Gravitational Wave Astronomy

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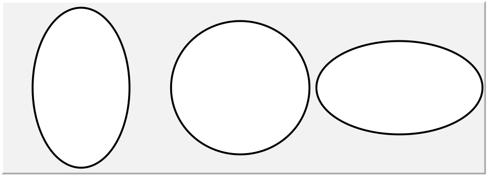
GW basics in 1 slide

Gauged fixed metric perturbations $h_{\mu\nu}$ after discarding $h_{0\mu}$ components, which are not radiative $\rightarrow$ transverse waves

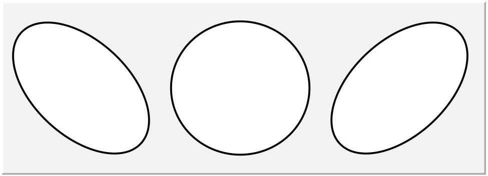
$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2 pol. state like any mass-less particle

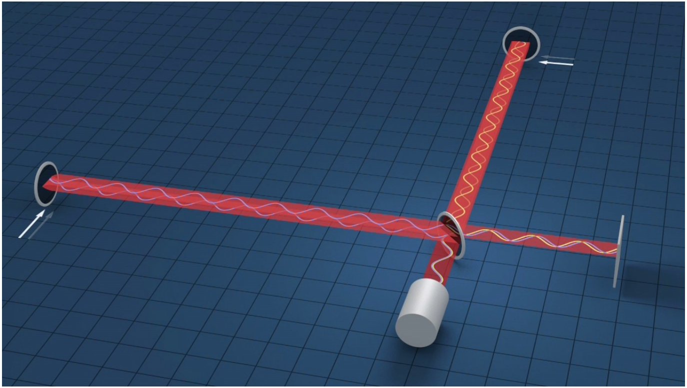
h_+



$h_\times$



LIGO and Virgo: very precise rulers



Robert Hurt (Caltech)

Light intensity $\propto$ light travel difference in perpendicular arms
Effective optical path increased by factor $N \sim 500$ via Fabry-Perot cavities
Phase shift $\Delta\phi \sim 10^{-8}$ can be measured $\sim 2\pi N \Delta L/\lambda \rightarrow \Delta L \sim 10^{-15}/N$ m

Almost omnidirectional detectors

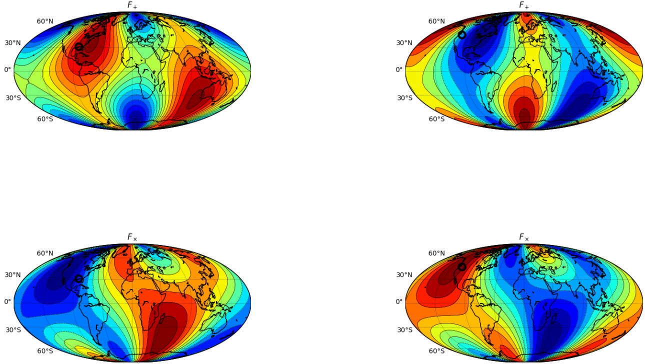
Detectors measure h_{det}: linear combination $F_+ h_+ + F_\times h_\times$

L *H*

-1 0 1

F_+

$F_\times$



$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

Almost omnidirectional detectors

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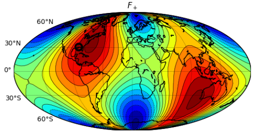
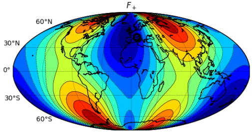
L V

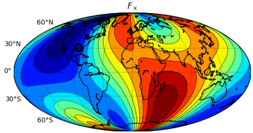
-1 0 1

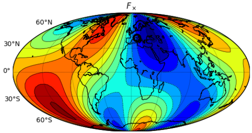
F_+





$F_\times$

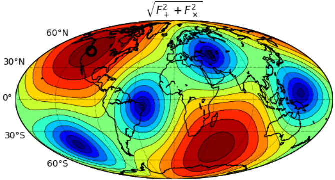


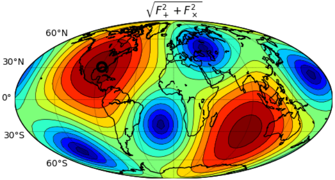


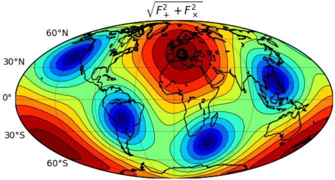
$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

Pattern functions: $\sqrt{F_+^2 + F_\times^2}$







The LIGO and Virgo observatories

- Observation run **O1** Sept '15 - Jan '16

$\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 detectors, 3BBH

- **O2** Dec. '16 – Jul'17 2 det's + Aug '17 3 det's

3(+4) BBH + 1BNS in double (triple) coinc.

- **O3a**: 3 detectors, Apr 2nd - Oct 1st 2019
- **O3b**: Nov 1st – 6 months

In $\sim$ start of 2020 Japanese KAGRA and $\sim$ 2025 Indian INDIGO will join the collaboration

Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right)\,, \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1m_2/M^2 \\
h_{ij} &\sim g(\theta_{LN})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 6kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{LN})$ takes account of transversality projection (angular momentum L of the binary, observation direction N)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\,\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
\end{aligned}
$$

Amplitudes of 2 polarizations modulated by θ_{LN} ($h \nearrow$ for $\theta_{LN} \searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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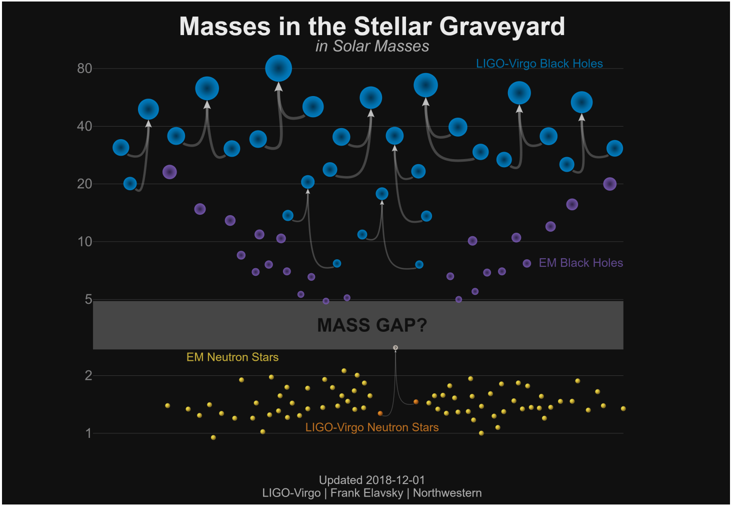
Amplitudes of 2 polarizations modulated by θ_{LN} ($h\nearrow$ for $\theta_{LN}\searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

h sensitive to red-shifted masses $M\to M(1+z)\equiv\mathcal{M}$

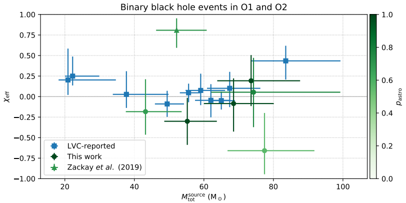
Binary systems and compact detected so far



