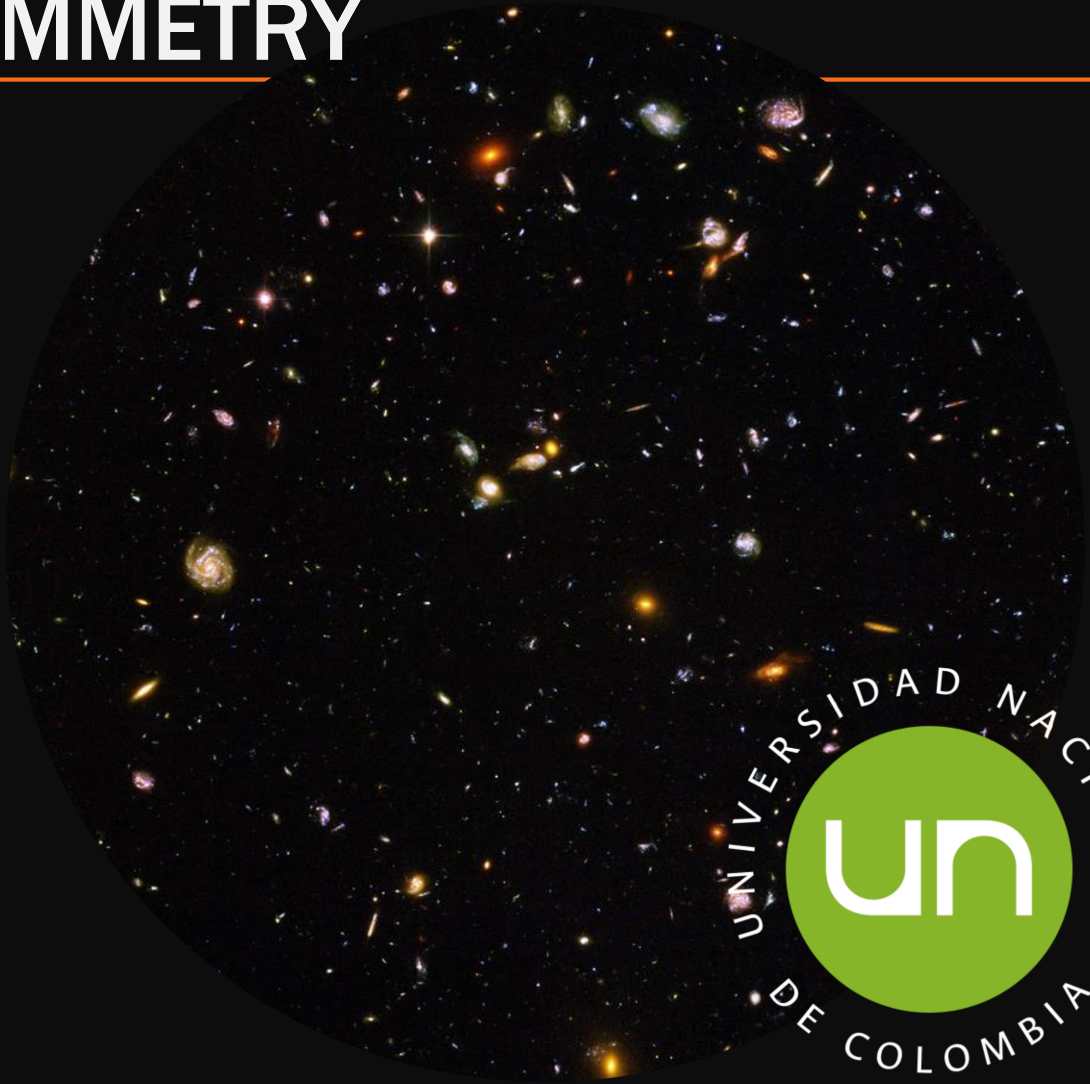
SPONTANEOUS BREAKING AND FLAT DIRECTIONS IN SUPERSYMMETRY

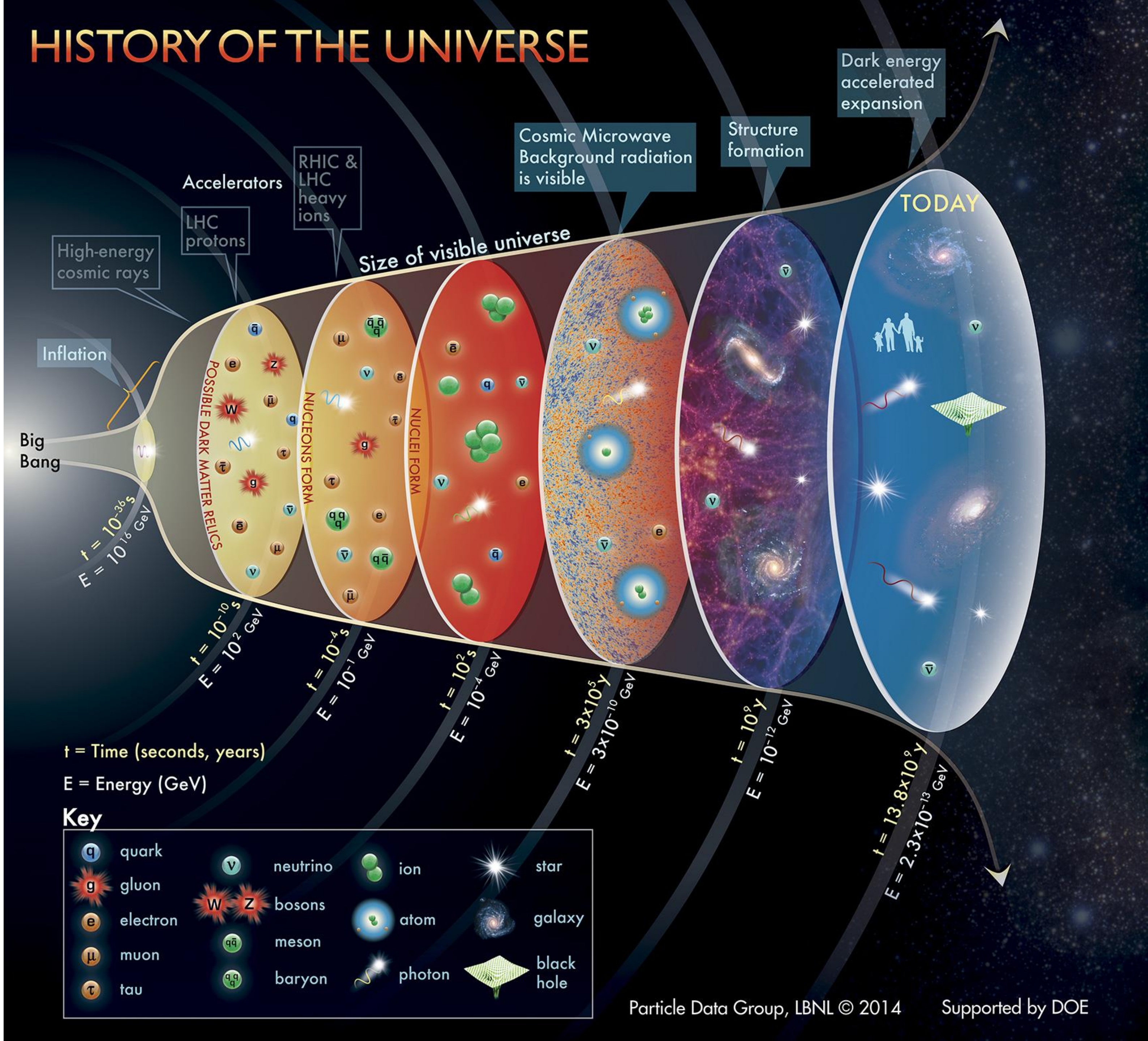


Diana García Sandoval

Universidad Nacional de Colombia

magarciasa@unal.edu.co

INFLATION


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
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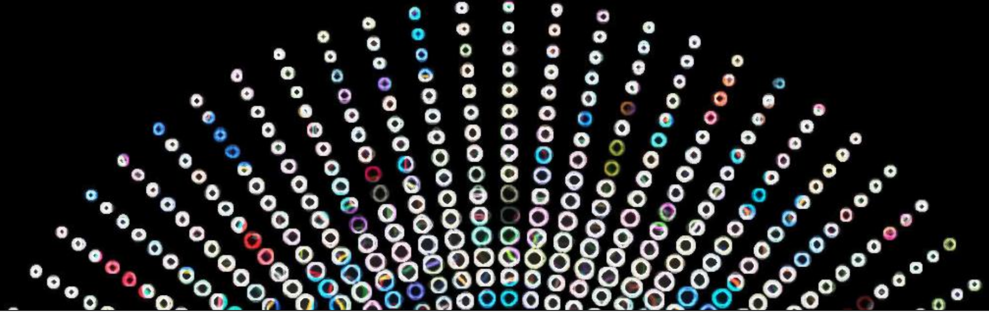
INFLATION



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$$\frac{d^2a}{dt^2} > 0$$

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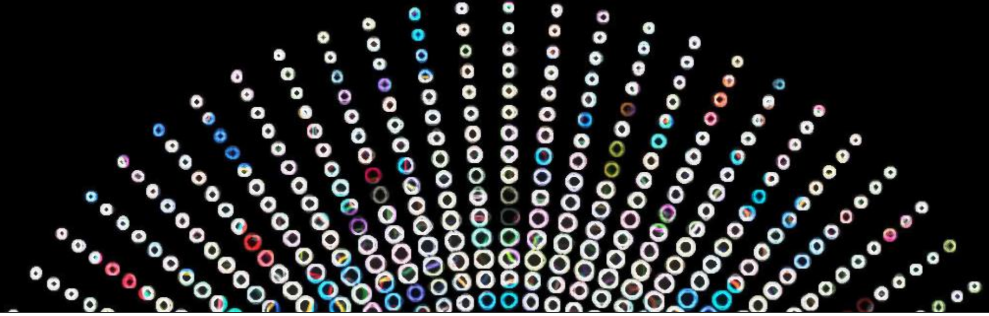

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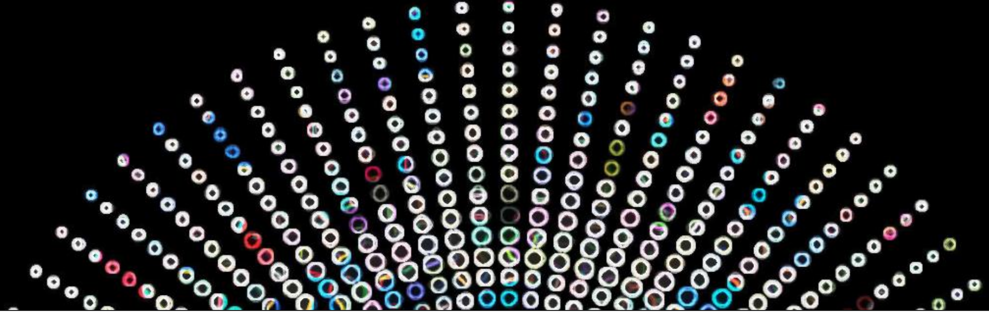
Negative Pressure

$$p < -\frac{1}{3}\rho$$

INFLATION



Inflaton → Scalar field

Action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

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$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

