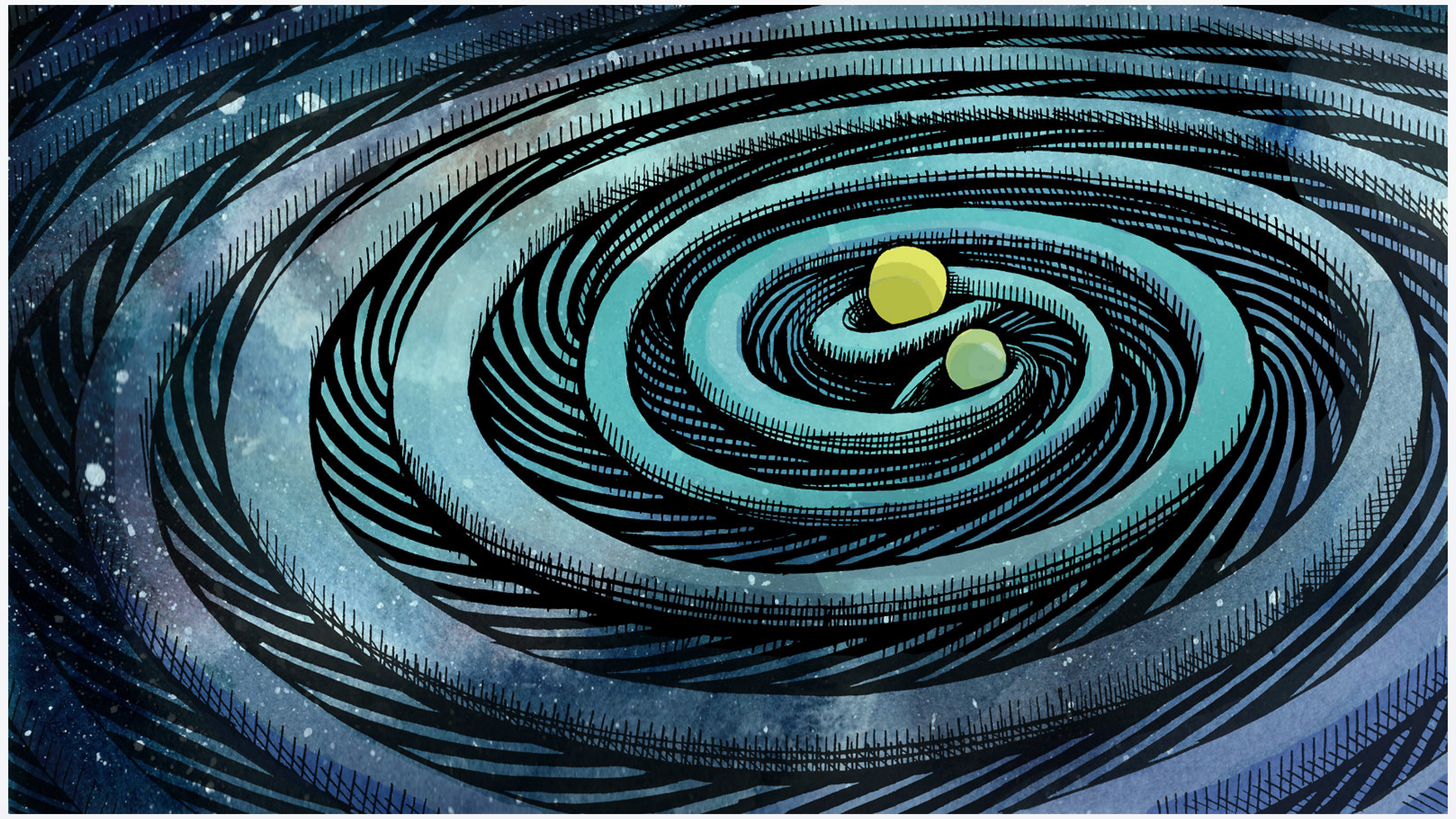
Unraveling the origin of Black Holes from effective spin measurements

Nicolas Fernandez

COMHEP 2020

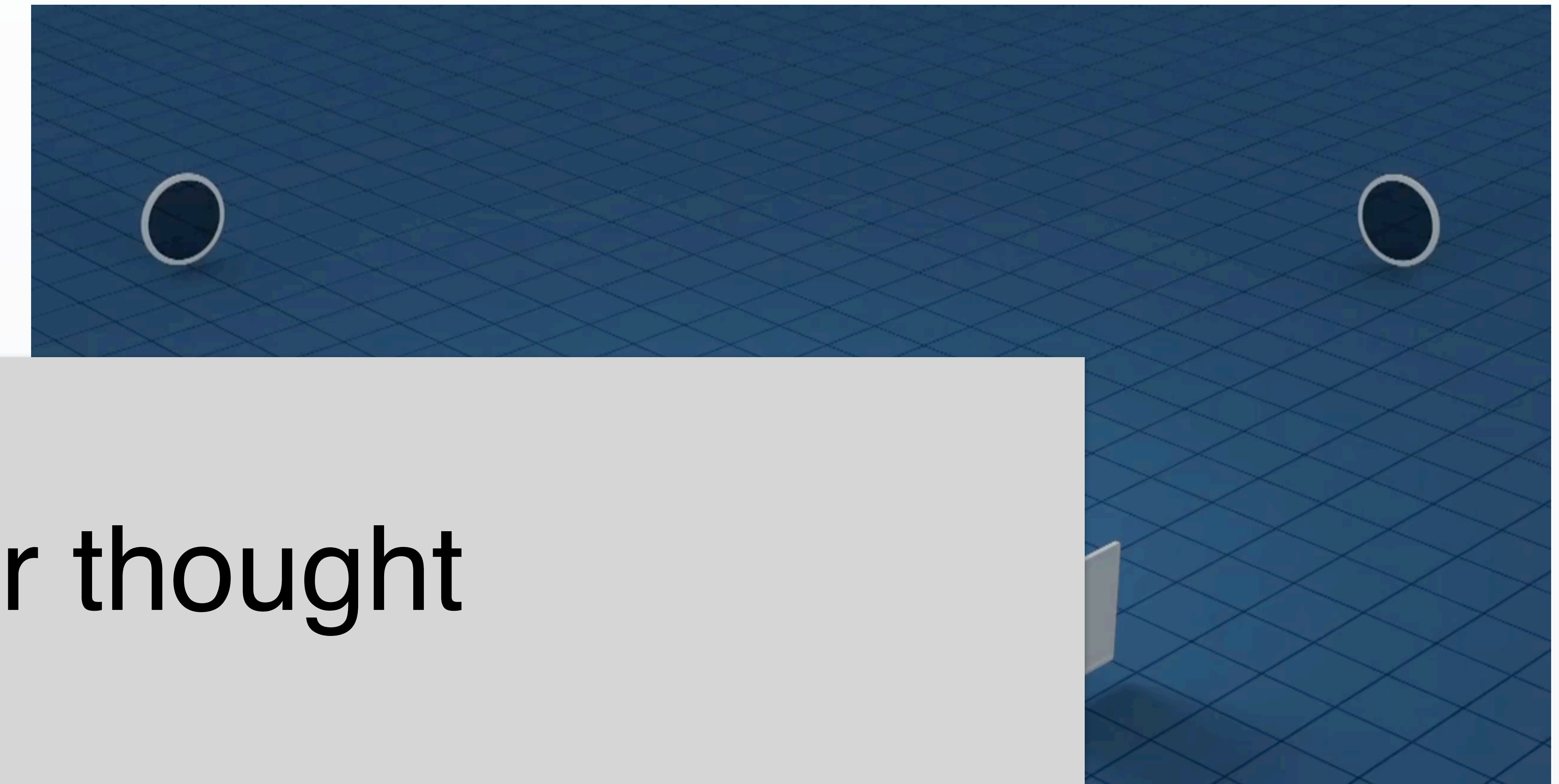


Collaborator: Stefano Profumo

ILLINOIS

LIGO

Livingston



Movie courtesy LIGO

Food for thought

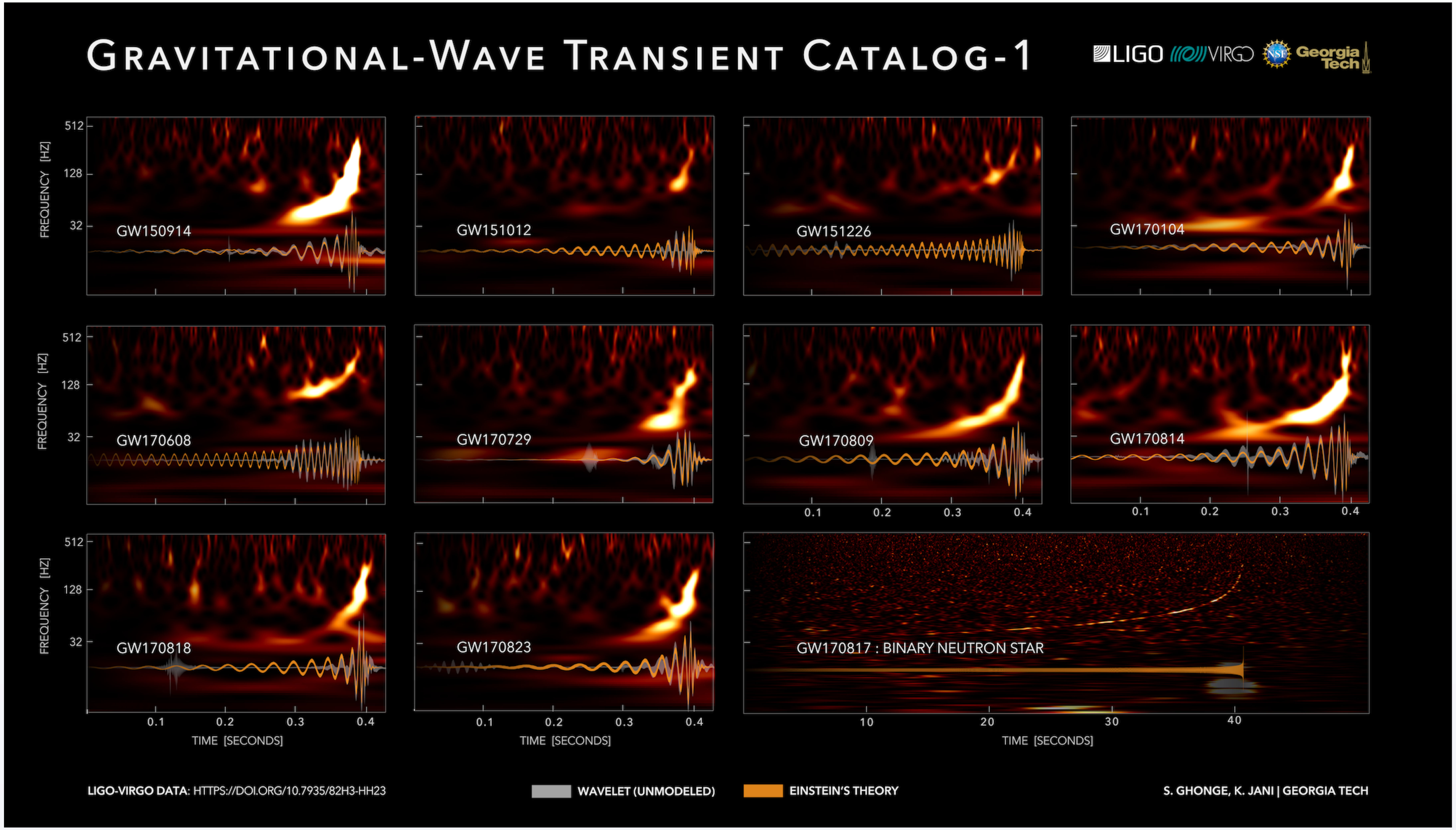
How can LIGO measure changes of 10^{-18} meters with wavelength of 10^{-6} meters?

Strain:

Arms change by: $\Delta L \sim 10^{-18}\mathrm{m}$

Laser: $\lambda \sim 10^{-6}\mathrm{m}$

~36 Gravitational Wave Detections



But before 2015…
0 Gravitational Wave Detections

- BHs expected to be 5 ~10 solar masses.

[Mandel, et al. arXiv:0912.1074]

- Effective Spin ~ 0.5.

- Rates: 33 ~ 100 more NS-NS binaries events compare to BH-BH binaries.

[LIGO Collaboration arXiv:1003.2480]

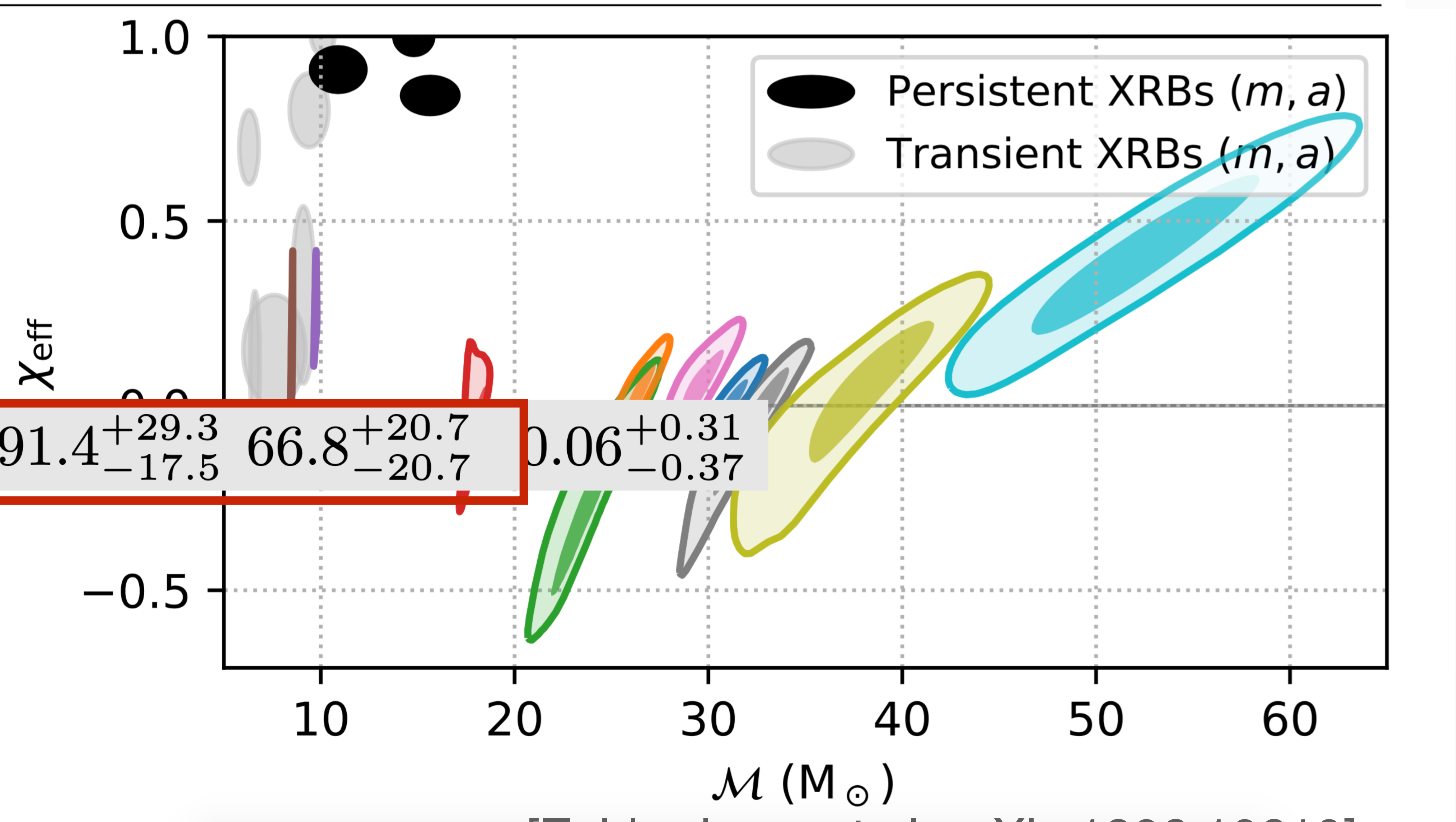
LIGO Parameters

[LIGO/Virgo Collaboration arXiv:1811.12940]

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	χ_{eff}
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$.186^{+0.00}_{-0.00}$	$0.00^{+0.02}_{-0.01}$
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$

GW190521 $157.9^{+37.4}_{-20.9}$ $66.9^{+15.5}_{-9.2}$ $91.4^{+29.3}_{-17.5}$ $66.8^{+20.7}_{-20.7}$ $0.06^{+0.31}_{-0.37}$

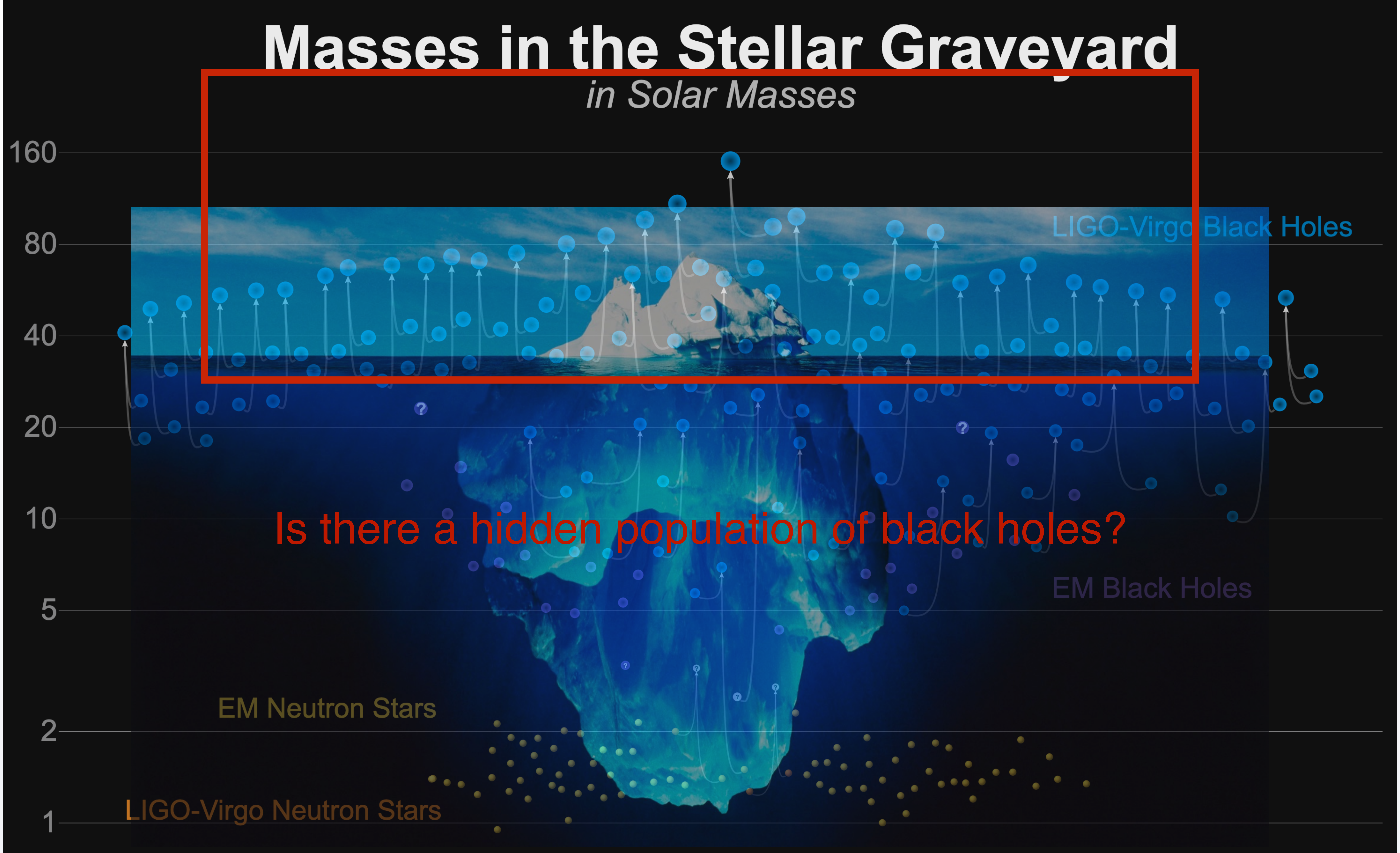
Rate Masses Spin



[Zaldarriaga, et al. arXiv:1806.10610]

"…The black holes were a little heavier than I expected…"

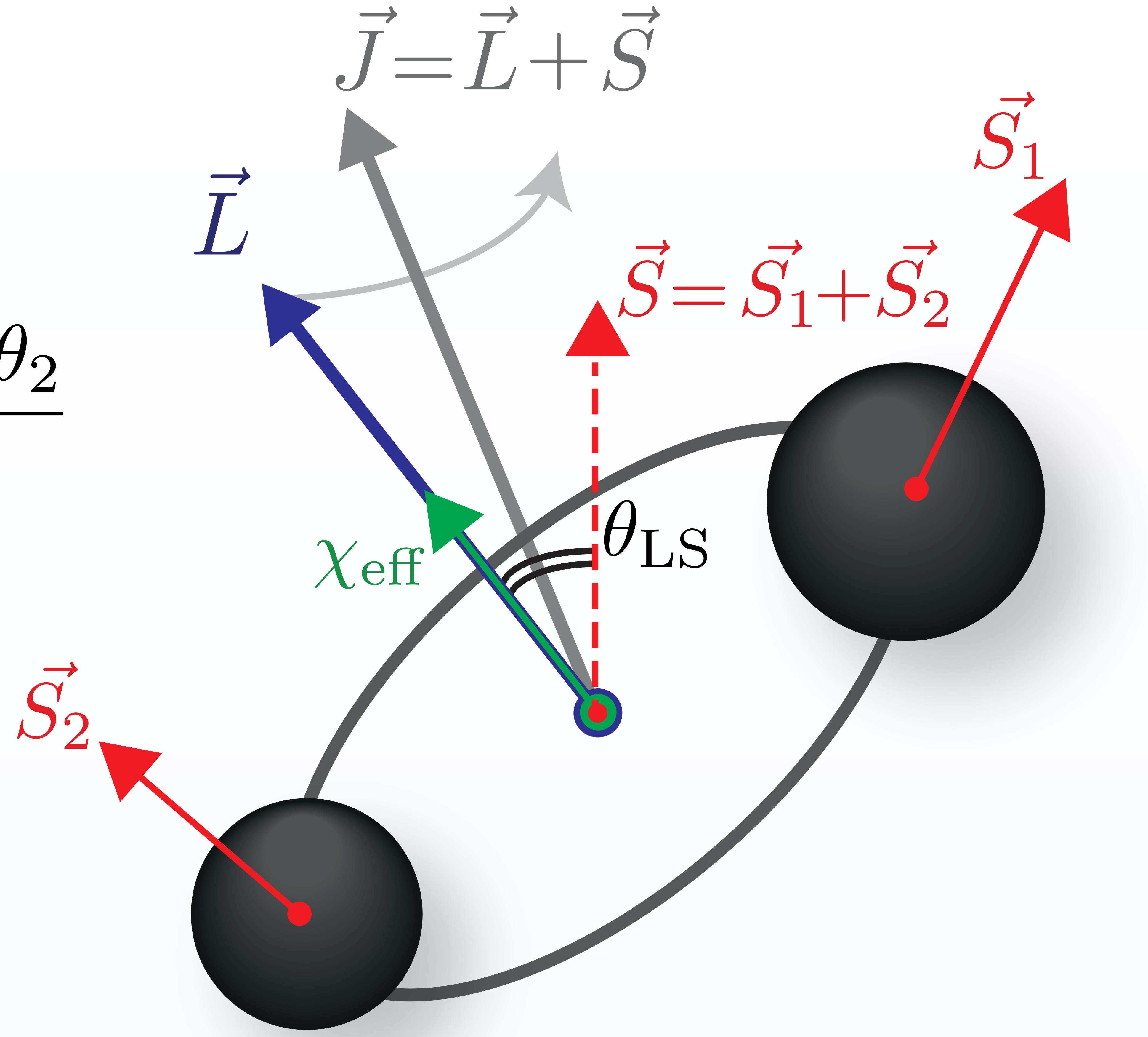
[Kip Thorne on Mindscape: Ep 24.]


Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
Is there a hidden population of black holes?
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars

Effective Spin

$$\chi = \frac{|\vec{S}|}{Gm^2}$$ Dimensionless spin parameter

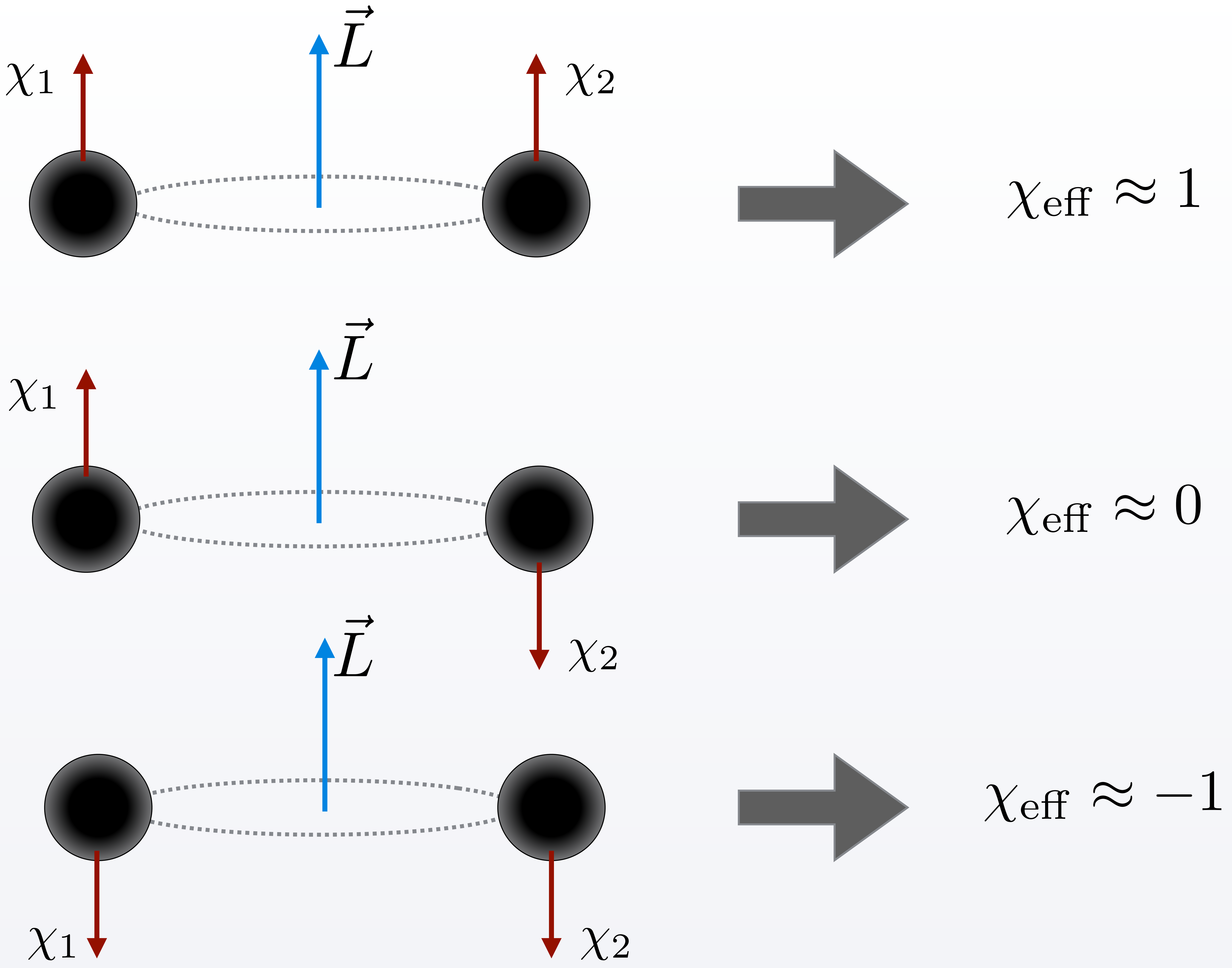
$$\chi_{\rm eff} = \frac{m_1\chi_1\cos\theta_1 + m_2\chi_2\cos\theta_2}{m_1+m_2}$$



It gives information about:

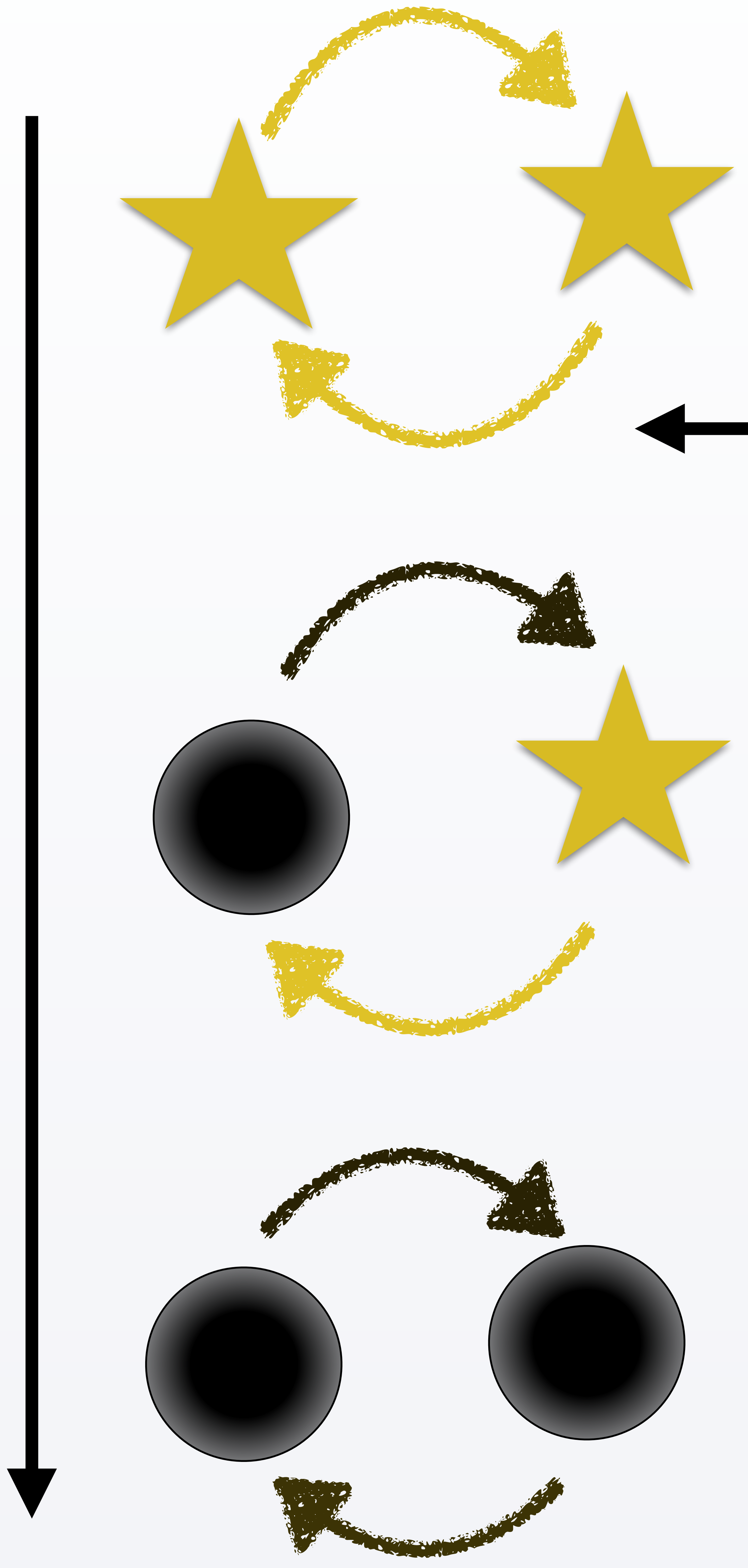
- Direction. +++
- Spin magnitude. ++
- masses. +

[Rodriguez, et al. arXiv:1609.05916]



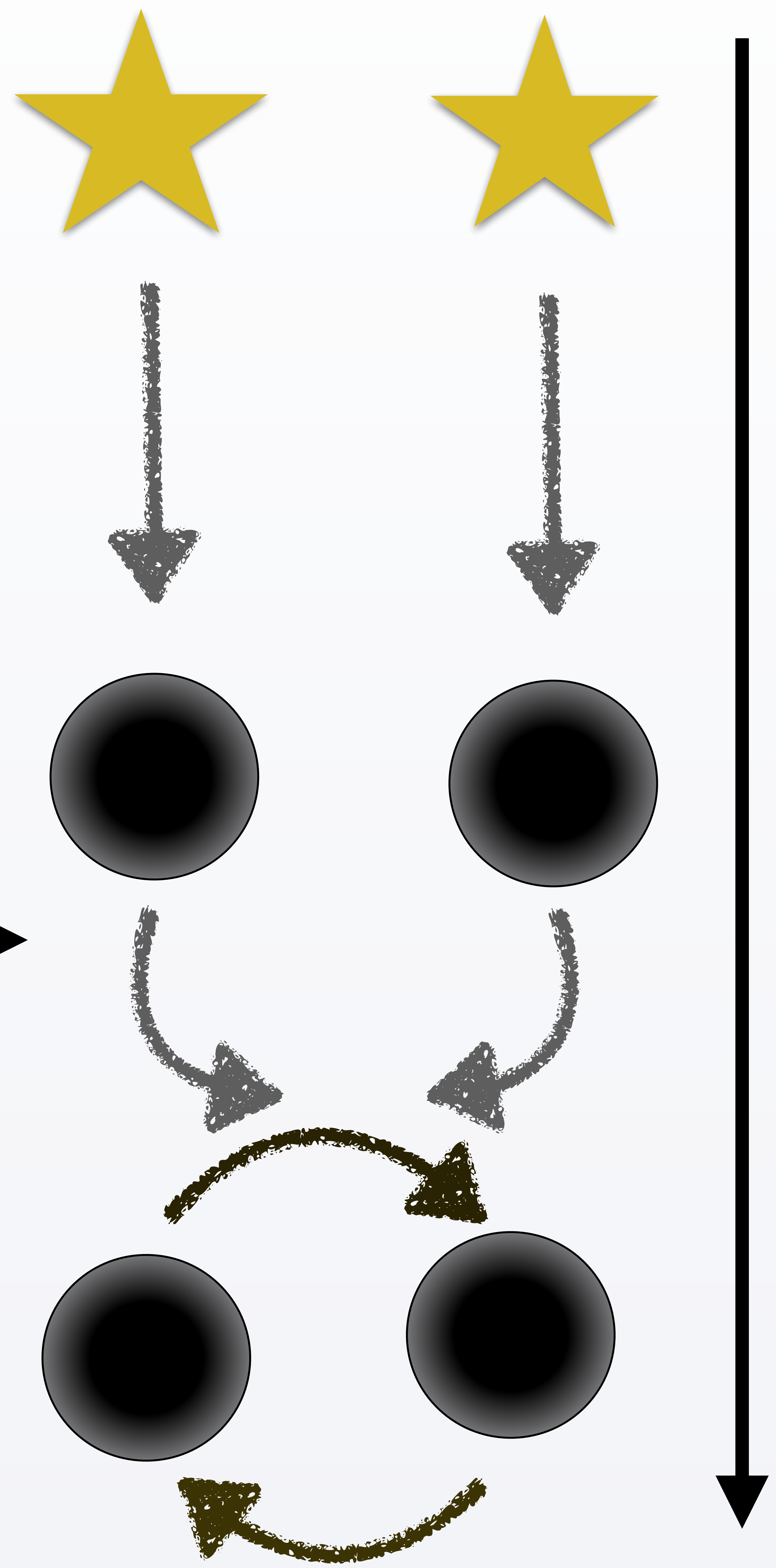
$\vec{L}$
χ_1
χ_2
$\chi_{\rm eff} \approx 1$
$\vec{L}$
χ_1
χ_2
$\chi_{\rm eff} \approx 0$
$\vec{L}$
χ_1
χ_2
$\chi_{\rm eff} \approx -1$

Possible formation channels



Isolated field binary scenario.

Dynamical assembling. Cluster evolution. (Many three body interactions)



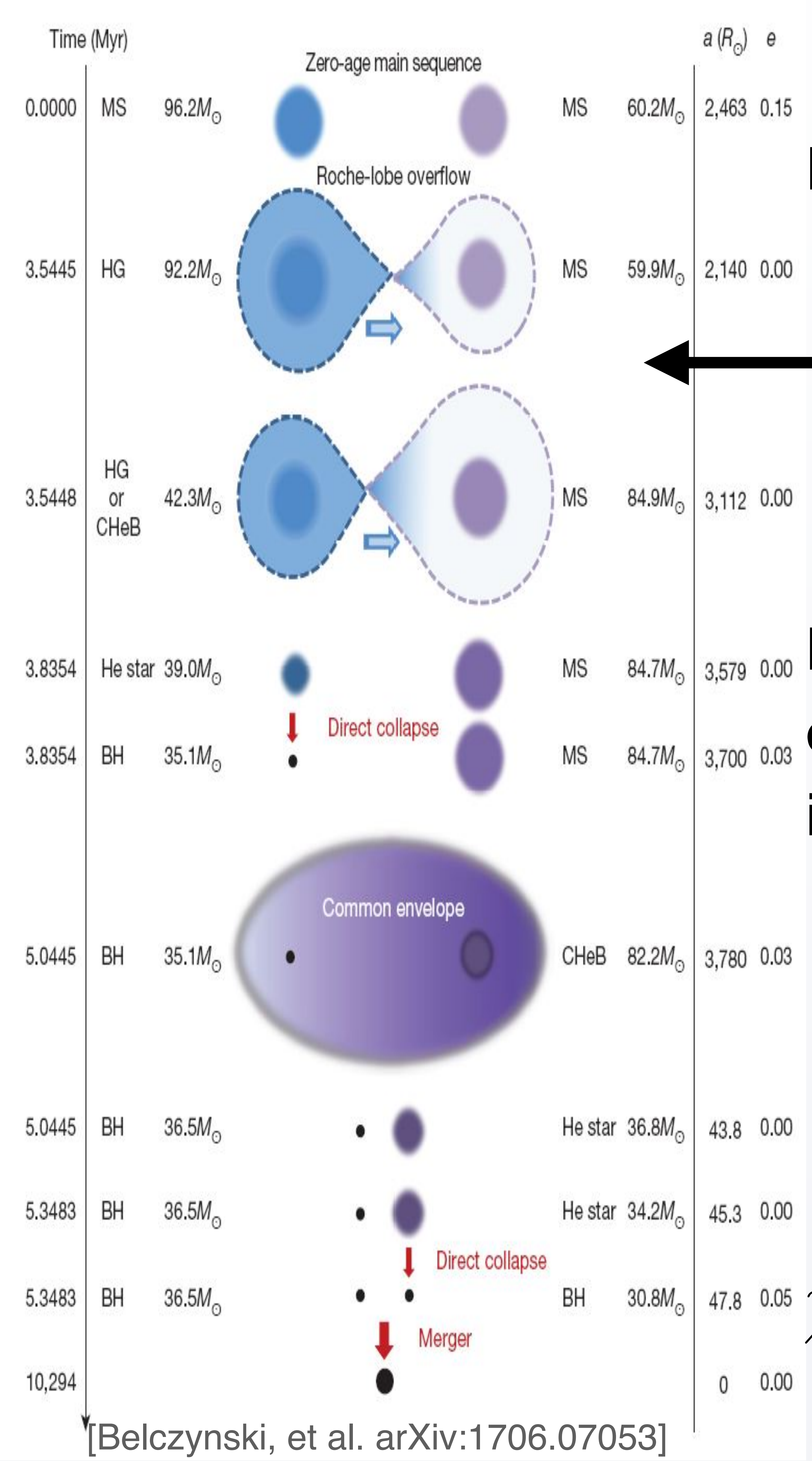
$\chi_{\text{eff}} \gtrsim 0$

$\chi_{\text{eff}} \sim 0$

[Belczynski, et al. arXiv:1706.07053]

[Rodriguez, et al. arXiv:1604.04254]

Possible formation channels



[Belczynski, et al. arXiv:1706.07053]

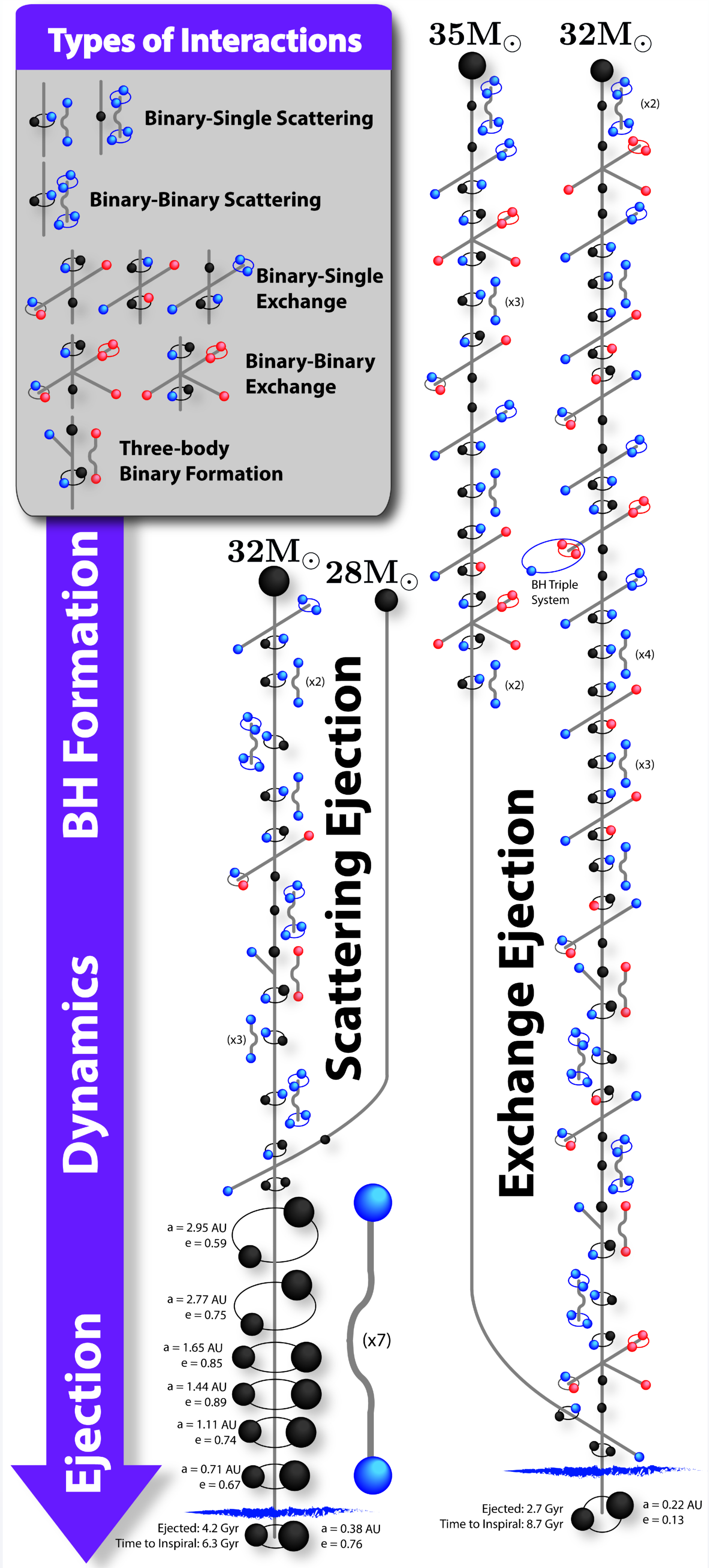
Isolated field binary scenario.