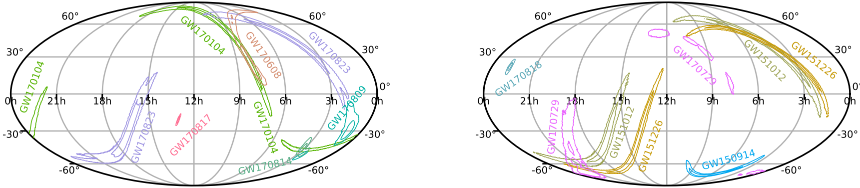
Event	Primary mass (M_sun)	Secondary mass (M_sun)	Effective inspiral spin	chirp mass (M_sun)	Final spin	Final mass (M_sun)	Luminosity distance (Mpc)	Sky localization (deg^2)
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$-0.01^{+0.12}_{-0.13}$	$28.6^{+1.6}_{-1.5}$	$0.69^{+0.05}_{-0.04}$	$63.1^{+3.3}_{-3.0}$	430^{+150}_{-170}	179
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$0.04^{+0.28}_{-0.19}$	$15.2^{+2.0}_{-1.1}$	$0.67^{+0.13}_{-0.11}$	$35.7^{+9.9}_{-3.8}$	1060^{+540}_{-480}	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$0.18^{+0.20}_{-0.12}$	$8.9^{+0.3}_{-0.3}$	$0.74^{+0.07}_{-0.05}$	$20.5^{+6.4}_{-1.5}$	440^{+180}_{-190}	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$-0.04^{+0.17}_{-0.20}$	$21.5^{+2.1}_{-1.7}$	$0.66^{+0.08}_{-0.10}$	$49.1^{+5.2}_{-3.9}$	960^{+430}_{-410}	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$0.03^{+0.19}_{-0.07}$	$7.9^{+0.2}_{-0.2}$	$0.69^{+0.04}_{-0.04}$	$17.8^{+3.2}_{-0.7}$	320^{+120}_{-110}	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$0.36^{+0.21}_{-0.25}$	$35.7^{+6.5}_{-4.7}$	$0.81^{+0.07}_{-0.13}$	$80.3^{+14.6}_{-10.2}$	2750^{+1350}_{-1320}	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$0.07^{+0.16}_{-0.16}$	$25.0^{+2.1}_{-1.6}$	$0.70^{+0.08}_{-0.09}$	$56.4^{+5.2}_{-3.7}$	990^{+320}_{-380}	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$0.07^{+0.12}_{-0.11}$	$24.2^{+1.4}_{-1.1}$	$0.72^{+0.07}_{-0.05}$	$53.4^{+3.2}_{-2.4}$	580^{+160}_{-210}	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$0.00^{+0.02}_{-0.01}$	$1.186^{+0.001}_{-0.001}$	≤ 0.89	≤ 2.8	40^{+10}_{-10}	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$-0.09^{+0.18}_{-0.21}$	$26.7^{+2.1}_{-1.7}$	$0.67^{+0.07}_{-0.08}$	$59.8^{+4.8}_{-3.8}$	1020^{+430}_{-360}	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$0.08^{+0.20}_{-0.22}$	$29.3^{+4.2}_{-3.2}$	$0.71^{+0.08}_{-0.10}$	$65.6^{+9.4}_{-6.6}$	1850^{+840}_{-840}	1651

https://www.gw-openscience.org/catalog/

See however Princeton's group results arXiv:1904.07214 and Nitz et al. arXiv:1910.05331



Sky localization of GW binary systems detected so far



Sky localization regions shrinks when Virgo in: 3rd detector matters!

LIGO/Virgo 1811.12907

Astro sources emitting transient GWs/EM/ν radiation

- Coalescing binary systems of compact objects: $\eta \equiv m_1 m_2/M^2$

$$\Delta E_{GW} \sim 4\times 10^{-2} M_\odot \eta \left(\frac{M}{3M_\odot}\right)^{5/3} \left(\frac{f_{max}}{1\,\mathrm{kHz}}\right)^{2/3}$$

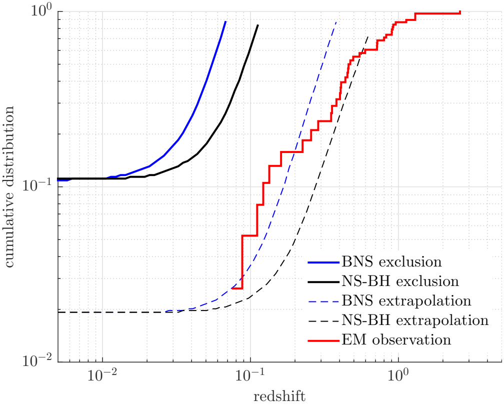
  A good proxy of the f_{max} (inspiral) is the Innermost Stable Circular Orbit frequency

$$f_{ISCO} = \frac{1}{6\sqrt{6}}\frac{1}{M} \simeq 700\mathrm{Hz}\left(\frac{3M}{M_\odot}\right)^{-1} \qquad f_{RD} = \frac{1}{2M} \simeq 6\mathrm{kHz}\left(\frac{3M}{M_\odot}\right)^{-1}$$

  For NS-NS/BH ejected material supposed to produce short ($<$ 2 sec) GRBs
  $E_\gamma \sim$keV-MeV, ejected sub-relativistic material may produce radio signals
  ν: baryon loaded jets may emit νs: TeV-PeV ν @ γ emission; PeV-EeV ν @ optical afterglow
- NS-NS/BH $\rightarrow$ energetic outflows at different timescale/wavelength
- Rapid neutron capture in the sub-relativistic ejecta can produce a kilonova or macronova, with optical and near-infrared signal (hours - weeks)
- Eventually radio blast from sub-relativistic outflow (months to years)

LIGO/Virgo + EM partners ApJ Let. 2016

Association with 20 short GRB during O1 (before GW170817)

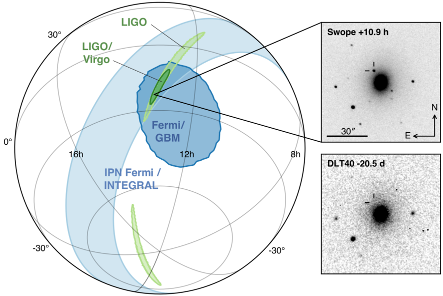


GW searched in a [-5,+1) sec window with GRB
GW constraints far from EM exclusion distance, but extrapolation to design Advanced LIGO sensitivity (2 years of data) will lead to detection/non-trivial constraints