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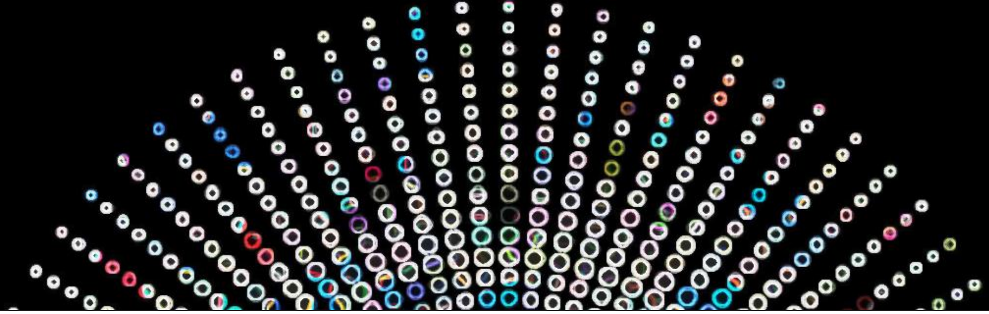

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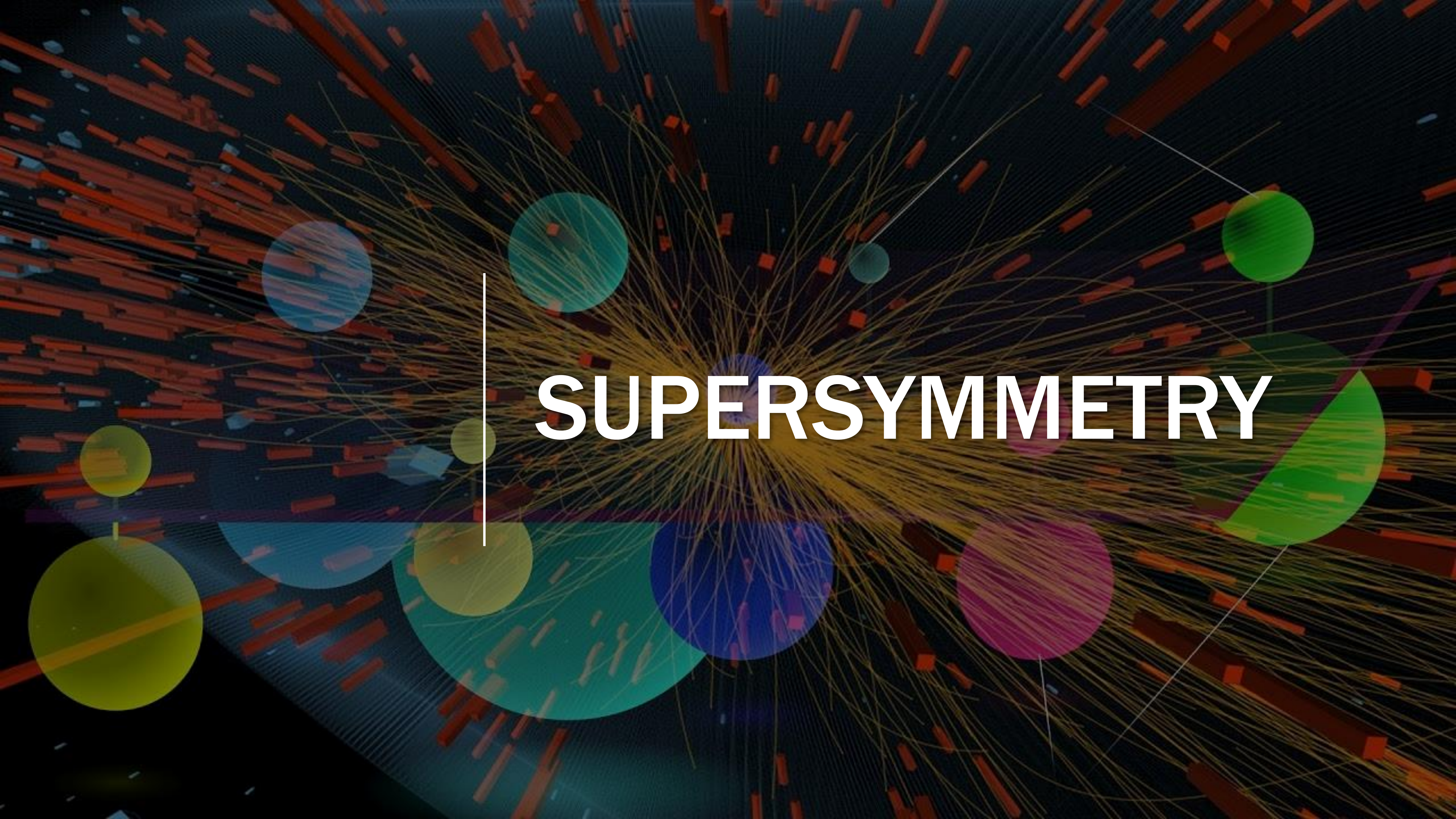
Asymptotic behaviour:

$V(\phi = 0) \approx \text{constant}$

$\text{Large} \quad V(\phi_{min})$

Small coupling parameters



SUPERSYMMETRY

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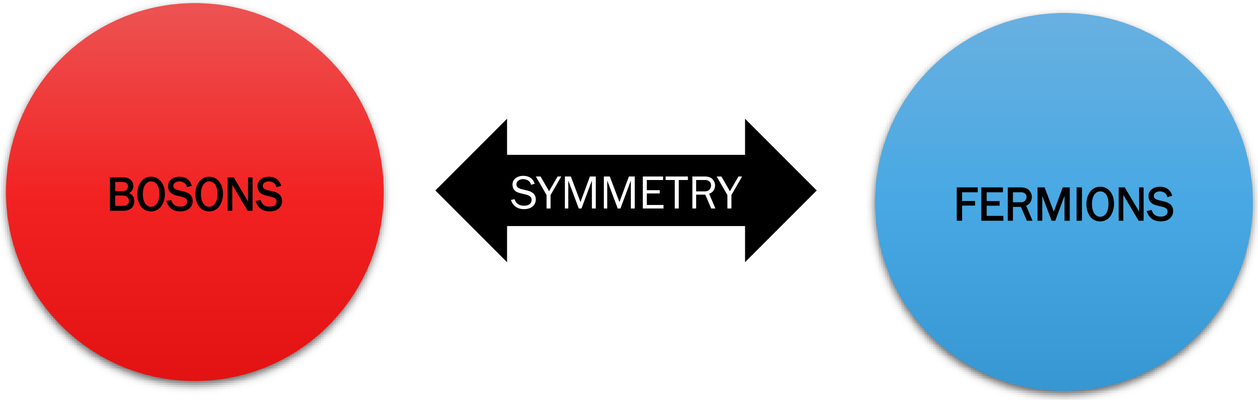
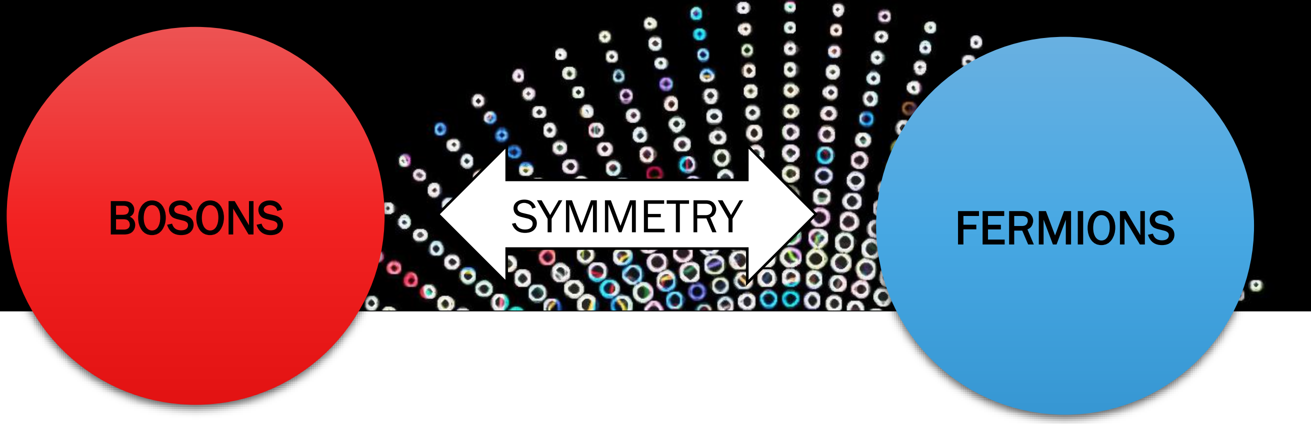


BOSONS

SYMMETRY

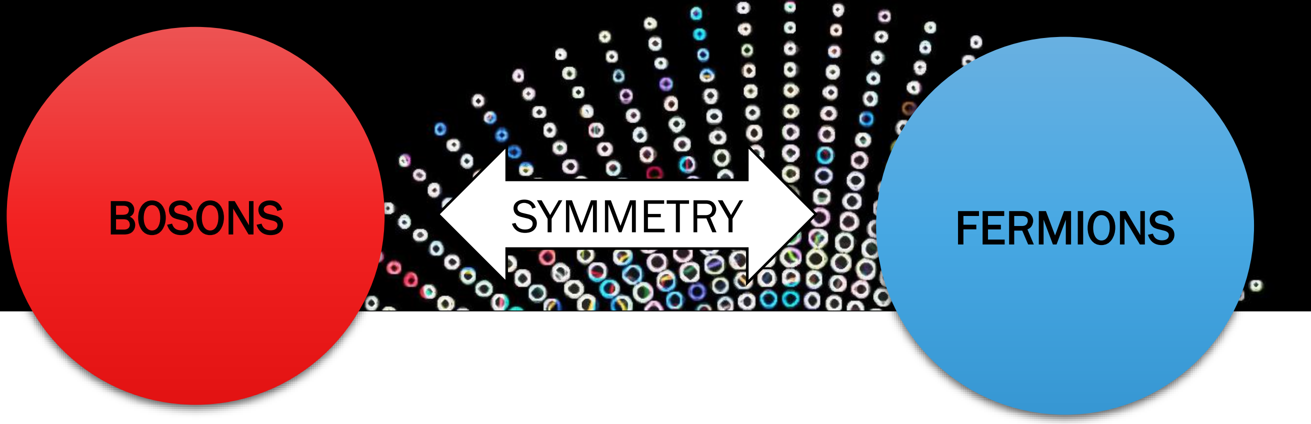
FERMIONS

SUPERSYMMETRY



Given any generator $G = a^\dagger * K * a,$

SUPERSYMMETRY



BOSONS ⟷ SYMMETRY ⟷ FERMIONS

Given any generator $G = B + Q;$

$$B = b^\dagger * K_{bb} * b + q^\dagger * K_{qq} * q;$$

B → B F → F

INTERNAL SYMMETRY

$$Q = q^\dagger * K_{qb} * b + b^\dagger * K_{bq} * q;$$

B → F F → B

SUPERSYMMETRY

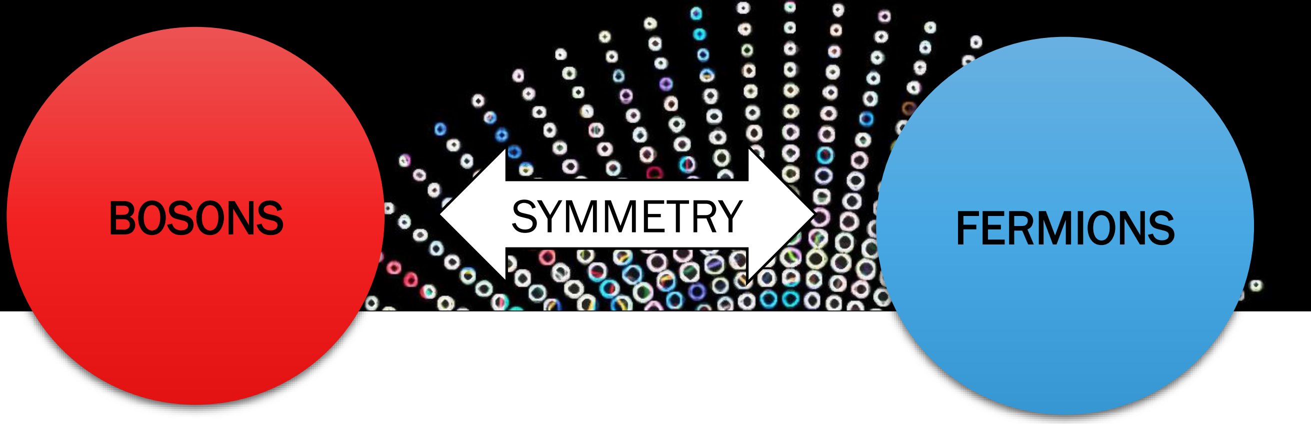
$$to$$

$$[b_i(\mathbf{p}), b_j^\dagger(\mathbf{q})] = \delta_{ij}\delta^3(\mathbf{p}-\mathbf{q}) \; ; \quad [b,b] = [b^\dagger, b^\dagger] = 0$$

$$\{q_i(\mathbf{p}), q_j^\dagger(\mathbf{q})\} = \delta_{ij}\delta^3(\mathbf{p}-\mathbf{q}) \; ; \quad \{q,q\} = \{q^\dagger, q^\dagger\} = 0$$

$$[b,q] = [b,q^\dagger] = [b^\dagger, q] = [b^\dagger, q^\dagger] = 0 \qquad (2)$$