Dynamical assembling. Cluster evolution. (Many three body interactions)

There are a lot of uncertainties involve

$\chi_{\rm eff} \gtrsim 0$

$\chi_{\rm eff} \sim 0$



[Rodriguez, et al. arXiv:1604.04254]

Primordial Black Holes

Why Primordial Black Holes (PBHs)?

- Non-particle candidate of DM or a fraction of DM.
- Candidate of gravitational wave events observed by LIGO
- Seeds of SMBHs

How do you produce them?

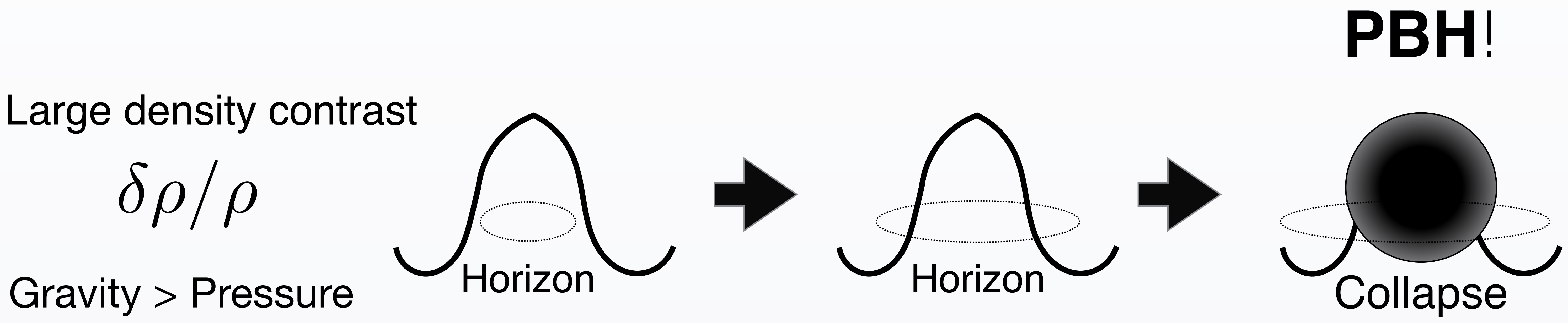
- Bubble collisions
- Cosmic Strings Loops
- Large density fluctuation

Smoking gun?

- BH less than one solar mass
- Mergers at high redshift $z > 40$

Formation of PBHs

[Carr, ’75]



Mass:

$$M_{PBH} \approx M_H \approx \gamma\rho\frac{4\pi}{3}H^{-3} \approx 10M_\odot\left(\frac{10^6\,\mathrm{Mpc}^{-1}}{k}\right)^2 \approx 10M_\odot\left(\frac{100\,\mathrm{MeV}}{T}\right)^2$$

Abundance of PBHs:

$$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}} = \int_{\delta_c} P(\delta)\,\mathrm{d}\delta$$

Gaussian

A precise value of the threshold for PBH formation has been extensively investigated

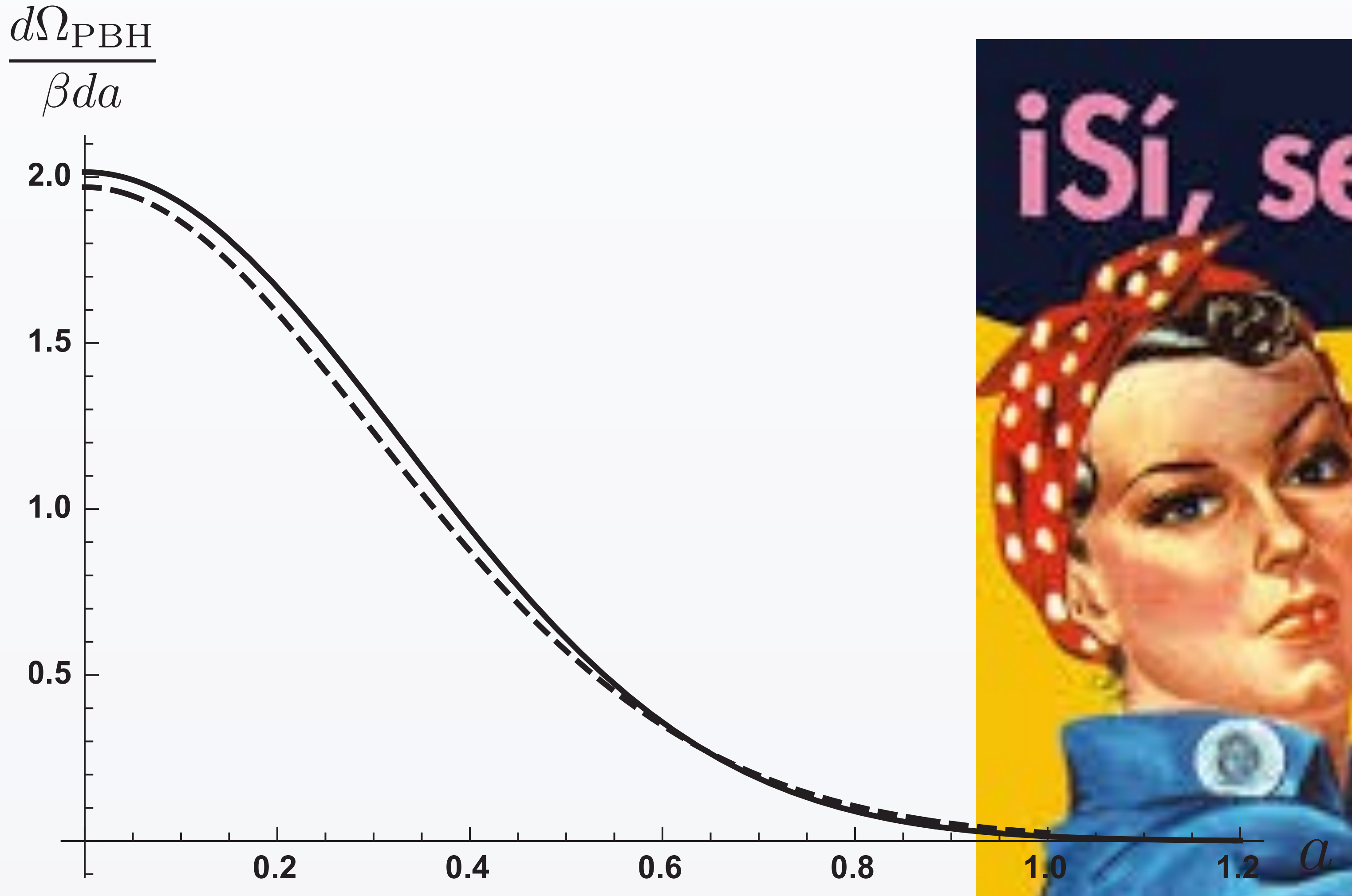
$$\delta_c \approx 0.45$$

Can we discriminate primordial versus astrophysical black holes using the LIGO-Virgo Events?

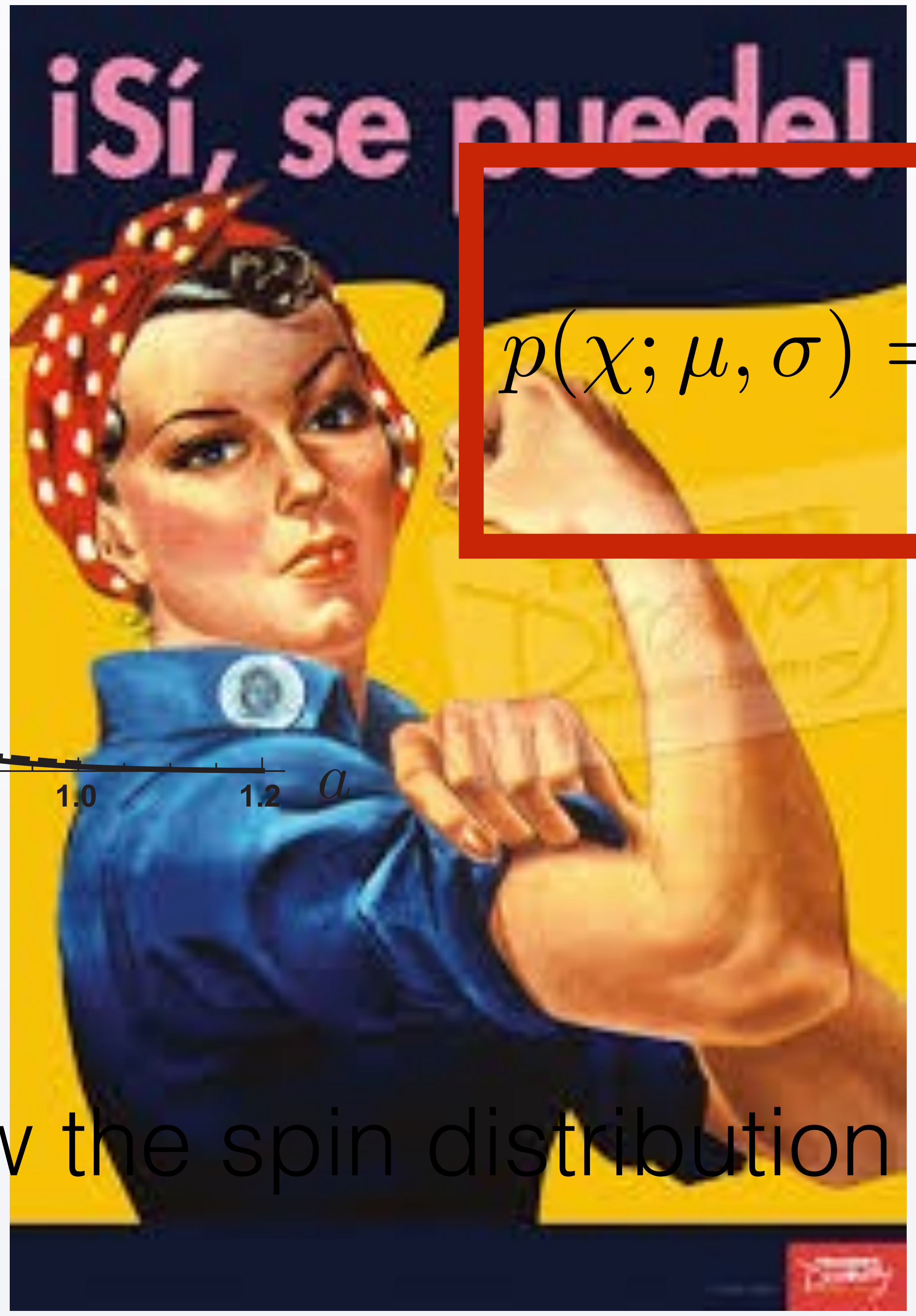
Spin Distribution for PBH:

[T. Chiba, et al: arXiv: 1704.06573]

Yes we can!



$$\mu = 0$$

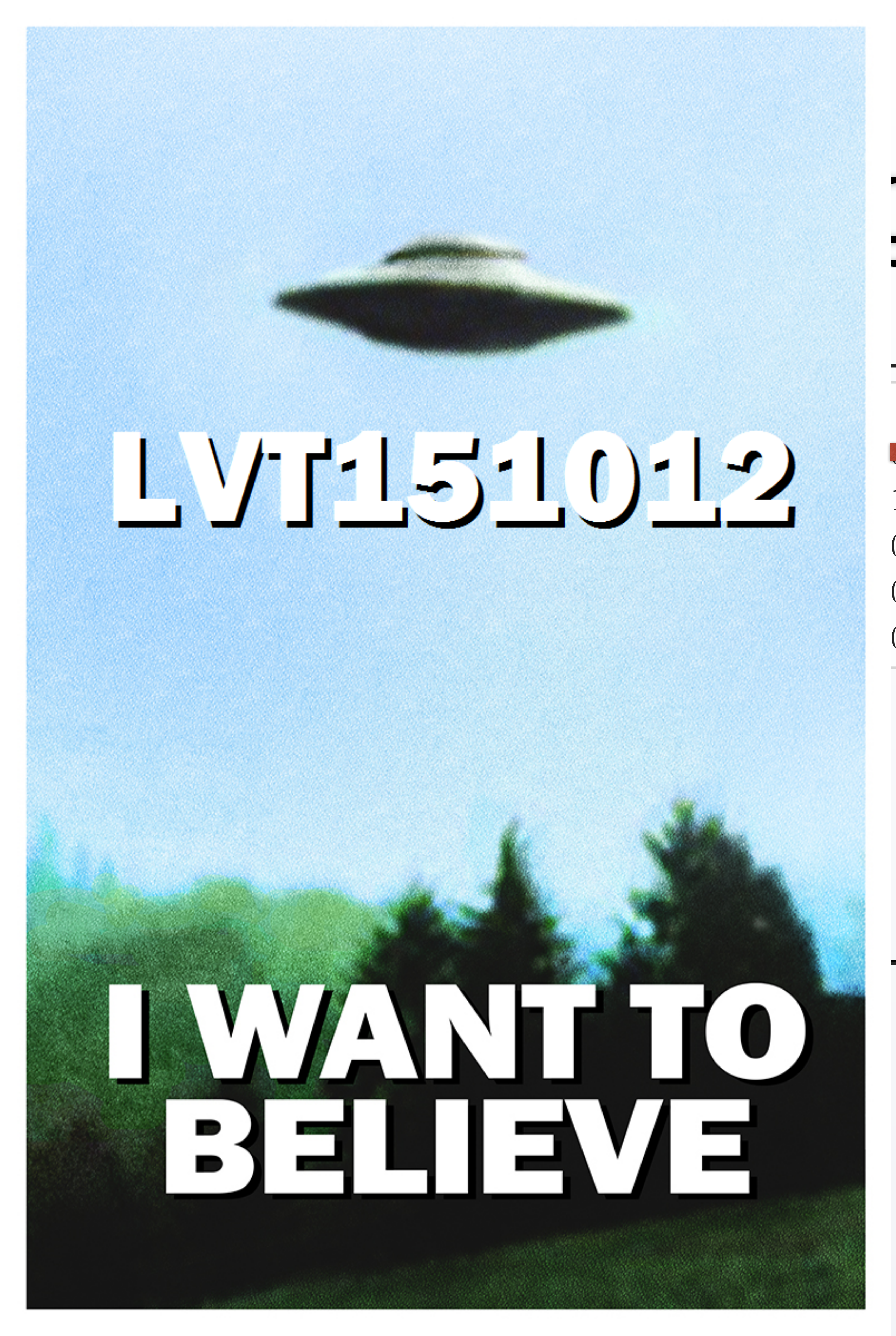


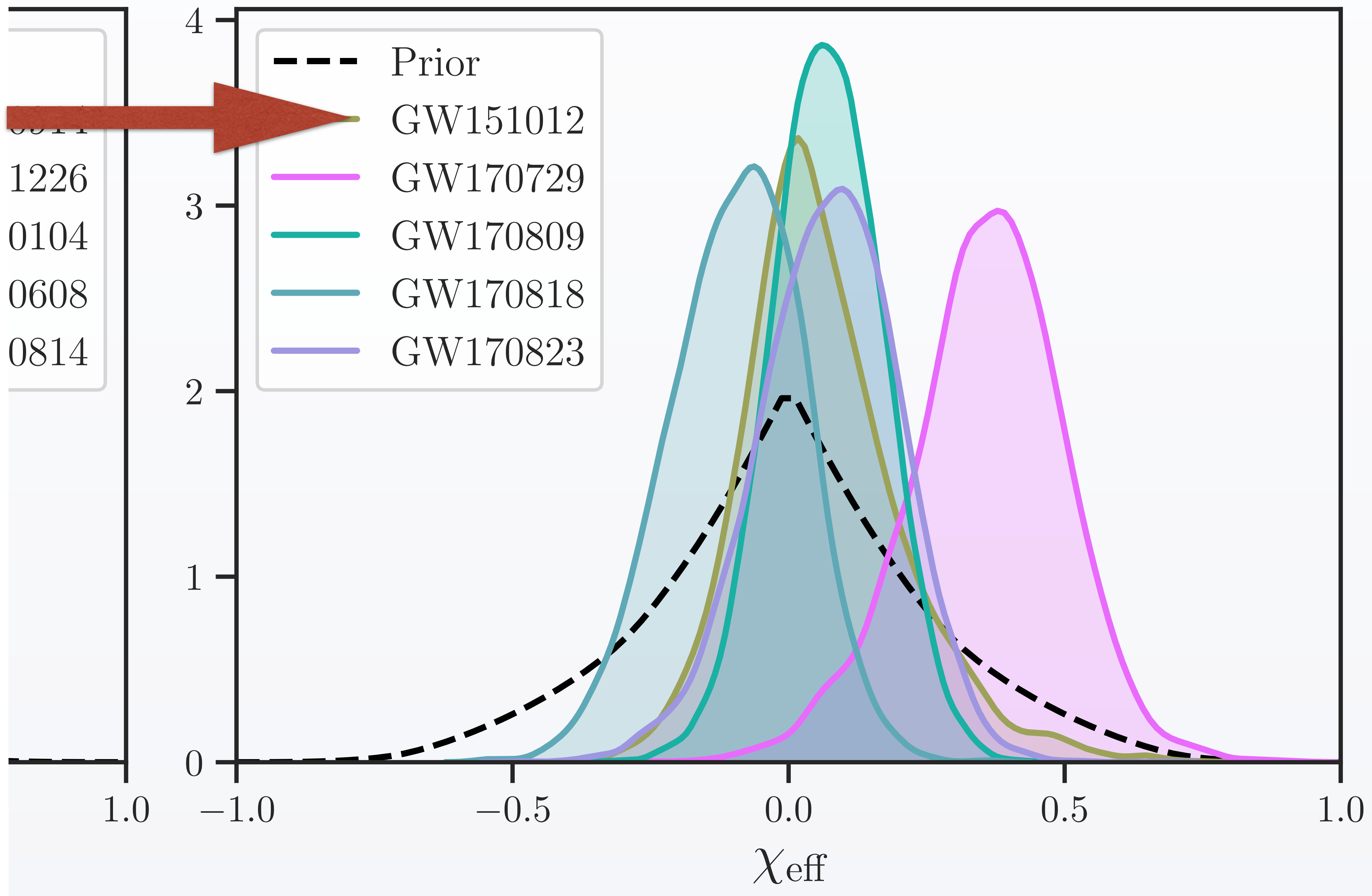
$$p(\chi;\mu,\sigma) = \frac{\mathcal{N}(\mu,\sigma)}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(\chi-\mu)^2}{2\sigma^2}\right]$$

More careful studies show the spin distribution is smaller, but they are model dependen

[De Luca et al: arXiv: 1903.01179]

go 10 events





After 1st Dec 2018

Bayes' Theorem

Likelihood: how probable is the data given some parameters?

Prior: how probable is a set of parameters before getting new data?

$$P(\theta|d) = \frac{P(d|\theta)\,p(\theta)}{\int P(d|\theta)p(\theta)\,d\theta}$$

Posterior: how probable are these parameters given the observed data?

Evidence: how probable is the data under all possible parameters? (normalization factor)

Why it is important to talk about it?

- The results of LIGO-Virgo already have some assumptions (**priors**) about the nature of the BHs.

- LIGO has not provided the full likelihoods to the community yet.

LIGO PRIORS:

- Spin: **Uniform** in magnitude and **isotropic** in direction.

- Masses: **Uniform** in individual masses.

But if we have *prior* knowledge…?

Does it matter?

