

Prediction for Neutrino Masses, CP Violation, Leptogenesis and Neutrinoless Double Beta Decay from \mathcal{T}_{13} Flavor Symmetry

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- ▶ **An Asymmetric Yukawa Texture**
 - ▶ bottom-up approach using $SU(5)$ GUT
arXiv:1805.10684, M.H.R., P. Ramond, B. Xu
- ▶ **Yukawa Texture from a Discrete Family Symmetry**
 - ▶ based on $\mathcal{T}_{13} = \mathcal{Z}_{13} \rtimes \mathcal{Z}_3$
arXiv:1907.10698, M.J. Pérez, M.H.R., P. Ramond, A.J. Stuart, B. Xu
- ▶ **Tribimaximal Seesaw Mixing from \mathcal{T}_{13}**
 - ▶ predicts “normal ordering” of light neutrino masses
arXiv:2001.04019, M.J. Pérez, M.H.R., P. Ramond, A.J. Stuart, B. Xu
- ▶ **Leptogenesis from the Asymmetric Texture**
 - ▶ low energy \mathcal{CP} phase explains baryon asymmetry
arXiv:2008.04204, M.H.R.

A Marriage between the EW and Seesaw Sectors?

Lepton mixing angles in the PMNS matrix are

$$\theta_{12} = 33.65^\circ, \quad \theta_{23} = 47.58^\circ, \quad \theta_{13} = 8.49^\circ \quad [\text{PDG 2020}]$$

$$U_{PMNS} = \underbrace{\mathcal{U}^{(-1)\dagger}}_{\text{EW sector}} \underbrace{\mathcal{U}_{TBM}}_{\text{Seesaw sector}}$$

$$U_{TBM} \equiv \mathcal{R}(\theta_{12} = 35.3^\circ), \mathcal{R}(\theta_{23} = 45^\circ), \mathcal{R}(\theta_{13} = 0^\circ), \delta = 0^\circ$$

Very unlike quark mixing:

$$U_{CKM} = \underbrace{\mathcal{U}^{(\frac{2}{3})\dagger}}_{\text{EW sector}} \underbrace{\mathcal{U}^{(-\frac{1}{3})}}_{\text{EW sector}}$$

Can the Electroweak sector generate enough “Cabibbo Haze”?

Hunting the ‘Minimal’ Texture

What is the ‘minimal’ Yukawa texture that

- ▶ reproduces the GUT-scale mass relations

$$m_d : m_s : m_b \approx \lambda^4/3 : \lambda^2/3 : 1, \quad m_e : m_\mu : m_\tau \approx \lambda^4/9 : \lambda^2 : 1, \quad m_b \approx m_\tau,$$

the CKM mixing angles $s_{12} = \lambda$, $s_{23} = A\lambda^2$, $s_{13} = A\lambda^3\sqrt{\rho^2 + \eta^2}$

- ▶ generates *enough* “Cabibbo haze” for the reactor angle?

The Asymmetric Texture [arXiv:1805.10684](https://arxiv.org/abs/1805.10684), M.H.R. et al.

$$Y(\frac{2}{3}) \sim \text{diag} (\lambda^8, \lambda^4, 1),$$

$$Y(-\frac{1}{3}) \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \quad Y(-1) \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

$$\mathcal{U}_{\text{Seesaw}} = \text{diag} (1, 1, e^{i\delta}) \mathcal{U}_{\text{TBM}}$$

$$a = c = \frac{1}{3}, \quad g = A, \quad b = A\sqrt{\rho^2 + \eta^2}, \quad d = \frac{2a}{g} = \frac{2}{3A}, \quad \cos \delta = 0.2$$

$$Y^{(-\frac{1}{3})} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \quad Y^{(-1)} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

- ▶ One phase to rule them all:

$|\delta| \approx 78^\circ$: all angles within 1σ of PDG 2020 fit

Predicts leptonic CP violation: $|\delta_{CP}| = 1.32\pi$

consistent with PDG 2020 fit $\delta_{CP} = 1.36 \pm 0.17 \pi$, to be measured at DUNE and Hyper-K in ~ 10 years [arXiv:2002.02967, 1805.04163].