SUPERSYMMETRY



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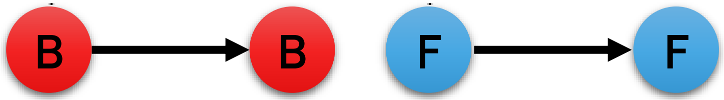
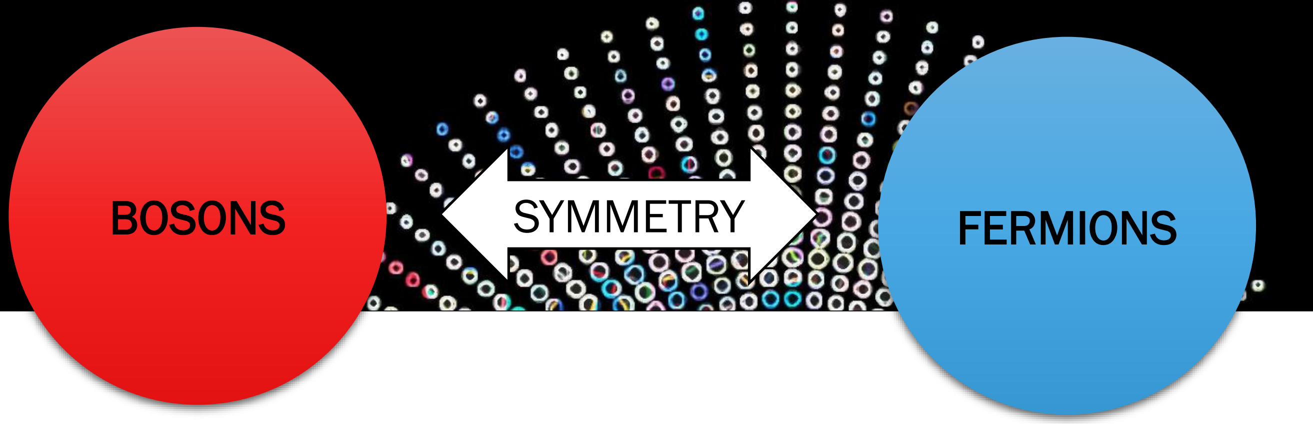


INTERNAL SYMMETRY

SUPERSYMMETRY



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Given any generator $G = B + Q;$

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INTERNAL SYMMETRY

$$Q = q^\dagger * K_{qb} * b + b^\dagger * K_{bq} * q;$$

SUPERSYMMETRY

Graded Lie algebra:

$$\begin{aligned}
[B_i, B_j] &= ic_{ij}^k B_k \\
[Q_\alpha, B_i] &= s_{\alpha i}^\beta Q_\beta \\
\{Q_\alpha, Q_\beta\} &= \gamma_{\alpha\beta}^i B_i .
\end{aligned}$$

$$\begin{aligned}
[[B_i, B_j], B_k] + [[B_j, B_k], B_i] + [[B_k, B_i], B_j] &= 0 \\
[[Q_\alpha, B_i], B_j] + [[B_i, B_j], Q_\alpha] + [[B_j, Q_\alpha], B_i] &= 0 \\
[\{Q_\alpha, Q_\beta\}, B_i] + \{[B_i, Q_\alpha], Q_\beta\} - \{[Q_\beta, B_i], Q_\alpha\} &= 0 \\
[\{Q_\alpha, Q_\beta\}, Q_\gamma] + [\{Q_\gamma, Q_\alpha\}, Q_\beta] + [\{Q_\beta, Q_\gamma\}, Q_\alpha] &= 0
\end{aligned}$$

COLEMAN-MANDULA THEOREM

Given G a group of **bosonic** symmetry. If:

1. G contains a subgroup locally isomorphic to Poincaré
2. For M>0, particles with m<M are finite.
3. Non trivial and analytic S-matrix

G is a direct product of Poincaré and an internal symmetry group.

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[P_\mu, M_{\nu\sigma}] &= i(\eta_{\mu\nu}P_\sigma - \eta_{\mu\sigma}P_\nu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\rho}M_{\nu\sigma} + \eta_{\mu\sigma}M_{\nu\rho}) \\
[B_i, B_j] &= ic_{ij}^k B_k \\
[B_i, P_\mu] &= [B_i, M_{\mu\nu}] = 0
\end{aligned}$$

HAAG-LOPUSZANSKI-SOHNIUS THEOREM

The generators of supersymmetry satisfy the positive metric condition

$$\langle ..|\{Q,Q^\dagger\}|..\rangle = |Q^\dagger|..\rangle|^2 + |Q|..\rangle|^2 > 0 \quad Q \neq 0$$

And belong to the representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ of Lorentz group.

$$\begin{array}{rcl}
[Q_{\alpha i}, M_{\mu\nu}] & = & \frac{1}{2}(\sigma_{\mu\nu})_{\alpha}^{\beta} Q_{\beta i} \quad , \quad [\bar{Q}^i_{\dot{\alpha}}, M_{\mu\nu}] = -\bar{Q}^i_{\dot{\beta}}\frac{1}{2}(\sigma_{\mu\nu})_{\alpha}^{\beta} \\
\{Q_{\alpha i}, \bar{Q}^i_{\dot{\beta}}\} & = & 2\delta_i^j(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu \\
[Q_{\alpha i}, P_\mu] & = & [\bar{Q}^i_{\dot{\alpha}}, P_\mu] = 0 \\
[Q_{\alpha i}, B_r] & = & (b_r)_i^j Q_{\alpha j} \quad , \quad [\bar{Q}^i_{\dot{\alpha}}, B_r] = -\bar{Q}^j_{\dot{\alpha}}(b_r)_j^i \\
\{Q_{\alpha i}, Q_{\beta j}\} & = & 2\epsilon_{\alpha\beta}Z_{ij} \quad , \quad Z_{ij} = a^r_{ij}B_r \\
\{\bar{Q}^i_{\dot{\alpha}}, \bar{Q}^j_{\dot{\beta}}\} & = & -2\epsilon_{\dot{\alpha}\dot{\beta}}Z^{ij} \quad , \quad Z^{ij} = (Z_{ij})^\dagger \\
[Z_{ij}, \text{any generator}] & = & 0.
\end{array}$$

GRASSMAN NUMBERS

$$\{a_i, a_j\} = 0 \quad \forall i, j = 1, ..., n.$$

$$\frac{\partial a_i}{\partial a_j} = \delta_{ij}$$

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Grassman Variables: $\text{Weyl spinors } \theta \text{ y } \bar{\theta}$

$$\partial_\alpha := \frac{\partial}{\partial \theta^\alpha}, \qquad \partial^\alpha := \frac{\partial}{\partial \theta_\alpha}$$

$$\bar{\partial}_{\dot{\alpha}} := \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}, \qquad \bar{\partial}^{\dot{\alpha}} := \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}}$$

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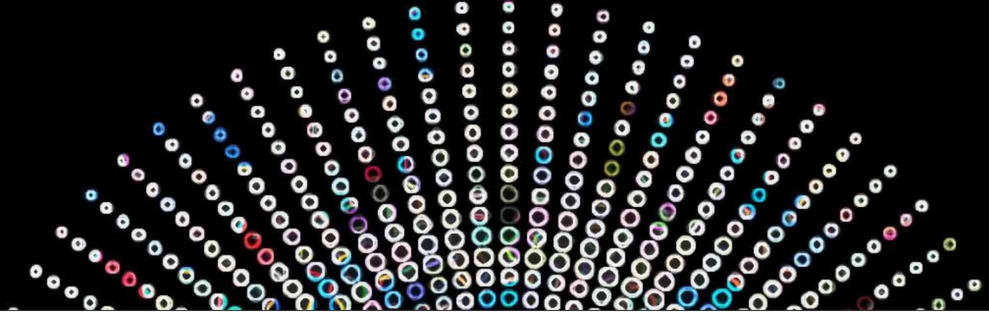
$$\int d\theta \equiv 0 \qquad \int \theta d\theta \equiv 1$$

GRADED ALGEBRA → **LIE ALGEBRA**

$$Q, \quad \bar{Q}$$

$$\theta Q, \quad \bar{Q}\bar{\theta}$$

REPRESENTATIONS ON SUPERFIELDS



Superspace

Symmetry Group:
Super-Poincaré

$$g = e^{ix\cdot P + \theta Q + i\bar{Q}\bar{\theta} + \frac{i}{2}\lambda\cdot M}.$$

$$L(x,\theta,\bar{\theta}) = e^{ix\cdot P + \theta Q + i\bar{Q}\bar{\theta}}$$

$$\phi(x) = L(x)\phi(0)L^{-1}(x)$$

8 coordinates: 4 bosonic, 4 Fermionic

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Most general Superfield:

$$\begin{aligned} V(x,\theta,\bar{\theta}) \quad &= \quad C - i\theta\chi + i\bar{\xi}\bar{\theta} - \frac{i}{2}\theta^2(M - iN) + \frac{i}{2}\bar{\theta}^2(M + iN) - \theta\sigma^\mu\bar{\theta}A_\mu \\ &\quad + i\bar{\theta}^2\theta(\lambda - \frac{i}{2}\partial\bar{\xi}) - i\theta^2\bar{\theta}(\bar{\kappa} - \frac{i}{2}\partial\chi) - \frac{1}{2}\theta^2\bar{\theta}^2(D + \frac{1}{2}\Box C) \end{aligned}$$

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Real Vector Superfield:

$$\left[V(x,\theta,\bar{\theta})\right]^{\dagger} = V(x,\theta,\bar{\theta})$$

$$\mathrm{C}{=}\mathrm{C}^{\dagger}, \quad M = M^{\dagger}, \quad N = N^{\dagger}, \quad D = D^{\dagger}, \quad \bar{\xi} = \chi^{\dagger}, \quad \bar{\kappa} = \lambda^{\dagger}$$

