physics, (2003).

Notation, basics and generalities

P-forms in coordinates

Field strength $\longrightarrow$ $F_{(p)\mu_1\mu_2\cdots\mu_{p+1}} = (p+1)\partial_{[\mu_1}A_{(p)\,\mu_2\mu_3\cdots\mu_{p+1}]}\,.$

Hodge dual $\longrightarrow$ $\tilde{F}_{(p)\nu_1\cdots\nu_{D-p-1}} = \frac{\sqrt{-g}}{(p+1)\,!}\epsilon_{\mu_1\cdots\mu_{p+1}\nu_1\cdots\nu_{D-p-1}}F_{(p)}^{\;\mu_1\cdots\mu_{p+1}}\,.$

Exterior product $\longrightarrow$ $(A_{(r)} \wedge B_{(q)})_{\mu_1\cdots\mu_r\,\nu_1\cdots\nu_q} = \frac{(p+r)\,!}{p\,!r\,!}A_{(r)[\mu_1\cdots\mu_r}B_{(q)\nu_1\cdots\nu_q]}\,.$

Some particular cases

1. A scalar field ϕ

0-form $\longrightarrow$ ϕ Field strength $\longrightarrow$ $\partial_\mu\phi$

2. D-form. Non dynamical. Proportional to the volume element.

D-form $\longrightarrow$ $A_D\sqrt{-g}\epsilon_{1\ldots D}\mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^D$ Field strength $\longrightarrow$ 0

Notation, basics and generalities

Some remarks. Constraints of the model

1. Gauge invariance: $A_{(p)\mu_1\cdots\mu_p} \to A_{(p)\mu_1\cdots\mu_p} + \partial_{[\mu_1}\xi_{(p-1)\mu_2\cdots\mu_p]}$.

2. Covariant derivatives in the field strength become ordinary derivatives due to total antisymmetry (torsion free): $\nabla_\mu \to \partial_\mu$.

3. Minimal coupling with gravity. In 1st order derivative theories, there is no need to introduce non-minimal coupling to gravity. Only needed in higher than two derivatives p-form Galileons.

Previous studies (an incomplete list).

1. "Vanishing/Neutralization" of the cosmological constant with 3-forms: Duncan & Jensen NPB336 (1990), Bousso and Polchinski, JHEP 06 (2000).
2. 3-form/psudoscalar mixing: "natural framework" for chaotic inflation and string theory landscape: Kaloper & Sorbo, PRD79 (2009), PRL102 (2009).
3. "P-nflation". Inflation with "massive p-forms" with non minimal couplings to gravity: Germani & Kehagias, JCAP 0903 (2009), Koivisto, Mota & Pitrou, JHEP 0909 (2009).
4. Inflation & DE from 3-forms: Koivisto & Nunes, PLB685 (2010), PRD80 (2009).
5. p-form Galileons: Deffayet, Deser & Esposito-Farese, PRD82 (2010).
6. Anisotropic primordial signatures: Ohashi, Soda & Tsujikawa, PRD87 (2013), JCAP 1312 (2013).
7. Anisotropic inflation: Ito & Soda. PRD92 (2015).
8. Cosmology in Bianchi spacetimes: Normann, Hervik, Ricciardone & Thorsrud, CQG35, Thorsrud CQG35 (2018).
9. …

Galileon models as a motivation

Galileons

General Lagrangians involving 1st and 2nd order derivatives of a scalar field

$$\mathscr{L} = \int \mathrm{d}^D x \mathscr{A}^{\alpha_1 \cdots \alpha_{n+1} \beta_1 \cdots \beta_{n+1}} (\partial_{\alpha_{n+1}} \pi)(\partial_{\beta_{n+1}} \pi)(\partial_{\alpha_1 \beta_1} \pi) \cdots (\partial_{\alpha_n \beta_n} \pi)$$

$$\mathscr{A}^{\alpha_1 \cdots \alpha_m \beta_1 \cdots \beta_m} = \frac{1}{(D-n)!} \epsilon^{\alpha_1 \cdots \alpha_m \sigma_1 \cdots \sigma_{D-m}} \epsilon^{\beta_1 \cdots \beta_m}{}_{\sigma_1 \cdots \sigma_{D-m}}$$

$$\epsilon^{\alpha_1 \cdots \alpha_D} = -\,\delta_1^{[\mu_1} \delta_2^{\mu_2} \ldots \delta_D^{\mu_D]}$$