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta)\, d\theta}$$

Data is strong

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta)\, d\theta}$$

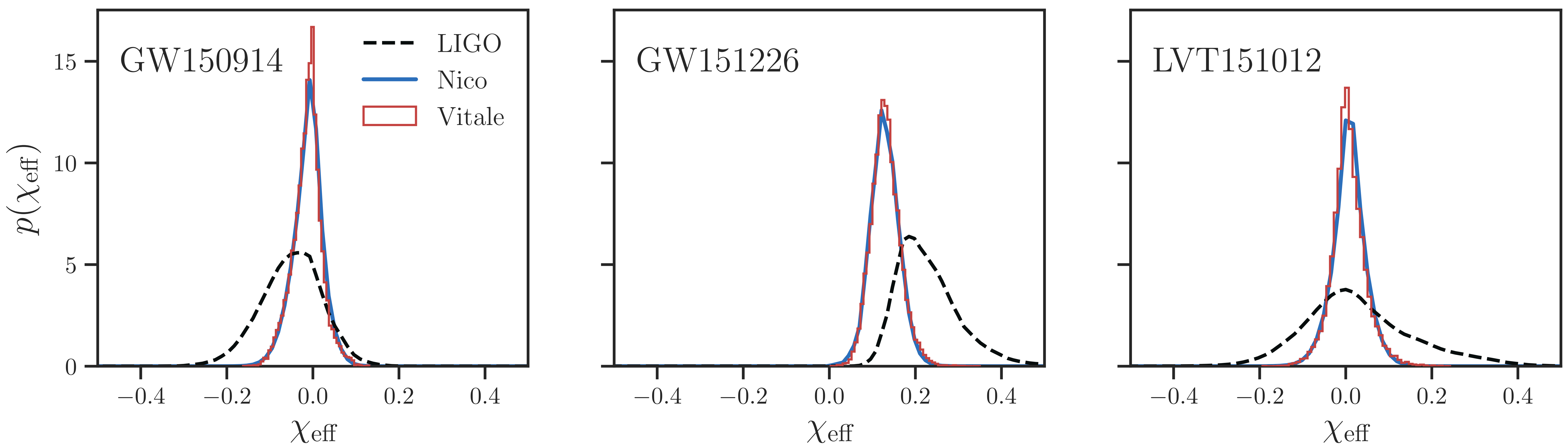
LIGO data is middle informative

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta)\, d\theta}$$

Data are weakly informative

Priors are important for current data

* old data



[Vitale, et al: arXiv:17007.04637]

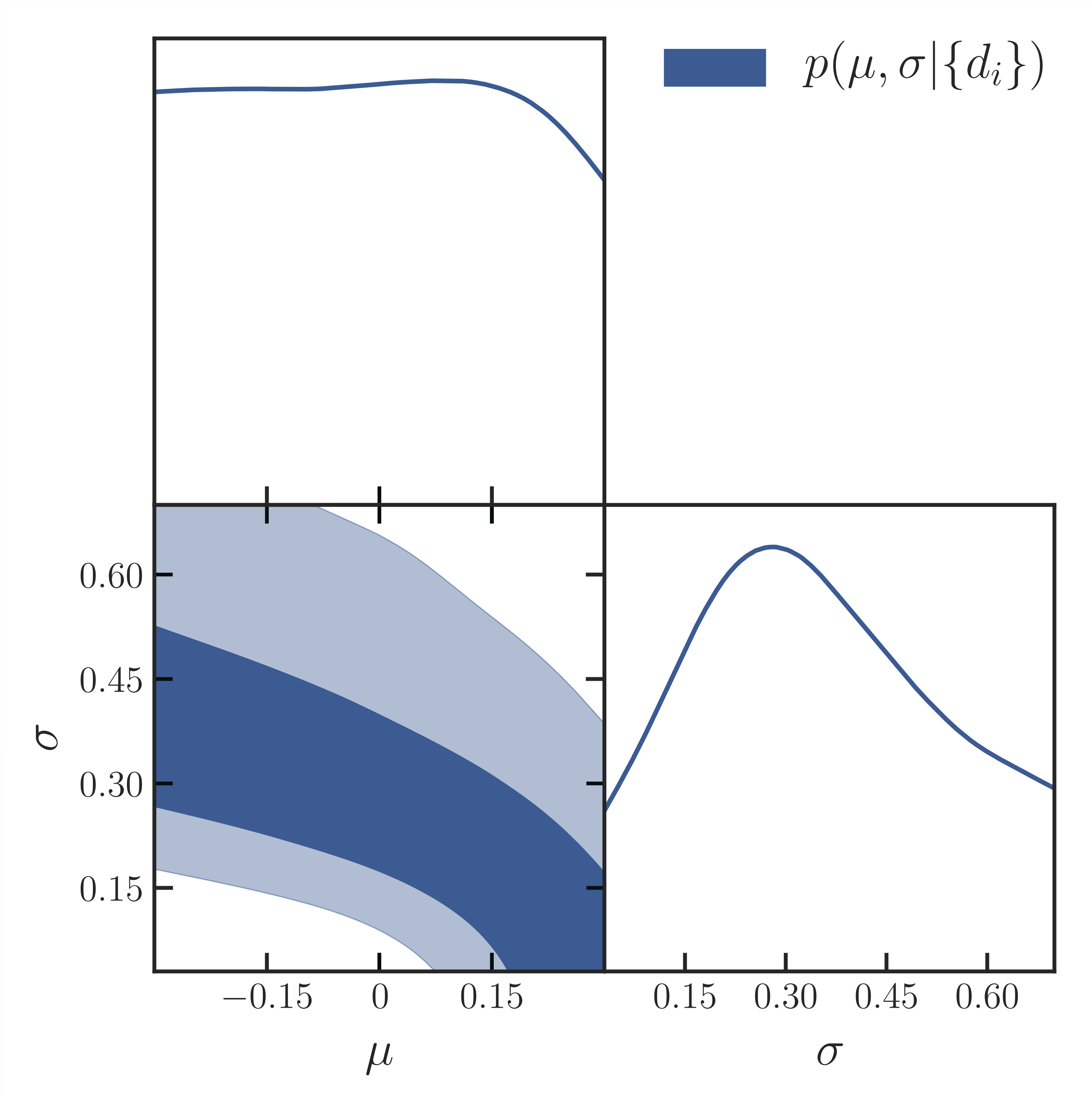
High agreement with Vitale, et al.

- Isotropic
- Spin $\propto \mathcal{N}(0, 0.1)$
- Masses Uniform

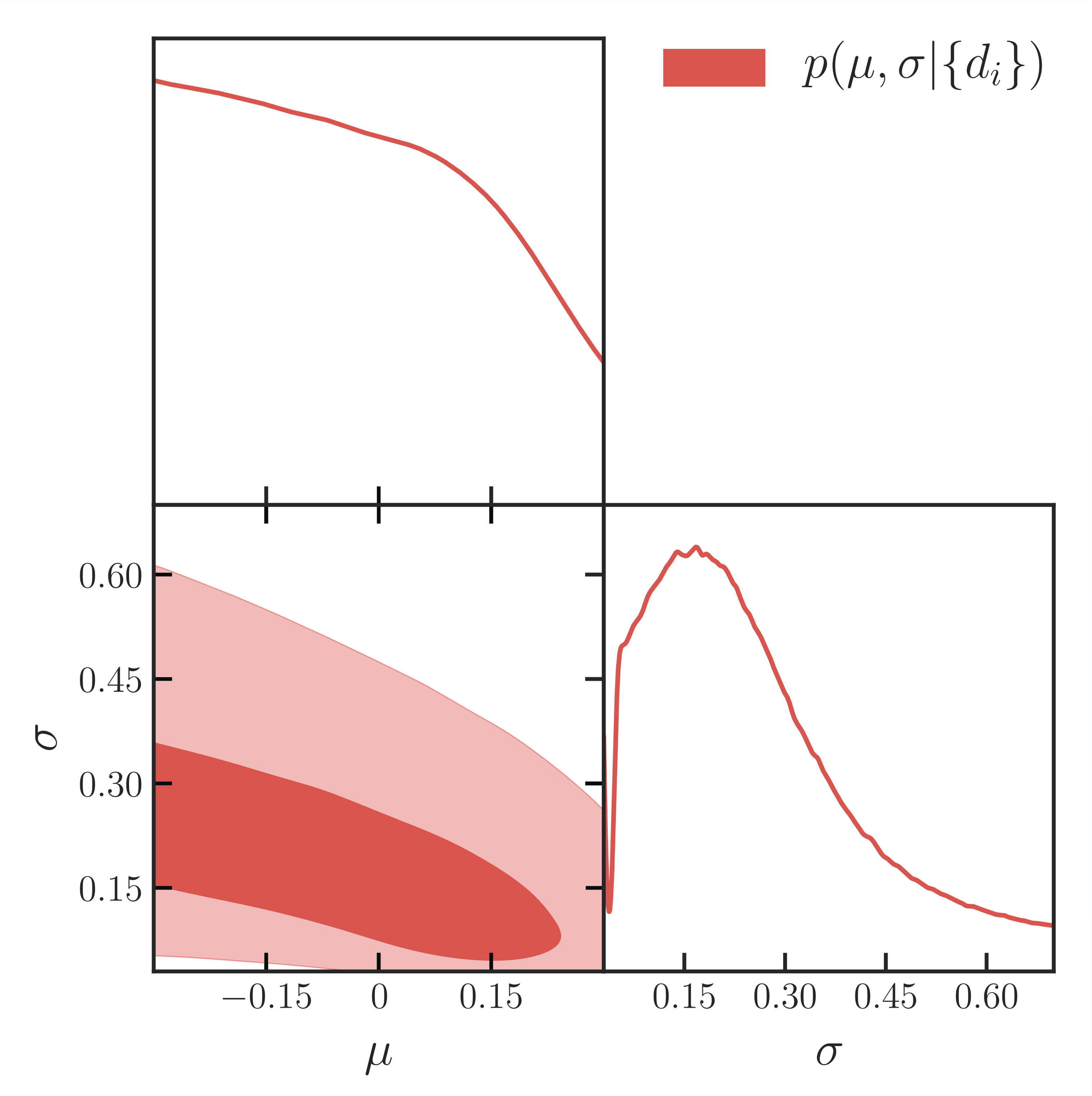
Re-weight posterior works!

Probability density functions for μ and σ parameters

$$p(\chi;\mu,\sigma) = \frac{\mathcal{N}(\mu,\sigma)}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(\chi-\mu)^2}{2\sigma^2}\right]$$

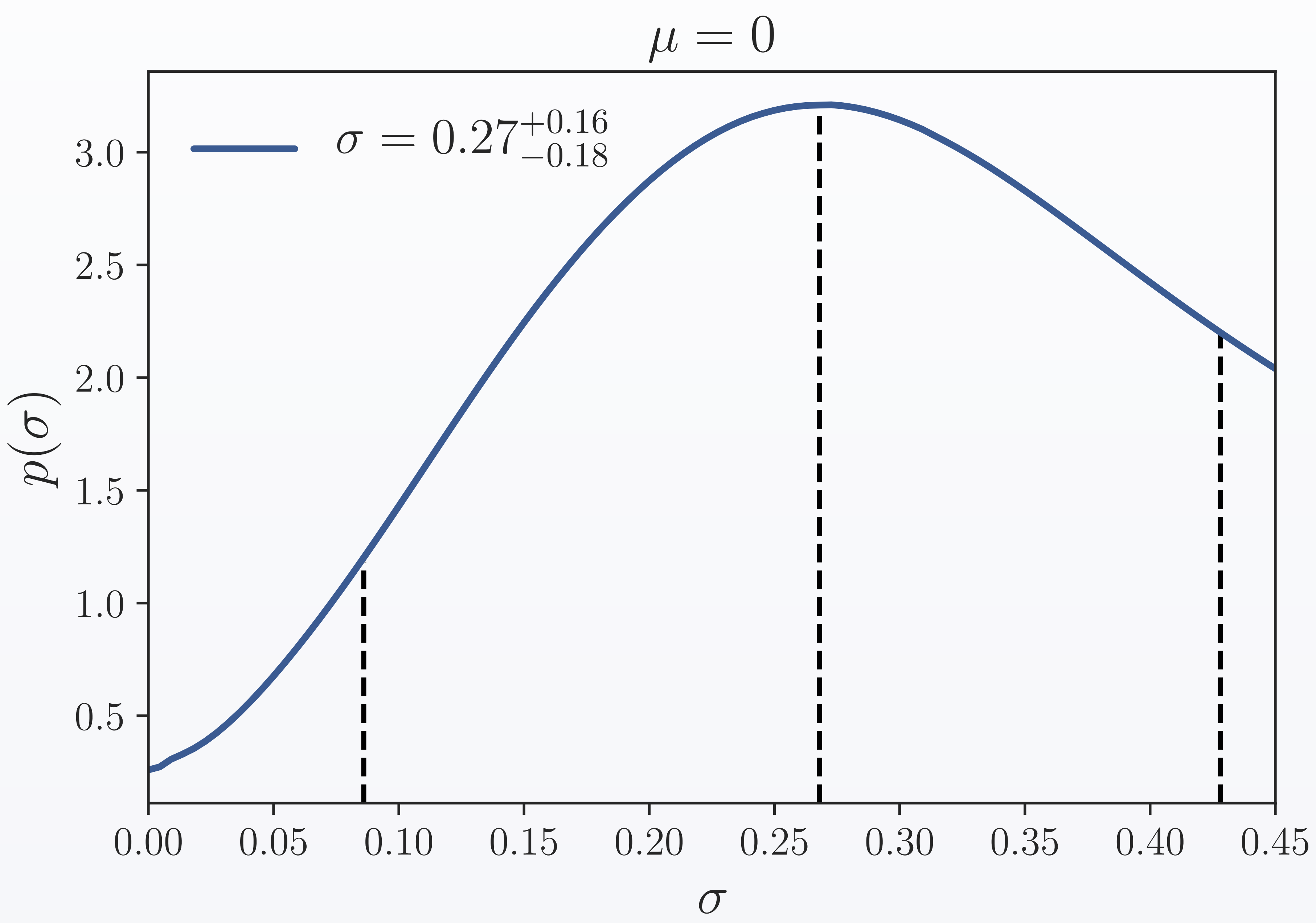


10 events



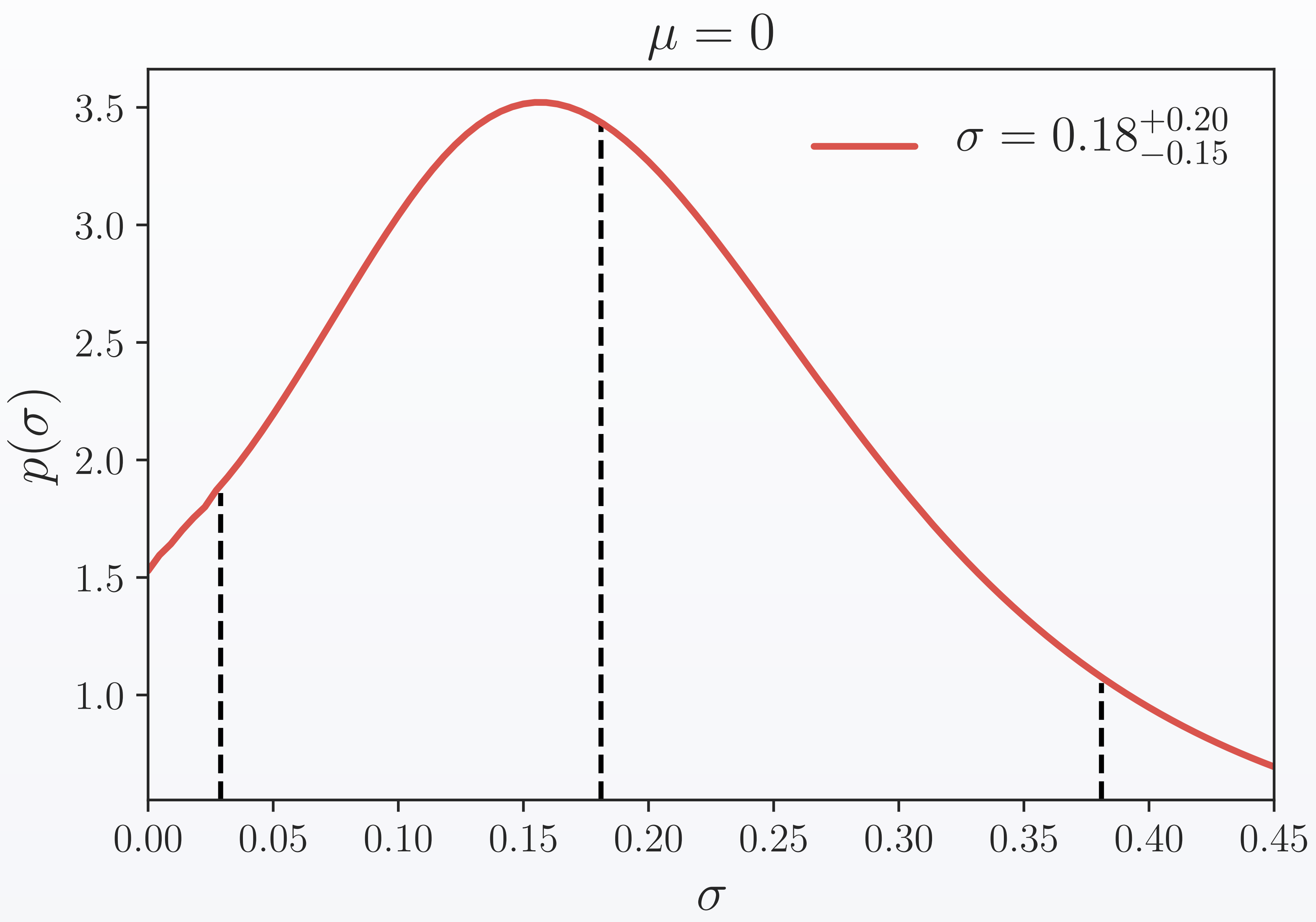
Excluding GW170729

Probability density functions for σ with μ=0



10 events

$\sigma = 0.27$



Excluding GW170729

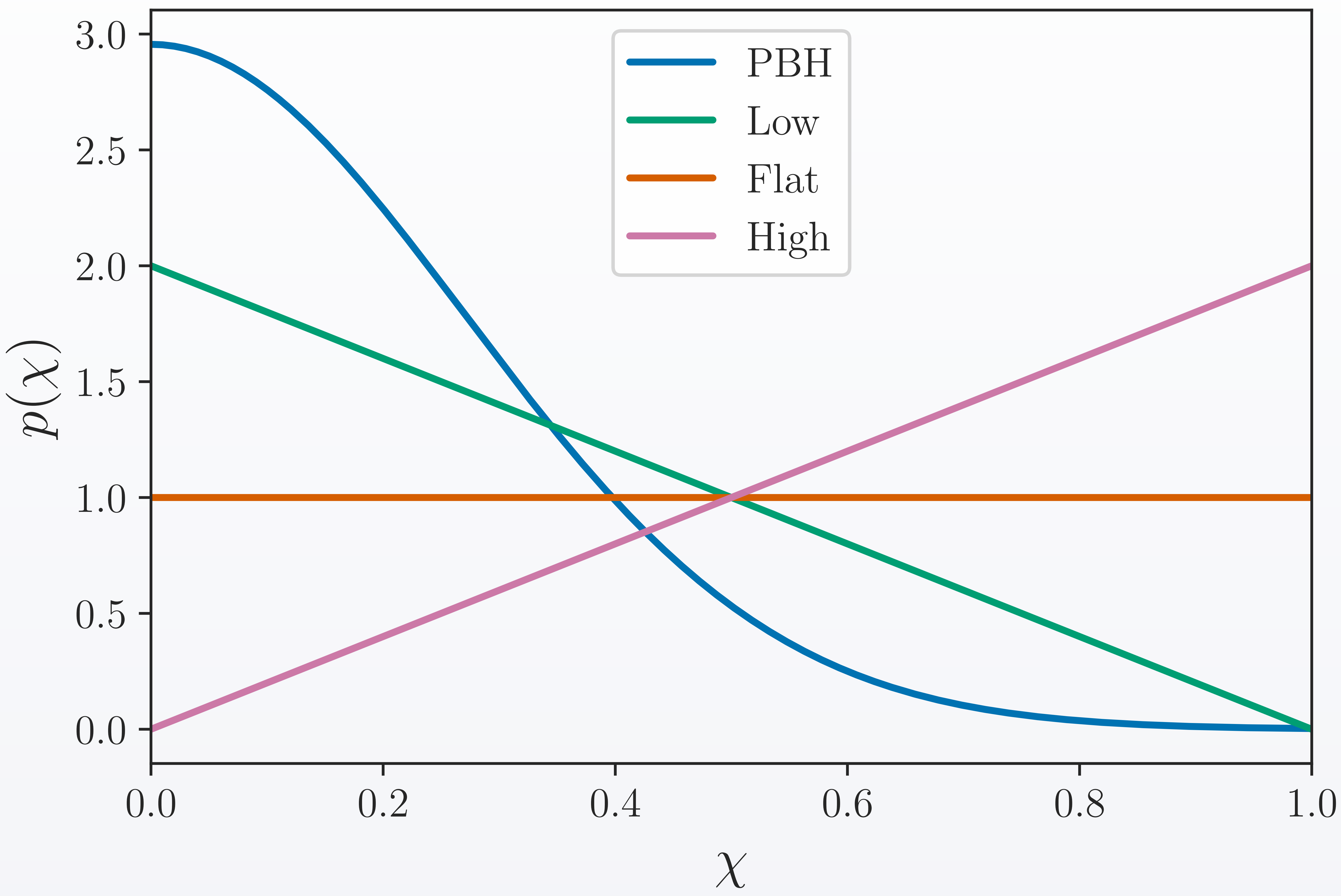
$\sigma = 0.18$

Model Selection

- Spin magnitude: Low (**L**), Flat (**F**) , High (**H**) and PBH
- Spin orientations: Isotropic (**I**) and Aligned (**A**)

$$\chi_{\rm eff} = \frac{m_1 \chi_1 \cos\theta_1 + m_2 \chi_2 \cos\theta_2}{m_1 + m_2}$$

Spin Magnitude Distribution



Low (**L**), Flat (**F**), High (**H**) and PBH

Let's build some toy models

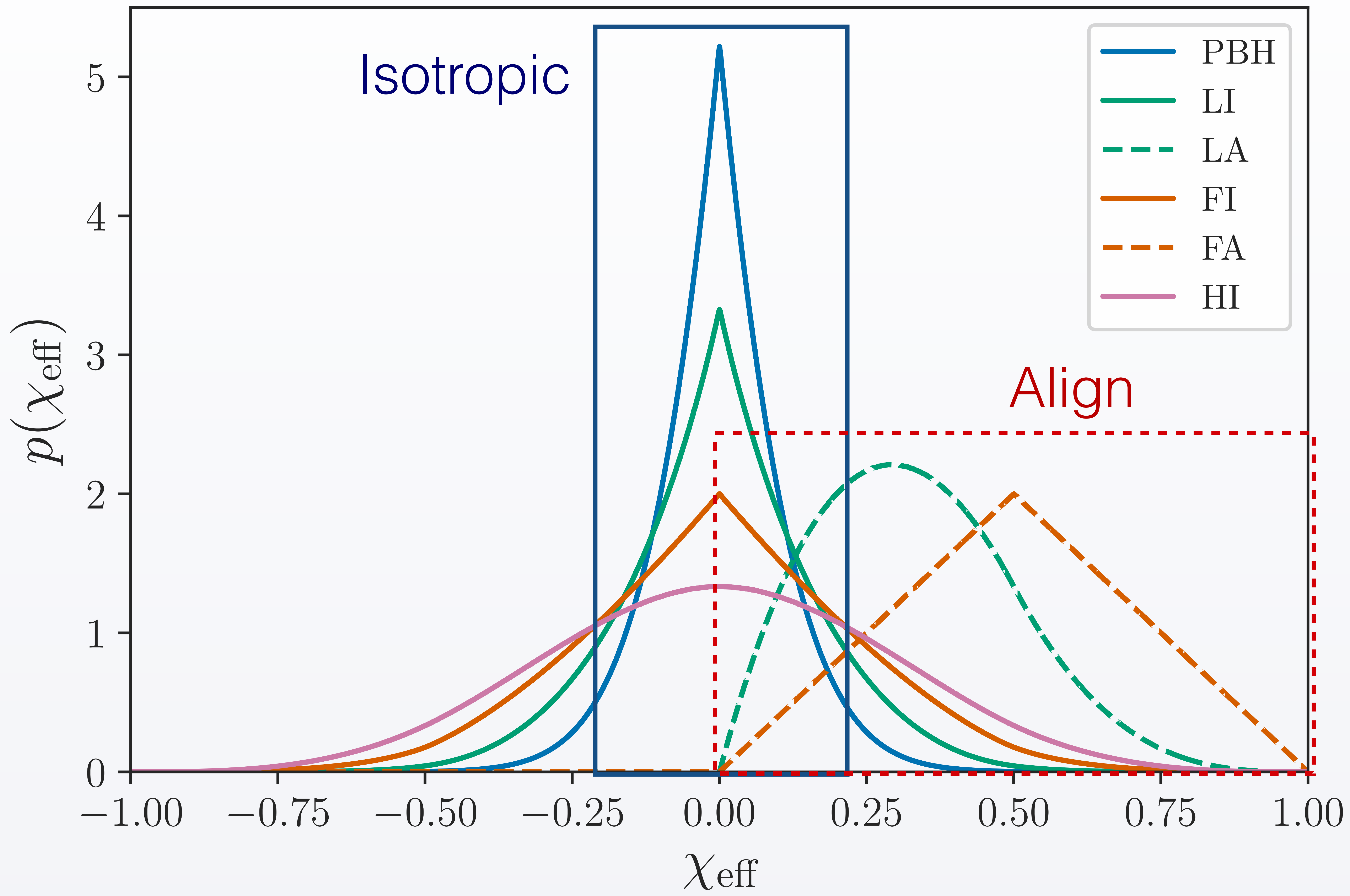
Dynamical:

LI = Low spin magnitude and isotropic spins.
FI = Flat spin magnitude and isotropic spins (LIGO).
HI = High spin magnitude and isotropic spins.