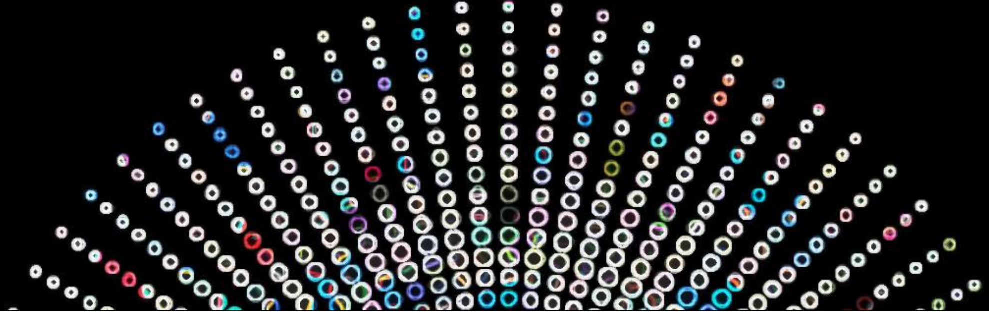
$$\mathrm{V}(\mathrm{x},\theta,\bar{\theta}) = C - i\theta\chi + i\bar{\chi}\bar{\theta} - \frac{i}{2}\theta^2(M - iN) + \frac{i}{2}\bar{\theta}^2(M + iN) - \theta\sigma^{\mu}\bar{\theta}A_{\mu}$$
$$+\mathrm{i}^2\theta(\lambda - \frac{i}{2}\partial\bar{\chi}) - i\theta^2\bar{\theta}(\bar{\lambda} - \frac{i}{2}\partial\chi) - \frac{1}{2}\theta^2\bar{\theta}^2(D + \frac{1}{2}\Box C).$$

REPRESENTATIONS ON SUPERFIELDS

Covariant derivatives

$$
\begin{aligned}
D_\mu &= \partial_\mu, \\
D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \\
\bar D_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu,
\end{aligned}
$$

REPRESENTATIONS ON SUPERFIELDS



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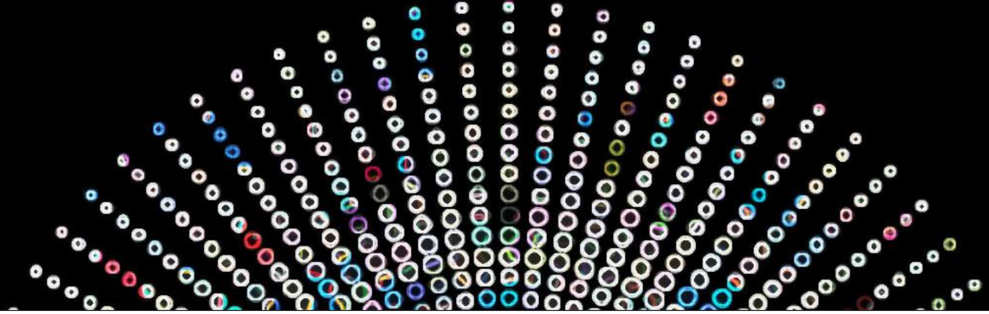
Chiral Field

$$\bar{D}_{\dot{\alpha}}\Phi = 0$$

$$\begin{aligned} \Phi(x,\theta,\bar{\theta}) &= \exp(-i\theta\partial\bar{\theta})\phi(x,\theta) \\ \phi(x,\theta) &= A + 2\theta\psi - \theta^2 F \end{aligned}$$

REPRESENTATIONS ON SUPERFIELDS



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Anti-Chiral Field

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Kinetic Field

$$T\phi = \frac{1}{4}\bar{D}^2\bar{\phi}$$

$$TT = -\Box$$

Chiral Field

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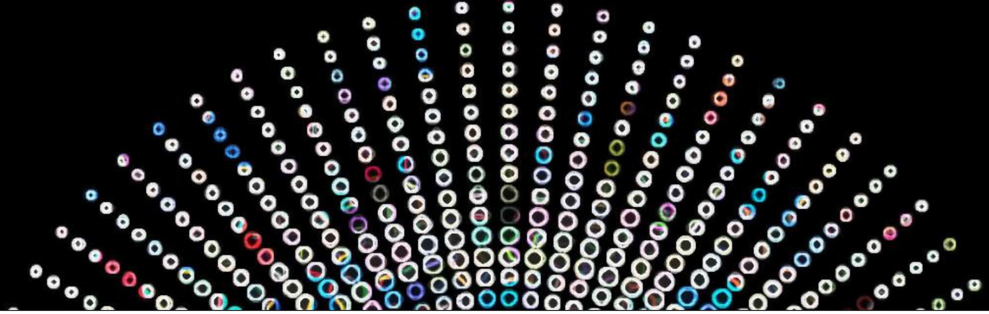
LAGRANGIANS



$$\begin{aligned}
f(\theta) &= f_0 + f_1\theta \\
\int d\theta f(\theta) &= \int d\theta f_0 + \int d\theta\theta f_1 = f_1 \\
\int d\theta_1 d\theta_2 f(\theta_1, \theta_2) &= \int d\theta_1 \int d\theta_2 (f_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_1\theta_2 f_{12}) \\
&= \int d\theta_1 \int d\theta_2 \theta_1 \theta_2 f_{12} = f_{12}
\end{aligned}$$

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INTEGRATION OVER GRASSMAN VARIABLES = PROYECTION OF HIGHEST ORDER COMPONENT

LAGRANGIANS

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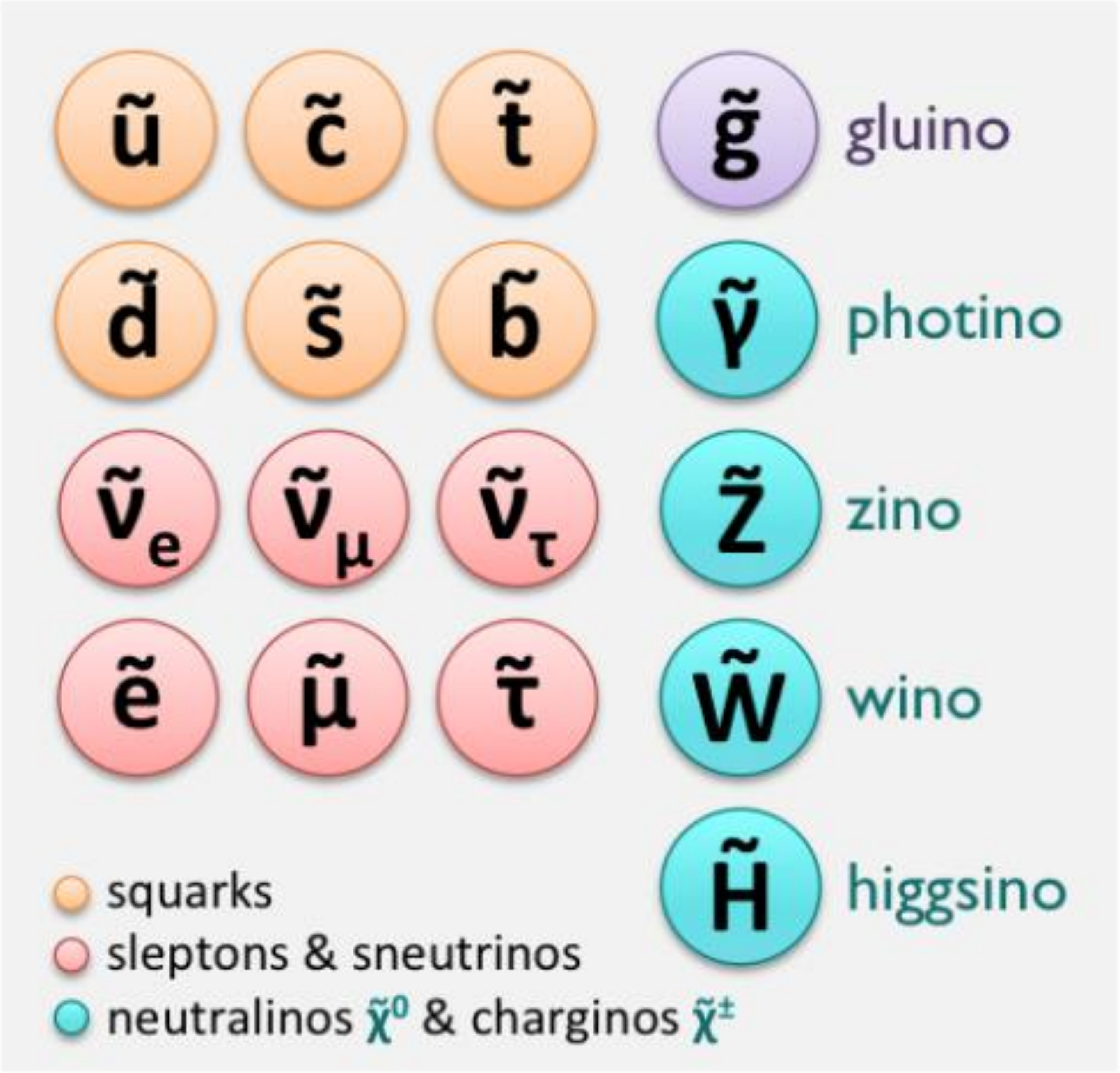
$$\boxed{\begin{aligned} \frac{1}{4}\int d^2\theta\phi + \text{h.c.} &= [\phi]_F + \text{4-div} \\ -\frac{1}{2}\int d^4x d^2\theta d^2\bar{\theta} V &= [V]_D + \text{4-div} \end{aligned}}$$

INTEGRATION OVER GRASSMAN VARIABLES = PROYECTION OF HIGHEST ORDER COMPONENT

Standard Model particles

Supersymmetric partners



SUPERSYMMETRY BREAKING

$$\langle ..|\{Q, Q^\dagger\}|..\rangle = |Q^\dagger|..\rangle|^2 + |Q|..\rangle|^2 > 0 \quad Q \neq 0$$

$$\{Q_{\alpha i}, \bar{Q}^i_{\dot{\beta}}\} \quad = \quad 2\delta_i^j (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$E_{min} = \langle 0|E|0\rangle = \frac{1}{4}\sum_{\alpha=1}^{4} |Q_\alpha|0\rangle|^2 = \langle 0|U|0\rangle.$$

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SUPERSYMETRY BREAKING

↕

SCALAR POTENTIAL V.E.V
STRICTELY POSITIVE

SUPERSYMMETRY BREAKING

$$\langle ..|\{Q,Q^\dagger\}|..\rangle = |Q^\dagger|..\rangle|^2 + |Q|..\rangle|^2 > 0 \quad Q \neq 0$$

$$\{Q_{\alpha i}, \bar{Q}^i_{\dot{\beta}}\} \quad = \quad 2\delta^j_i (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$E_{min} = \langle 0|E|0\rangle = \frac{1}{4}\sum_{\alpha=1}^{4} |Q_\alpha|0\rangle|^2 = \langle 0|U|0\rangle.$$