The particular structure of those terms produce up to 2nd order e.o.m. C. Deffayet, G. Esposito-Farese and A. Vikman, PRD 79 (2009) 084003 [arXiv:0901.1314], C. Deffayet, S. Deser and G. Esposito-Farese, PRD 80 (2009) 064015 [arXiv:0906.1967]

Galileon models as a motivation

Galileons

Galileon generalization of electromagnetism and a no-go theorem

$$\mathscr{L} = \int \mathrm{d}^D x \mathscr{A}^{\alpha_1 \cdots \alpha_{n+1} \beta_1 \cdots \beta_{n+1}} (F_{\alpha_1 \alpha_2})(F_{\beta_1 \beta_2})(\partial_{\alpha_k} F_{\beta_s \beta_{s+1}})(\partial_{\beta_k} F_{\alpha_m \alpha_{m+1}})$$

Assuming a gauge and Lorentz invariant theory, this kind of generalization produce the same equations of motion of ordinary electromagnetism derived from the action:

$$\mathscr{L}_F = -\frac{1}{4} \int \mathrm{d}^D x F^{\mu\nu} F_{\mu\nu}$$

C. Deffayet, E. Gümrükçouglu , S. Mukohyama Y. and Wang",
JHEP04(2014)082, hep-th [arXiv 1312.6690].

Interacting p-form Lagrangian

A general Lagrangian with coupled p-forms.

$$\mathscr{L} = \mathscr{L}_\phi(\phi, K) + \mathscr{L}_p(\phi, A_p) \quad \text{with} \quad K = \partial_\mu\phi\partial^\mu\phi\,.$$

Coupling with the scalar is needed to isotropize the evolution of the system.

Not so general. Some simplifying restrictions.

1. Gauge invariance. "Massless" p-forms.
2. Up to 1st order derivatives of the p-forms. No Galileons.
3. Minimal coupling to gravity. Standard gravity.
4. Up to quadratic terms in the field strength of each p-form.
5. Abelian fields.

Scalar and p-forms mixing

The p-form interacting Lagrangian $\mathcal{L}_p(\phi, A_p)$

Mixing of p-forms:

$$\mathcal{L}_{\rm mix} = g_{p_1\cdots p_r}(\phi) X_{(p_1)} \wedge \cdots \wedge X_{(p_r)}$$

$$(p_1 + 1) + \cdots (p_r + 1) = D\,.$$

$$= g_{p_1\cdots p_r}(\phi) \frac{\epsilon^{\mu_1\cdots\mu_D}}{\sqrt{-g}} X_{(p_1)\mu_1\cdots\mu_{p_1+1}} \cdots X_{(p_r)\mu_{p_r+1}\cdots\mu_D},$$

Where $X_{(p)} = F_{(p)}$ or $X_{(p)} = \tilde{F}_{(p)}$.

$g_{p_1\cdots p_r}(\phi) \longrightarrow$ General coupling functions of ϕ.

In principle, the couplings can depend on $K = \partial_\mu\phi\partial^\mu\phi$.

$g_{p_1\cdots p_r}(\phi, K) \longrightarrow$ Hamiltonian unbounded by below!
Fleury, JPBA, Pitrou and Uzan, JCAP1411 (2014).

Scalar and p-forms mixing

Standard kinetic terms (Maxwell-like):

$$F_{(p)} \wedge \star F_{(p)} \equiv \frac{1}{(p+1)!}\sqrt{-g}F^2_{(p)},$$

with $F^2_{(p)} \equiv F_{(p)\mu_1\mu_2\ldots\mu_{p+1}}F_{(p)}{}^{\mu_1\mu_2\ldots\mu_{p+1}},$

Coupling with the scalar field $\mathscr{L}_{mix} \propto f(\phi)F_{(p)} \wedge \star F_{(p)}\,.$

$$X_{(p_1)} \wedge \cdots \wedge X_{(p_r)} \quad \begin{cases} F_{(p)}F^{(n)}F^{(m)} \equiv F_{(p)\mu_1\mu_2\ldots\mu_{p+1}}F_{(n)}{}^{\mu_1\mu_2\ldots\mu_{n+1}}F_{(m)}{}^{\mu_{n+2}\ldots\mu_{p+1}}, \\ \tilde{F}_{(p)}F^{(n)}F^{(m)} \equiv \tilde{F}_{(p)\mu_1\mu_2\ldots\mu_{D-(p+1)}}F_{(n)}{}^{\mu_1\mu_2\ldots\mu_{n+1}}F_{(m)}{}^{\mu_1\mu_2\ldots\mu_{m+1}}, \\ \ldots \end{cases}$$

Scalar and p-forms mixing

Further possible mixing terms.