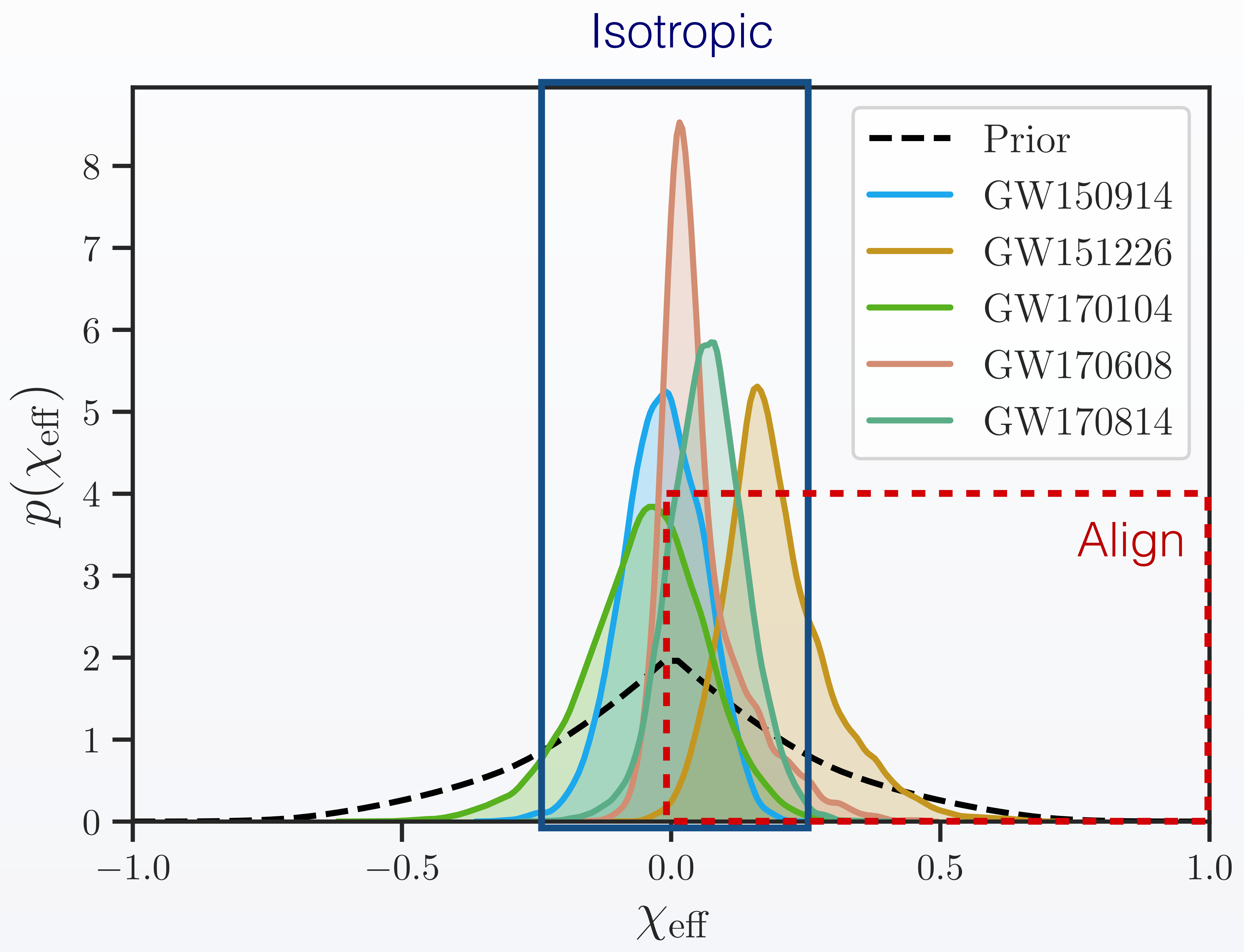
Binary star (Field evolution):

LA = Low spin magnitude and align spins.
FA = Flat spin magnitude and align spins.

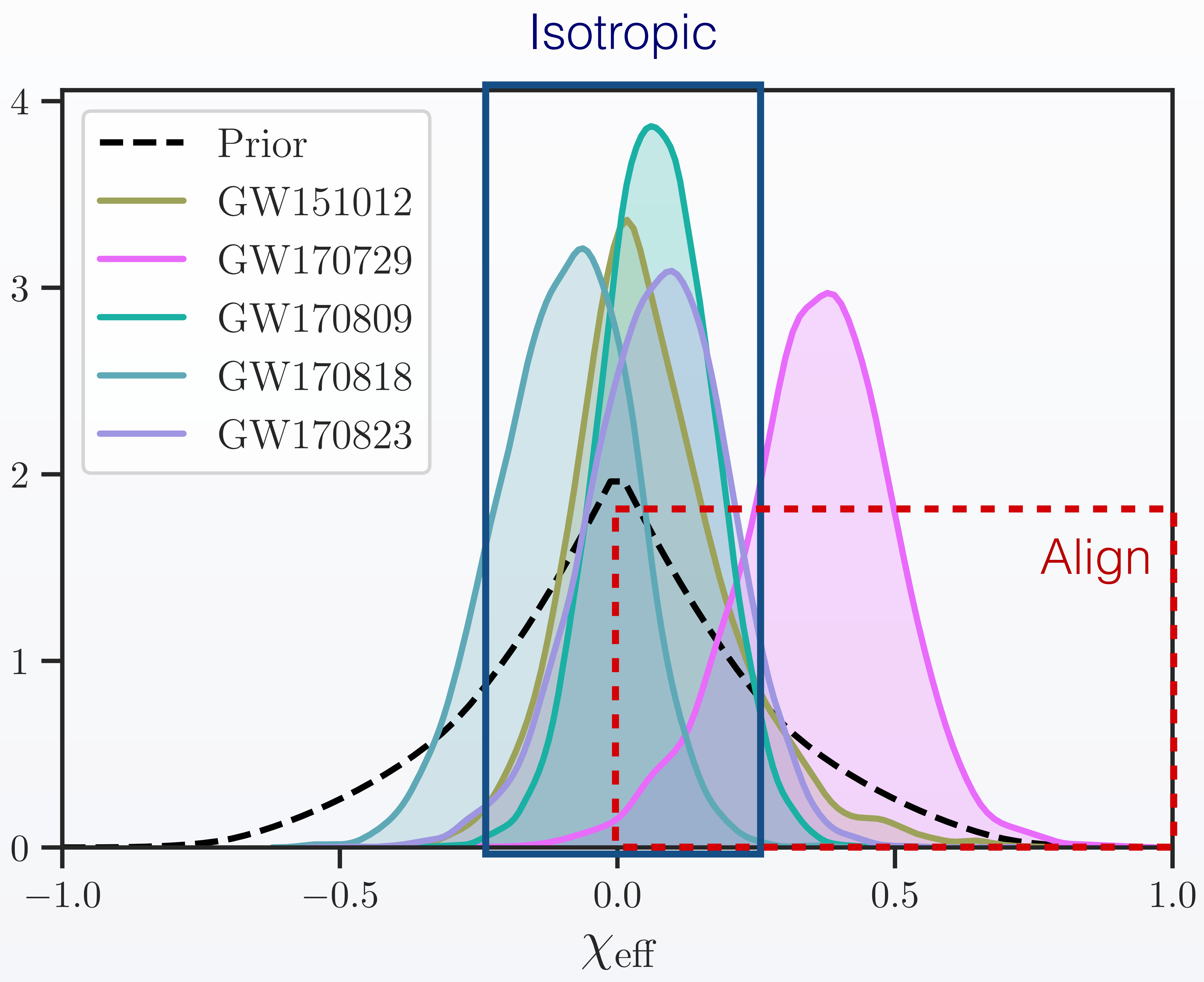
Effective Spin Distribution (Priors)



LIGO-Virgo 10 events (again)



Before 1st Dec 2018



After 1st Dec 2018

Odds ratios

Which model explains better the data?

Nico	Low	Flat	High	PBH
Isotropic	0.0	-1.18	-2.49	0.39
Aligned	-6.07	-14.65	-36.41	

$\sigma = 0.27$

Slightly preferred

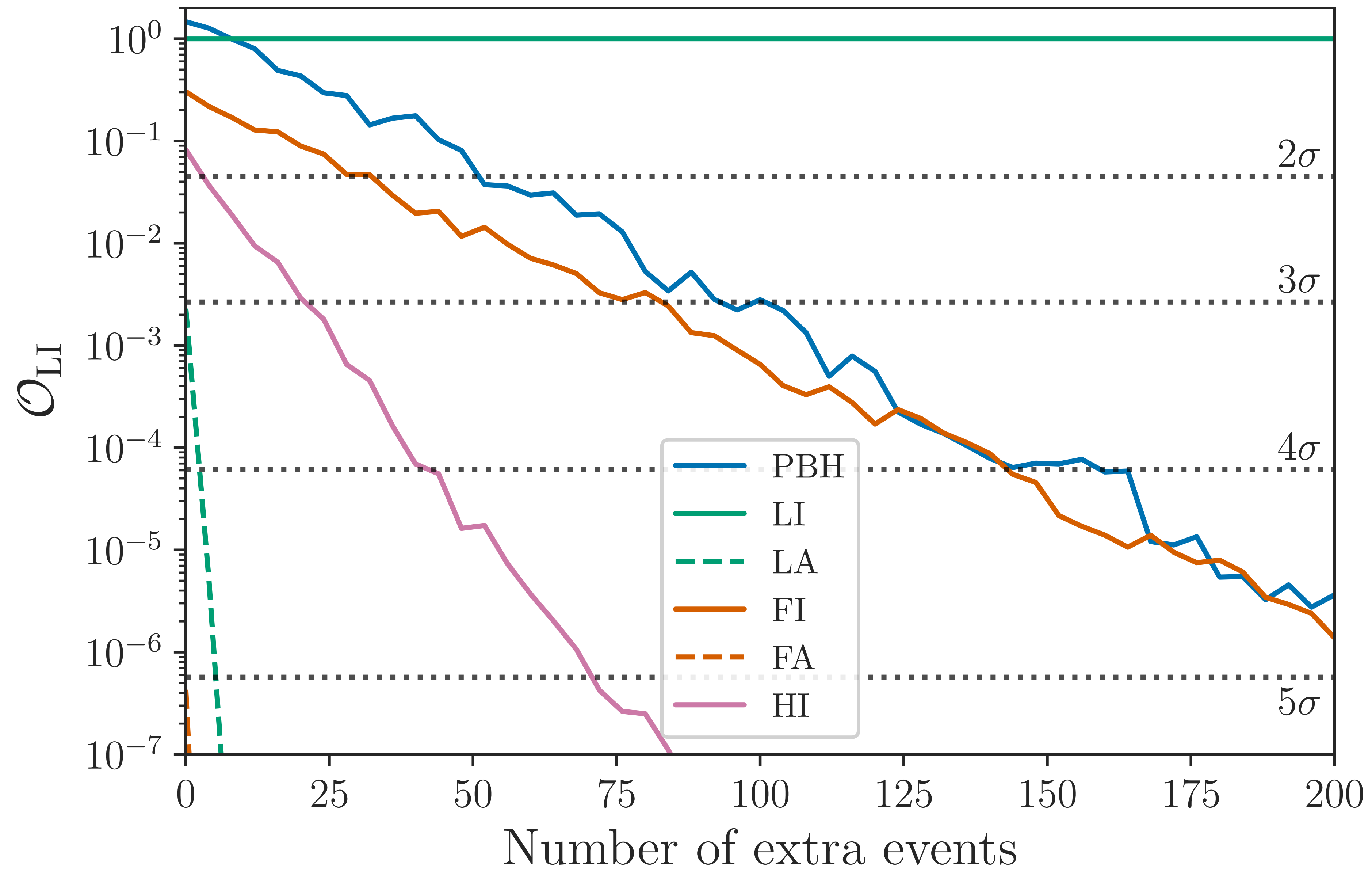
In good agreement with LIGO

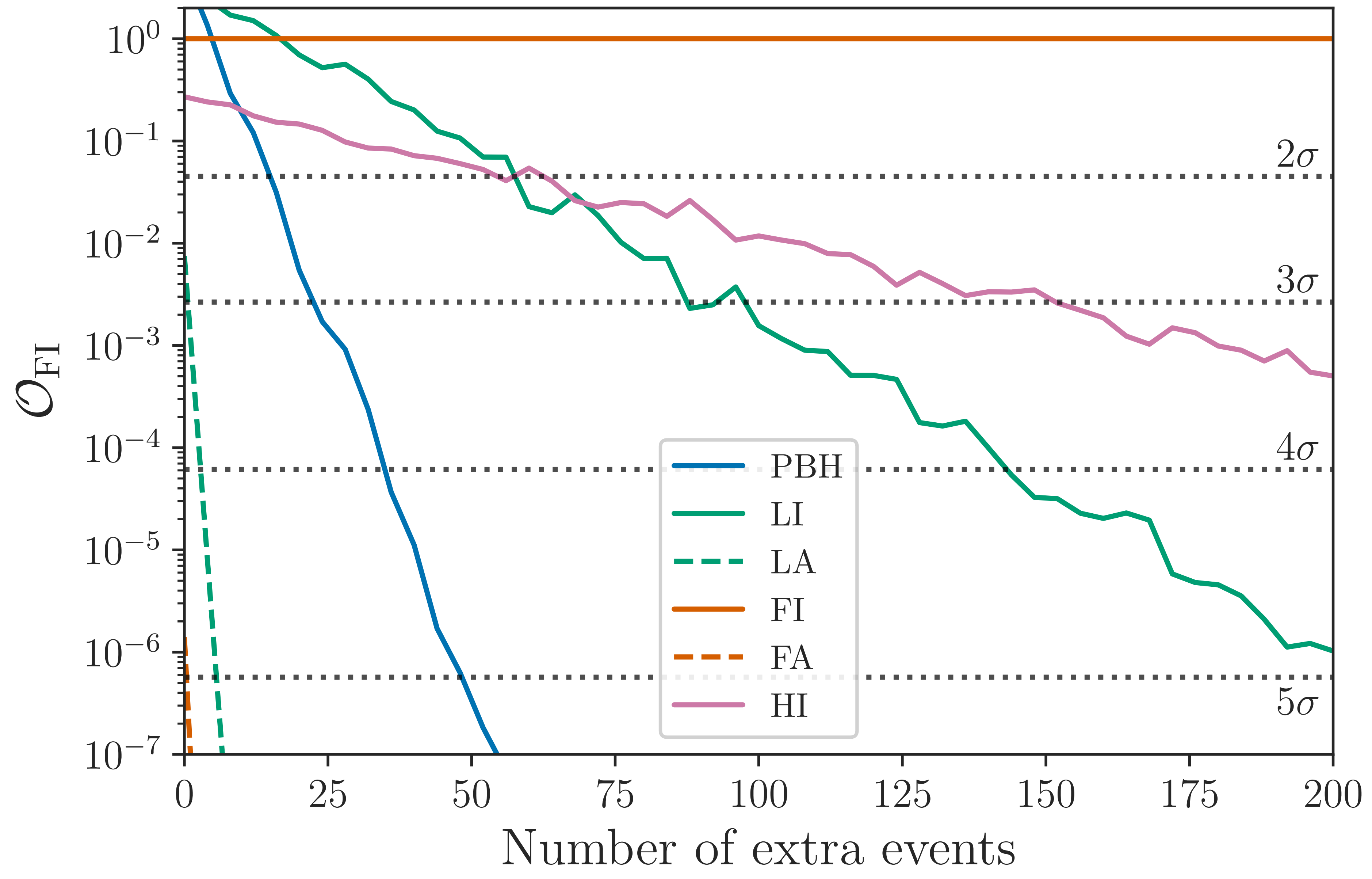
LIGO	Low	Flat	High
Isotropic	0.0	-0.93	-2.07
Aligned	-4.12	-12.92	-32.37

[LIGO/Virgo Collaboration arXiv:1811.12940]

Evolution of the Odds ratios with number of events for **LI** and **FI**.

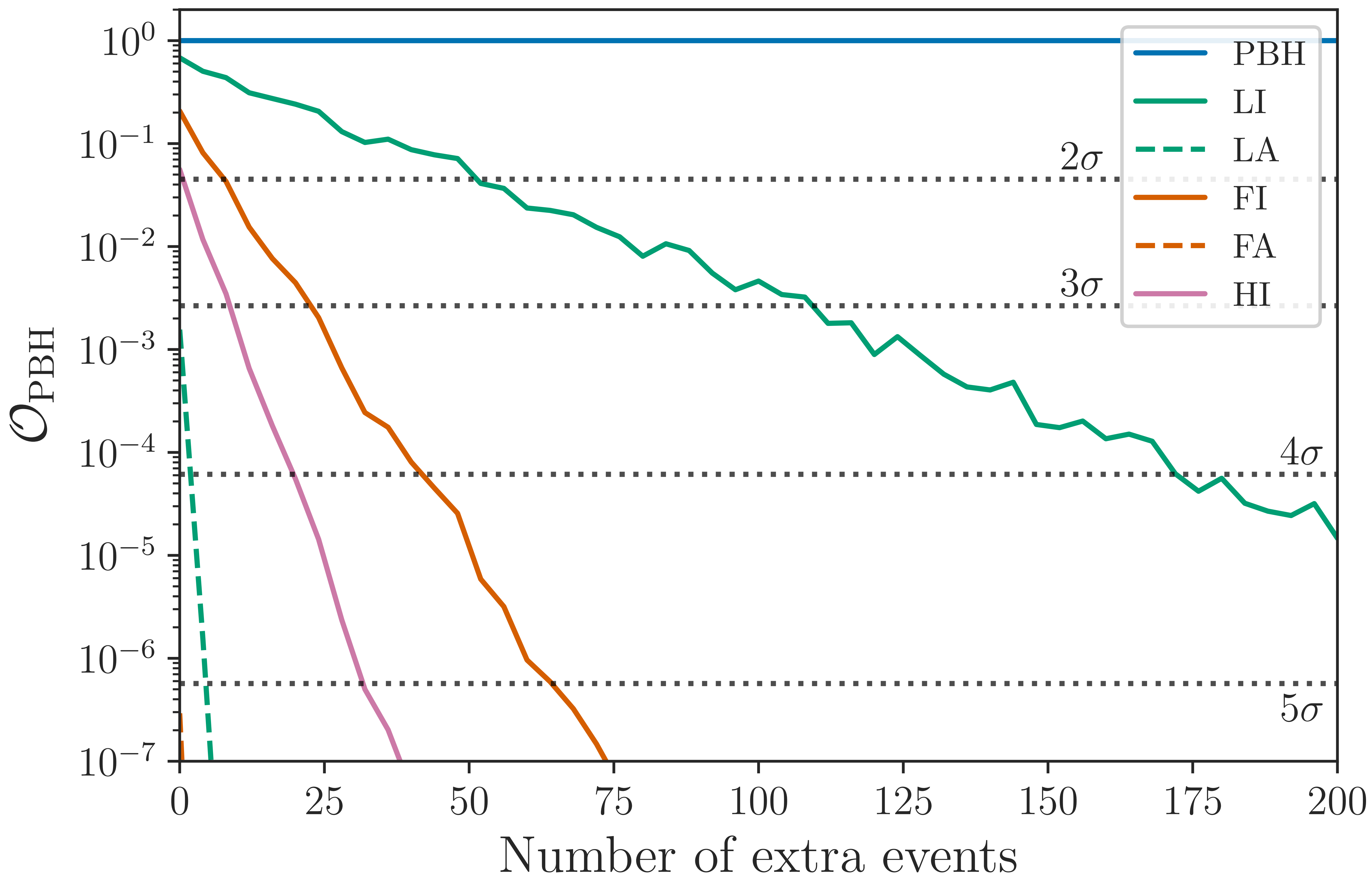
Q: How many more event do we need to be sure?