SUPERSYMMETRY BREAKING

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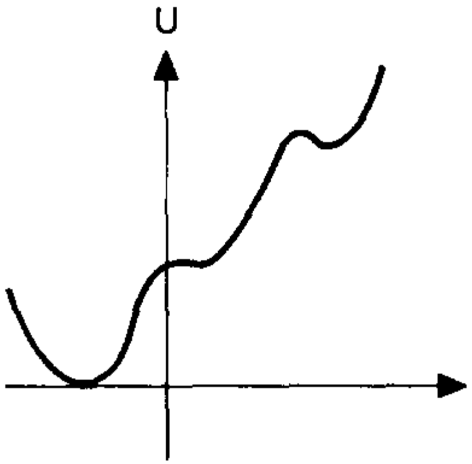
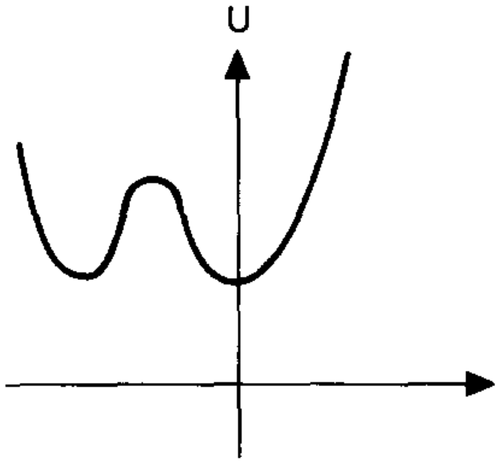
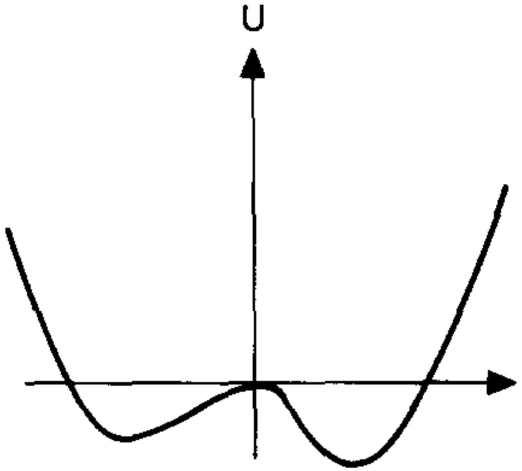
SCALAR POTENTIAL V.E.V
STRICTELY POSITIVE



supersymmetric
ground state



non-supersymmetric
ground state



non-supersymmetric
potential

WESS-ZUMINO'S MODEL

$$\mathcal{L} = \overbrace{\frac{1}{2}(\phi \cdot T\phi)_F}^{\text{Kinetic}} - \overbrace{[V(\phi)]_F}^{\text{Superpotential}}$$

$$V(\phi) = \lambda\phi + \frac{m}{2}\phi \cdot \phi + \frac{g}{3}\phi \cdot \phi \cdot \phi$$

WESS-ZUMINO'S MODEL

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Equations of motion $T\phi = V'(\phi)$

$$\left.\begin{aligned}F &= mA + g(A^2 - B^2) \\ G &= mB + 2gAB\end{aligned}\right\} \text{Auxiliary Fields}$$

$$\begin{aligned}i\partial\psi &= m\psi + 2g(A - \gamma_5 B)\psi \\ -\Box A &= mF + 2g\left(AF + BG + \frac{1}{2}\bar{\psi}\psi\right) \\ -\Box B &= mG + 2g\left(AG - BF - \frac{1}{2}\bar{\psi}\gamma_5\psi\right)\end{aligned}$$

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$$V(\phi) = \lambda\phi + \frac{m}{2}\phi\cdot\phi + \frac{g}{3}\phi\cdot\phi\cdot\phi$$

Scalar Potential

$$U = \frac{1}{2}(F^2 + G^2) + \frac{1}{2}\bar{\psi}[V'(\phi)]_\psi$$

$$U_{\min} = U(F = G = 0) \longleftrightarrow U_{\min} = 0$$

Equations of motion $T\phi = V'(\phi)$

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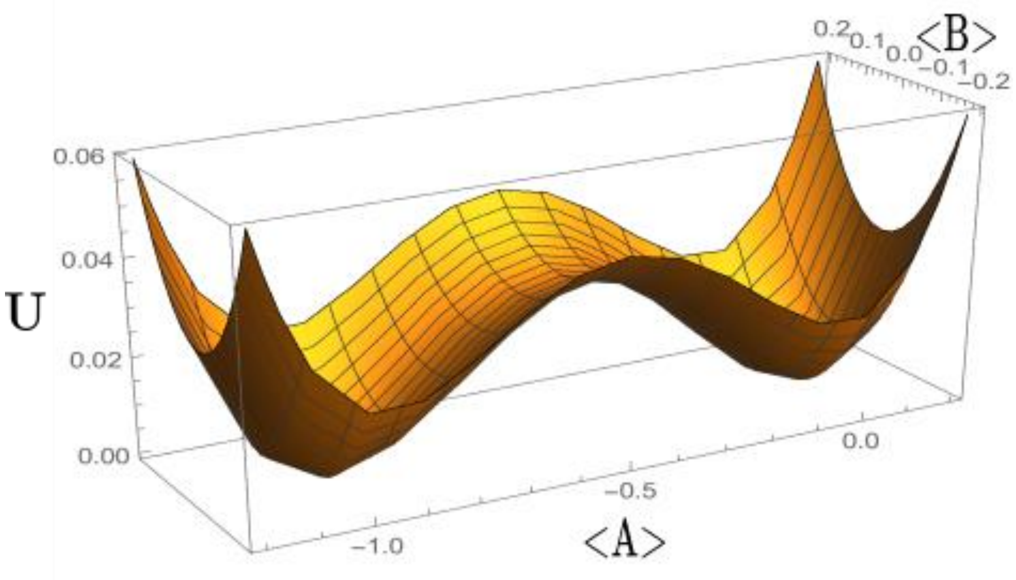
SUPERSYMMETRY REMAINS UNBROKEN

WESS-ZUMINO'S MODEL

$$\begin{cases} m^2 \geq 4g\lambda \\ \langle A \rangle = -\frac{1}{2g}(m \pm \sqrt{m^2 - 4g\lambda}) \\ \langle B \rangle = 0 \end{cases}$$



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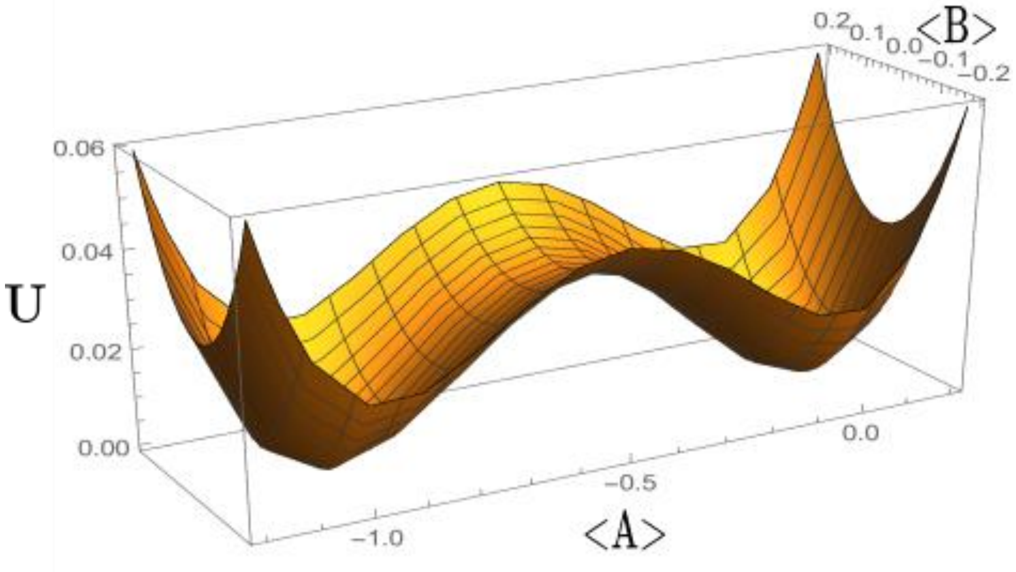
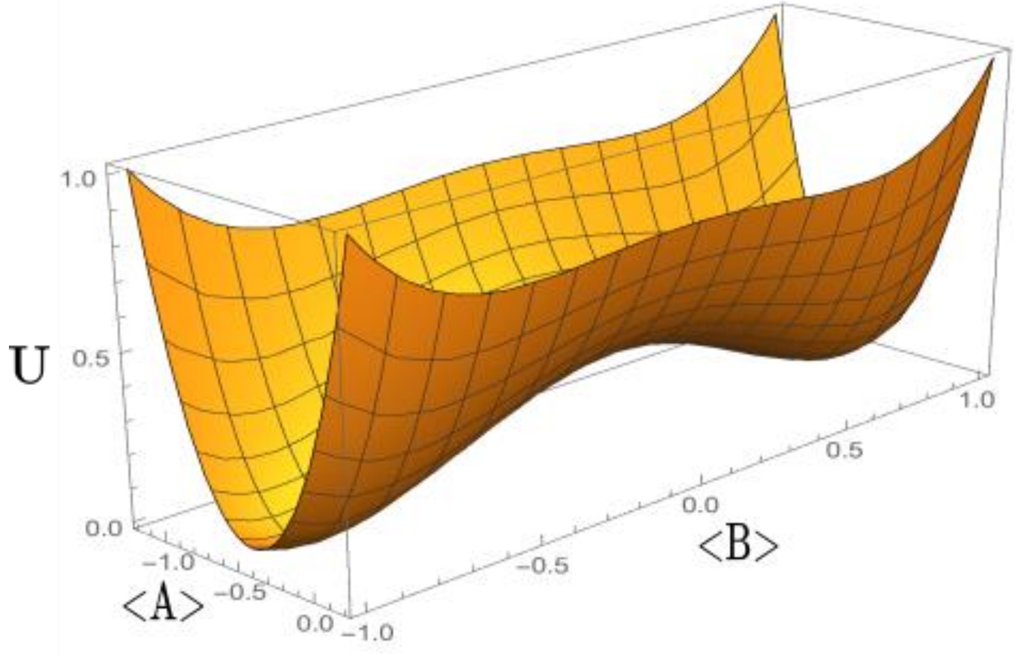
$$\begin{cases} m^2 \leq 4g\lambda \\ \langle A \rangle = -\frac{m}{2g} \\ \langle B \rangle = \pm\frac{1}{2g}(\sqrt{-m^2 - 4g\lambda}) \end{cases}$$





O’RAIFEARTAIGH’S MODEL

$$\mathcal{L} = \overbrace{\left(\frac{1}{2}\phi_a \cdot T\phi_a\right)_F}^{\text{Kinetic}} - \overbrace{(V)_F}^{\text{Superpotential}}$$

$$V = \lambda_a\phi_a + \frac{1}{2}m_{ab}\phi_a \cdot \phi_b + \frac{1}{3}g_{abc}\phi_a \cdot \phi_b \cdot \phi_c$$

Scalar Potential

$$U = \frac{1}{2}F_aF_a + \frac{1}{2}G_aG_a + \frac{1}{2}\bar{\psi}_a\left[\frac{\partial}{\partial\phi_a}V\right]_\phi .$$

Unbroken supersymmetry: $\langle F\rangle_a = \langle G\rangle_a = 0$

O’RAIFEARTAIGH’S MODEL

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Unbroken supersymmetry: $\langle F\rangle_a = \langle G\rangle_a = 0$

$$\lambda_3 = \lambda, \quad m_{12} = m_{21} = m, \quad g_{113} = g_{131} = g_{311} = g$$

Equations of motion

$$\begin{aligned}
F_1 &= mA_2 + 2g(A_3A_1 - B_3B_1)\\
G_1 &= mB_2 + 2g(A_3B_1 + B_3A_1)\\
F_2 &= mA_1\\
G_2 &= mB_1\\
F_3 &= \lambda + g(A_1^2 - B_1^2),\\
G_3 &= 2gA_1B_1
\end{aligned}$$

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F_1 &= mA_2 + 2g(A_3A_1 - B_3B_1) \\
G_1 &= mB_2 + 2g(A_3B_1 + B_3A_1) \\
F_2 &= mA_1 \\
G_2 &= mB_1 \\
F_3 &= \lambda + g(A_1^2 - B_1^2), \\
G_3 &= 2gA_1B_1
\end{aligned}$$

$$2U_{min} = \lambda^2 \quad \text{for } |2g\lambda| \leq m^2$$

$$\langle A_1\rangle = \langle A_2\rangle = \langle B_1\rangle = \langle B_2\rangle = 0$$