Kinetic mixing with the scalar derivatives.

$\phi F^{\mu\nu}_{(1)}\partial_\mu\phi\partial_\nu\phi = \phi F^{\mu\nu\sigma}_{(2)}\partial_\mu\phi\partial_\nu\phi\partial_\sigma\phi = 0$ -> trivially zero

Cubic terms are zero (in the Abelian case) --> $F^{\alpha}{}_{\nu}F^{\nu}{}_{\beta}F^{\beta}{}_{\alpha} = 0$

$(\partial_\alpha\phi F^{\alpha\nu}_{(1)})(\partial_\beta\phi F^{\beta}_{(1)\nu})$ or $(\partial_\alpha\phi \tilde{F}^{\alpha\nu}_{(1)})(\partial_\beta\phi \tilde{F}^{\beta}_{(1)\nu})$ -> lead to non-hyperbolic (gradient instability) equations of motion!

Fleury, JPBA, Pitrou and Uzan, JCAP1411 (2014)

$\partial_\alpha\phi F^{\beta\gamma}_{(1)}F^{\alpha}_{(2)\beta\gamma}$ etc. --> Not considered here. They are suspicious!

Scalar and p-forms mixing

General Lagrangian

$$\mathscr{L}_p = -\frac{1}{2}\sum_{n=1}^{D-1} f_n(\phi) F_{(n)} \wedge \star F_{(n)} + \sum_{(p_1 p_2 \cdots p_r)} g_{p_1 p_2 \cdots p_r}(\phi) X_{(p_1)} \wedge \cdots \wedge X_{(p_r)},$$

$$= -\frac{1}{2}\sum_{n=1}^{D-1} \frac{f_n(\phi)}{(n+1)!} F_{(n)}^2 + \sum_{(p_1 p_2 \cdots p_r)} g_{p_1 p_2 \cdots p_r}(\phi) X_{(p_1)} \wedge \cdots \wedge X_{(p_r)}$$

$(p_1 p_2 \cdots p_r) \longrightarrow$ All posible combinations of F and *F

Topologic terms

Topologic -> Metric independent. For quantum metric independence, see Blau & Thompson PLB255 (1991).

Chern-Simons. D=2p+1.

$$S_{\phi \mathrm{CS}} = \int A_{(p)} \wedge F_{(p)} \to \int f(\phi) A_{(p)} \wedge F_{(p)}$$

—> Gauge invariant only if f is constant

Chern-Pontryagin. D=2(p+1).

$$S_{\mathrm{CP}} = -\frac{1}{2}\int F_{(p)} \wedge F_{(p)} \to -\frac{1}{2}\int f(\phi) F_{(p)} \wedge F_{(p)}$$

—> Dynamical if f is field dependent

BF-model. Blau & Thompson, PLB228 (1989), Horowitz, CMP125 (1989)

$$S_{\phi \mathrm{BF}} = \int f(\phi) A_{(p)} \wedge F_{(D-p-1)} \quad \text{alternatively} \quad \int f(\phi) A_{(D-p-1)} \wedge F_{(p)}$$

p-forms in 4-dimensions

Basic elements. We have 1,2, and 3-forms. 4-form is non dynamical.

1,2,3-forms

$$A_{(1)} = A_{(1)\,\mu_1}\mathrm{d}x^{\mu_1}, \quad A_{(2)} = \frac{1}{2}A_{(2)\,\mu_1\mu_2}\mathrm{d}x^{\mu_1}\wedge\mathrm{d}x^{\mu_2}, \quad A_{(3)} = \frac{1}{6}A_{(3)\,\mu_1\mu_2\mu_3}\mathrm{d}x^{\mu_1}\wedge\mathrm{d}x^{\mu_2}\wedge\mathrm{d}x^{\mu_3}\,.$$

Field strengths

$$F_{(1)\mu_1\mu_2} = 2\partial_{[\mu_1}A_{(1)\mu_2]}, \quad F_{(2)\mu_1\mu_2\mu_3} = 3\partial_{[\mu_1}A_{(2)\mu_2\mu_3]}, \quad F_{(3)\,\mu_1\mu_2\mu_3\mu_4} = 4\partial_{[\mu_1}A_{(3)\mu_2\mu_3\mu_4]}\,.$$