LIGO/Virgo Astrophys.J. 841 (2017) no.2, 89

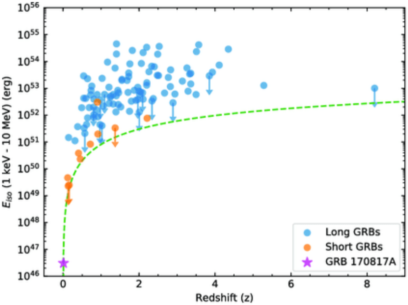
GW170817

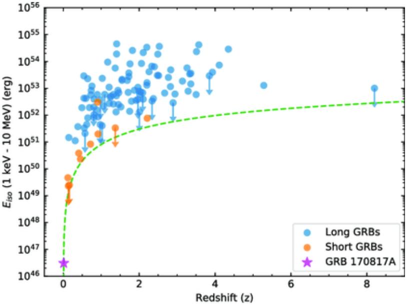
- GW trigger on Aug 17th, 2017, ended at 12h 41' 04.4" UTC, first in in LIGO Hanford, then confirmed as a triple coincidence $\rightarrow$ localized in an area of ~ 28 deg^2
- GRB trigger from Fermi-GBM 1.7" after
- first optical image 10.87 hr afterwards by One-Meter Two Hemisphere team with Swope telescope at Las Campanas Observatory in Chile
- X obtained by the X-Ray Telescope on Swift after 14.9 h (NuSTAR 16.8 h)
- radio ($\sim 3, 6$ GHz) by VLA 16 days after GW event



LIGO/Virgo & Partner Astronomy groups, Astrophys.J. 848 (2017) no.2, L12

GRB170817A: a faint GRB

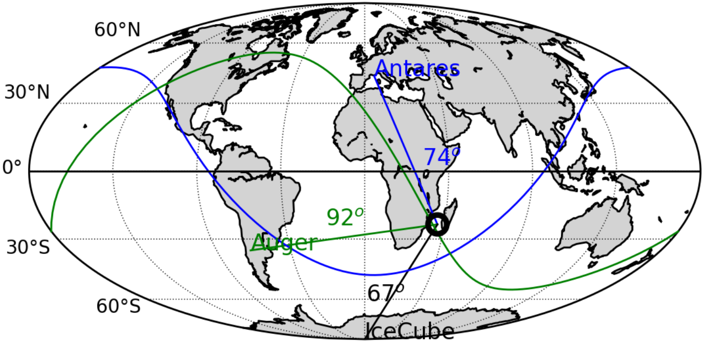




A dim outlier both in energy and luminosity ($T_{\rm short} < 2{\rm sec} < T_{\rm long}$)

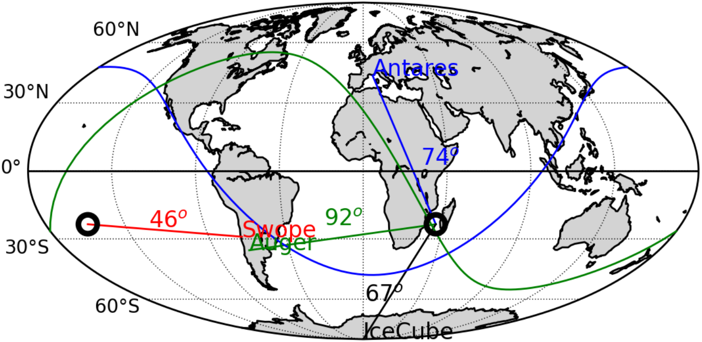
Astrophys.J. 848 (2017) no.2, L13

GW170817 and ν detectors



Potential νs down-going for Antares and IceCube, grazing for Auger

GW170817 and ν detectors



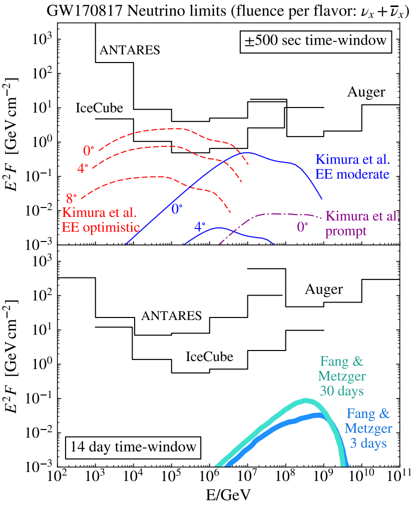
Potential νs down-going for Antares and IceCube, grazing for Auger
Source location at optical detection

ANTARES and ICECube look for ν

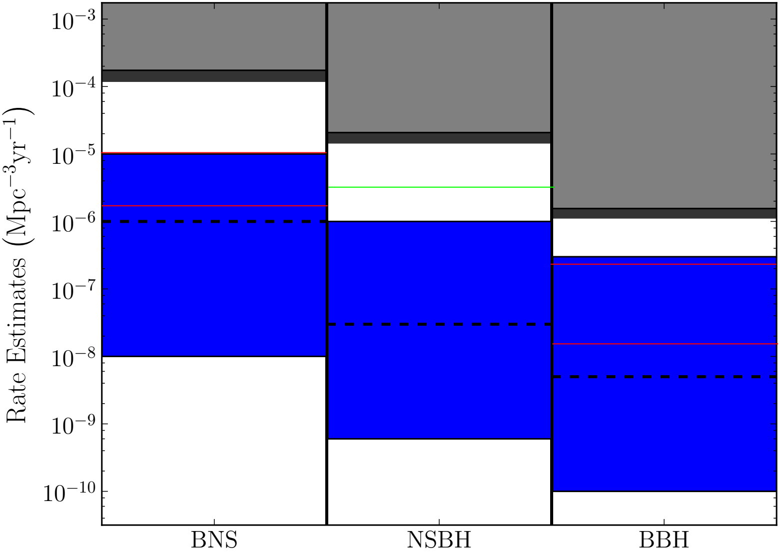
IceCube and ANTARES looked for $\mu - \nu$ at $\pm 500 sec$, observation for 2 weeks

no detection

Pierre Auger for $> 10^{17}$ eV, grazing good for τ production at 10^{18} eV



LIGO/Virgo, ANTARES, ICECUBE and Pierre Auger, Astrophys.J. 850 '17 2, L35



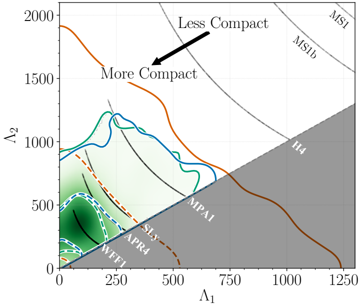
Astrophysical predictions, upper limit from S5/6/O1 and observation from O1/O2

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)

Fundamental physics of Neutron stars

Tidal deformability $\Lambda \simeq R^{-5} Q_{ab} / (\partial_a \partial_b U_{tidal})$ depends on Equation of State

Determinaton of $\Lambda_{1,2}$
(assuming same EoS for 1,2)



Shading Common EoS for 2 bodies and constrain for $m_{max} > 1.97 M_\odot$
Lines 50%, 90% countours, No mimimum m_{max}, independent EoSs

LIGO/Virgo collaboration, arXiv:1805.11581, PRL 121 ('18)