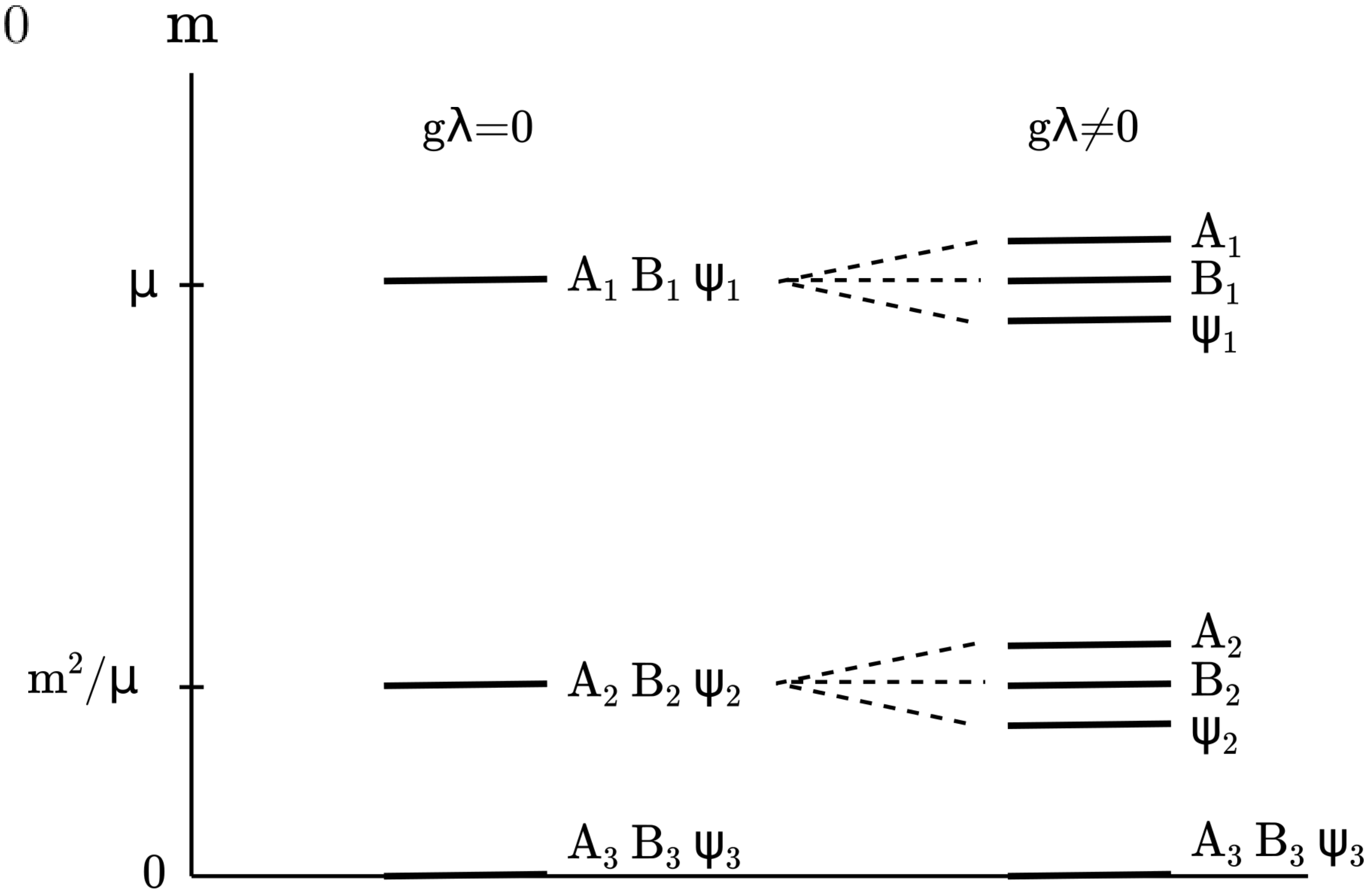
SUPERSYMMETRY BREAKS

O’RAIFEARTAIGH’S MODEL

$$\langle A_3 \rangle = \mu/2g\,, \quad \langle B_3 \rangle = 0$$

Large Mass Scale

$$\sqrt{|2g\lambda|} \leq |m| << \mu$$



FAYET-ILLIOPOULOS’S MODEL

$$\begin{aligned}
L_{\mathrm{FI}} \quad = \quad & -\frac{1}{2}\sum_{i=1}^{2}\Big[(\partial A_i)^2+(\partial B_i)^2-F_i^2-G_i^2+i\bar{\psi}_i\partial\psi_i+m(F_iA_i+G_iB_i-\frac{1}{2}\bar{\psi}_i\psi_i)\Big] \\
& +g\Big[D(A_1B_2-A_2B_1)-A_\mu(A_1\partial^\mu A_2-A_2\partial^\mu A_1+B_1\partial^\mu B_2-B_2\partial^\mu B_1-i\bar{\psi}_1\gamma^\mu\psi_2) \\
& -i\bar{\lambda}\Big((A_1+\gamma_5B_1)\psi_2-(A_2+\gamma_5B_2)\psi_1\Big)\Big]-\frac{1}{2}g^2A_\mu^2(A_1^2+B_1^2+A_2^2+B_2^2) \\
& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}\bar{\lambda}\partial\lambda+\frac{1}{2}D^2+\xi D
\end{aligned}$$

FAYET-ILLIOPOULOS'S MODEL

$$\begin{aligned} L_{\mathrm{FI}} \quad = \quad & -\frac{1}{2}\sum_{i=1}^{2}\left[(\partial A_i)^2 + (\partial B_i)^2 - F_i^2 - G_i^2 + i\bar{\psi}_i\partial\psi_i + m(F_iA_i + G_iB_i - \frac{1}{2}\bar{\psi}_i\psi_i)\right] \\ & + g\Big[D(A_1B_2 - A_2B_1) - A_\mu(A_1\partial^\mu A_2 - A_2\partial^\mu A_1 + B_1\partial^\mu B_2 - B_2\partial^\mu B_1 - i\bar{\psi}_1\gamma^\mu\psi_2) \\ & -i\bar{\lambda}\Big((A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1\Big)\Big] - \frac{1}{2}g^2A_\mu^2(A_1^2 + B_1^2 + A_2^2 + B_2^2) \\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar{\lambda}\partial\lambda + \frac{1}{2}D^2 + \xi D \end{aligned}$$

FAYET-ILLIOPOULOS'S MODEL

$$L_{\rm pot} \;=\; \frac{1}{2}D^2 - g(A_1B_2 - B_1A_2)D + \xi D + \sum_{i=1}^{2}\Big(\frac{1}{2}F_i^2 + \frac{1}{2}G_i^2 - mA_iF_i - mB_iG_i\Big).$$

Equations of motion

$$\begin{aligned} D &= g(A_1B_2 - B_1A_2) - \xi \\ F &= mA_i, \quad G_i = mB_i \end{aligned}$$