Hodge duals

$$\tilde{F}_{(1)\mu_1\mu_2} = \frac{\sqrt{-g}}{2!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}F_{(1)}{}^{\mu_3\mu_4}, \quad \tilde{F}_{(2)\mu_1} = \frac{\sqrt{-g}}{3!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}F_{(2)}{}^{\mu_2\mu_3\mu_4},$$

$$\tilde{F}_{(3)} = \frac{\sqrt{-g}}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}F_{(3)}{}^{\mu_1\mu_2\mu_3\mu_4}\,.$$

p-forms in 4-dimensions

Maxwell like terms

$$\mathscr{L}_p^{\mathrm{M}}(\phi, A_p) = -\frac{f_1(\phi)}{4} F_{(1)\mu_1\mu_2} F_{(1)}{}^{\mu_1\mu_2} - \frac{f_2(\phi)}{12} F_{(2)\mu_1\mu_2\mu_3} F_{(2)}{}^{\mu_1\mu_2\mu_3} - \frac{f_3(\phi)}{48} F_{(3)\,\mu_1\mu_2\mu_3\mu_4} F_{(3)}{}^{\mu_1\mu_2\mu_3\mu_4}\,.$$

Mixing terms. The only possible independent contractions:

$$\mathscr{L}_p^{\mathrm{mixing}}(\phi, A_p) = -\frac{g_1(\phi)}{4} F_{(1)\mu_1\mu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2} - \frac{g_2(\phi)}{2} A_{(2)\mu_1\nu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2} - g_3(\phi) \tilde{F}_{(3)}\,.$$

Other possible combinations:

$$F_{(3)}{}^{\mu_1\mu_2\mu_3\mu_4} F_{(1)\mu_1\mu_2} F_{(1)\mu_3\mu_4} \propto F_{(1)\mu_1\mu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2}$$

$$F_{(3)}{}^{\mu_1\mu_2\mu_3\mu_4} F_{(1)\mu_1\mu_2} \tilde{F}_{(1)\mu_3\mu_4} \propto F_{(1)\mu_1\mu_2} F_{(1)}{}^{\mu_1\mu_2}$$

↓

Already included

$$F_{(2)}{}^{\mu_1\mu_2\mu_3} \tilde{F}_{(2)\mu_1} \tilde{F}_{(1)\mu_2\mu_3} \propto \eta^{\sigma\mu_1\mu_2\mu_3} \tilde{F}_{(2)\sigma} \tilde{F}_{(2)\mu_1} \tilde{F}_{(1)\mu_2\mu_3} = 0$$

$$F_{(2)}{}^{\mu_1\mu_2\mu_3} \tilde{F}_{(2)\mu_1} F_{(1)\mu_2\mu_3} \propto \eta^{\sigma\mu_1\mu_2\mu_3} \tilde{F}_{(2)\sigma} \tilde{F}_{(2)\mu_1} F_{(1)\mu_2\mu_3} = 0$$

↓

Identically zero!

p-forms in 4-dimensions

Final Lagrangian

$$\mathscr{L}_p(\phi, A_p) = -\frac{f_1(\phi)}{4} F_{(1)\mu_1\mu_2} F_{(1)}{}^{\mu_1\mu_2} - \frac{f_2(\phi)}{12} F_{(2)\mu_1\mu_2\mu_3} F_{(2)}{}^{\mu_1\mu_2\mu_3} - \frac{f_3(\phi)}{48} F_{(3)\,\mu_1\mu_2\mu_3\mu_4} F_{(3)}{}^{\mu_1\mu_2\mu_3\mu_4}$$

$$-\frac{g_1(\phi)}{4} F_{(1)\mu_1\mu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2} - \frac{g_2(\phi)}{2} A_{(2)\mu_1\nu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2} - f_4(\phi) \tilde{F}_{(3)}\,.$$

$$= -\frac{1}{2} \sum_{n=1}^{3} \frac{f_n(\phi)}{(n+1)!} F_{(n)}^2 - \frac{f_4(\phi)}{24} \tilde{F}_{(3)} - \frac{g_1(\phi)}{4} F_{(1)\mu_1\mu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2} - \frac{g_2(\phi)}{2} A_{(2)\mu_1\nu_2} \tilde{F}_{(1)}{}^{\mu_1\mu_2}$$

Don't break parity! (two epsilons)

Breaks parity!