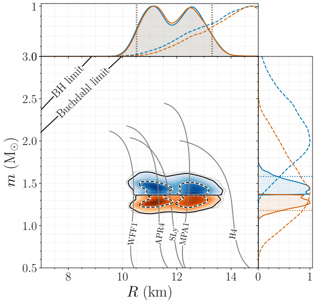
Fundamental physics of Neutron stars II

Parametrizing EoS with $\Gamma \equiv \frac{\epsilon+p}{p}\frac{dp}{d\epsilon}$, $\Gamma(x) = \exp\left(\sum_k \gamma_k x^k\right)$, $\qquad x \equiv \log \frac{p}{p_0}$

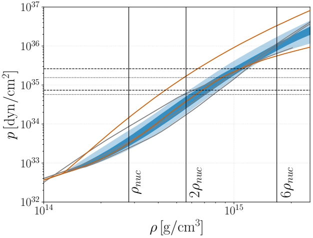
$$\gamma_0 \in [0.2, 2], \gamma_1 \in [1.6, 1.7], \gamma_2 \in [0.6, 0.6] \gamma_3 \in [0.02, 0.02], \Gamma(p) \in [0.6, 4.5]$$

L. Lindblom PRD 82, 103011 (2010)

Constraints on Mass vs- radius and p vs. rho



lighter, heavier



90% prior, posterior

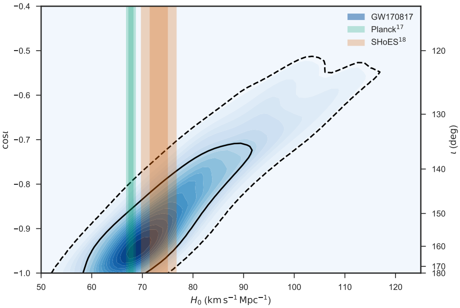
LIGO/Virgo collaboration, arXiv:1805.11581, PRL 121 ('18)

Cosmology

- From GWs only: luminosity distance $d_L \sim 40^{+8}_{-14}$Mpc, viewing angle $\theta_{LN} > 125^o$
- Fixing d_L from info from EM location + Hubble relation $d_L H_0 = z$
  1. Hubble constant from SuperNovae (SHoES) $148^o < \theta_{LN} < 166^o$

     Riess et al. ApJ 826, 56 '16
  2. from Planck $157 < \theta_{LN} < 177^o$

     Planck A&A, 594, A13 '16
- GW almost coincident with EM $\implies \Delta v_{GW}/c \sim 10^{-15}$ LIGO/Virgo, Fermi Gamma-Ray Burst Monitor, INTEGRAL Ap.J. 848 (2017) 2, L13



LIGO/Virgo Nature 551 '17 7678, 85

$$h''_{+\times} + 2\frac{a'}{a}h'_{+,\times} + c_s^2\nabla^2 h_{+,\times} = 0$$

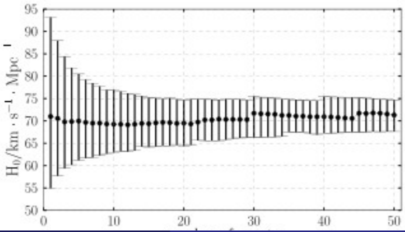
Determining H_0 without EM counterpart

Hubble law: $z = H_0 d_L$

D_L can be measured, z degenerate with M, however if

- the source in the sky has been localised (α, δ)
- GW sources are in the galaxy catalogue with known red-shift

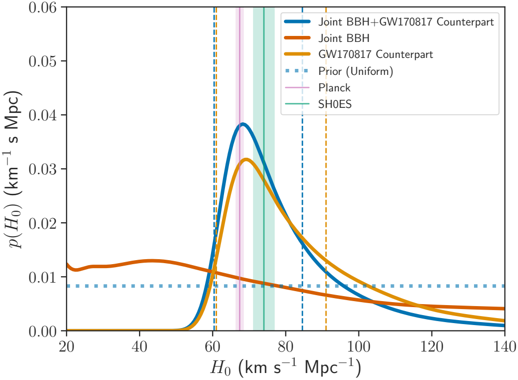
$$P(z, D_L|c_i) = \int d\mathcal{M}\, d\vec{\theta}\, d\alpha\, d\delta\, P(D_L \mathcal{M}, \vec{\theta}, \alpha, \delta|c_i)\pi(z, |\alpha, \delta)$$



Schutz, Nature (1986) 323 310

W. Del Pozzo, Phys.Rev.D86 (2012) 043011

Galaxy catalogs and H_0

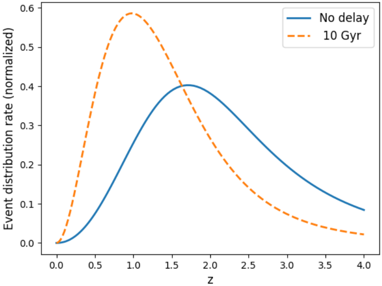


sky localization from DES-Y1 (GW170814)
GWENS (GW170818)
GLADE (GW150914, GW151226, GW170608)

LIGO/Virgo, arXiv:1908.06060

Ongoing O3: towards measuring merger distribution

At https://gracedb.ligo.org/superevents/public/O3
Summary of candidate detections from O3a:
35 candidate events, $\sim$ 8 of which possibly involving at least a NS



Expected merger distribution from Star Formation Rate (—) or SFR+Poissian distributed delay (– –)

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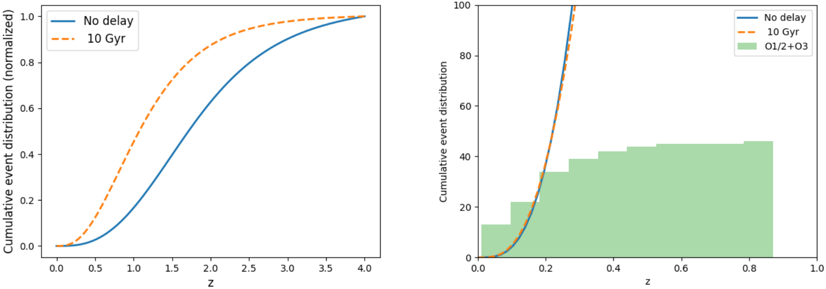

Expected merger distribution from Star Formation Rate (—) or
SFR+Poissian distributed delay (– –)

Fundamental GR: inspiral analytic model

Inspiral $h = A\cos(\phi(t))$ $\quad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta)v^2 + \#(\eta)v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\eta)v^2 + \#(\eta)v^3 + \ldots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)}dv \\ &\sim \int \left(1 + \#(\eta)v^2 + \ldots + \#(\eta)v^6 + \ldots\right)\frac{dv}{v^6} \end{aligned}$$

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PN Coefficients (tidal $\sim v^{10}$)

How to compute effective potential?