A: only ~ 100 events

Evolution of the Odds ratios with number of events for PBH.



With only ~ 100 events we could discriminate PBH!

LIGO O3 ~ 10-200 Events/Gpc^3/yr

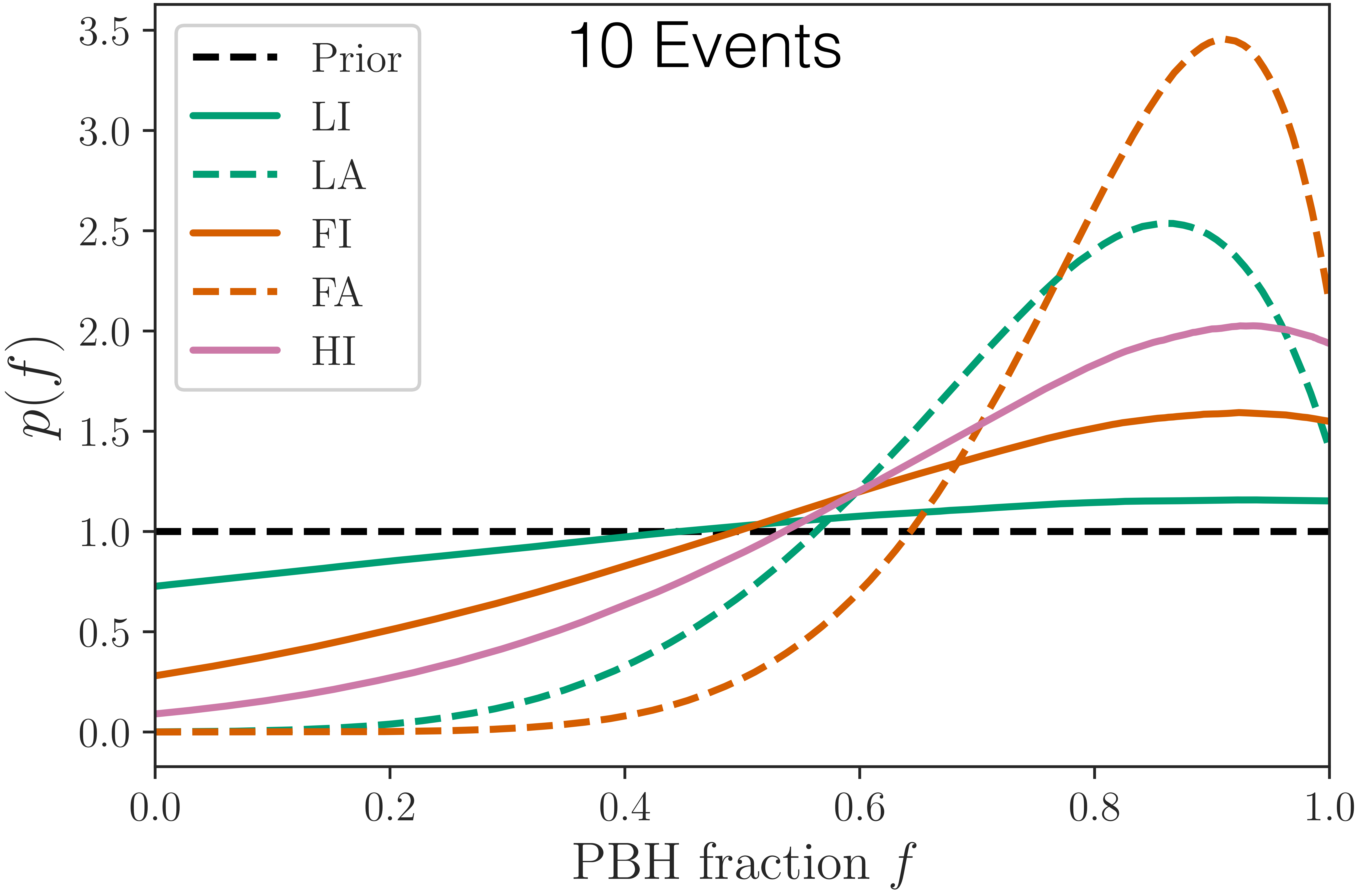
What if there are 2 populations?

- Can we differentiate 2 different populations?
- Can we know how many events do we need?

Yes we can!

Mix Models: PBH + a second population.

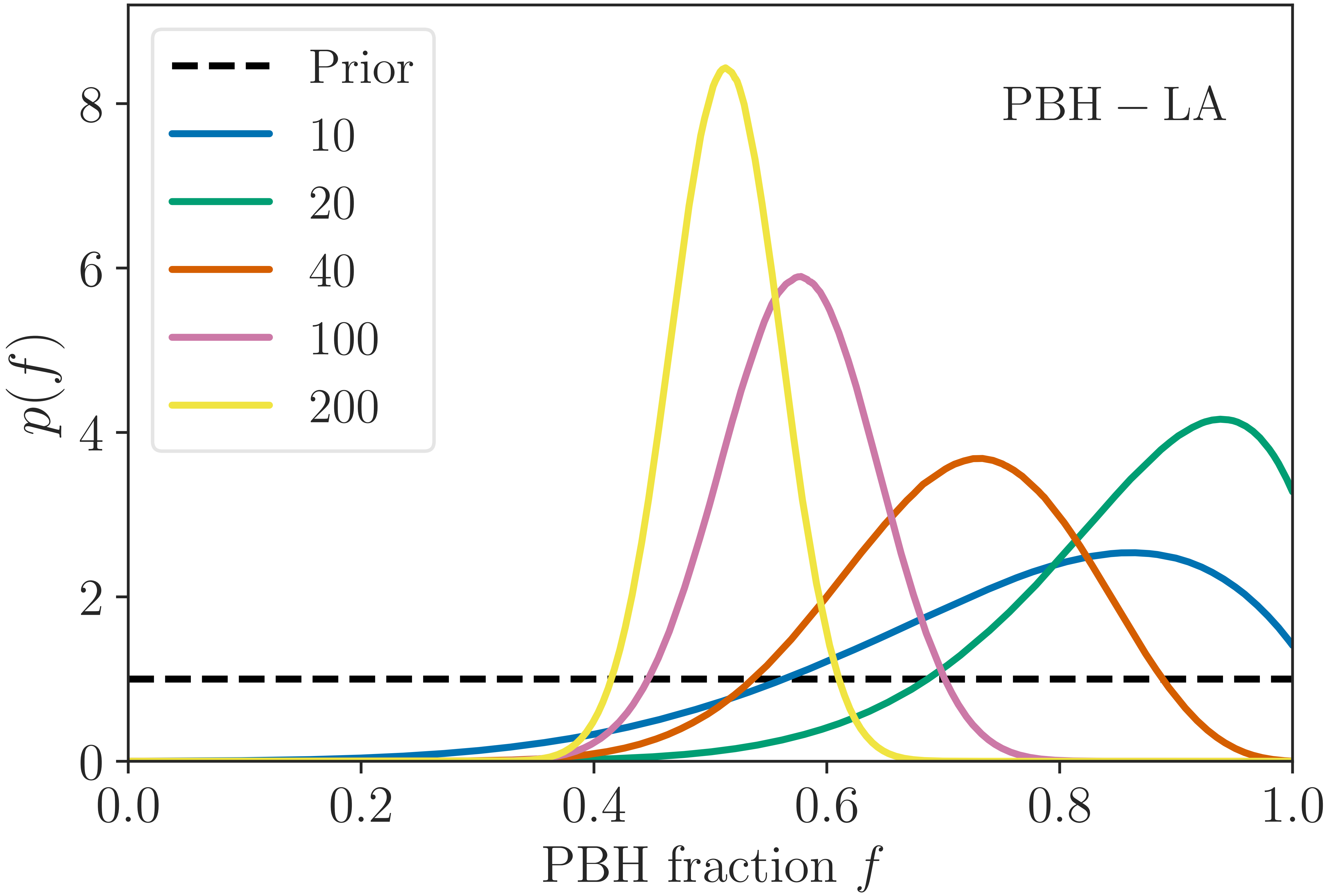
$$f_X = (1 - f_{\mathrm{PBH}})$$



Mix Models: PBH + Low Aligned

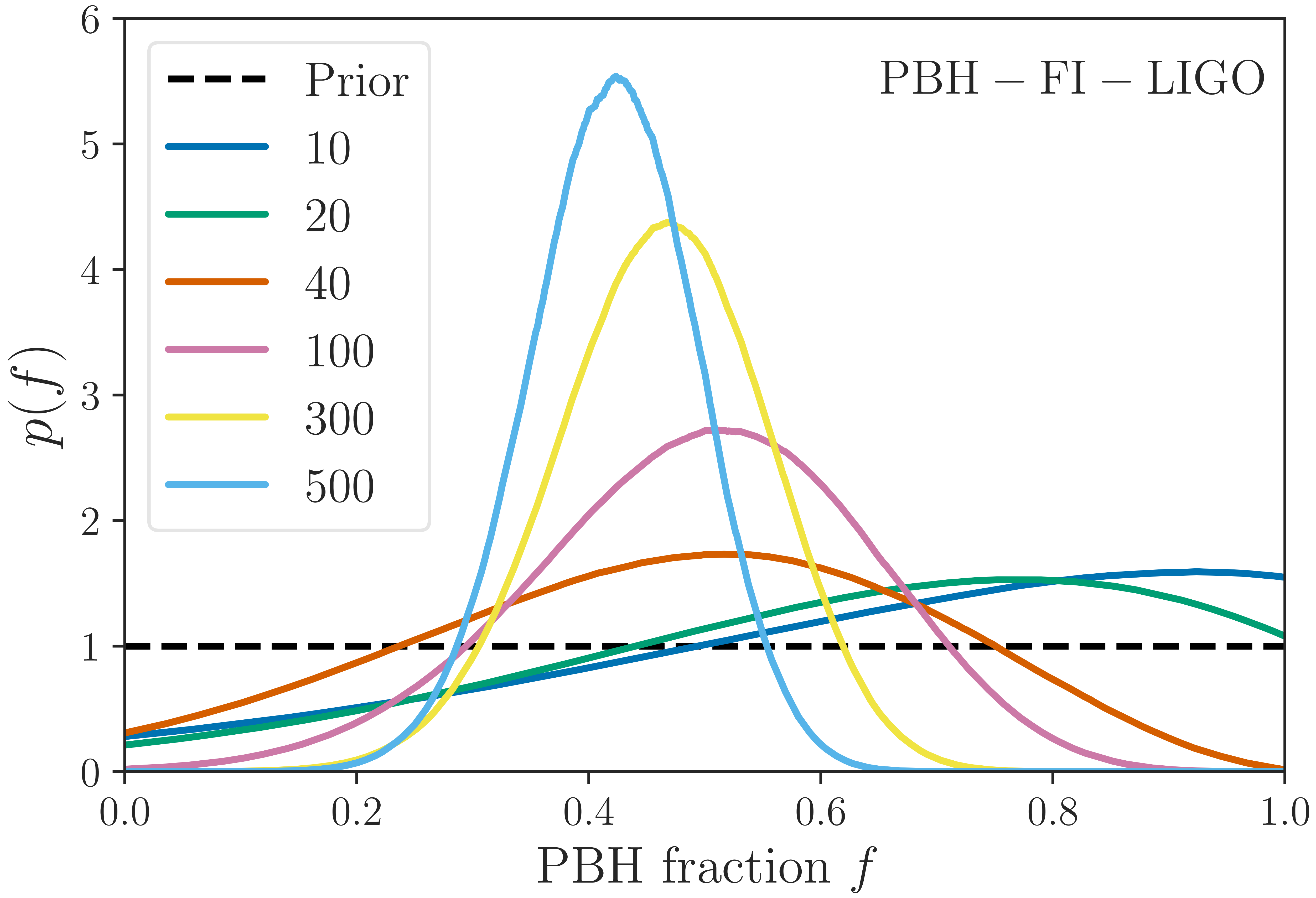
Q: How many more event do we need to be sure?

Simulated:
95 **PBH**
95 **LA**



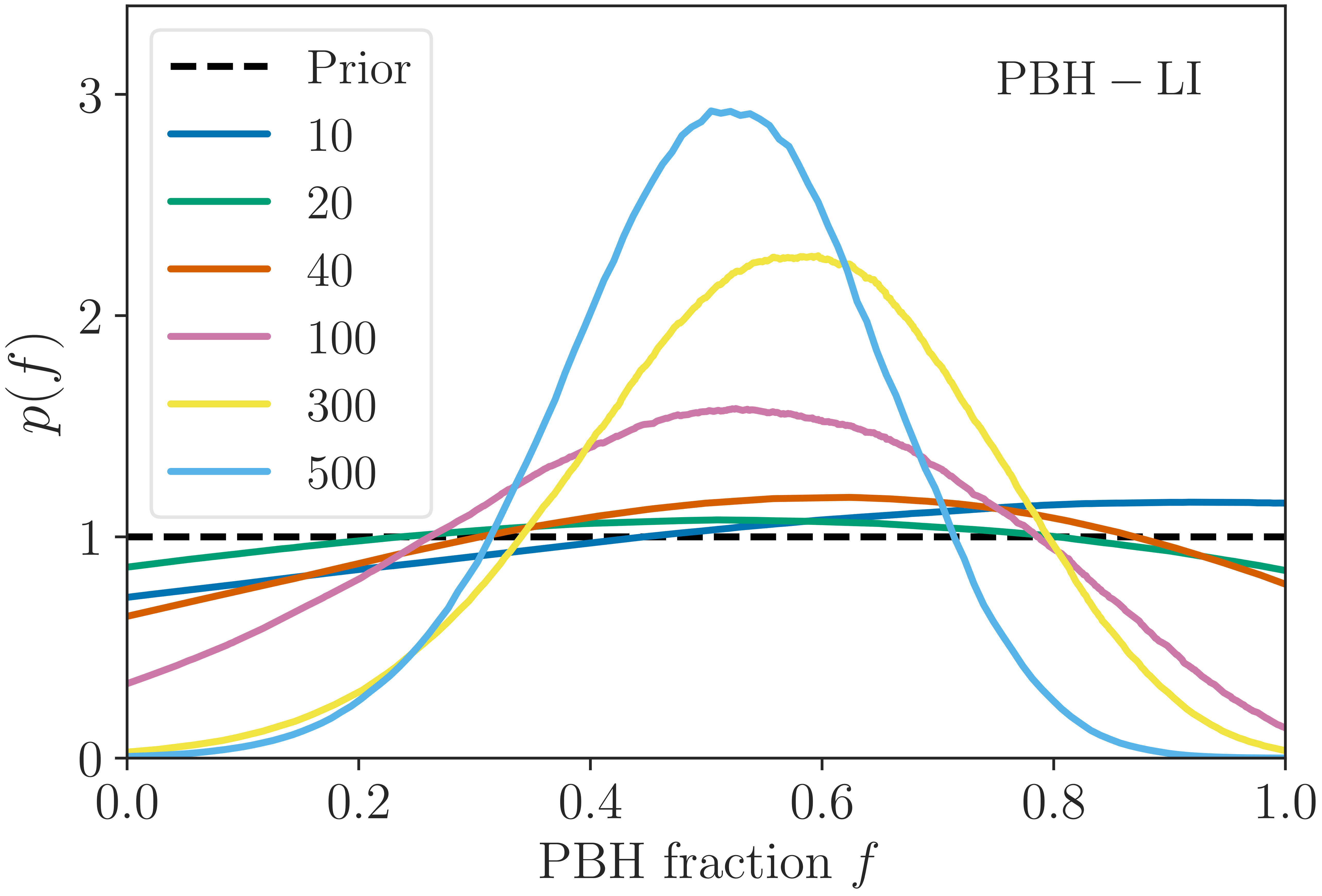
Mix Models: PBH + Flat Isotropic

Simulated:
245 **PBH**
245 **FI**



Mix Models: PBH + Low Isotropic

Simulated:
245 **PBH**
245 **FI**



What about GWTC-2?

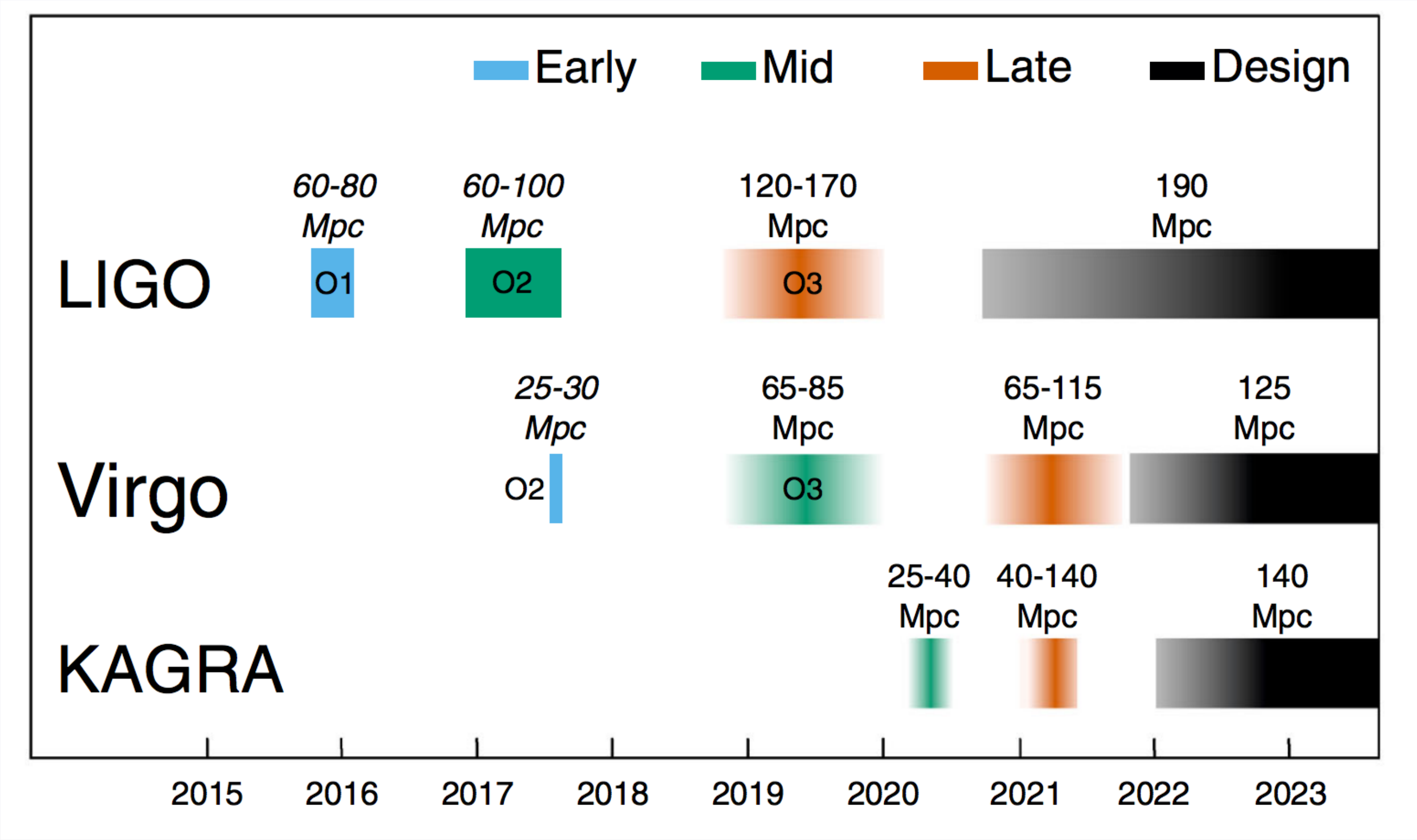
- LIGO has presented an updated catalog +26 more events

- LALSuite is the LIGO Scientific Collaboration Algorithm Library for gravitational-wave analysis.

