Light Pseudoscalar and Axial Meson Spectroscopy via an AdS/QCD Modified Soft Wall Model

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In Collaboration with

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Low-Energy QCD Phenomena

1. QCD perturbative regime: $E > 1$ GeV.
2. Non-Perturbative QCD ($E \leq 1$ GeV):
   - $\alpha_{QCD} \geq 1$.
   - Strongly bounded quarks.
     - Hadron states with light flavors (u, d and s).
     - Mass generation and chiral symmetry breaking (χSB).
3. Possible solutions:
   - Effective Field Theories (χPT) and Effective Models (NJL, -N-LSM).
   - Nonconformal AdS/QCD holographic approaches.
     - AdS/CFT-like formulation without scale invariance.
     - Nonperturbative FT $\rightarrow$ Perturbative gravitational approach.

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1. AdS Poincare patch with an extra UV cutoff:

$$dS^2 = g_{MN}\, dx^M\, dx^N = \frac{R^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right] \Theta\left(z - z_0\right). \tag{1}$$

2. Associated action:

$$I = I_{\text{Scalar}} + I_{\text{Vector}}, \tag{2}$$

3. where

$$I_{\text{Scalar}} = -\frac{1}{2\, g_S^2}\int d^5 x\, \sqrt{-g}\, e^{-\Phi(z)}\left[g^{MN}\, \partial_M S\, \partial_N S + M_5^2\, S^2\right], \tag{3}$$

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Modified Soft-Wall Model - Hadronic Identity

1. Hadronic dimension Δ and Bulk Mass:

$$M_5^2\, R^2 = \Delta\,(\Delta - 4) - s\,(s-4)\,. \tag{6}$$

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$$M_5^2\, R^2 = (\Delta_{\text{Phys}} + \Delta_P)\,(\Delta_{\text{Phys}} + \Delta_P - 4) - s\,(s-4)\,. \tag{7}$$

3. Table:

Meson Identity	Δ_P	$M_5^2\, R^2$
Scalar meson	0	-3
Vector meson	0	0
Pseudoscalar meson	-1	-4
Axial vector meson	-1	-1

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Modified Soft-Wall Model - Two-Point Functions

1. 5D scalar/vector action.
2. Small variations in the fields involved.
3. Euler-Lagrange Equations of Motion.
   - Boundary/On-shell action term evaluated at $z = z_0$.
   - Bulk-to-boundary propagator (BtBP).
4. BtBP into On-shell action with source terms.
5. Two-point scalar/vector function (2PF).
   - 2PF poles = mass spectra.
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Holographic Description of Pseudoscalar Mesons

1. Equation of Motion (after taking $S(z,q) = S_0(q)\mathcal{V}(z,q)$):

$$\partial_z \left[\frac{e^{-\kappa^2 z^2}}{z^3} \partial_z \mathcal{V} \right] + \frac{e^{-\kappa^2 z^2}}{z^3} q^2 \mathcal{V} + \frac{4\, e^{-\kappa^2 z^2}}{z^5} \mathcal{V} = 0. \quad (8)$$

2. Normalized pseudoscalar propagator:

$$\mathcal{V}_\eta(z,q) = \frac{z^2\, {}_1F_1\left(1 - \frac{q^2}{4\kappa^2}, 1, \kappa^2 z^2\right)}{z_0^2\, {}_1F_1\left(1 - \frac{q^2}{4\kappa^2}, 1, \kappa^2 z_0^2\right)}. \quad (9)$$

3. η-meson two-point function:

$$\Pi_\eta(q^2) = -\frac{R^3}{g_S^2} \frac{e^{-\kappa^2 z_0^2}}{z_0^3} \left[\frac{2}{z_0} + \left(1 - \frac{q^2}{4\kappa^2}\right) \frac{2\kappa^2 z_0\, {}_1F_1\left(2 - \frac{q^2}{4\kappa^2}, 2, \kappa^2 z_0^2\right)}{{}_1F_1\left(1 - \frac{q^2}{4\kappa^2}, 1, \kappa^2 z_0^2\right)} \right]. \quad (10)$$

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η Mesons

Table: Mass spectrum for η pseudoscalar mesons with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}.