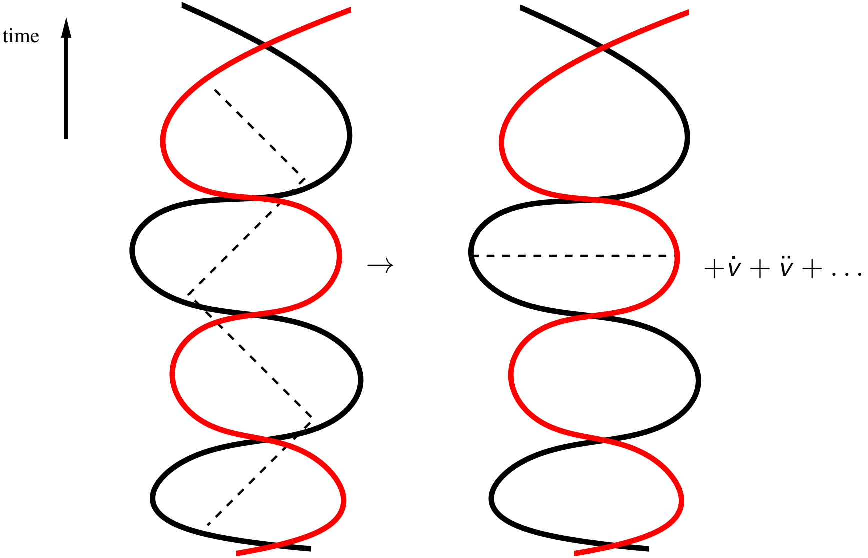
Iteratively solve Einstein equations for point particles ($J(x) \sim \delta^3(x-\bar{x})$):

$$
\begin{aligned}
-\int dt V_{1-2}(t,r) &\sim \int dt\, dt'\, d^3x\, d^3x'\, J(x) G(x-x') J(x') \\
&= 32\pi G_N m_1 m_2 \int dt\, dt' \frac{d^4k}{(2\pi)^4} \frac{e^{ik^\mu (x_1(t_1)-x_2(t_2))_\mu}}{k^2-\omega^2} \\
&\simeq G_N \int dt\, dt' \delta(t-t') \frac{m_1 m_2}{|x_1(t)-x_2(t')|}
\end{aligned}
$$

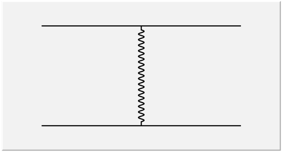
where k_0 has been neglected. Considering effects of v

$$
\begin{aligned}
V &\propto \int d\omega d^3k \frac{e^{-i\omega t_{12}+ikx_{12}}}{k^2-\omega^2} = \int d\omega d^3k \frac{e^{-i\omega t_{12}+ikx_{12}}}{k^2}\left(1+\frac{\omega^2}{k^2}+\ldots\right) \\
&= \delta(t_{12}) \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1+\frac{\partial_{t_1}\partial_{t_2}}{k^2}+\ldots\right) = \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1-\frac{k\cdot v_1\, k\cdot v_2}{k^2}+\ldots\right)
\end{aligned}
$$

Trading knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time

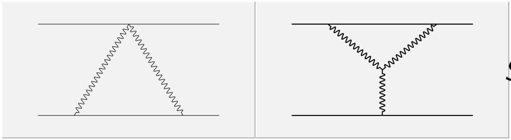


Newtonian potential:

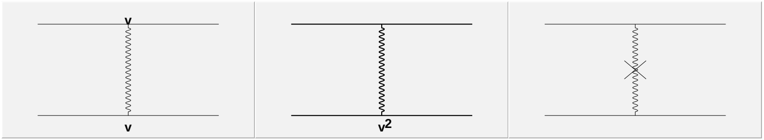


$$S_{eff} \sim \int dt \, \frac{G_N m_1 m_2}{r} \sim L$$

At 1PN:



$$S_{eff} = \int dt \, \frac{G_N^2 m_1^2 m_2}{r^2} \sim L v^2$$



$$V_{1PN} = -\frac{G m_1 m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r}\right] + 1 \leftrightarrow 2$$

Effective potential from integration over regions

Internal graviton momentum can be expanded upon the following scaling:

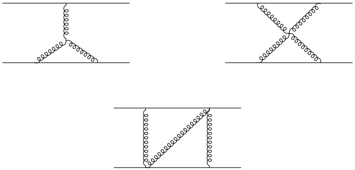
hard	(m, m)	quantum	×
soft	$(\|\vec{q}\|, \|\vec{q}\|)$	quantum	×
potential	$(v/r, 1/r)$	classical	✓
radiation	$(v/r, v/r)$	classical	next section

and then integrated over the full phase space

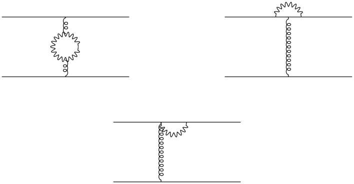
Only potential and radiation gravitons exchanged in classical processes: theory in terms of world lines selects diagrams that do not send source off-shell

Potential graviton → small change in energy wrt momentum, dominate classically

Ex. of classical connected diagrams



Ex. of quantum diagrams



Gravitational interactions do not create anti-BHs!

Graviton loop are negligible in astronomy!

From relativistic scattering amplitudes to 2-body potential

Equivalently, loops with massive particles contain a classical piece for $m \gg |\vec{q}|$:

$$V(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} A(\vec{q}, m_1, m_2, \vec{v}_1, \vec{v}_2, \ldots) e^{i\frac{\vec{q}\cdot\vec{r}}{\hbar}}$$

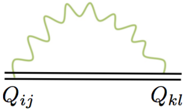
$$\supset \frac{1}{m^2 - \vec{q}^2} \quad \rightarrow \quad \underbrace{\frac{1}{m^2}}_{classical} \quad + \quad \underbrace{\frac{\vec{q}^2}{m^4}}_{quantum} \quad + \quad \ldots$$

E.g.: $\frac{G_N}{q^2}, \frac{G_N^2}{|q|}, G_N^3 \log|q| \ldots$ are classical contributions

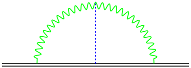
Conservative-dissipative interplay (Lamb-shift)

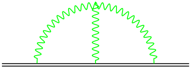
Self-energy diagrams of composite object

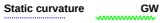
purely imaginary:



real and imaginary:

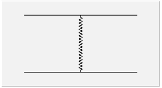






Foffa & RS PRD ('13), 1907.02869, Galley & al. PRD ('15),Damour & al. PRD ('13)

The 4PN sector

- G: : 3 diagrams
- G^2:  21 diagrams, RS and S. Foffa, PRD '12
- G^3: 212 diagrams
- G^4: 317 diagrams RS and S. Foffa PRD'19
- G^5: 50 diagrams, RS et al. PRD '18

Also derived with other methods by Damour, Jaranowski, Schäfer, Blanchet, Bernard, Marsat, Faye, Marchand, Bohé

2 body dynamics expansions (spin-less)