Looks like parity breaking but it is actually parity conserving

p-forms in 4-dimensions

A short digression on bibliography

$$\frac{f_1(\phi)}{4}F_{(1)\mu_1\mu_2}F_{(1)}{}^{\mu_1\mu_2}$$ ⟶ Ratra, AJ391(1992); Yokoyama & Soda, JCAP0808(2008); Watanabe, Kanno and Soda, PRL102(2009); Dimopoulos, Karciauskas & Wagstaff PLB683(2010); Bartolo, Matarrese, Peloso & Ricciardone, PRD87(2013); Shiraishi, Komatsu, Peloso & Barnaby, JCAP1305(2013); Abolhasani, Emami, Firouzjaee & Firouzjahi, JCAP1308(2013), …

$$\frac{g_1(\phi)}{4}F_{(1)\mu_1\mu_2}\tilde{F}_{(1)}{}^{\mu_1\mu_2}$$ ⟶ Anber & Sorbo, PRD81(2010), PRD85(2012); Dimopoulos & Karciauskas, JHEP06(2012); Sorbo JCAP1106(2011); Caprini & Sorbo, JCAP1410(2014); Bartolo, Matarrese, Peloso & Shiraishi, JCAP1507(2015); JPBA, Motoa-Manzano & Valenzuela-Toledo, JCAP1711(2017), …

p-forms in 4-dimensions

A short digression on bibliography

$\frac{f_2(\phi)}{4} F_{(2)\mu_1\mu_2\mu_3} F_{(2)}{}^{\mu_1\mu_2\mu_3}$ ——> Ohashi, Soda & Tsujikawa, JCAP1312(2013), PRD87(2013), Ito & Soda, PRD92(2015).

$\phi \tilde{F}_{(3)}$ ——> Kaloper and Sorbo, PRD79(2009), PRL102(2009).

Related models

Massive p-forms. Including non minimal couplings ——> Germani & Kehagias, JCAP 0903 (2009), Koivisto, Mota & Pitrou, JHEP 0909 (2009).

Massive 3-forms ——> Koivisto & Nunes, PLB685 (2010), PRD80 (2009).

p-forms in 4-dimensions

Non-Abelian extensions

1-form generalization

$$F_{\mu\alpha} \to \mathbf{F}^{(a)}{}_{\mu\alpha} = \partial_{[\mu}\mathbf{A}^{(a)}_{\nu]} + f^{a}{}_{bc}\left[\mathbf{A}^{(b)}_{\mu}, \mathbf{A}^{(c)}_{\nu}\right]$$

Gauge-flation and Chromo-Natural Inflation. Maleknejad & Sheikh-Jabbari PLB723(2013); Adshead & Wyman, PRL108 (2012)

$$(\mathrm{Tr}[\mathbf{F}^{(a)}_{\mu\nu}\tilde{\mathbf{F}}^{(a)\mu\nu}])^2 \quad \text{and} \quad \phi\mathrm{Tr}[\mathbf{F}^{(a)}_{\mu\nu}\tilde{\mathbf{F}}^{(a)\mu\nu}]$$

Cubic terms. Maleknejad, PRD90(2014), JCAP1612(2016). "Gaugid inflation", Piazza, Pirtskhalava, Rattazzi, Simon, JCAP1711(2017).

$$f_{abc}\mathbf{F}^{(a)}{}_{\mu\alpha}\mathbf{F}^{(b)\alpha}{}_{\beta}\mathbf{F}^{(c)\beta\mu} \quad \text{and} \quad f_{abc}\mathbf{F}^{(a)}{}_{\mu\alpha}\mathbf{F}^{(b)\alpha}{}_{\beta}\tilde{\mathbf{F}}^{(c)\beta\mu},$$

Massive Gauge-flation and Chromo-Natural Inflation. Nieto & Rodríguez MPLA31(2016); Adshead, Martinec, Sfakianakis & Wyman, JHEP12(2016).

Equations of motion and 1-2 forms system

1 and 2-form system. Introduce the notation:

$$A_{(1)\mu} = A_\mu, \;\; F_{(1)\mu\nu} = F_{(1)\mu\nu}, \;\; A_{(2)\mu\nu} = B_{\mu\nu}, \;\; F_{(2)\mu\nu\rho} = H_{\mu\nu\rho}$$

$$\nabla^\mu \left(f_1(\phi) F_{\mu\nu} + g_1(\phi) \tilde{F}_{\mu\nu} + g_2 \tilde{B}_{\mu\nu} \right) = 0,$$

$$\nabla^\mu \left(f_2(\phi) H_{\mu\nu\alpha} \right) + \frac{g_2}{2} \tilde{F}_{\nu\alpha} = 0.$$

Plus the Bianchi identities --> $\nabla^\mu \tilde{F}_{\mu\nu} = 0, \quad \nabla^\mu \tilde{H}_\mu = 0$

Equations of motion and 1-2 forms system

1 and 2-form system.