η trajectory with $\Delta_P = -1$				
n	State	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$
1	$\eta(550)$	547.86 ± 0.017	975.25	43.8
2	$\eta(1295)$	1294 ± 4	1233.6	4.9
3	$\eta(1405)$	1408.8 ± 1.8	1455.3	3.2
4	$\eta(1475)$	1476 ± 4	1652.9	10.7
5	$\eta(1760)$	1760 ± 11	1829.2	3.8
6	$\eta(2225)$	2216 ± 21	1992.7	11.3

Ref: Martín Contreras, Miguel Ángel *et al*, Chin. J. Phys. 66 (2020) 715-723; Tanabashi, M. *et al*, Review of Particle Physics Phys. Rev. D98 (2018) 030001; Wang, Li-Ming *et al*, Phys. Rev. D96 (2017) 034013.

a_1 Mesons

Table: Mass spectrum for a_1 axial mesons with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}

a_1 trajectory with $\Delta_P = -1$				
n	State	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$
1	$a_1(1260)$	1230 ± 40	808.1	52.2
2	$a_1(1420)$	1414^{+15}_{-13}	1114.7	26.8
3	$a_1(1640)$	1654 ± 19	1351.3	22.4
4	$a_1(1930)$	1930^{+19}_{-70}	1558.7	23.8
5	$a_1(2095)$	$2096 \pm 17 \pm 121$	1744.3	20.1
6	$a_1(2270)$	2270^{+55}_{-40}	1913.4	18.6

Ref: Martín Contreras, Miguel Ángel *et al*, Chin. J. Phys. 66 (2020) 715-723; Tanabashi, M. *et al*, Review of Particle Physics Phys. Rev. D98 (2018) 030001; Adolph, C. *et al*, Phys. Rev. Lett. 115 (2015) 082001.

Scalar Mesons

Table: Mass spectrum for $f_0(500)$ mesons with $\kappa = 0.45$ GeV and $z_0 = 5$ GeV^{-1}.

f_0	M_{th} (MeV)	M_{exp} (MeV)	$\%M$
$f_0(980)$	1.070	0.99	7.46
$f_0(1370)$	1.284	1.370	5.11
$f_0(1500)$	1.487	1.504	1.13
$f_0(1710)$	1.674	1.723	2.93
$f_0(2020)$	1.846	1.992	7.94
$f_0(2100)$	2.153	2.101	2.39
$f_0(2200)$	2.292	2.189	4.49
$f_0(2330)$	2.424	2.314	4.52

Ref: Cortés, Santiago *et al*, Phys. Rev. D (2017) 106002; C. Patrignani *et al.*, [Particle Data Group] Chin. Phys. C **40**, no. 10, 100001 (2016).

Vector Mesons

Table: Mass spectrum for ρ and mesons with $\kappa = 0.45$ GeV and $z_0 = 5$ GeV^{-1}.

ρ	M_{th} (GeV)	M_{exp} (GeV)	$\%M$
$\rho(775)$	0.975	0.775	20.53
$\rho(1450)$	1.455	1.465	0.66
$\rho(1570)$	1.652	1.570	4.96
$\rho(1700)$	1.829	1.720	5.97
$\rho(1900)$	1.992	1.909	4.15
$\rho(2150)$	2.142	2.153	0.50

Ref: Cortés, Santiago *et al*, Phys. Rev. D (2017) 106002; C. Patrignani *et al.*, [Particle Data Group] Chin. Phys. C **40**, no. 10, 100001 (2016).

Conclusions

1. We have fitted the radial trajectory for both the η and the a_1 mesons. The first excited states, i. e., $\eta(550)$ and $a_1(1260)$, were not well predicted by the model. However, we could fit 26 states (among scalar, pseudoscalar, vector and axial particles) by just taking three parameters within a δ_{RMS} error close to 21.1%.
2. We infer that the small amount of theoretical parameters taken in our approach must have had an important influence in our results for the masses described in this work. Something similar happens with the radial states in non-conformal models of scalar and vector mesons, as shown previously with the first vector state.
3. We could confirm that chiral symmetry is restored at highly orbital excited states; this was checked after confirming the rising of some degeneracy in their respective masses.

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