Post-Minkowskian expansion parameter is $G_N M/r$, vs PN expansion

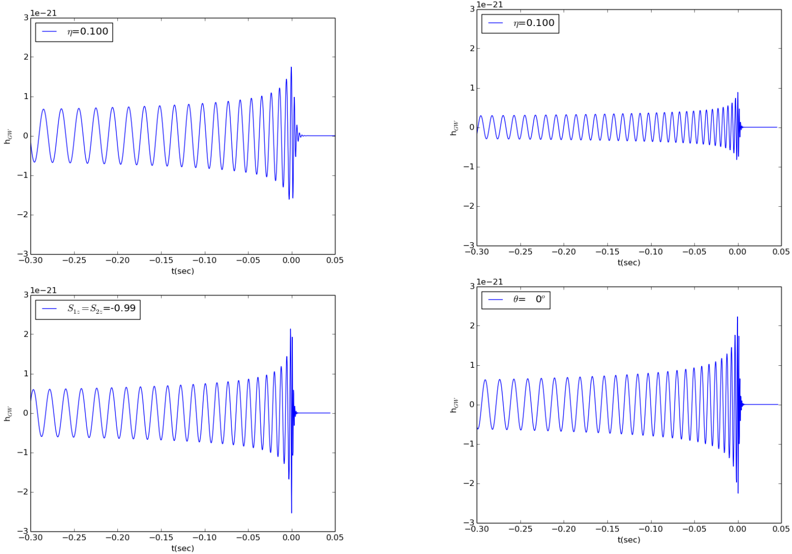
$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

Terms known so far

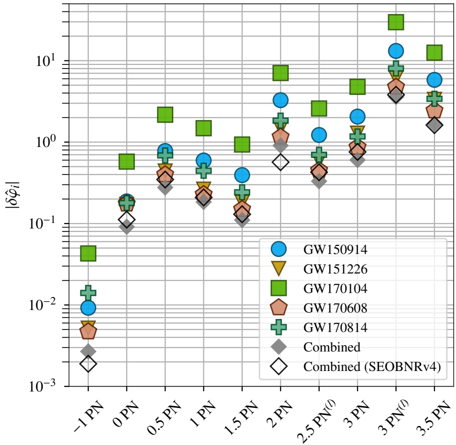
		N	1PN	2PN	3PN	4PN	5PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	...
5PM						$1/r^5$	v^2/r^5	...
...							...	...

3PM recently computed (!) by Z. Bern, C. Cheung et al.
arXiv:1901.04424PRL 122 ('19)
What's next? Amplitude program with modern methods: generalized unitarity, gravity as a double copy of gauge theory...

Parameter estimation from $\phi(\mathbf{t})$



Testing the PN series with O1/2



Precision in measuring the PN coefficients ∼ 100%: one cannot both measure the astro-parameters and test GR with same detection! Unique probe of high PN-orders

LIGO/Virgo arXiv:1903.04467, Phys.Rev. D100 ('19)

Precision gravity

Future 3rd generetaion detectors (Einstein Telescope, Cosmic Explore) / space telescope LISA will detect BBH binary signals with SNR $10 - 10^2$, with few golden events with SNR $\sim 10^3$.

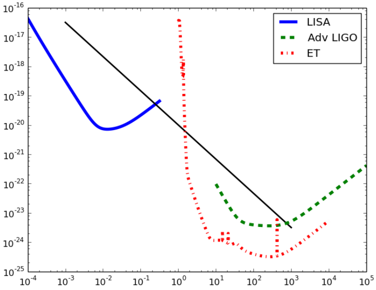
Templates few % accurate OK for characterising a source with SNR $O(10)$ (typical for LIGO/Virgo so far)

for SNR $\sim 10^3$ residual after extracting that source will have SNR $\sim O(10)$

1. baising parameter estimation
2. contaminating the extraction of additional sources.

$$h(f) \propto f^{-7/6} M_c^{5/6}$$

$$\Delta t_{i\to m} \propto M_c^{-5/3}(f_i^{-8/3} - f_m^{-8/3})$$

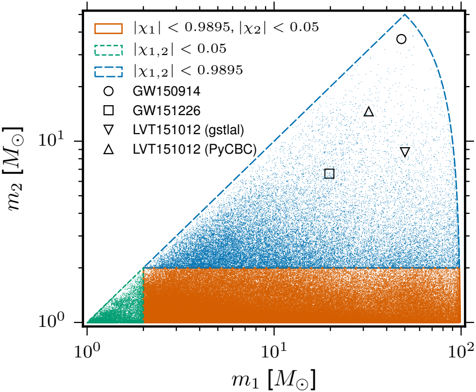


Conclusions (Actually just the beginning!)

- After the beginning of GW astronomy, we have witnessed the start of multi-messenger astronomy
- New era for Cosmology: H_0 from GWs! (and EM)
- New input for fundamental physics: test of gravity in the strong field at short scale, radiative gravity sector tested at large distances and soon precision gravity is going to be needed

Spare slides

Template Bank O1/2

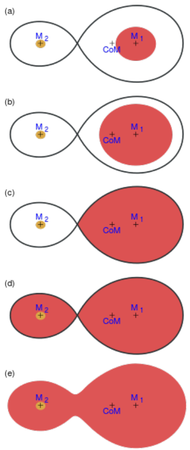


Bank with over 200k templates is prepared, matched-filtering computed against all wfs. Spin somehow neglected for low masses (anyway non-precessing) because

- astrophysically spin of neutron stars $\chi \equiv |Spin|/m^2 < 0.1$
- impractical as number of templates would explode!

LIGO/Virgo PRX (2016) 4, 041014

How binary systems progenitors formed? I



Izzard et al. Proc. IAU 2012

A binary system of massive stars ($M_{binary} < 100 M_{odot}$) can go through a common envelope phase that shrinks considerably the orbit and align the spins $\rightarrow$ collapse to compact objects

BNS requires complex evolution from massive binaries + mass transfer and 2 core-collapse SN explosions through which the binary system survives

Asymmetries in collapse imparts natal kick to the (galactic pulsar proper motion)

LIGO/Virgo Ap.J. 850 (2017) L40

vs.

For BBH another mechanism is possible

Globular Cluster: medium with high density of black holes/stars, when 3 black holes meet one is ejected and the binary shrinks

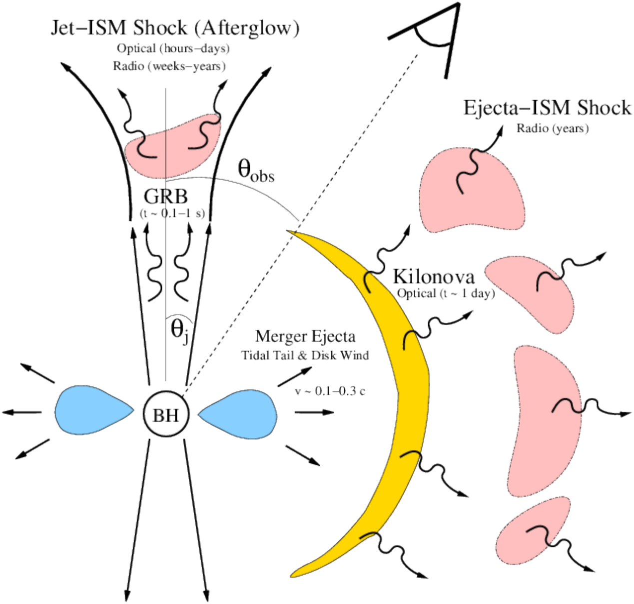
LIGO/VirgoAstrophys.J. 818 (2016) no.2, L22

Remnants of the first stars, produced at $z \sim 6$ can give only a small contribution to the total rate