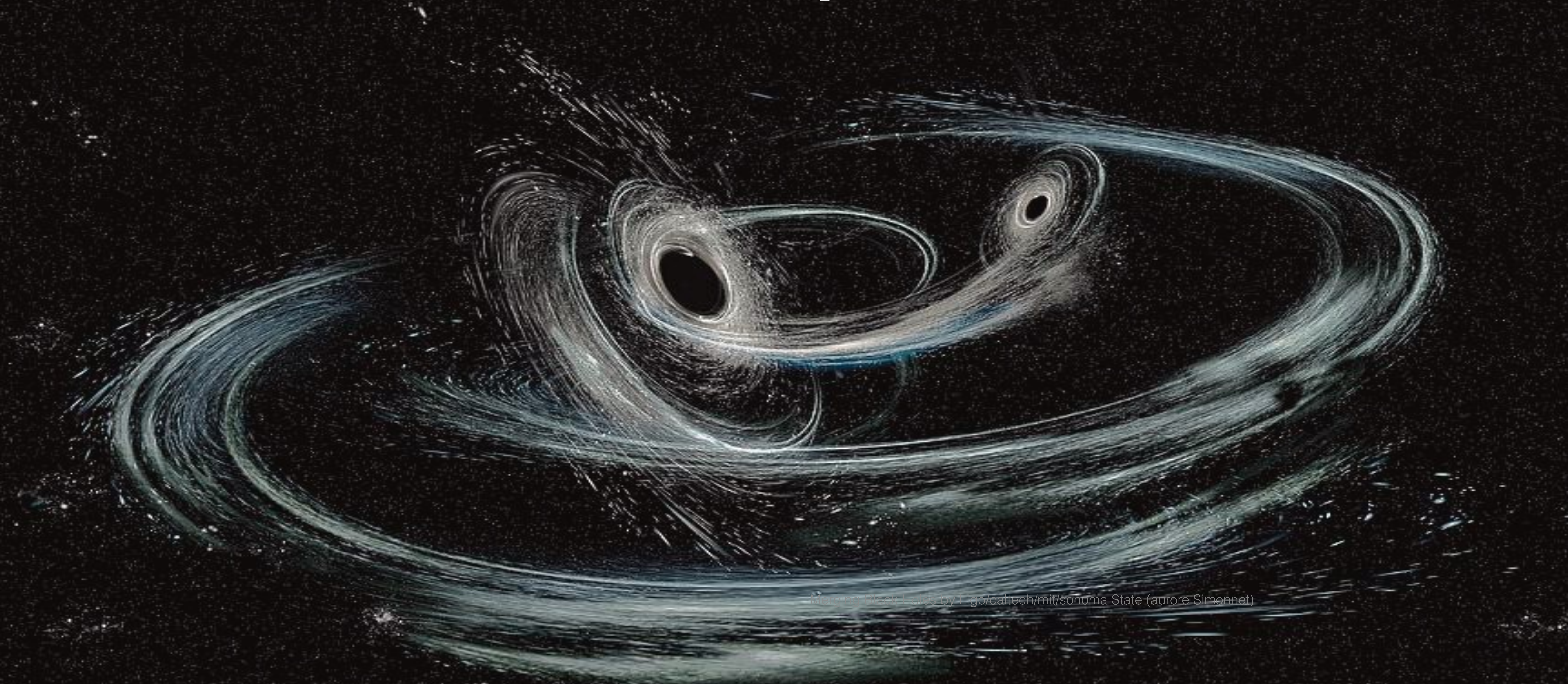
- Better analysis but computational expensive

Conclusions

- Most likely LIGO has discovered a new population of black holes
- PBH could explain the nature of this new BH population
- More events coming from O3b running. We can test PBH hypothesis in the near future



Thank you!



Merging Black Holes by Ligo/caltech/mit/sonoma State (aurore Simonnet)

Critical collapse

[Musco, et al. arXiv:0811.1452]

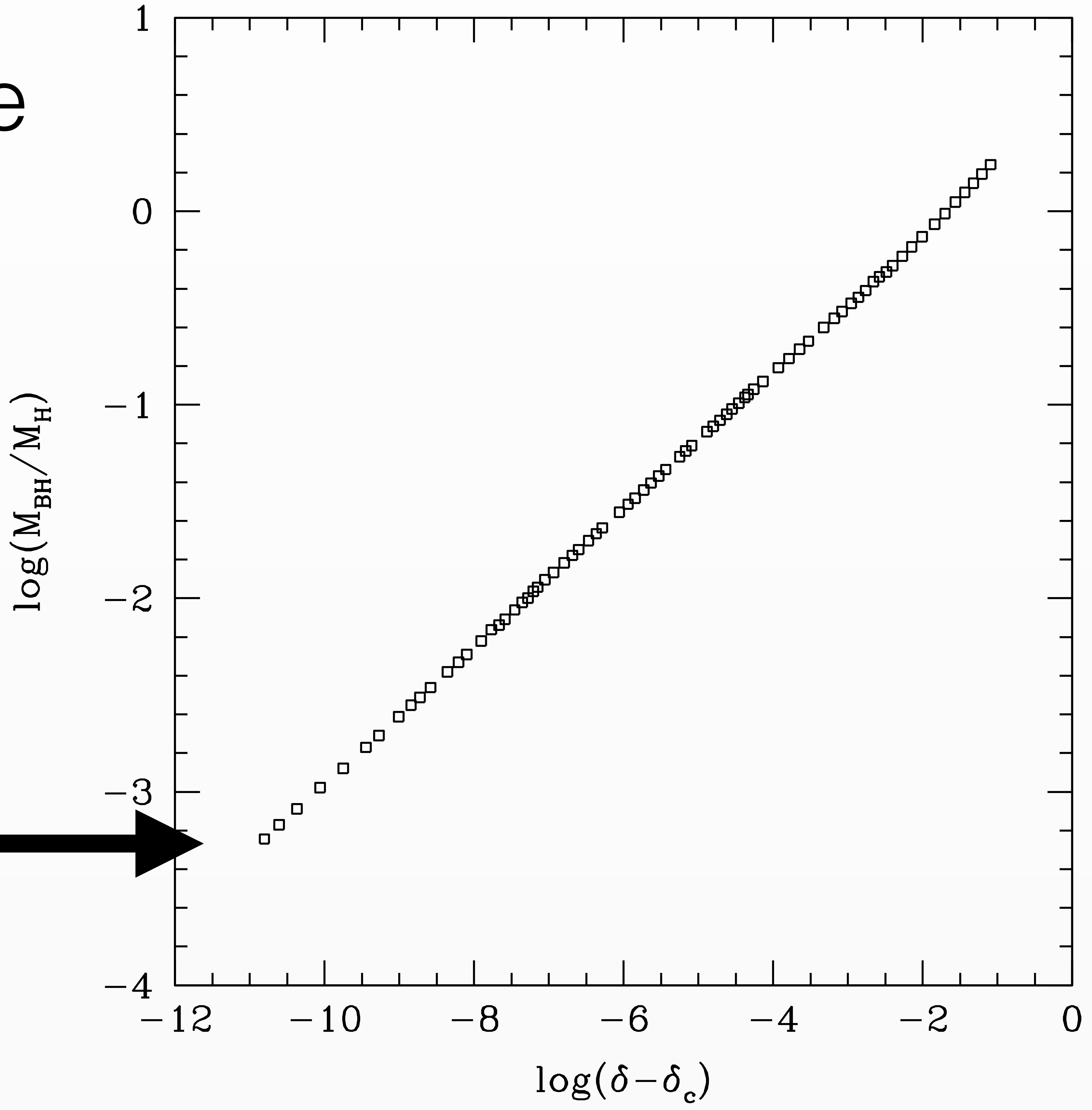
Numerical results show that density contrast slightly above the threshold:

$$M_{\rm PBH} = C_M |\delta - \delta_c|^{\gamma_M}$$

$$\gamma_M \approx 0.3558$$

$$C_M = 5.117 M_H$$

PBH Mass can be order of magnitude less than the horizon mass



[T. Chiba, et al: arXiv: 1704.06573]

Critical collapse

Numerical simulations of the collapse of a rotating radiation fluid. Angular momentum also feature critical behavior

$$J = C_J|\delta - \delta_c(q)|^{\gamma_J} q$$

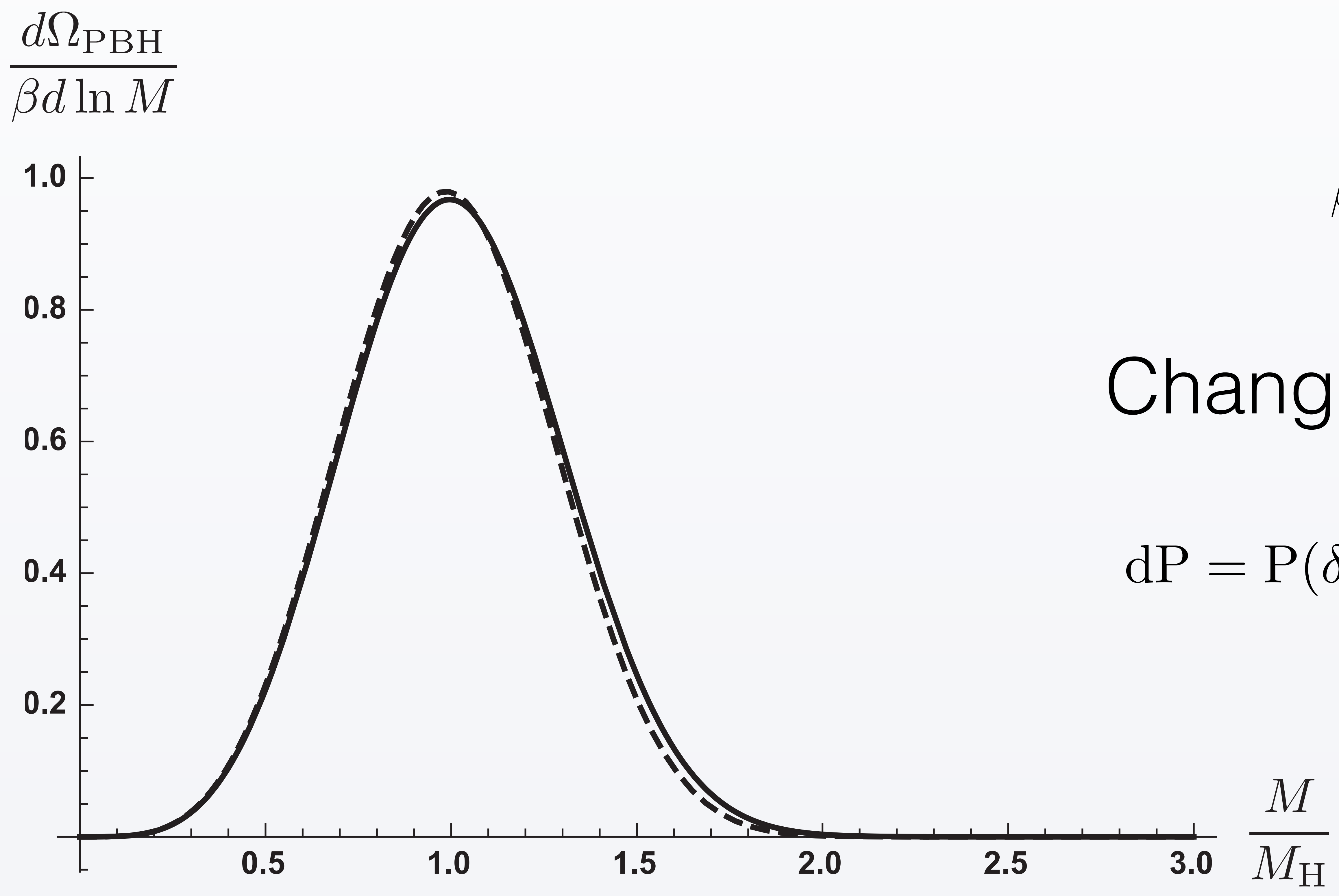
$$\gamma_J \approx 0.8895$$

$$C_J \approx 26.19 M_H^2$$

[Gundlach, et al. arXiv:1608.0049, 1603.04373]

[T. Chiba, et al: arXiv: 1704.06573]

Mass Distribution



[J.C. Niemeyer, et al: arXiv: 9709072]

$$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\delta^2}{2\sigma^2}\right]$$

$$\beta = \int_0^\infty Q(q) \int_{\delta_c(q)}^\infty P(\delta) d\delta$$

Change of variables:

$$\mathrm{dP} = \mathrm{P}(\delta, \mathrm{q})\mathrm{d}\delta\mathrm{dq} = \mathrm{F}(\mathrm{M}_{\mathrm{PBH}}, \chi)\mathrm{dM}_{\mathrm{PBH}}\mathrm{d}\chi$$

[T. Chiba, et al: arXiv: 1704.06573]

Zaldarriaga Parameters

[Zaldarriaga, et al. arXiv:1904.07214]

Name	Bank	$\mathcal{M}^{\rm det}(\mathrm{M}_\odot)$	$\chi_{\rm eff}$	z	GPS time[a]	$\rho^2_{\rm H}$	$\rho^2_{\rm L}$	$\mathrm{FAR}^{-1}(\mathrm{O2})$[b]	$\frac{W(\rm event)}{\mathcal{R}(\rm event \vert \mathcal{N})}$ (O2)	$p_{\rm astro}$
GW170121	BBH (3,0)	29^{+4}_{-3}	$-0.3^{+0.3}_{-0.3}$	$0.24^{+0.14}_{-0.13}$	1169069154.565	29.4	89.7	2.8×10^3	>30	>0.99
GW170304	BBH (4,0)	47^{+8}_{-7}	$0.2^{+0.3}_{-0.3}$	$0.5^{+0.2}_{-0.2}$	1172680691.356	24.9	55.9	377	13.6	0.985
GW170727	BBH (4,0)	42^{+6}_{-6}	$-0.1^{+0.3}_{-0.3}$	$0.43^{+0.18}_{-0.17}$	1185152688.019	25.4	53.5	370	11.8	0.98
GW170425	BBH (4,0)	47^{+26}_{-10}	$0.0^{+0.4}_{-0.5}$	$0.5^{+0.4}_{-0.3}$	1177134832.178	28.6	37.5	15	0.65	0.77
GW170202	BBH (3,0)	$21.6^{+4.2}_{-1.4}$	$-0.2^{+0.4}_{-0.3}$	$0.27^{+0.13}_{-0.12}$	1170079035.715	26.5	41.7	6.3	0.25	0.68
GW170403	BBH (4,1)	48^{+9}_{-7}	$-0.7^{+0.5}_{-0.3}$	$0.45^{+0.22}_{-0.19}$	1175295989.221	31.3	31.0	4.7	0.23	0.56

$$m_1 \approx [62 - 70]\mathrm{M}_\odot$$

$$m_2 \approx [42 - 47]\mathrm{M}_\odot$$

Tough luck

LIGO has not provided the full likelihoods to the community yet.

You need this

To calculate this

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta)\, d\theta}$$

- Build your own likelihood [M. Zaldarriaga, et al: arXiv: 1806.10610]
- Be an insider [Vitale, et al: arXiv:1707.04637]

Impact of Bayesian Priors on the Characterization of Binary Black Hole Coalescences

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In a regime where data are only mildly informative, prior choices can play a significant role in Bayesian statistical inference, potentially affecting the inferred physics. We show this is indeed the case for some of the parameters inferred from current gravitational-wave measurements of binary black hole coalescences. We reanalyze the first detections performed by the twin LIGO interferometers using alternative (and astrophysically motivated) prior assumptions. We find different prior distributions can introduce deviations in the resulting posteriors that impact the physical interpretation of these systems. For instance, (i) limits on the 90% credible interval on the effective black hole spin χ_{eff} are subject to variations of $\sim$10% if a prior with black hole spins mostly aligned to the binary's angular momentum is considered instead of the standard choice of isotropic spin directions, and (ii) under priors motivated by the initial stellar mass function, we infer tighter constraints on the black hole masses, and in particular, we find no support for any of the inferred masses within the putative mass gap $M \lesssim 5\ M_\odot$.

[Vitale, et al: arXiv:1707.04637]

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The parameters of all LIGO/Virgo events were estimated using powerful statistical pipelines which inevitably include prior assumptions. Here we present a critical reanalysis of the three O1 detections (GW150914, GW151226, and LVIT151012) under different Bayesian priors, summarizing results presented by Vitale et al. (2017). We repeat some references and context from Vitale et al. (2017) but refer the reader to that work for full details. This was the first independent reanalysis of the public LIGO data that made astrophysical inferences about the sources of the signals. This study pioneered the use of the scientific products released by the LIGO and Virgo Collaboration (losc.ligo.org, Vallisneri et al. 2015) to uncover finer details of these landmark discoveries.

[Vitale, et al: arXiv:1712.06635]

Third way:
Re-weight posterior

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta)\, \mathrm{d}\theta}$$

One model
with some
assumptions

$$\tilde{P}(\theta|d) \propto P(\theta|d) \frac{\tilde{p}(\theta)}{p(\theta)}$$

$$\tilde{P}(\theta|d) = \frac{P(d|\theta)\, \tilde{p}(\theta)}{\int P(d|\theta) \tilde{p}(\theta)\, \mathrm{d}\theta}$$

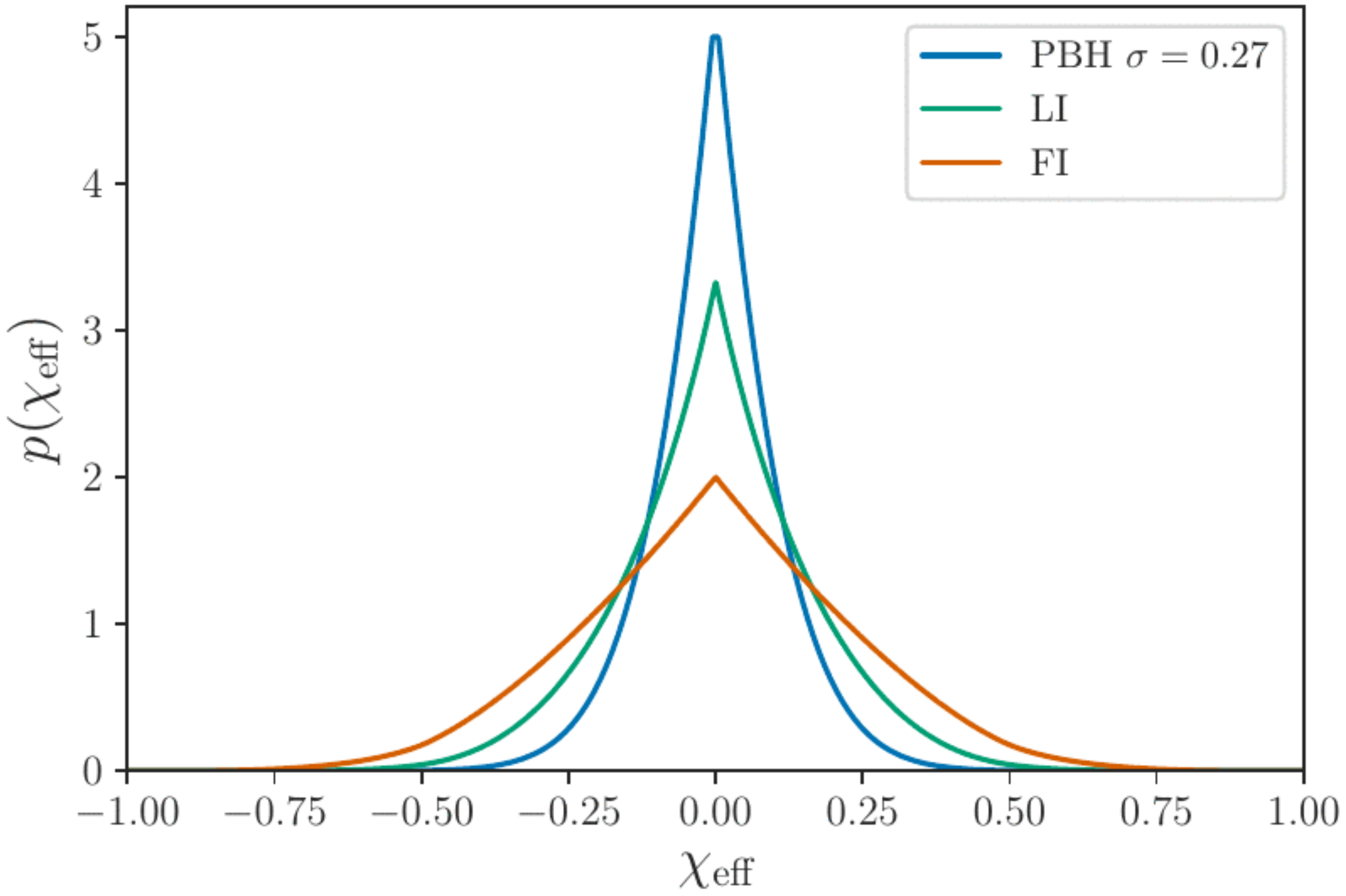
Other model with
some other
assumptions

Hierarchical Inference

- Ultimately we would like to calculate $p(\mu,\sigma|\{d\}) \propto \mathcal{N}(\mu, \sigma)$

So assuming that the LIGO events are from PBHs. What are the best parameters that describe them?

