Neutrino mixing matrix in the μ-τ symmetry framework

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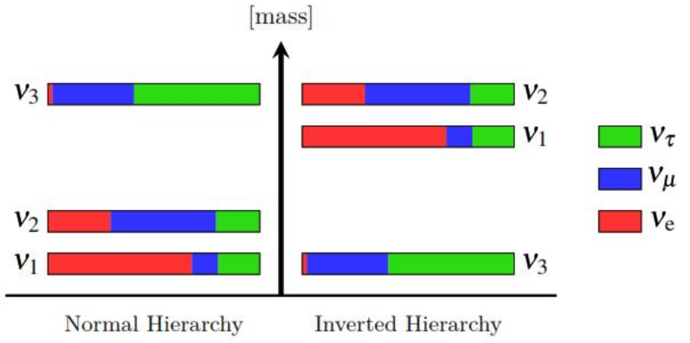
Introduction

- Neutrinos can be written in two different bases:
- Flavor states: $\nu_\alpha = \nu_e , \nu_\mu , \nu_\tau$
- Mass states: $\nu_i = \nu_1 , \nu_2 , \nu_3$
- They are related by an unitary transformation

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

Where $U_{\alpha i}$ are the elements of the PMNS matrix.



PMNS matrix

- The PMNS matrix depends on 6 independent parameters
- 3 mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$
- 1 Dirac CP violating phase δ_{CP}
- 2 Majorana phases λ_1, λ_2

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} + s_{12}s_{23}s_{13}e^{i\delta_{CP}} & -s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \times diag(1, e^{-i\lambda_1/2}, e^{-i\lambda_2/2})$$

μ-τ symmetry

- Measurements of these parameters have shown a remarkable result:

$$|U_{\mu i}| \simeq |U_{\tau i}|$$

$$U_{\mu-\tau} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Equality is obtained when $\theta_{23} = \frac{\pi}{4}$ and $\theta_{13} = 0$

- In this framework the different patterns are classified according the value given to s_{12}

- One of the patterns which used to dominate in the model building community is the Tri-Bi-Maximal mixing pattern where:

$$s_{12} = \frac{1}{\sqrt{3}}$$

Deviations from TBM pattern

- The values predicted by this mixing pattern are in conflict with the most recent experimental data especially $\theta_{13} \neq 0$

- This motivated the study of deviations from the TBM mixing pattern in a way that fits experimental results, for this we consider parametrizations in the form

$$U_{PMNS} = U_{TBM} U_{Corr}$$

where U_{Corr} is a correction matrix.

Deviations from TBM pattern

- We study cases where U_{Corr} is a product of unitary, orthogonal or both kinds of matrices, plus some additional phases to obtain predictions for Dirac and Majorana phases.

$$U_{Corr} = \begin{cases} U_{ij}(\phi,\alpha)\ \mathrm{Diag}\left(e^{-i\frac{\sigma_1}{2}}, e^{-i\frac{\sigma_2}{2}}, e^{-i\frac{\sigma_3}{2}}\right) \\ U_{ij}(\phi,\alpha)\ O_{kl}(\phi')\mathrm{Diag}\left(e^{-i\frac{\sigma_1}{2}}, e^{-i\frac{\sigma_2}{2}}, e^{-i\frac{\sigma_3}{2}}\right) \\ O_{ij}(\phi)\ O_{kl}(\phi')\mathrm{Diag}\left(e^{-i\frac{\sigma_1}{2}}, e^{-i\frac{\sigma_2}{2}}, e^{-i\frac{\sigma_3}{2}}\right) \end{cases}$$

$$U_{12} = \begin{pmatrix} \cos(\phi) & \sin(\phi)e^{i\alpha} & 0 \\ -\sin(\phi)e^{-i\alpha} & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{13} = \begin{pmatrix} \cos(\phi) & 0 & \sin(\phi)e^{i\alpha} \\ 0 & 1 & 0 \\ -\sin(\phi)e^{-i\alpha} & 0 & \cos(\phi) \end{pmatrix}$$

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi)e^{i\alpha} \\ 0 & -\sin(\phi)e^{-i\alpha} & \cos(\phi) \end{pmatrix}$$

Method

The $\sin^2\theta_{ij}$ are written in terms of the correction parameters from U_{Corr} using:

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2}, \qquad \sin^2\theta_{23} = \frac{|U_{\mu 2}|^2}{1-|U_{e3}|^2}, \qquad \sin^2\theta_{13} = |U_{e3}|^2.$$

Normal Ordering

$$0{,}273 < \sin^2\theta_{12} < 0{,}379,$$
$$0{,}0199 < \sin^2\theta_{13} < 0{,}0244,$$
$$0{,}453 < \sin^2\theta_{23} < 0{,}598.$$