T. Hartwig et al. Mon.Not.Roy.Astron.Soc. 460 (2016) L74

Primordial black-holes? viable if $10^{-2} \lesssim M_{BH}/M_{\odot} \lesssim 1$ (constraints from evaporation, microlensing and CMB) but could grow later to 20-100 $M_{\odot}$ by merger
If present in globular cluster may also explain their cosmological abundance as dark matter

Sasaki M. et al. PRL 2016, Bird S. et al. PRL 2016, Clesse S. and García-Bellido Phys. Dark Univ. 2017



Metzer & Berger Astrophys.J. 746 (2012) 48

NS merger $\rightarrow$ short γRB $\rightarrow$ kilonova

- Jet models for the ejecta (spherical fireball model)

$$\downarrow$$

BNS merger emitted γ rays, expected opening angles $\theta_j \simeq 3^o - 10^o$:
early time $\theta_j \sim 1/\gamma$ (jet collimated and relativistic), prompt emission from energy dissipation inside relativistic jet: X rays $\sim$ energy output and geometry
Shock wave from sGRB jet hitting surrounding medium, $\theta_j < 1/\gamma_{late}$

$$\downarrow$$

broadband synchrotron afterglow, with radio source $\sim$ months and UV, optical & IR emission from sub-relativistic ejecta
νs expected only from near GRB (or in late emission by flares and plateaus)

Alexander et al., Astrophys. J. **848** (2017) no.2, L21

NS merger frees neutron-rich radioactive species, decaying in kilonova in $\sim$ days

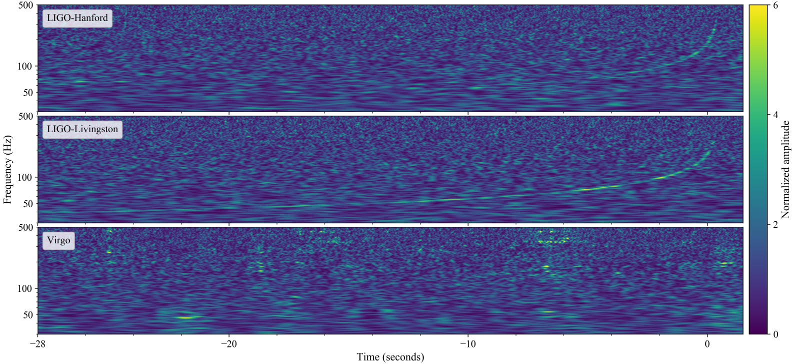
- Observation: Faint gamma ray emission: 10^{46} erg (isotropic), raising Xray flux 2.4 - 5.4 day, continued up to 15 days (Chandra), delayed afterglow

$$\downarrow$$

short GRB viewed off-axis $\theta_{obs} \gtrsim 20^o$

Berger E, 2014, ARA&A, 52, 43

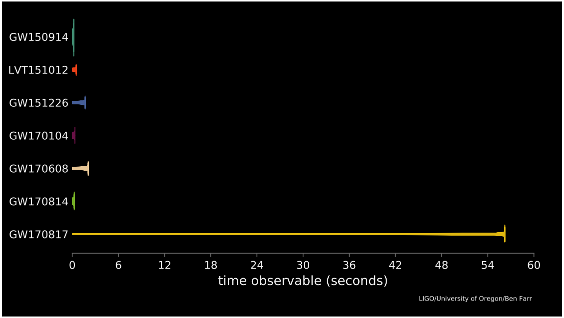
GW170817



BNS horizon (Mpc) was 218 for LL, 107 for LH, 58 for V

LIGO/Virgo Phys.Rev.Lett. 119 (2017) 161101

Detected waves (best fit)

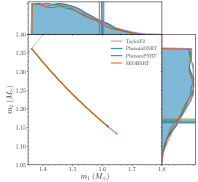


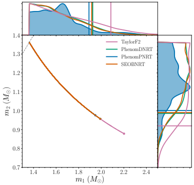
Smaller objects can get closer $\Longrightarrow$ reach higher frequency before merger

Parameter estimation for GW170817

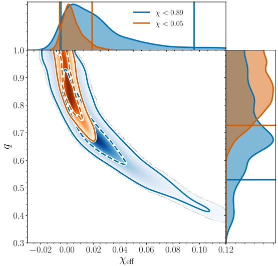
Low spin prior | High spin prior

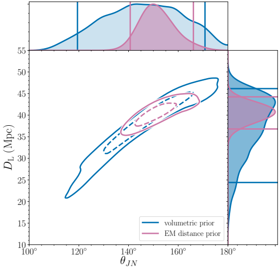
$m_{1,2}$





Spin $\chi_{eff} = (m_1 S_{1z} + m_2 S_{2z})/M$ - mass ratio q, distance - inclination





LIGO/Virgo PRL 119, 161101 (2017), arXiv:1805.11579

- Templates need to be 2 orders of magnitude more accurate for use in LISA than in Advanced LIGO/Virgo.

$$
\begin{aligned}
SNR &\propto \frac{G_N M_c^{5/6}}{D}\left(\int \frac{f^{-4/3}}{S_n(f)}\frac{df}{f}\right)^{1/2} \\
&= \frac{(G_N M_c)^{5/6}}{D} S_n^{-1/2}(\bar{f})\left(f_i^{-4/3} - f_f^{-4/3}\right)^{1/2} \\
&= \frac{(G_N M_c)^{5/6}}{D} S_n^{-1/2}(f_i)\left\{\begin{array}{l} f_i^{-2/3} \\ \sqrt{\frac{\Delta f}{f_i^{7/3}}} \end{array}\right.
\end{aligned}
$$

$$
\begin{aligned}
&\simeq 200\left(\frac{S_n^{1/2}\mathrm{Hz}^{1/2}}{10^{-24}}\right)^{-1}\left(\frac{\eta}{0.25}\right)^{1/2}\left(\frac{M}{10M_\odot}\right)^{5/6}\left(\frac{f}{150\mathrm{Hz}}\right)^{-2/3}\left(\frac{D}{500Mpc}\right)^{-1} \\
&\simeq 40\left(\frac{S_n^{1/2}\mathrm{Hz}^{1/2}}{10^{-21}}\right)^{-1}\left(\frac{\eta}{0.25}\right)^{1/2}\left(\frac{M}{70M_\odot}\right)^{5/6}\left(\frac{f}{10^{-4}\mathrm{Hz}}\right)^{-7/6}\left(\frac{D}{100Mpc}\right)^{-1} \\
&\times\left(\frac{\Delta f}{10^{-5}Hz}\right)^{1/2}
\end{aligned}
$$

