Algebraic approach to scattering amplitudes

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L_∞-algebras

Remembering a Lie algebras, vector space with a product

$$[[T^i, T^j], T^k] + [[T^k, T^i], T^j] + [[T^j, T^k], T^i] = 0$$

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renaming $T^i \to a_i$ and $[,] \to l_2$

$$l_2(l_2(a_i, a_j), a_k) + l_2(l_2(a_k, a_i), a_j) + l_2(l_2(a_j, a_k), a_i) = 0$$

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Now the a_i's are the fields of the theory
The grading for the space comes from the ghost number (BRST) and the products...

BV formalism

Off-shell BRST, antibracket formalism,...

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gauge parameters $\longrightarrow$ ghost fields (field content Φ^A)

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Antibracket of two general functionals $F[\Phi, \Phi^*]$ and $G[\Phi, \Phi^*]$ by

$$(F, G) = F\left(\frac{\overleftarrow{\partial}}{\partial\Phi^A}\frac{\partial}{\partial\Phi^*_A} - \frac{\overleftarrow{\partial}}{\partial\Phi^*_A}\frac{\partial}{\partial\Phi^A}\right)G$$

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We want a particular functional: the action

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The classical master action $S[\Phi, \Phi^*]$ must satisfy:

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BV nilpotent transformations

$$\begin{aligned} \delta_{\text{BV}} \Phi^A &= -(S, \Phi^A) = \frac{\partial_r S}{\partial \Phi^*_A}, \\ \delta_{\text{BV}} \Phi^*_A &= -(S, \Phi^*_A) = -\frac{\partial_r S}{\partial \Phi^A} \end{aligned}$$

BV formalism and L_∞-algebras

Taking the field content as
$(A, \psi, c) \longrightarrow (A, \psi, c, A^*, \psi^*, c^*)$

BV formalism and L_∞-algebras

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From the master action $S[A, \psi, c, A^*, \psi^*, c^*]$ we have the BV transformations

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\begin{aligned}
\delta_{\mathrm{BV}} c^a &= -\tfrac{1}{2} l_2(c,c)^a, \\
\delta_{\mathrm{BV}} A^a_\mu &= l_1(c)^a_\mu + l_2(A,c)^a_\mu + \tfrac{1}{2} l_3(A,A,c)^a_\mu + \tfrac{1}{2} l_3(\psi+\bar\psi, \psi+\bar\psi, c)^a_\mu + \tfrac{1}{2} l_3(c,c,A^*)^a_\mu, \\
\delta_{\mathrm{BV}} \psi^i &= l_2(\psi,c)^i + l_3(A,\psi,c)^i + l_3(c,c,\psi^*)^i, \\
\delta_{\mathrm{BV}} \bar\psi_i &= l_2(\bar\psi,c)_i + l_3(A,\bar\psi,c)_i + l_3(c,c,\bar\psi^*)_i, \\
\delta_{\mathrm{BV}} A^{*a}_\mu &= -l_1(A)^a_\mu - \tfrac{1}{2} l_2(A,A)^a_\mu - \tfrac{1}{2} l_2(\psi+\bar\psi, \psi+\bar\psi)^a_\mu - l_2(c,A^*)^a_\mu + \cdots, \\
\delta_{\mathrm{BV}} \bar\psi^*_i &= -l_1(\psi+\bar\psi)_i - l_2(A,\bar\psi)_i - l_2(c,\bar\psi^*)_i + \cdots, \\
\delta_{\mathrm{BV}} \psi^{*i} &= -l_1(\psi+\bar\psi)^i - l_2(A,\psi)^i - l_2(c,\psi^*)^i + \cdots, \\
\delta_{\mathrm{BV}} c^{*a} &= l_1(A^*)^a + l_2(A,A^*)^a - l_2(c,c^*)^a + l_2(\psi+\bar\psi, \bar\psi^* + \psi^*)^a + \cdots,
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\end{aligned}
$$