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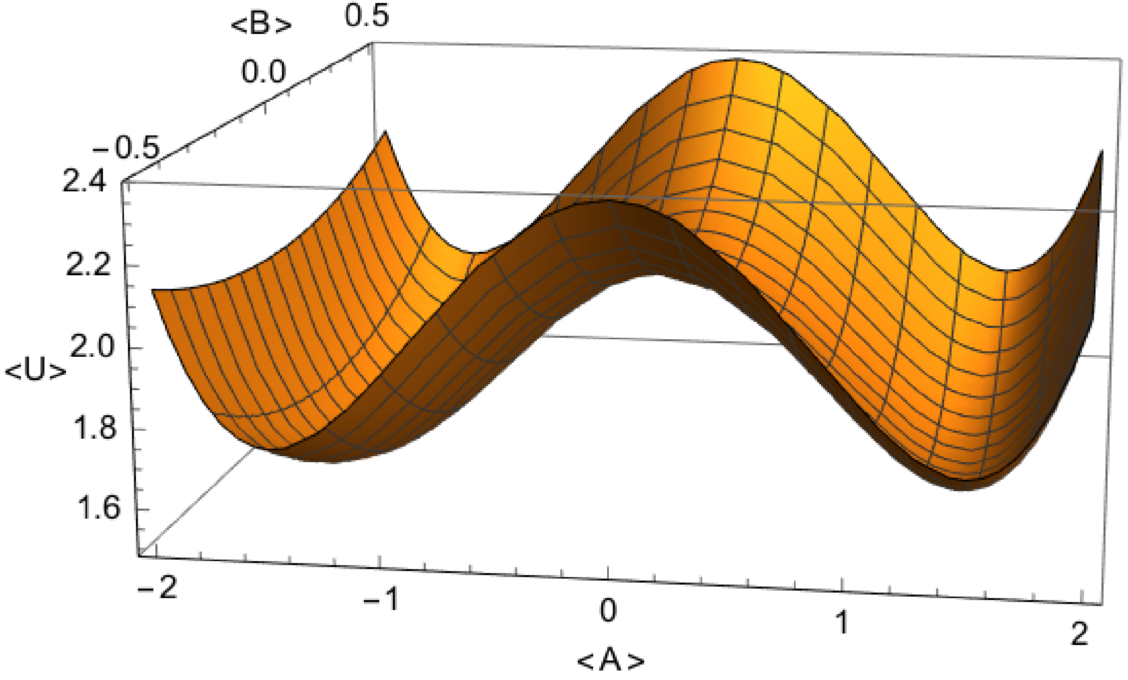
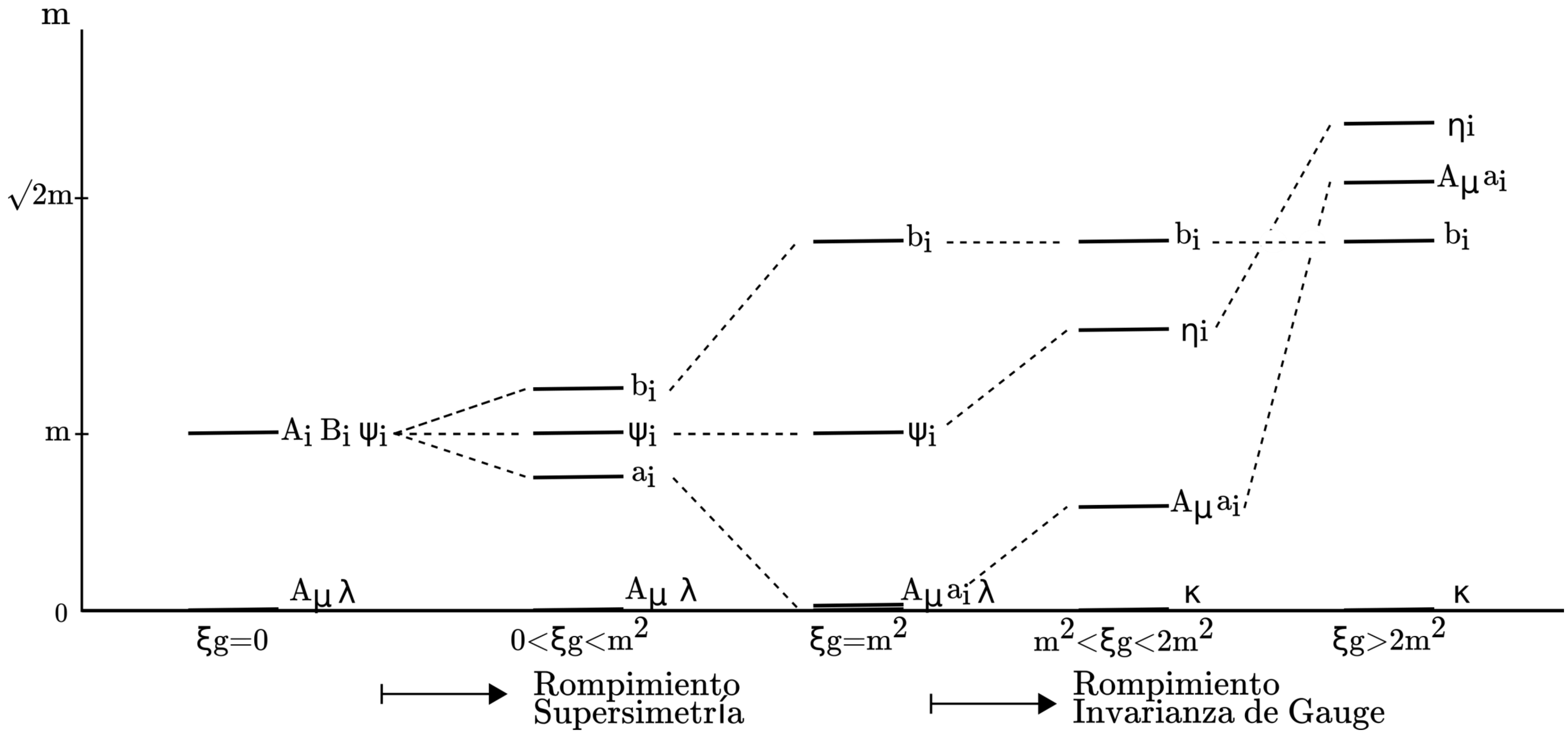
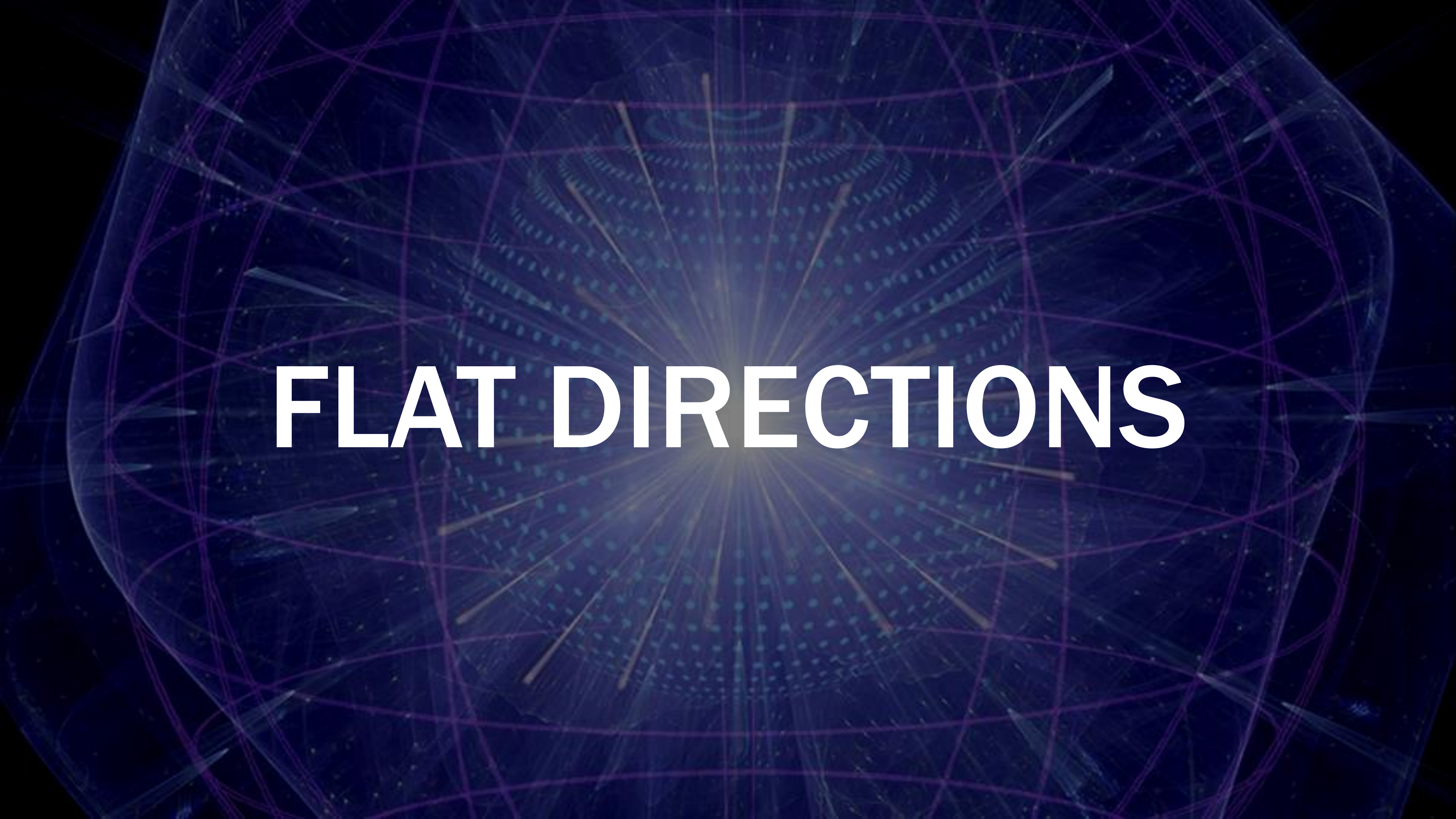
SUPERSYMMETRY IS BROKEN



FAYET-ILLIOPOULOS'S MODEL





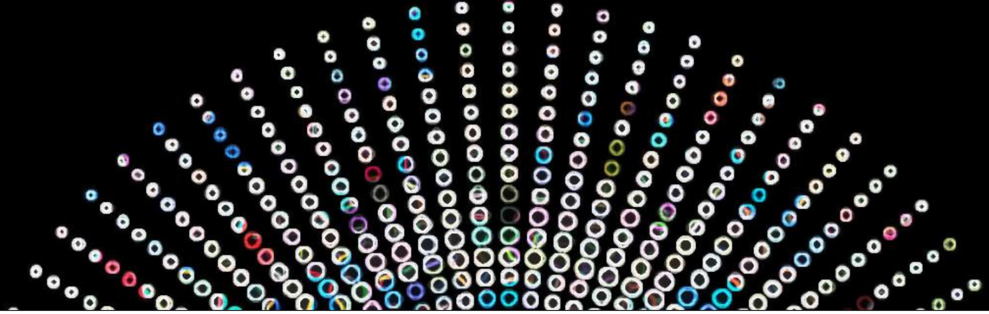
FLAT DIRECTIONS

SUPERFIELD-SINGLET

Superpotential

$$W(\phi, X) = \frac{1}{2} f X \phi^2 - X \mu^2$$

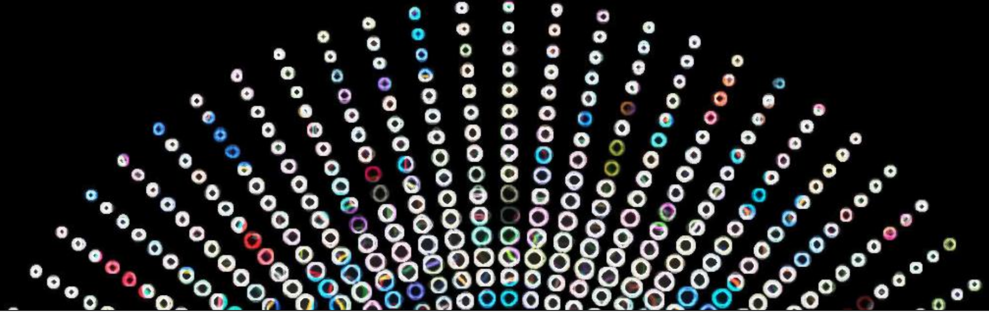
SUPERFIELD-SINGLET



Superpotential

$$W(\phi, X) = \frac{1}{2} f X \phi^2 - X \mu^2 \xrightarrow{\text{F-TERMS}} \frac{\partial W}{\partial \phi} = f X \bar{\phi}, \quad \frac{\partial W}{\partial X} = \frac{1}{2} f \bar{\phi} \phi - \mu^2$$

SUPERFIELD-SINGLET



Superpotential

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Scalar Potential

$$V(\phi, X) = |\frac{1}{2} f \phi^2 - \mu^2|^2 + f^2 |X|^2 |\phi|^2 + D\text{-terms}$$

SUPERFIELD-SINGLET

Superpotential

$$W(\phi, X) = \frac{1}{2} f X \phi^2 - X \mu^2$$

Scalar Potential

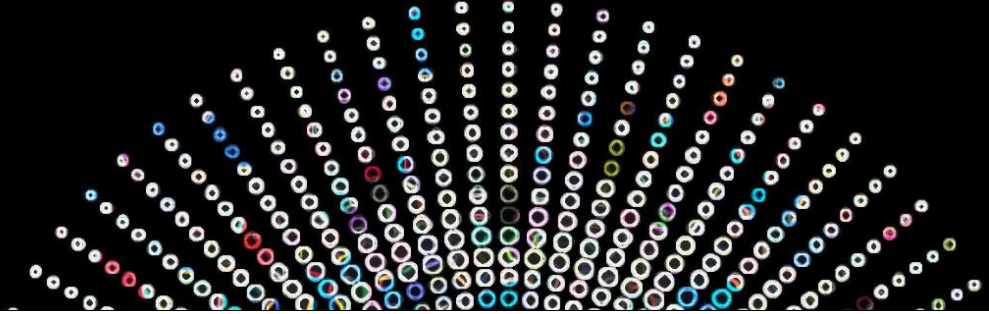
$$V(\phi, X) = |\frac{1}{2} f \phi^2 - \mu^2|^2 + f^2 |X|^2 |\phi|^2 + D\text{-terms}$$

Minimization

$$\frac{\partial V}{\partial \phi} = \left(\frac{1}{2} f \phi^2 - \mu^2\right) f \phi + f^2 X^2 \phi = 0$$

$$\frac{\partial V}{\partial X} = 2 f^2 X \phi^2 = 0$$

SUPERFIELD-SINGLET



Superpotential

$$W(\phi, X) = \frac{1}{2} f X \phi^2 - X\mu^2$$

Scalar Potential

$$V(\phi, X) = |\frac{1}{2} f\phi^2 - \mu^2|^2 + f^2|X|^2|\phi|^2 + D\text{-terms}$$