Inverted Ordering

$$0{,}273 < \sin^2\theta_{12} < 0{,}379,$$
$$0{,}0196 < \sin^2\theta_{13} < 0{,}0241,$$
$$0{,}445 < \sin^2\theta_{23} < 0{,}599.$$

Rephasing invariants such as the Jarlskog invariant are use to relate the Dirac and Majorana phases with the correction parameters

$$\begin{aligned} J &= Im[U_{e1}U_{\mu 2}U_{e2}^{*}U_{\mu 1}^{*}] \\ &= \cos^2\theta_{13}\sin\theta_{13}\sin\theta_{23}\cos\theta_{12}\cos\theta_{23}\boxed{\sin\delta_{CP}} \end{aligned}$$

$$\mathcal{I}_1 = Im[U_{e2}^{2}U_{e1}^{*2}] = -\cos^2\theta_{12}\cos^4\theta_{13}\sin^2\theta_{12}\boxed{\sin\lambda_1}$$

$$\mathcal{I}_2 = Im[U_{e3}^{2}U_{e1}^{*2}] = -\cos^2\theta_{12}\cos^2\theta_{13}\sin^2\theta_{13}\boxed{\sin(\lambda_2+2\delta_{CP})}$$

Results

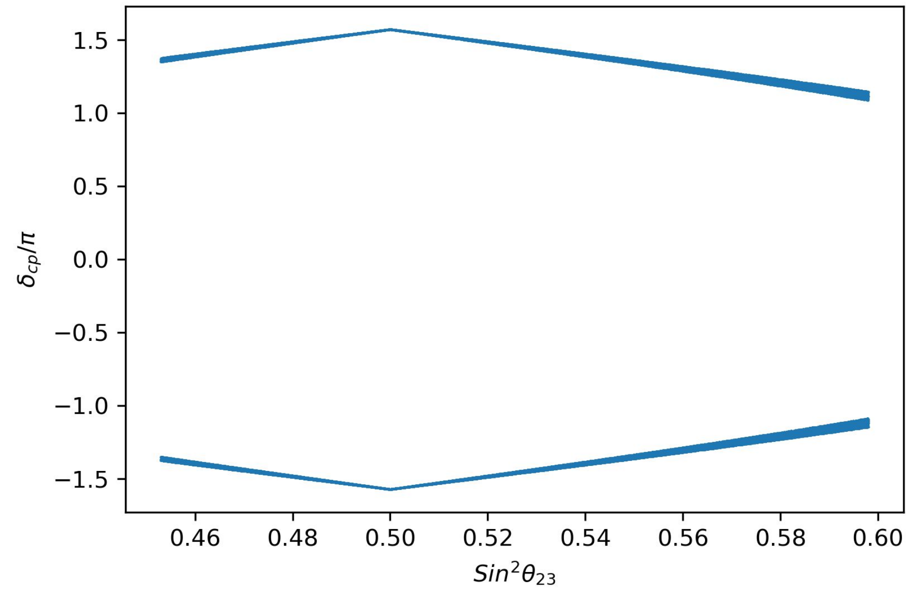
- For the cases of purely orthogonal rotation we found that they are either not compatible with the experimental ranges or did not constrain the phases.

$$U_{Corr} = O_{ij} O_{kl} \times diag\left(e^{-i\frac{\sigma_1}{2}}, e^{-i\frac{\sigma_2}{2}}, e^{-i\frac{\sigma_3}{2}}\right)$$

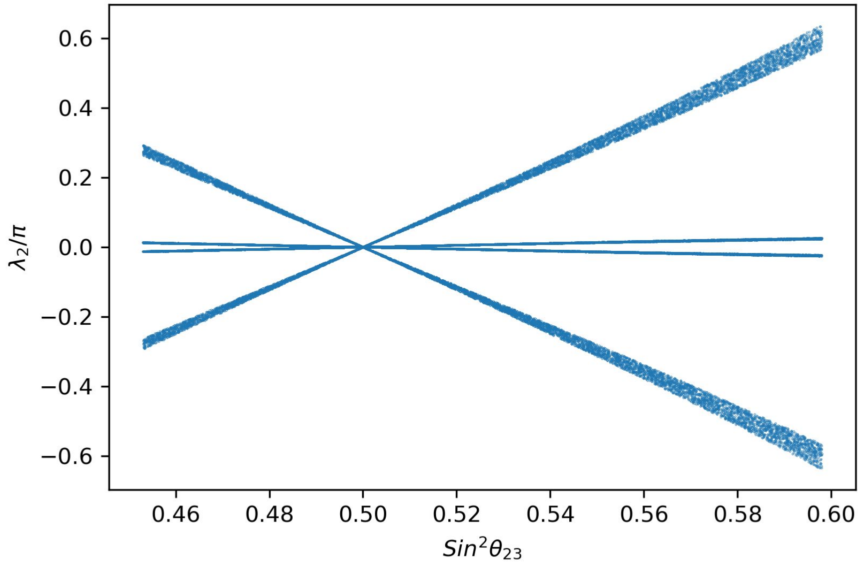
$$O_{12} = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad O_{13} = \begin{pmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{pmatrix} \qquad , O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}.$$