and from the BV transformations the L_∞-algebra products

BV formalism and L_∞-algebras

Back to the graded vector field with elements x_i

BV formalism and L_∞-algebras

Back to the graded vector field with elements x_i

The equivalent of the Jacobi identity for an L_∞-algebra

$$\sum_{i=1}^{n}(-1)^{n-i}\sum_{\sigma\in\mathfrak{S}_{i,n-i}}\chi(\sigma;x_1,\ldots,x_n)l_{n-i+1}(l_i(x_{\sigma(1)},\ldots,x_{\sigma(i)}),x_{\sigma(i+1)},\ldots,x_{\sigma(n)})=0$$

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Nilpotency of the BV transformations lead exactly to a L_∞-algebra!

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Calling the the physical fields a_i's with degree 1, we can recover the classical action (Maurer-Cartan)

$$S_{\mathrm{MC}}[a]=\sum_{n\geq 1}\frac{1}{(n+1)!}\langle a,l_n(a,\dots,a)\rangle$$

L_∞-algebras and scattering amplitudes

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The elements in the cohomology are plane waves and due to to f:

$$\left\{ \begin{matrix} \text{plane} \\ \text{waves} \end{matrix} \right\} \to \left\{ \begin{matrix} \text{multi}-\text{particle} \\ \text{solutions} \end{matrix} \right\}$$

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Taking an infinite sum of plane waves, it give us the perturbiner expansion.

L_∞-algebras and scattering amplitudes

The construction of f is recursive and gives the Berends-Giele currents. e. g.

$$A'^{\mu} = \sum_{i\geq 1} \mathcal{A}_i^{\mu} \mathrm{e}^{\mathrm{i}k_i \cdot x} T^{a_i} \longrightarrow A^{\mu} = \sum_{n\geq 1} \frac{1}{n!} f_n(A', \ldots, A')^{\mu}$$

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Since we have a L_∞-algebra in $H^\bullet(L)$ we have an action

$$S'_{\mathrm{MC}}[A'] = \sum_{n\geq 2} \frac{1}{(n+1)!} \langle A', l'_n(A', \ldots, A')\rangle$$

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this action generates all the tree-level amplitudes

Examples

Yang-Mills

$$A^{\mu} = \sum_{n\geq 1}\sum_{I\in\mathcal{W}_n} \mathcal{A}_I^{\mu} e^{ik_I\cdot x} T^{a_I} = \sum_{i\geq 1} \mathcal{A}_i^{\mu} e^{ik_i\cdot x} T^{a_i} + \sum_{i,j\geq 1} \mathcal{A}_{ij}^{\mu} e^{ik_{ij}\cdot x} T^{a_i} T^{a_j} + \cdots$$

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where

$$\mathcal{A}_I^{\mu} = \frac{1}{s_I} \sum_{I=JK} \Big\{ (k_K \cdot \mathcal{A}_J) \mathcal{A}_K^{\mu} + \mathcal{A}_{J\nu} \mathcal{F}_K^{\mu\nu} - (k_J \cdot \mathcal{A}_K) \mathcal{A}_J^{\mu} - \mathcal{A}_{K\nu} \mathcal{F}_J^{\mu\nu} \Big\}$$

Examples

Yang-Mills

$$A^\mu = \sum_{n\geq 1}\sum_{I\in \mathcal{W}_n} \mathcal{A}^\mu_I e^{ik_I\cdot x} T^{a_I} = \sum_{i\geq 1} \mathcal{A}^\mu_i e^{ik_i\cdot x} T^{a_i} + \sum_{i,j\geq 1} \mathcal{A}^\mu_{ij} e^{ik_{ij}\cdot x} T^{a_i} T^{a_j} + \cdots$$