- ▶ Asymmetric terms: $(13 - 31)$, by $\mathcal{O}(\lambda)$, not negligible!
- ▶ Zero subdeterminant: GUT-scale mass relations
 $m_d : m_s : m_b \approx \lambda^4/3 : \lambda^2/3 : 1$, $m_e : m_\mu : m_\tau \approx \lambda^4/9 : \lambda^2 : 1$, $m_b \approx m_\tau$.
 The determinant of the $[13]$ submatrix vanishes, so that

$$\det Y^{(-\frac{1}{3})} = \det Y^{(-1)}$$

Suppose $SU(5)$ matter fields $F \sim \bar{\mathbf{5}} \equiv (\bar{d}, L)$ and $T \sim \mathbf{10} \equiv (Q, \bar{u}, \bar{e})$ are triplets of some discrete family symmetry group, $G_f \subset SU(3)_f$.

F and T must be different triplets of G_f

- ▶ $Y^{(-1/3)}$ and $Y^{(-1)}$ comes from $F \otimes T = (\bar{\mathbf{5}}, \mathbf{r}) \otimes (\mathbf{10}, \mathbf{s})$
- ▶ $3 \times 3 \Rightarrow$ either symmetric or antisymmetric
We need a group with at least two different triplets!
- ▶ Candidates: \mathcal{S}_4 (order 24), $\Delta(27)$ (order 27), \mathcal{T}_{13} (order 39)

$\mathbf{s} \otimes \mathbf{s}$ must distinguish diagonal from off-diagonal

- ▶ $Y^{(2/3)}$ comes from $T \otimes T = (\mathbf{10}, \mathbf{s}) \otimes (\mathbf{10}, \mathbf{s})$

Only \mathcal{T}_{13} survives!

Generating the Asymmetric Term

$$Y^{(-\frac{1}{3})} \leftarrow FTH\bar{5}\varphi^{(-\frac{1}{3})}$$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}_{\mathbf{3}_1} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} F_3 T_2 \\ F_1 T_1 \\ F_2 T_3 \end{pmatrix}_{\mathbf{3}_1} \oplus \begin{pmatrix} F_3 T_1 \\ F_1 T_3 \\ F_2 T_2 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} F_3 T_3 \\ F_1 T_2 \\ F_2 T_1 \end{pmatrix}_{\mathbf{3}_2}$$

T_{13} can dial *individual* matrix elements!

$$Y^{(\frac{2}{3})} \leftarrow TT\bar{H}_5\varphi^{(\frac{2}{3})}$$

$$\begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} T_3 T_3 \\ T_2 T_2 \\ T_1 T_1 \end{pmatrix}_{\bar{\mathbf{3}}_1} \oplus \begin{pmatrix} T_3 T_2 \\ T_2 T_1 \\ T_1 T_3 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} T_2 T_3 \\ T_1 T_2 \\ T_3 T_1 \end{pmatrix}_{\bar{\mathbf{3}}_2}$$

Diagonals are distinguished from off-diagonals!

Generating the Zero Subdeterminant

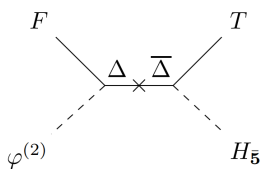
$$\begin{pmatrix} bd\lambda^4 & \times & b\lambda^3 \\ \times & \times & \times \\ d\lambda & \times & 1 \end{pmatrix} = \begin{pmatrix} b\lambda^3 \cdot d\lambda & \times & b\lambda^3 \cdot 1 \\ \times & \times & \times \\ d\lambda & \times & 1 \end{pmatrix}$$

$$(33) \text{ term} \rightarrow FT H_{\bar{5}} \varphi^{(1)}$$

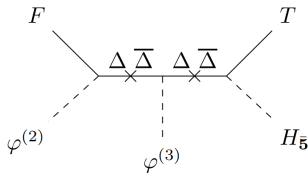
$$(31) \text{ term} \rightarrow FT H_{\bar{5}} \varphi^{(2)}$$