Minimization

$$\frac{\partial V}{\partial \phi} = \left(\frac{1}{2} f\phi^2 - \mu^2\right) f\phi + f^2 X^2 \phi = 0$$

$$\frac{\partial V}{\partial X} = 2f^2 X \phi^2 = 0$$

$$\phi = 0\,, \quad X > X_c = \mu/\sqrt{f}$$

$$V(0, \mu/\sqrt{f}) = \mu^4$$

Non-Supersymmetric Vacuum

Large curvature $\sim f|X|$

$$\phi^2 = 2\mu^2/f\,, \quad X = 0$$

$$V(\sqrt{2/f}\mu, 0) = 0$$

Supersymmetric Vacuum

Flat direction

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Superpotential

$$W = fS\phi_+\phi_- - \mu^2 S + X(a_+A_+\phi_- + a_-A_-\phi_+)$$

F-TERMS

$$\frac{\partial W}{\partial \phi_+} = fS\phi_- + Xa_-A_-, \quad \frac{\partial W}{\partial \phi_-} = fS\phi_+ + Xa_+A_+,$$

$$\frac{\partial W}{\partial A_+} = Xa_+\phi_-, \quad \frac{\partial W}{\partial A_-} = Xa_-\phi_+,$$

$$\frac{\partial W}{\partial S} = f\phi_+\phi_- - \mu^2 \quad \frac{\partial W}{\partial X} = a_+A_+\phi_- + a_-A_-\phi_+$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Superpotential

$$W = fS\phi_+\phi_- - \mu^2 S + X(a_+A_+\phi_- + a_-A_-\phi_+)$$

Scalar Potential

$$\begin{aligned} V \quad = \quad & |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ & + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Scalar Potential

$$\begin{aligned} V &= |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &\quad + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \\ \frac{\partial V}{\partial \phi_-} &= 2\left[a_-A_-a_+A_+\phi_+ + f(-\mu^2\phi_+ + f\phi_-(\phi_+^2 + S^2) + a_-A_-SX) + a_+^2\phi_-(A_+^2 + X^2)\right] \\ \frac{\partial V}{\partial \phi_+} &= 2\left[a_-A_-a_+A_+\phi_- + f(-\mu^2\phi_- + f\phi_+(\phi_-^2 + S^2) + a_+A_+SX) + a_-^2\phi_+(A_-^2 + X^2)\right] \\ \frac{\partial V}{\partial A_-} &= 2a_-(a_+A_+\phi_-\phi_+ + f\phi_-SX + a_-A_-(\phi_+^2 + X^2)) \\ \frac{\partial V}{\partial A_+} &= 2a_+(\phi_+(a_-A_-\phi_- + fSX) + a_+A_+(\phi_-^2 + X^2)) \\ \frac{\partial V}{\partial S} &= 2f(f(\phi_-^2 + \phi_+^2)S + (a_-A_-\phi_- + a_+A_+\phi_+)X) \\ \frac{\partial V}{\partial X} &= 2(a_-A_-f\phi_-S + a_-^2(A_-^2 + \phi_+^2)X + a_+(A_+f\phi_+S + a_+A_+^2X + a_+\phi_-^2X)) \end{aligned}$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Scalar Potential

$$\begin{aligned} V \quad &= \quad |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &\quad + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Minimization along ϕ_+, ϕ_-

$$\begin{aligned} \frac{\partial V}{\partial \phi_-} \quad &= \quad 2f\left[-\mu^2\phi_+ + f\phi_-\phi_+^2\right] = 0 \\ \frac{\partial V}{\partial \phi_+} \quad &= \quad 2f\left[-\mu^2\phi_- + f\phi_+\phi_-^2\right] = 0 \\ \frac{\partial V}{\partial A_-} \quad &= \quad \frac{\partial V}{\partial A_+} = \frac{\partial V}{\partial S} = \frac{\partial V}{\partial X} = 0 \end{aligned}$$

Supersymmetric Vacuum

$$\phi_+ = \phi_- = \frac{\mu}{\sqrt{f}}$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Scalar Potential

$$\begin{aligned} V \;&=\; |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &+ \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Minimization along ϕ_+, ϕ_-

Supersymmetric Vacuum

$$\phi_+ = \phi_- = \frac{\mu}{\sqrt{f}}$$

➡

$$\begin{aligned} V \;&=\; \frac{\mu^2}{f}\left[(a_-A_- + a_+A_+)^2 + (a_-^2 + a_+^2)X^2\right] \\ &+ (\sqrt{f}\mu S + a_-A_-X)^2 + (\sqrt{f}\mu S + a_+A_+X)^2 \end{aligned}$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Scalar Potential

$$\begin{aligned} V \quad = \quad & |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ & + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Minimization along ϕ_+, ϕ_-

Supersymmetric Vacuum

$$\phi_+ = \phi_- = \frac{\mu}{\sqrt{f}}$$

$$\begin{aligned} V \quad = \quad & \frac{\mu^2}{f}\left[(a_-A_- + a_+A_+)^2 + (a_-^2 + a_+^2)X^2\right] \\ & + (\sqrt{f}\mu S + a_-A_-X)^2 + (\sqrt{f}\mu S + a_+A_+X)^2 \end{aligned}$$

Mixed State $\dfrac{a_-A_- + a_+A_+}{\sqrt{a_-^2 a_+^2}}$ $\quad m = \sqrt{\dfrac{a_-^2 + a_+^2}{f}}\mu$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

Scalar Potential

$$\begin{aligned} V \;&=\; |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &+ \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Minimization along $A_+ = A_- = 0$

$$\begin{aligned} \frac{\partial V}{\partial \phi_-} &= 2\left[f(-\mu^2\phi_+ + f\phi_-(\phi_+^2 + S^2)) + a_+^2\phi_-X^2\right] = 0 \\ \frac{\partial V}{\partial \phi_+} &= 2\left[f(-\mu^2\phi_- + f\phi_+(\phi_-^2 + S^2)) + a_-^2\phi_+X^2\right] = 0 \\ \frac{\partial V}{\partial A_-} &= 2fa_-\phi_-S = 0 \\ \frac{\partial V}{\partial A_+} &= 2fa_+\phi_+SX = 0 \\ \frac{\partial V}{\partial S} &= 2f^2(\phi_-^2 + \phi_+^2)S = 0 \\ \frac{\partial V}{\partial X} &= 2f^2(a_+^2\phi_-^2 + a_-^2\phi_+^2)X = 0 \end{aligned}$$

$$X > X_c = \sqrt{\frac{f}{a_-a_+}}\mu$$

Non-supersymmetric Vacuum

$$\phi_+ = \phi_- = 0$$

$$S \text{ arbitrary}$$

$$V = \mu^4$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS

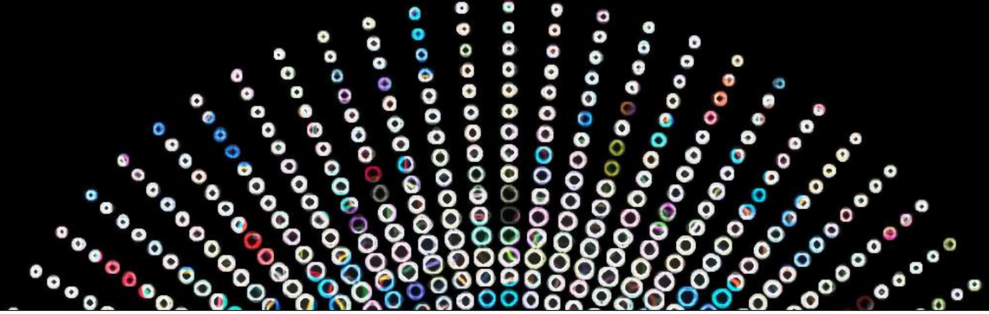
Scalar Potential

$$\begin{aligned} V &= |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &\quad + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Mass Matrix

$$M_{\phi_\pm} = \begin{pmatrix} 2a_-^2X^2 & -2f\mu^2 \\ -2f\mu^2 & 2a_+^2X^2 \end{pmatrix}$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS



Scalar Potential

$$V = |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2$$
$$+\left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2)$$

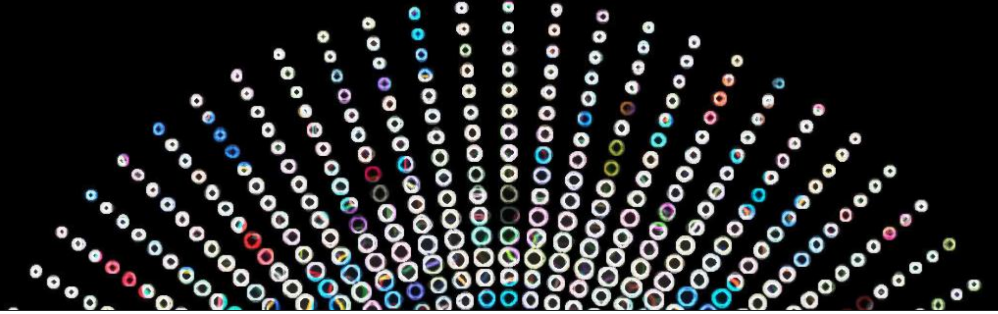
Mass Matrix

$$M_{\phi_\pm} = \begin{pmatrix} 2a_-^2X^2 & -2f\mu^2 \\ -2f\mu^2 & 2a_+^2X^2 \end{pmatrix}$$

⬇ Two complex Scalars

$$m_\pm^2 = a_-^2|X|^2 + a_+^2|X|^2 \mp \sqrt{4f^2\mu^4 + (a_-^2 - a_+^2)^2|X|^4}.$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS



Scalar Potential

$$\begin{aligned} V &= |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &\quad + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

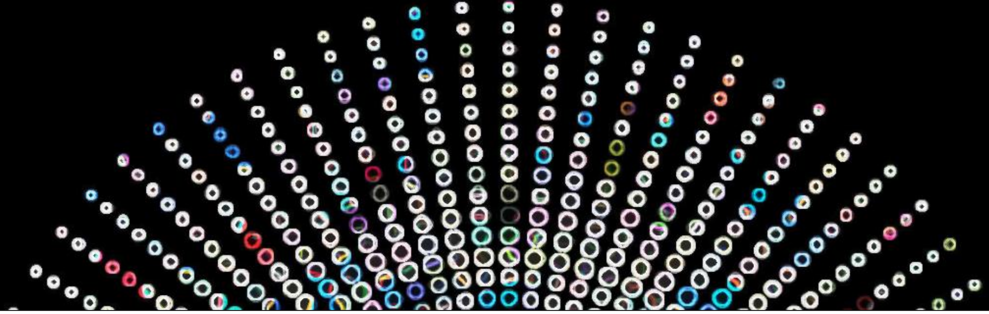
Mass Matrix

$$M_{\phi\pm} = \begin{pmatrix} 2a_-^2X^2 & -2f\mu^2 \\ -2f\mu^2 & 2a_+^2X^2 \end{pmatrix} \qquad M_{A\pm} = \begin{pmatrix} 2a_+^2X^2 & 0 \\ 0 & 2a_-^2X^2 \end{pmatrix}$$

Two complex Scalars

$$m_\pm^2 = a_-^2|X|^2 + a_+^2|X|^2 \mp \sqrt{4f^2\mu^4 + (a_-^2 - a_+^2)^2|X|^4}.$$

FOUR GAUGE SUPERFIELDS AND TWO SINGLETS



Scalar Potential

$$\begin{aligned} V \quad &= \quad |fS\phi_- + Xa_-A_-|^2 + |fS\phi_+ + Xa_+A_+|^2 + |Xa_+\phi_-|^2 + |Xa_-\phi_+|^2 \\ &\quad + \left|f\phi_+\phi_- - \mu^2\right|^2 + |a_+A_+\phi_- + a_-A_-\phi_+|^2 + \frac{g}{2}(|\phi_+|^2 - |\phi_-|^2 + |A_+|^2 - |A_-|^2) \end{aligned}$$

Mass Matrix

$$M_{\phi\pm} \quad = \quad \begin{pmatrix} 2a_-^2X^2 & -2f\mu^2 \\ -2f\mu^2 & 2a_+^2X^2 \end{pmatrix}$$

$$M_{A\pm} \quad = \quad \begin{pmatrix} 2a_+^2X^2 & 0 \\ 0 & 2a_-^2X^2 \end{pmatrix}$$

Two complex Scalars

Two Fermions

$$m_\pm^2 = a_-^2|X|^2 + a_+^2|X|^2 \mp \sqrt{4f^2\mu^4 + (a_-^2 - a_+^2)^2|X|^4}.$$

$$m = a_\pm|X|$$

CONCLUSIONS

- The smallest system of interacting quiral fields that exhibit spontaneous SUSY breaking contains three of them.
- Spontaneous breaking of SUSY in Super-QED gives non-masive photon and photino.
- SUSY breaking is **crucial** for the inflaton Flat directions.