Dynamic degrees of freedom and gauge symmetry.

$A_0 = B_{0i} = 0$ ⟶ Their time derivative dont appear in the Lagrangian.

$\partial_i A_i = \partial_i B_{ij} = \partial_j B_{ij} = 0$ ⟶ Gauge choice.

3 propagating d.o.f:
2 (1-form) + 1 (2-form) ⟶ $\{A_i, B_{ij}\}$

Equations of motion and 1-2 forms system

1 and 2-form system.

The coupled system in FLRW background

$$A_i'' - \nabla^2 A_i + \frac{f_1'}{f_1} A_i' + \frac{g_1'}{f_1} \epsilon_{0ijk} \partial_j A_k = \frac{g_2}{2f_1} \left[\partial_\tau - 2\frac{a'}{a} \right] \epsilon_{0ijk} B_{jk},$$

$$B_{ij}'' - \nabla^2 B_{ij} + \left(\frac{f_2'}{f_2} - 2\frac{a'}{a} \right) B_{ij}' = -\, a^2 \frac{g_2}{4f_2} \epsilon_{0ijk} A_k' .$$

The uncoupled system (g2=0):

$$A_i'' - \nabla^2 A_i + \frac{f_1'}{f_1} A_i' + \frac{g_1'}{f_1} \epsilon_{0ijk} \partial_j A_k = 0,$$

$$B_{ij}'' - \nabla^2 B_{ij} + \left(\frac{f_2'}{f_2} - 2\frac{a'}{a} \right) B_{ij}' = 0.$$

Topological generation of mass

Topological generation of mass

1. Allen, Bowick & Lahiri, "Topological mass generation in (3+1)-dimensions," MPLA6 (1991).
2. Dvali, Jackiw & Pi, "Topological mass generation in four dimensions," PRL96 (2006).

A curious and interesting feature.

$$\nabla^{\mu}\left(f_2(\phi)H_{\mu\nu\alpha}\right)+\frac{g_2}{2}\tilde{F}_{\nu\alpha}=0.$$

Formal Solution ⟶

$$F^{\mu\nu}=-\frac{1}{3g_2}\nabla^{[\mu}f_2(\phi)\tilde{H}^{\nu]}$$

$$V^{\mu}\equiv\frac{f_2(\phi)}{3g_2}\tilde{H}^{\mu}$$

$$m^2(\phi)\equiv\frac{3g_2^2}{2f_2}$$

⟶

$$\nabla^{\mu}\left(f_1(\phi)\,\nabla_{[\mu}V_{\nu]}+\frac{g_1(\phi)}{2}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\nabla^{[\alpha}V^{\beta]}\right)=m^2(\phi)V_{\nu}$$

↓

$$S_V=-\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\,\left[f_1(\phi)W_{\mu\nu}W^{\mu\nu}+g_1(\phi)W_{\mu\nu}\tilde{W}^{\mu\nu}+2m^2(\phi)V_{\mu}V^{\mu}\right]$$

Not invariant under $V_{\mu}\rightarrow V_{\mu}+\partial_{\mu}\xi$

Still invariant under $\tilde{H}_{\mu}\rightarrow\tilde{H}_{\mu}+\partial_{\mu}\xi$

Topological generation of mass

Topological generation of mass

Alternatively

$$F^{\mu\nu} = -\frac{1}{3g_2}\nabla^{[\mu} f_2(\phi)\tilde{H}^{\nu]}$$ Formal Solution ⟶ $$V_\mu = A_\mu + \partial_\mu v$$

gauge choice $\nabla_\nu \nabla_\mu A^\mu = -\frac{m^2}{f_1}\partial_\nu v\,.$

$$\Box A_\nu - R_{\nu\mu}A^\mu + \frac{\partial^\mu f_1}{f_1}(\nabla_\mu A_\nu - \nabla_\nu A_\mu) + \frac{\partial^\mu g_1}{f_1}\eta_{\mu\nu\alpha\beta}\nabla^\alpha A^\beta - \frac{m^2}{f_1}A_\nu = 0$$

↓

Proca field with kinetic couplings in a curved background!