$$(13) \text{ term} \rightarrow FT H_{\bar{5}} \varphi^{(1)} \varphi^{(3)}$$

$$(11) \text{ term} \rightarrow FT H_{\bar{5}} \varphi^{(2)} \varphi^{(3)}$$



(a) (31) term



(b) (11) term

- ▶ Introduce three right handed neutrinos $\bar{N} \sim (\mathbf{1}, \mathbf{3}_2)$
- ▶ Dirac Yukawa matrix, $Y^{(0)}$: $F\bar{N}\bar{H}_5\varphi_A$, $\varphi_A \sim (\mathbf{1}, \bar{\mathbf{3}}_2)$
- ▶ Majorana matrix, \mathcal{M} : $\bar{N}\bar{N}\varphi_B$, $\varphi_B \sim (\mathbf{1}, \mathbf{3}_2)$
- ▶ Seesaw matrix, $\mathcal{S} = Y^{(0)} \mathcal{M}^{-1} Y^{(0)T}$
- ▶ For $\langle\varphi_B\rangle \propto (1, 1, 1)$ and $\langle\varphi_A\rangle \propto (-e^{i\delta}, 1, 1)$,

$$\mathcal{S} \propto \mathcal{U}_{TBM}(\delta) \text{diag} \left(-1, \frac{1}{2}, -1 \right) \mathcal{U}_{TBM}^T(\delta).$$

- ▶ Introduce a fourth right handed neutrino $\bar{N}_4 \sim (\mathbf{1}, \mathbf{1})$ with no Yukawa interaction
- ▶ New Majorana operator $\bar{N}\bar{N}_4\varphi_z$, with $\langle\varphi_z\rangle \propto (1, -2, 1)$

Prediction: normal ordering

$$m_{\nu_1} = 27.6, \quad m_{\nu_2} = 28.9, \quad m_{\nu_3} = 57.8 \text{ meV}$$

- ▶ $\sum_i |m_{\nu_i}| = 114.3 \text{ meV}$ compared to **Planck**: $\sum_i |m_{\nu_i}| < 120 \text{ meV}$ [arXiv:1807.06209].
- ▶ Combining data from Euclid and LSST to DESI and WFIRST, the **error bound on $\sum_i |m_{\nu_i}|$ will be constrained to $8 - 11 \text{ meV}$.**
- ▶ **Normal ordering is preferred above 3σ** by Super-K, T2K and NOvA [arXiv:2006.11237].
- ▶ DUNE and Hyper-K will **resolve the correct mass ordering beyond 5σ** in $5 - 7 \text{ yrs}$ [arXiv:2002.02967, 1805.04163].

Neutrinoless Double Beta Decay

- ▶ Dirac \mathcal{CP} Jarlskog-Greenberg Invariant, $|\mathcal{J}| = 0.028$
- ▶ Majorana Invariants, $|\mathcal{I}_1| = 0.106$, $|\mathcal{I}_2| = 0.011$

Prediction for $0\nu\beta\beta$

$$|m_{\beta\beta}| = 13.02 \text{ or } 25.21 \text{ meV}$$

compared to $|m_{\beta\beta}| \leq 61 - 165 \text{ meV}$ by [KamLAND-Zen](#)
[arXiv:1605.02889]

- ▶ Our predictions are expected to be tested in several next generation experiments [J.Phys.Conf.Ser. 1390 (2019) 1, 012048]:

| Experiment | Sensitivity (meV) | Experiment | Sensitivity (meV) |
|------------|-------------------|------------|-------------------|
| LEGEND | 11 - 28 | SNO+-II | 20 - 70 |
| nEXO | 8 - 22 | AMoRE-II | 15 - 30 |
| CUPID | 6 - 17 | PandaX-III | 20 - 55 |

- ▶ Leptogenesis fails if light neutrino **flavor effects** are not considered.
- ▶ Since the right handed neutrino mass scale is yet unknown, consider the **density matrix formalism** instead of approximate Boltzmann equations.
- ▶ Low energy \mathcal{CP} phase δ_{CP} explains baryon asymmetry $\eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$ when right handed neutrino masses are $\mathcal{O}(10^{11})$ GeV.
- ▶ Sign of baryon asymmetry picks the **correct sign** of \mathcal{CP} phase $\delta_{CP} = 1.32\pi$.

- ▶ An $SU(5) \times \mathcal{T}_{13}$ model that covers both quarks and leptons
- ▶ Asymmetry and zero-subdeterminant conditions are implemented without *fine-tuning*
- ▶ Prediction for CP violation: $\delta_{CP} = 1.32\pi$
- ▶ Prediction for light neutrino masses: Normal ordering
 $m_{\nu_1} = 27.6$, $m_{\nu_2} = 28.9$, $m_{\nu_3} = 57.8$ meV
- ▶ Prediction for $0\nu\beta\beta$: $|m_{\beta\beta}| = 13.02$ or 25.21 meV

Thank You!