Long duration signals

Remember that:

$$\frac{dt}{df} = \frac{5}{96\pi\eta} \left(\pi G_N M f\right)^{-5/3} f^{-2}$$

$$t = 10yrs \left(\frac{M}{70 M_\odot}\right)^{-5/3} \left(\frac{f}{10^{-2} Hz}\right)^{-8/3}$$

$$\Delta f \simeq \frac{96\pi}{5} \left(\pi G_N M_c f\right)^{5/3} f^2 \Delta t \simeq 10^{-3} Hz \left(\frac{\eta}{0.25}\right) \left(\frac{M}{70 M_\odot}\right)^{5/3} \left(\frac{f}{10^{-2} Hz}\right)^2$$

$$f_{ISCO} \simeq 1.5kHz \left(\frac{M}{3 M_\odot}\right)^{-1} \qquad f_{max} \simeq 20kHz \left(\frac{M}{3 M_\odot}\right)^{-1}$$

$$f_{gw} \simeq 134 Hz \left(\frac{1.21 M_\odot}{M_c}\right)^{5/8} \left(\frac{1sec}{\tau}\right)^{3/8}$$

$$\tau \simeq 2.2sec \left(\frac{1.21 M_\odot}{M_c}\right)^{5/3} \left(\frac{100 Hz}{f_{gw}}\right)^{8/3}$$

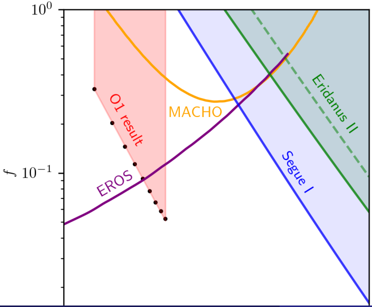
Need for *longer duration templates*, as for the third-generation ground based detectors, since the signals are present in the data stream for months

Upper bounds on sub-solar mass compact binaries coalescence rate

$\mathrm{Mass}/M_\odot$	$\mathrm{Rate}(Gpc^{-3}yr^{-1})$	Horizon(Mpc)
1	$< 10^6$	100
0.2	$< 2 \times 10^4$	30

$SNR \propto M_c^{5/6}$, $M_c \equiv \eta^{3/5} M$

LIGO/Virgo arXiv:1808.04771, PRL 121 ('18)



Fraction f of MACHOs in binary $\sim 10\% \times \Omega_{Macho} \implies \sim 10^{-3}\Omega_{Macho}$ in coalescing binaries coalescing within H_0^{-1}

T. Nakamura et al. in Astrophys.J. 487 (1997) L139-L142

Formula for the N of cycles $x \equiv v^2 \equiv (M\omega_s)^{2/3} = (\pi M f_{GW})^{2/3}$

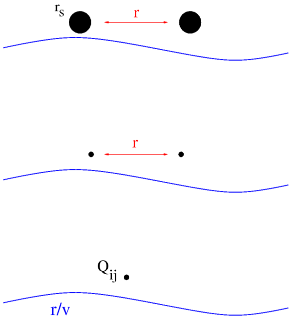
$$N = 2\int_{v_0}^{v} \omega(v')\frac{dE/dv'(v')}{F(v)}dv' = \frac{2\eta G_N M}{\eta^2 G_N M}\int \frac{1}{v^6}\left(1 + \# v^2 + \ldots\right) \propto \frac{1}{\eta}v^{-5}$$

$$v = 0.12\left(\frac{M}{10M_\odot}\frac{f_{GW}}{10\mathrm{Hz}}\right)^{1/3} \implies v^{-5} = 4.7\times 10^4\left(\frac{M}{10M_\odot}\frac{f_{GW}}{10\mathrm{Hz}}\right)^{-5/3} 5\times 10^4$$

The EFT point of view on the 2-body problem

Different scales in EFT (borrowing ideas from NRQCD):

- Very short distance $\lesssim r_s = G_N M$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $H_{\mu\nu}$, $k_\mu \sim (v/r, 1/r)$ with $r \sim r_s/v^2$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ GWs $h^{GW}_{\mu\nu}$ coupled to point particles with moments



M. Beneke and V. A. Smirnov, NPB '98; I. Stewart and W. Manohar, PRD '07; W. Goldberger and I. Rothstein, PRD '06

EFT for compact binary systems

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x) \exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m\int d\tau \\ S_{EH} &= \frac{1}{32\pi G_N}\int d^4x\left[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2\ldots\right] \end{aligned}$$

EFT for compact binary systems

Fundamental

- Fundamental gravitational fields
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$$\begin{aligned} S_{pp} &= -m \int dt\, d^3x\, \delta(\vec{x} - \vec{x}_a(t)) \left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{32\pi G_N} \int d^4x \left[(\partial_i h)^2 - (\partial_t h)^2 + h(\partial h)^2 + h^2(\partial h)^2 \ldots\right] \end{aligned}$$

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$$\begin{aligned} S_{pp} &= -m \int dt\, d^3x\, \delta(\vec{x} - \vec{x}_a(t)) \left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{2}\int d^4x \left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N} h (\partial h)^2 + G_N h^2 (\partial h)^2 \ldots\right] \end{aligned}$$

EFT for compact binary systems

Fundamental

- Fundamental gravitational fields
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Effective

- Generic v-dependent potential terms
- Instantaneous couplings between world-line particles

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x)\exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m\int dt\, d^3x\,\delta(\vec{x}-\vec{x}_a(t))\left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{2}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right] \end{aligned}$$

$$S_{eff} = m_1\int dt\, d^3x\left[\frac{1}{2}v_1^2 + \frac{G_N m_2}{2r} + \frac{1}{8}v_1^4 + \frac{G_N^2 m_2}{2r^2}\left(\frac{G_N m_1}{2r} + v_1^2 + v_{1r}v_{2r}\right) + 1 \leftrightarrow 2\right]$$

3PN: Jaranowski, Schäfer, Damour; Blanchet, Faye; Itoh Futamase; Foffa & RS
4PN: Jaranowski, Schäfer, Damour; Blanchet et al, RS Foffa; RS Foffa, Mastrolia, Sturm

Fundamental Gravity tests at cosmological scale

GWs travel at the speed of light, with deviations $\lesssim 10^{15}$
Model of dark energy/modified gravity with a single scalar degree of freedom drastically constrained

$$\downarrow$$

standard conformal coupling to the Ricci scalar (for Horndeski theories)

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EM/ν from binary black hole coalescences?

BHs with accretion disks may lead to relativistic outflows with TeV - PeV ν (hadrons accelerated by jets, if magnetic fields and long-lived debris from BH progenitor, or dense gaseous environment) exotic but not impossible

e.g. R. Perna et al. Ap.J. 821 (2016) 1, L18, I. Bartos et al., Ap.J. 835 (2017) 2, 165

- GW150914:
  - γ, X optical, radio counterparts not found

    Ap.J. 826 (2016) 1, L13
  - ν: $E \gg O(100)$ GeV, time window ± 500 sec, coincidences $\sim$ backg. Limit on ν spectral fluence (10% - 90%): $< 10^{51} - 10^{54}$ erg

    LIGO, ANTARES and ICECube Phys.Rev. D93 (2016) no.12, 122010
- GW151226 (LVT151012): $E_{iso} < 2 \times 10^{51} - 2 \times 10^{54}$ erg

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  ANTARES Eur.Phys.J. C77 (2017) 12, 911

For reference: $E_{GW150914} \sim 10^{54}$, long(short) GRB $E_{iso} \sim 10^{51}$, $(10^{49}$ erg)

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