A mechanism different from Higgs mechanism!
Alternative to Higgsed Gauge-flation and Chromo-Natural inflation.

3-form "dynamics"

3-form + 0-form system and e.o.m.

$$\mathscr{L}_{\phi A_{(3)}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f_3(\phi)}{48}F_{(3)}^2 - g_3(\phi)\tilde{F}_{(3)}\,.$$

To have well defined variations of general field configurations, we need a boundary term!

Brown and Teitelboim, Phys. Lett. B195 (1987) 177 and Nucl. Phys. B297 (1988) 787; Duncan and Jensen, Nucl. Phys. B336 (1990) 100; Duff, Phys. Lett. B226 (1989) 36.

$$\mathscr{L}_{\phi A_{(3)}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f_3(\phi)}{48}F_{(3)}^2 - g_3(\phi)\tilde{F}_{(3)}$$

Boundary term –>

$$+\partial_{\mu_1}\left[(g_3 - f_3\tilde{F}_{(3)})\frac{\epsilon^{\mu_1\mu_2\mu_3\mu_4}}{4!\sqrt{-g}}A_{(3)\,\mu_2\mu_3\mu_4}\right]\,.$$

E.o.m.

$$\nabla^\mu\left(f_3(\phi)F_{(3)\mu\nu\alpha\beta} + g_3(\phi)\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\right) = 0, \quad \Box\phi - V_{,\phi} - \frac{f_{3,\phi}}{48}F_{(3)}^2 - g_{3,\phi}\tilde{F}_{(3)} = 0$$

3-form "dynamics"

3-form e.o.m.

$$\nabla^\mu \left(f_3(\phi) F_{(3)\mu\nu\alpha\beta} + g_3(\phi)\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta} \right) = 0.$$

$$F_{(3)\mu\nu\alpha\beta} = X(x)\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta} \quad \longrightarrow \quad X(x^\mu) = \frac{c - g_3(\phi)}{f_3(\phi)}.$$

Trivial in absence of coupling functions (f3= const. and g3=0)!

Plugging back in the scalar Lagrangian and using

$F^2_{(3)} = -4!X^2, \tilde{F}_{(3)} = -X$, we get:

$$\mathscr{L}_{\phi A_{(3)}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V_{eff}(\phi),$$

with ⟶

$$\boxed{V_{eff}(\phi) = V(\phi) + \frac{\left(c - g_3(\phi)\right)^2}{2f_3(\phi)}}$$

The scalar field "eats" the 3-form!

3-form "dynamics". Alternate take

3-form e.o.m.

$$\nabla^{\mu}\Big(f_3(\phi)F_{(3)\mu\nu\alpha\beta} + g_3(\phi)\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\Big) = 0.$$

Dynamic degrees of freedom and gauge symmetry.

1 d.o.f —> $A_{(3)ijk}$. Gauge choice —> $A_{(3)0ij} = \nabla^k A_{(3)0ik} = 0$

"Shift" (gauge) symmetry —>

$$A_{(3)\,ijk} \to A_{(3)\,ijk} + \nabla_{[i}\xi_{jk]}$$

$$\downarrow$$

$$A_{(3)\,ijk} \to A_{(3)\,ijk} + b(x^i)\varepsilon_{0ijk}$$

$F_{(3)0ijk} = \sqrt{-g}X(x^{\alpha})\epsilon_{0ijk} = \partial_0 A_{(3)ijk}$, or $\sqrt{-g}X(x^{\alpha}) = \partial_0 A_{(3)}$.

3-form "dynamics". Alternate take

3-form e.o.m.

$$\sqrt{-g}X(x^\alpha) = \partial_0 A_{(3)}$$

$\Downarrow$

$$A_{(3)}(\tau) = \int \mathrm{d}\tau' \sqrt{-g}\left(\frac{c - g_3(\phi)}{f_3(\phi)}\right) = \int \mathrm{d}\tau' \sqrt{-g}\left(\frac{c - g_3(\tau')}{f_3(\tau')}\right).$$

Energy momentum tensor:

$$T^{(3)}_{\alpha\beta} = -\left(V_{\rm eff}(\phi) - V(\phi)\right) g_{\alpha\beta} = -\frac{\left(c - g_3(\phi)\right)^2}{2f_3(\phi)} g_{\alpha\beta},$$

Equation of state parameter ⟶ $w_{(3)} = -1.$

Applications to inflation and DE!