Backup slides

- ▶ $\mathcal{T}_{13} = \mathcal{Z}_{13} \rtimes \mathcal{Z}_3$ has two generators a and b
- ▶ Presentation: $\langle a, b \mid a^{13} = b^3 = I, bab^{-1} = a^3 \rangle$
- ▶ Order = $13 \times 3 = 39$
- ▶ Three one dimensional irreps $\mathbf{1}, \mathbf{1}', \bar{\mathbf{1}}'$ and four three dimensional irreps $\mathbf{3}_1, \bar{\mathbf{3}}_1, \mathbf{3}_2, \bar{\mathbf{3}}_2$

$$1^2 + 1^2 + 1^2 + 3^2 + 3^2 + 3^2 + 3^2 = 39$$

- ▶ $\mathbf{1} : a = 1, b = 1; \mathbf{1}' : a = 1, b = \omega; \bar{\mathbf{1}}' : a = 1, b = \omega^2$, where $\omega = e^{i2\pi/3}$

\mathcal{T}_{13} Group Theory (contd.)

- ▶ b permutes the components of the triplets
- ▶ a assigns \mathcal{Z}_{13} charges:

$$\mathbf{3}_1 : (\rho, \rho^3, \rho^9), \quad \mathbf{3}_2 : (\rho^2, \rho^6, \rho^5)$$

where $\rho^{13} = 1$.

- ▶ Diagonal elements are naturally singled out in Kronecker products of triplets:

$$\mathbf{3}_1 \otimes \mathbf{3}_1 = \bar{\mathbf{3}}_1 \oplus \mathbf{3}_2 \oplus \bar{\mathbf{3}}_1,$$

$$\mathbf{3}_2 \otimes \mathbf{3}_2 = \bar{\mathbf{3}}_2 \oplus \bar{\mathbf{3}}_1 \oplus \bar{\mathbf{3}}_2,$$

$$\mathbf{3}_1 \otimes \mathbf{3}_2 = \mathbf{3}_2 \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_2.$$

Consider a triplet $\psi = \{\psi^1, \psi^2, \psi^3\} \sim \mathbf{3}_1$

Under the generator b ,

$\{\psi^1, \psi^2, \psi^3\} \rightarrow \{\psi^2, \psi^3, \psi^1\}$ cyclic permutation of the components

Under the generator a ,

$\{\psi^1, \psi^2, \psi^3\} \rightarrow \{\rho\psi^1, \rho^3\psi^2, \rho^9\psi^3\}$ the components gain \mathcal{Z}_{13} charges

We can determine the Clebsch-Gordan decompositions by simply following the action of the a generator!

\mathcal{T}_{13} Clebsch-Gordan Decomposition Table

$$\mathbf{3}_1 \otimes \mathbf{3}_1 \rightarrow \begin{pmatrix} \psi^{11} \\ \psi^{22} \\ \psi^{33} \end{pmatrix}_{\mathbf{3}_2}, \quad \begin{pmatrix} \psi^{23} \\ \psi^{31} \\ \psi^{12} \end{pmatrix}_{\bar{\mathbf{3}}_1}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\bar{\mathbf{3}}_1}$$

$$\mathbf{3}_2 \otimes \mathbf{3}_2 \rightarrow \begin{pmatrix} \psi^{22} \\ \psi^{33} \\ \psi^{11} \end{pmatrix}_{\bar{\mathbf{3}}_1}, \quad \begin{pmatrix} \psi^{23} \\ \psi^{31} \\ \psi^{12} \end{pmatrix}_{\bar{\mathbf{3}}_2}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\bar{\mathbf{3}}_2}$$

$$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_1 \rightarrow \begin{pmatrix} \psi_2^1 \\ \psi_3^2 \\ \psi_1^3 \end{pmatrix}_{\bar{\mathbf{3}}_2}, \quad \begin{pmatrix} \psi_1^2 \\ \psi_2^3 \\ \psi_3^1 \end{pmatrix}_{\mathbf{3}_2}, \quad (\psi_1^1 + \psi_2^2 + \psi_3^3)_{\mathbf{1}},$$

$$(\psi_1^1 + \omega\psi_2^2 + \omega^2\psi_3^3)_{\mathbf{1}'}, \quad (\psi_1^1 + \omega^2\psi_2^2 + \omega\psi_3^3)_{\bar{\mathbf{1}}'}$$