Results

$$U_{Corr} = U_{23}O_{12} \times diag(e^{\frac{-i\sigma_1}{2}}, e^{\frac{-i\sigma_1}{2}}, e^{\frac{-i\sigma_1}{2}})$$



The Dirac phase, δ_{CP}, was found to be different from π and zero, thus contributing to CP violation from the neutrino sector.



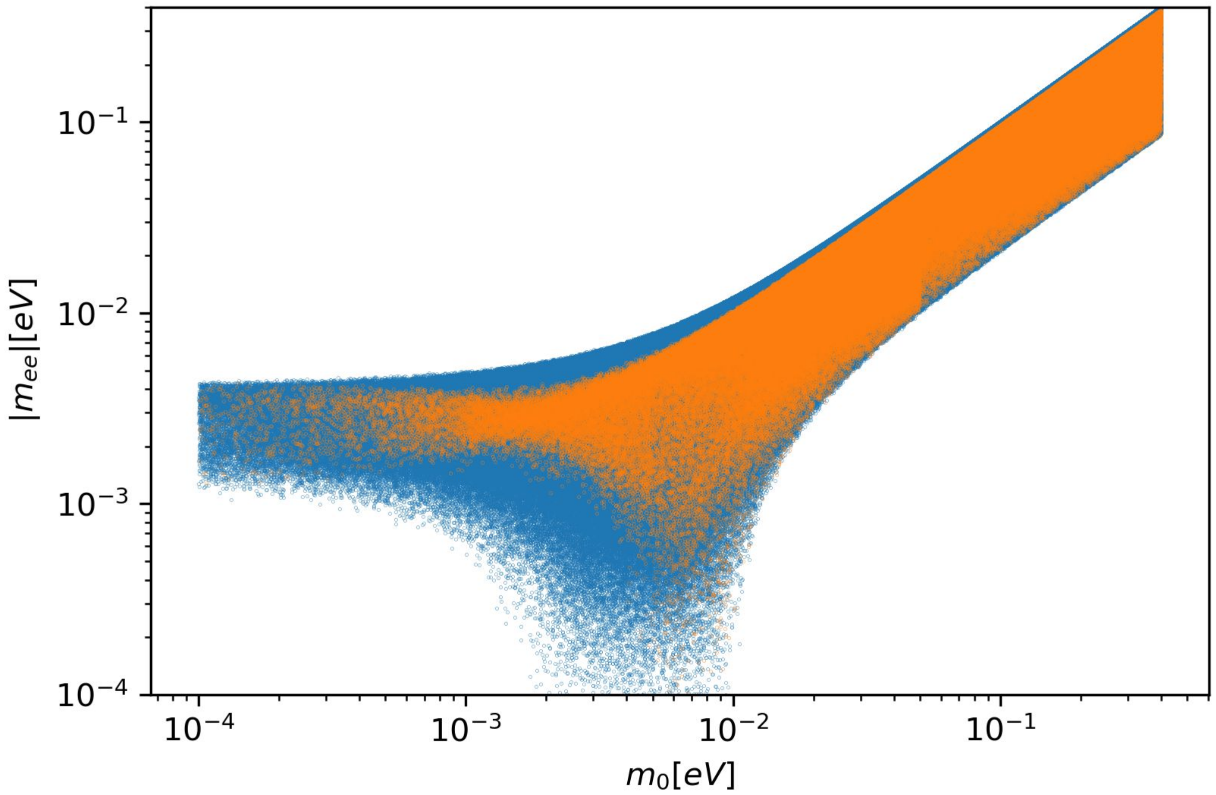
For the Majorana phase λ_2 CP conservation can't be ruled out, since it is zero when $\sin^2\theta_{23} = 0.5$

The Majorana λ_1 couldn't be constrained in this case.

Results

- Since it is possible to limit the possible values of the phases of the PMNS matrix, it is also possible to make predictions about the effective Majorana mass.

$$|m_{ee}| = \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right|$$



Conclusions

- It is possible to find prediction for the Majorana phases and relate them to the experimental angles.

- New experiments would help improve predictions and potentially confirm or exclude some parametrizations used.

- We expect that this type of work will serve as a guide for model building studies of mass generation and neutrino mixing and as a guideline to explore other possible parametrizations of the PMNS mixing matrix.