where

$$\mathcal{A}^\mu_I = \frac{1}{s_I}\sum_{I=JK}\Big\{(k_K\cdot\mathcal{A}_J)\mathcal{A}^\mu_K + \mathcal{A}_{J\nu}\mathcal{F}^{\mu\nu}_K - (k_J\cdot\mathcal{A}_K)\mathcal{A}^\mu_J - \mathcal{A}_{K\nu}\mathcal{F}^{\mu\nu}_J\Big\}$$

and

$$\mathcal{F}^{\mu\nu}_I = k^\mu_I \mathcal{A}^\nu_I - k^\nu_I \mathcal{A}^\mu_I - \sum_{I=JK}\Big(\mathcal{A}^\mu_J\mathcal{A}^\nu_K - \mathcal{A}^\mu_K\mathcal{A}^\nu_J\Big)$$

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and

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Amplitudes

$$S'_{\mathrm{MC}}[A'] = \sum_{n\geq 3} \frac{1}{n} \sum_{i\geq 1} \sum_{I\in\mathcal{W}_{n-1}} \delta(k_{iI}) s_I \mathcal{A}_i \cdot \mathcal{A}_I \,\mathrm{tr}(T^{a_{iI}})$$

Examples

QCD

$$\mathcal{A}_P^{a\mu} = \frac{1}{s_P}\mathcal{J}_P^{a\mu} + \frac{\mathrm{i}}{s_P}\sum_{P=Q\cup R}\{-\mathrm{i}\tilde{f}_{bc}{}^{a}(k_Q\cdot\mathcal{A}_R^b)\mathcal{A}_Q^{c\mu} + \mathcal{A}_{Q\nu}^b\mathcal{F}_R^{c\nu\mu}\},$$

$$\mathcal{J}_P^{a\mu} = \sum_{P=Q\cup R}\bar{\Psi}_{Qi}\gamma^\mu(T^a)^i{}_j\Psi_R^j,$$

$$\mathcal{F}_P^{a\mu\nu} = \mathrm{i}k_P^\mu\mathcal{A}_P^{a\nu} - \mathrm{i}k_P^\nu\mathcal{A}_P^{a\mu} + \mathrm{i}\tilde{f}_{bc}{}^{a}\sum_{P=Q\cup R}\mathcal{A}_Q^{b\mu}\mathcal{A}_R^{c\nu},$$

$$\Psi_P^i = -\left(\frac{\not{k}_P + m}{s_P - m^2}\right)\sum_{P=Q\cup R}\not{\mathcal{A}}_Q^a(T_a)^i{}_j\Psi_R^j,$$

$$\bar{\Psi}_{Pi} = -\sum_{P=Q\cup R}\bar{\Psi}_{Rj}(T_a)^j{}_i\not{\mathcal{A}}_Q^a\left(\frac{\not{k}_P - m}{s_P - m^2}\right)$$

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Amplitudes

$$S'^{\mathrm{QCD}}_{\mathrm{MC}} = \sum_{n\geq 1}\sum_{\substack{P\in\mathcal{OW}_n\\ P=Q\cup R}}\{\tilde{f}_{abc}\big((k_Q\cdot\mathcal{A}_R^b)(\mathcal{A}_Q^c\cdot\mathcal{A}_p^a) + \mathcal{A}_{Q\nu}^b\mathcal{A}_{p\mu}^a\mathcal{F}_R^{\mu\nu c}\big)$$

$$+\bar{\Psi}_{Qi}\not{\mathcal{A}}_p^a(T^a)^i{}_j\Psi_R^j - \bar{\Psi}_{Ri}\not{\mathcal{A}}_Q^a(T_a)^i{}_j\Psi_p^j - \bar{\Psi}_{pi}\not{\mathcal{A}}_Q^a(T_a)^i{}_j\Psi_R^j\}$$

Conclusions

- Deeper knowledge about the structure of scattering amplitudes (relations, identities)

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- Loop case (algebraic and mixed approach)
- Associahedron, Amplituhedron,...

Thanks for your attention!