1-2 and 3-forms toghether

1 and 2-forms system

$$S_{1,2} = -\frac{1}{4}\int d^4x\sqrt{-g}\left[f_1(\phi)W_{\mu\nu}W^{\mu\nu} + g_1(\phi)W_{\mu\nu}\tilde{W}^{\mu\nu} + 2m^2(\phi)V_\mu V^\mu\right]$$

Equivalent to a massive 1-form theory

0 and 3-forms system

$$S_{0,3} = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V_{eff}(\phi)\right]$$

Equivalent to a scalar theory with an effective potential

(1,2,3)-forms system

$$S_p = S_\phi - \frac{1}{4}\int d^4x\sqrt{-g}\left[f_1(\phi)W_{\mu\nu}W^{\mu\nu} + g_1(\phi)W_{\mu\nu}\tilde{W}^{\mu\nu} + 2m^2(\phi)V_\mu V^\mu\right]$$

Equivalent to a massive 1-form model with ϕ dependent couplings

Main conclusion. A kind of no-go theorem

This is a kind of no-go theorem which states:

Don't try to find more general p-form like "Galileon" models in 4D, you will fail. The most general that you would get is a massive vector theory.

We can test this statement for massive 2-forms

L. Heisenberg & G. Trenkler, JCAP 05 (2020) 019 e-Print: 1908.09328 [hep-th]

$$\mathcal{L}_4^H = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu B_{\nu\rho}\partial_\alpha B_{\beta\gamma} \propto H^2$$

$$\mathcal{L}_4^R = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu B_{\nu\beta}\partial_\alpha B_{\rho\gamma} \subset f(H^2, \mathcal{L}_4^T)$$

$$\mathcal{L}_4^T = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu B_{\alpha\rho}\partial_\nu B_{\beta\gamma}$$ --> New kind of interaction

Where

$$\mathcal{L}_4^T = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu B_{\alpha\rho}\partial_\nu B_{\beta\gamma} = \partial_\mu B^{\mu\nu}\partial_\alpha B_\nu{}^{\alpha} + \partial_\nu B_{\mu\alpha}\partial^\alpha B^{\mu\nu}$$

Main conclusion. A kind of no-go theorem

Testing the systematic construction with 2-forms

New interaction term

$$\mathscr{L}_4^T = \epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\partial_\mu B_{\alpha\rho}\partial_\nu B_{\beta\gamma} = \partial_\mu B^{\mu\nu}\partial_\alpha B_\nu{}^\alpha + \partial_\nu B_{\mu\alpha}\partial^\alpha B^{\mu\nu}$$

JPBA, A. Guarnizo, L. Heisenberg, C. Valenzuela-Toledoand J. Zosso: Phys. Rev. D **102**, 063521 (2020), arXiv:2003.11736 [hep-th]. Using the solution of the coupled 1+2-forms system:

$$B \wedge F = B_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{m}B_{\mu\nu}\partial^\alpha(H_{\alpha\mu\nu}) = -\frac{1}{m}\mathscr{L}_4^T - \frac{1}{3m}H^2$$

This new interaction term is a combination of B^F and H^2 terms!

What else we can do?

Adding non minimal coupling with gravity

A unique non minimal coupling[1]

$$\mathscr{L}^{nmc} = \sqrt{-g}L^{\mu\nu\alpha\beta}B_{\mu\nu}B_{\alpha\beta} \quad \textbf{where} \quad L^{\mu\nu\alpha\beta} = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\kappa\delta}R_{\rho\sigma\kappa\delta}$$

So, the full Lagrangian for massive 2-forms non minimally coupled to gravity becomes

$$\mathscr{L}^{2-forms} = \frac{M_p^2 R}{2} + \mathscr{L}_2(B, H, \tilde{H}) + \gamma L^{\mu\nu\alpha\beta}B_{\mu\nu}B_{\alpha\beta}$$

[1] L. Heisenberg & G. Trenkler, JCAP 05 (2020) 019 e-Print: 1908.09328 [hep-th]

Conclusions and Remarks

1. The most general version of the model discussed here is equivalent to a massive vector model with field dependent couplings.
2. Topological generation of mass mechanism results interesting for cosmological applications. New anisotropic solutions during DE domination and during inflation. Distinctive signatures.
3. Topologic terms like F~F acquire non trivial dynamics when coupled to a scalar field. Parity breaking signatures come from term like F^F and nothing else. Signatures can be traced in gravitational waves.
4. Non minimal coupled 2-form offers interesting possibilities. It is worth to do further analysis and applications of this model.