\mathcal{T}_{13} Clebsch-Gordan Decomposition Table

$$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_2 \rightarrow \begin{pmatrix} \psi_2^3 \\ \psi_3^1 \\ \psi_1^2 \end{pmatrix}_{\bar{\mathbf{3}}_1}, \quad \begin{pmatrix} \psi_3^2 \\ \psi_1^3 \\ \psi_2^1 \end{pmatrix}_{\mathbf{3}_1}, \quad (\psi_1^1 + \psi_2^2 + \psi_3^3)_{\mathbf{1}},$$

$$(\psi_1^1 + \omega\psi_2^2 + \omega^2\psi_3^3)_{\mathbf{1}'}, \quad (\psi_1^1 + \omega^2\psi_2^2 + \omega\psi_3^3)_{\bar{\mathbf{1}}'}$$

$$\mathbf{3}_1 \otimes \mathbf{3}_2 \rightarrow \begin{pmatrix} \psi^{33} \\ \psi^{11} \\ \psi^{22} \end{pmatrix}_{\mathbf{3}_1}, \quad \begin{pmatrix} \psi^{31} \\ \psi^{12} \\ \psi^{23} \end{pmatrix}_{\bar{\mathbf{3}}_2}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\mathbf{3}_2}$$

$$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_2 \rightarrow \begin{pmatrix} \psi_1^1 \\ \psi_2^2 \\ \psi_3^3 \end{pmatrix}_{\bar{\mathbf{3}}_1}, \quad \begin{pmatrix} \psi_3^2 \\ \psi_1^3 \\ \psi_2^1 \end{pmatrix}_{\bar{\mathbf{3}}_2}, \quad \begin{pmatrix} \psi_1^2 \\ \psi_2^3 \\ \psi_3^1 \end{pmatrix}_{\mathbf{3}_1}$$

$$\mathbf{1}' \otimes \mathbf{1}' = \bar{\mathbf{1}}'$$

$$\bar{\mathbf{1}}' \otimes \bar{\mathbf{1}}' = \mathbf{1}'$$

$$\mathbf{1}' \otimes \bar{\mathbf{1}}' = \mathbf{1}$$

$$\mathbf{3}_1 \otimes \mathbf{3}_1 = \bar{\mathbf{3}}_1 \oplus \bar{\mathbf{3}}_1 \oplus \mathbf{3}_2$$

$$\mathbf{3}_2 \otimes \mathbf{3}_2 = \bar{\mathbf{3}}_2 \oplus \bar{\mathbf{3}}_1 \oplus \bar{\mathbf{3}}_2$$

$$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_1 = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3}_2 \oplus \bar{\mathbf{3}}_2$$

$$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_2 = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$$

$$\mathbf{3}_1 \otimes \mathbf{3}_2 = \bar{\mathbf{3}}_2 \oplus \mathbf{3}_1 \oplus \mathbf{3}_2$$

$$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_2 = \bar{\mathbf{3}}_2 \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$$

$$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_1 = \mathbf{3}_2 \oplus \mathbf{3}_1 \oplus \bar{\mathbf{3}}_1$$

Table 1: $Z_{13} \times Z_3$ Kronecker products

Why do F and T transform differently under \mathcal{T}_{13} ?

$$SU(5) \times \mathcal{T}_{13} : \quad \overbrace{(\mathbf{10}, \mathbf{3}_2)}^T \oplus \overbrace{(\bar{\mathbf{5}}, \mathbf{3}_1)}^F \oplus \overbrace{(\mathbf{1}, \mathbf{3}_2)}^{\bar{N}} \oplus \overbrace{(\mathbf{1}, \mathbf{1})}^{\bar{N}_4}$$

$$SO(10) \times \mathcal{T}_{13} : \quad (\mathbf{16}, \mathbf{3}_2) \oplus (\mathbf{10}, \mathbf{3}_1) \oplus (\mathbf{1}, \mathbf{1}).$$

$$\mathbf{16} = \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}, \quad \mathbf{10} = \mathbf{5}_3 \oplus \bar{\mathbf{5}}_{-3},$$

$$E_6 \supset SO(10) \times U(1) : \quad \mathbf{27} = \mathbf{16}_1 \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_4,$$

Does the $U(1)$ charge dictate which is $\mathbf{3}_1$ and which is $\mathbf{3}_2$?