Effective Spin Prior for PBHs



Odds ratios

$$\mathcal{O}_{ij} = \frac{p(M_i|d)}{p(M_j|d)}$$

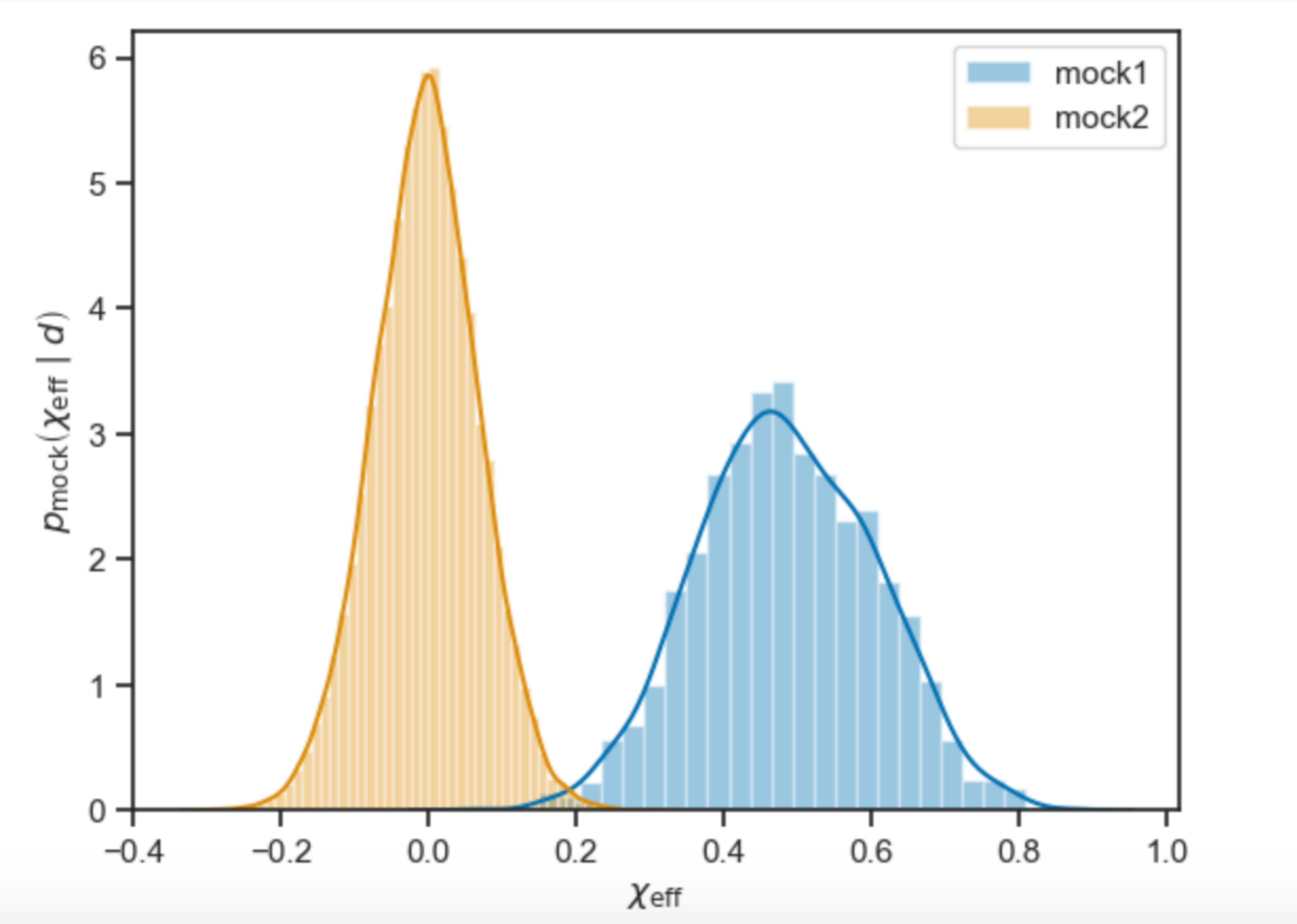
Compare competing models, for example, the 10 events are coming from **PBH** vs **FI**

$$\mathcal{O}_{\mathrm{FI,PBH}} = \frac{P(\mathrm{FI}|\mathrm{d})}{P(\mathrm{PBH}|\mathrm{d})}$$

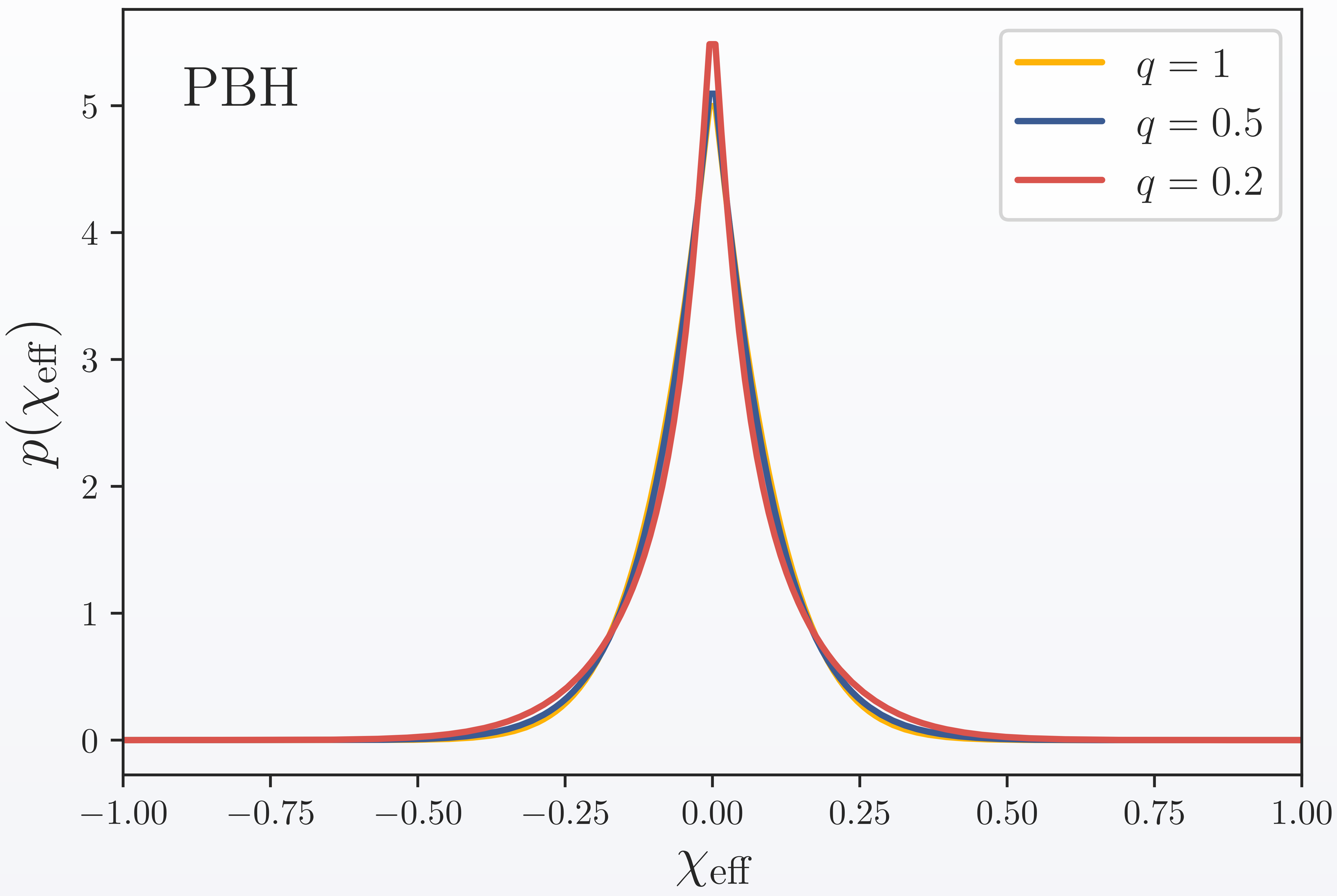
The odds ratio is equivalent to the Bayes factor if the prior probability for each model are equal

Simulated Events

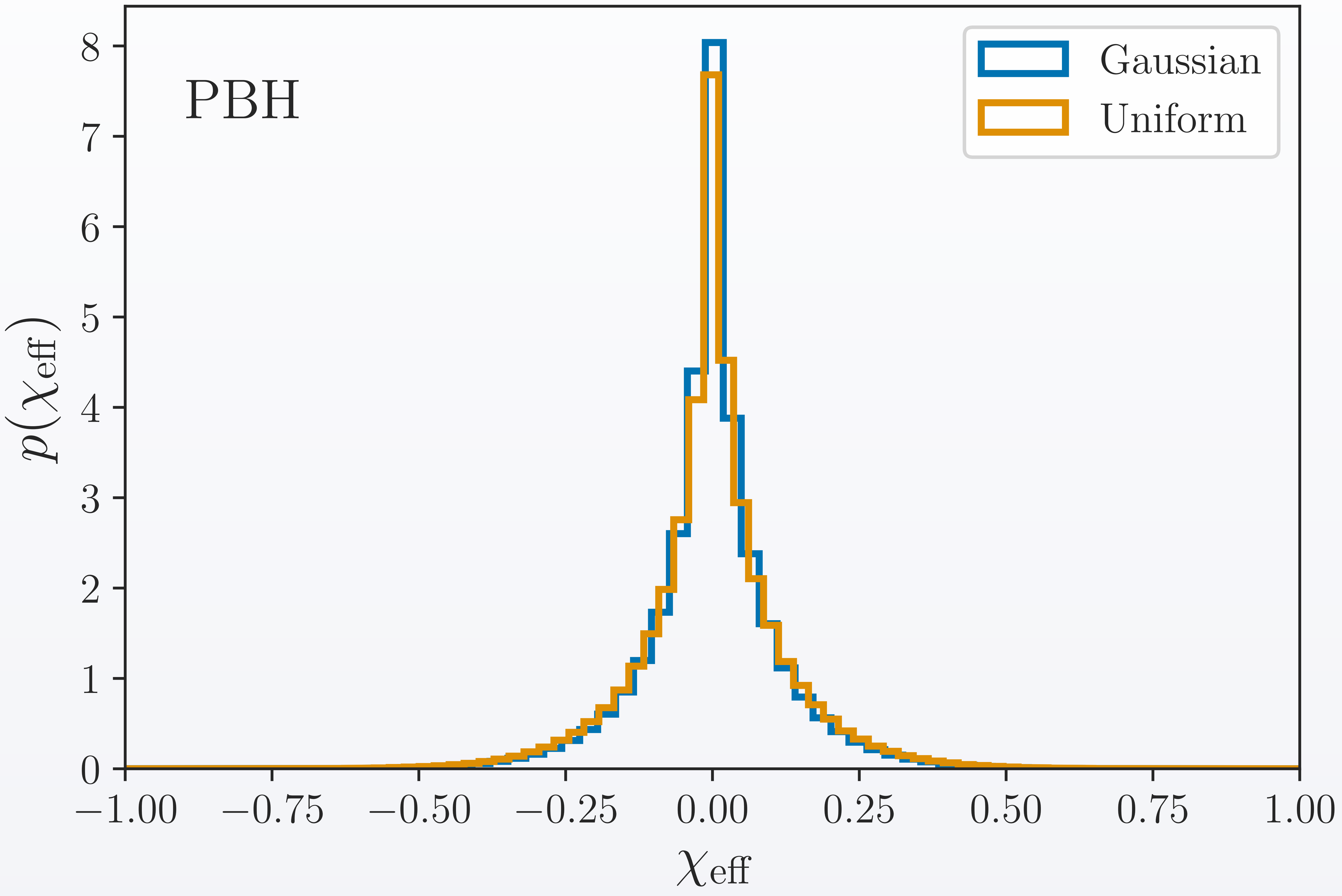
Choosing a random uncertainty from one of the ten events, picking a $\chi_{\rm eff}^{\rm true}$ from the flat-isotropic distribution, generating an "observation" of this via $\chi_{\rm eff}^{\rm obs} = \chi_{\rm eff}^{\rm true} + N(0, \sigma)$, from which the posterior is $\chi_{\rm eff} \sim N\left(\chi_{\rm eff}^{\rm obs}, \sigma\right) p_{\rm iso}\left(\chi_{\rm eff}\right)$.



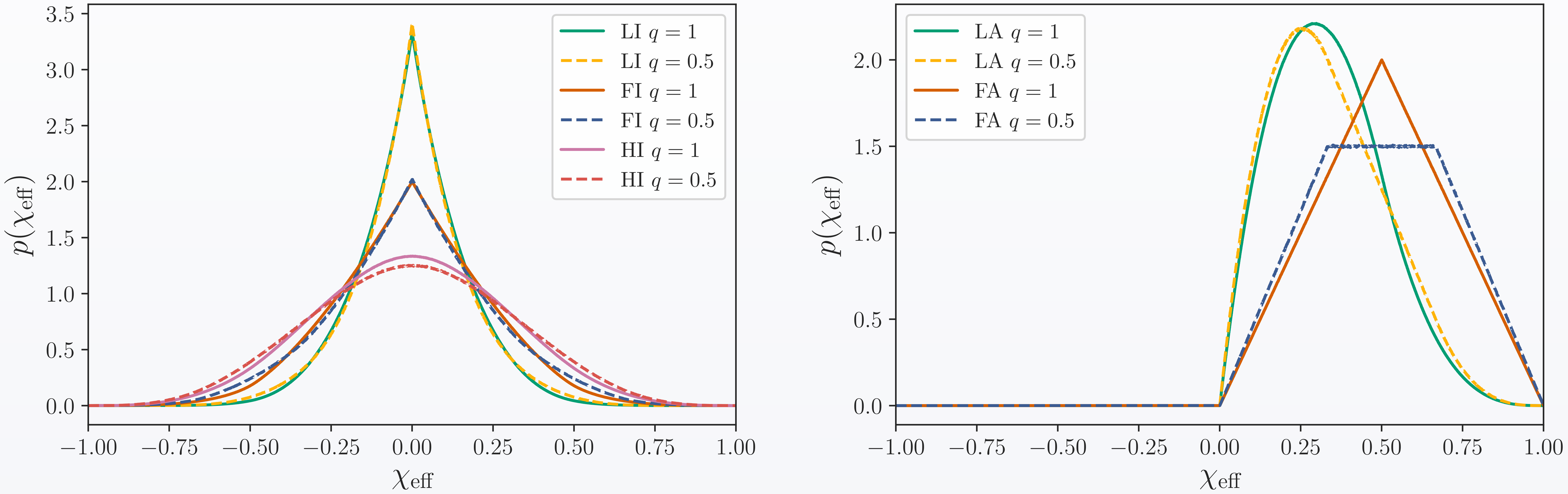
The distribution of the effective spin is not very sensitive to the mass ratio q



The distribution of the effective spin is not very sensitive to the distribution of the masses

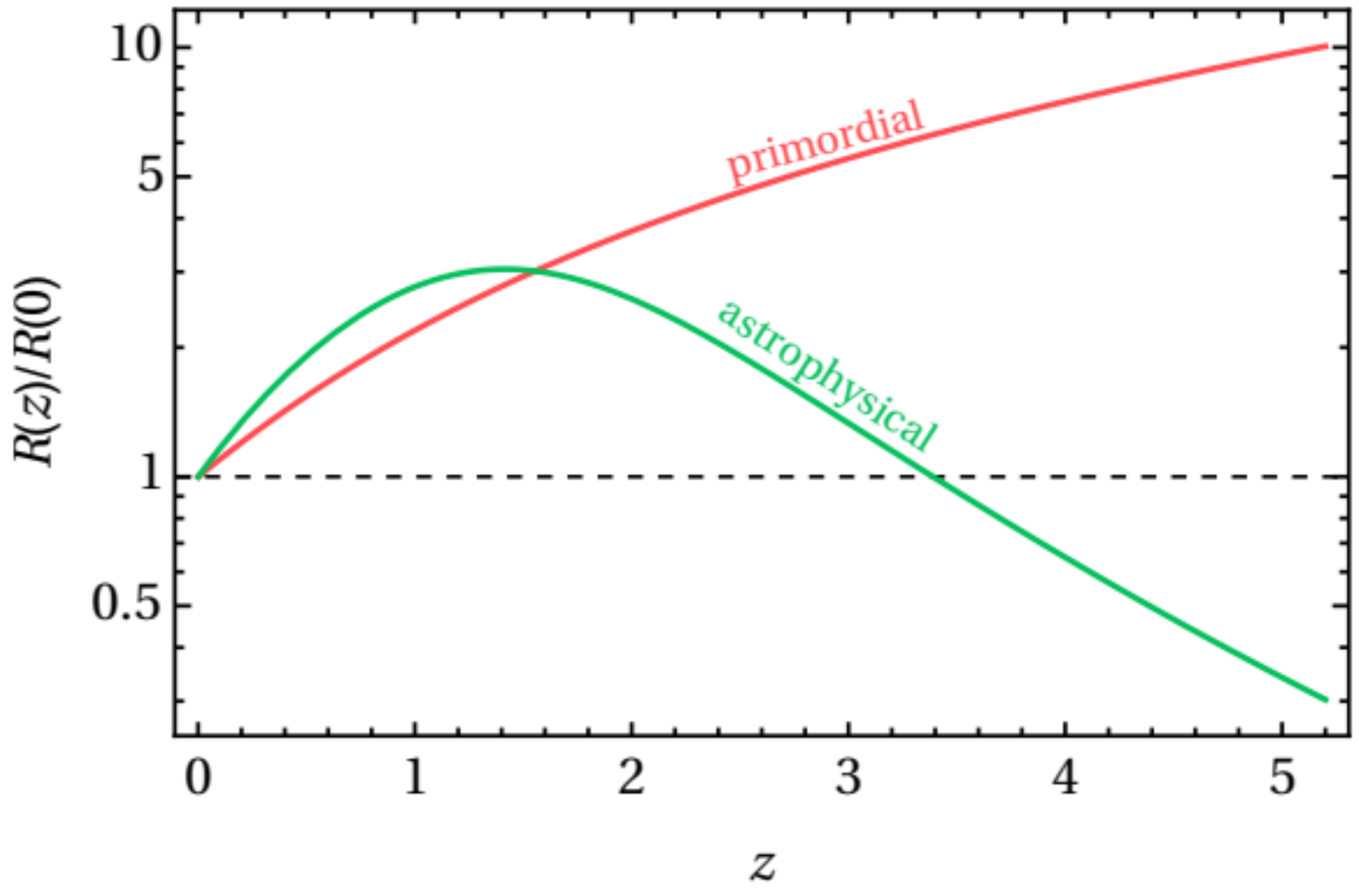


The distribution of the effective spin is not very sensitive to the mass ratio q



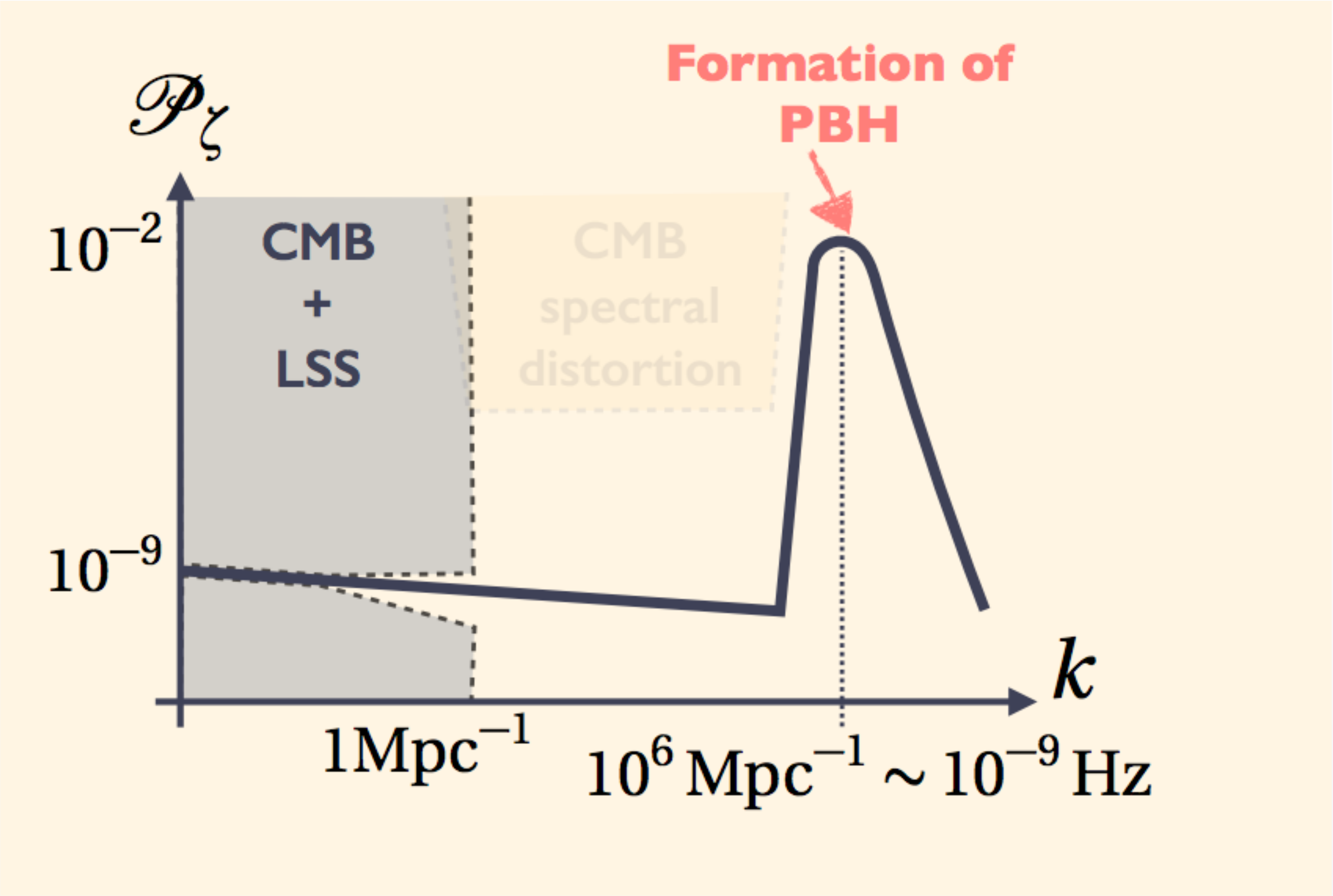
Marger Rate

The lines show the z dependence of the BH merger rates normalised to one at z = 0 for early PBH binary formation and for the astrophysical BH binaries.



[Belczynski, et al. arXiv:1706.07053]

Curvature Perturbation



[Inomata, et al. arXiv:1611.06130]