Models of fermion masses and fermion hierarchies

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Overview

Introduction

The origin of fermion masses and mixings is not explained by the SM.

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	(0–0.13)×10^{-9}	0
e electron	0.000511	–1
ν_M middle neutrino*	(0.009–0.13)×10^{-9}	0
μ muon	0.106	–1
ν_H heaviest neutrino*	(0.04–0.14)×10^{-9}	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

$$\sqrt{\left|\Delta m_{13}^2\right|} \sim \lambda^{20} m_t, \quad \sqrt{\Delta m_{12}^2} \sim \lambda^{21} m_t,$$

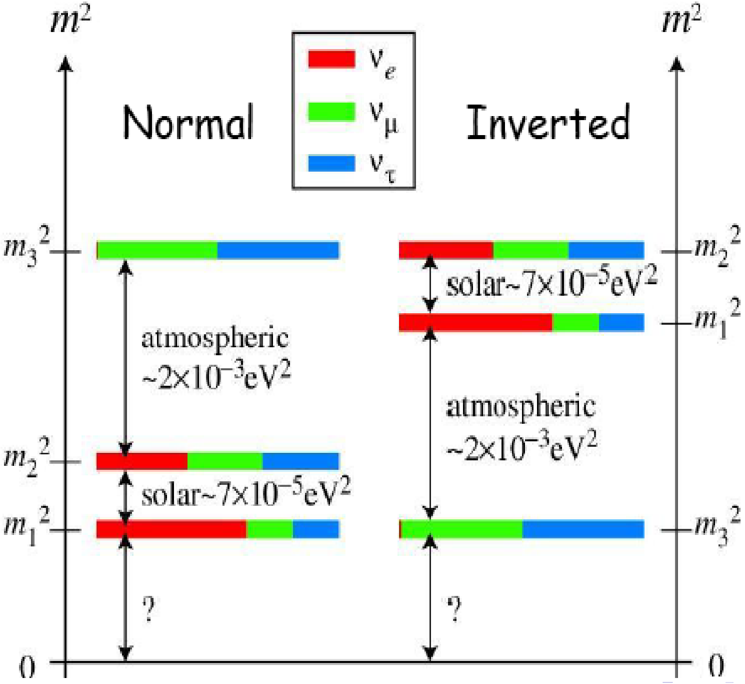
$$m_e \sim \lambda^9 m_t, \quad m_u \sim m_d \sim \lambda^8 m_t,$$

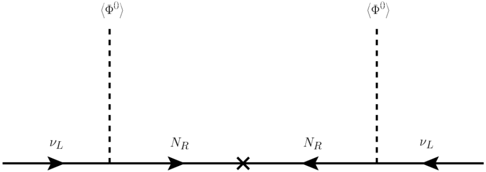
$$m_s \sim m_\mu \sim \lambda^5 m_t, \quad \lambda = 0.225,$$

$$m_c \sim \lambda^4 m_t, \quad m_b \sim m_\tau \sim \lambda^3 m_t,$$

$$\sin\theta_{12}^{(q)} \sim \lambda, \quad \sin\theta_{23}^{(q)} \sim \lambda^2, \quad \sin\theta_{13}^{(q)} \sim \lambda^4,$$

$$\sin\theta_{12}^{(l)} \sim \sqrt{\frac{1}{3}}, \quad \sin\theta_{23}^{(l)} \sim \sqrt{\frac{1}{2}}, \quad \sin\theta_{13}^{(l)} \sim \frac{\lambda}{\sqrt{2}}.$$


m^2
Normal
ν_e
ν_μ
ν_τ
Inverted
m^2
$m_3{}^2$
atmospheric
~2×10⁻³eV²
$m_2{}^2$
solar~7×10⁻⁵eV²
$m_1{}^2$
?
0
$m_2{}^2$
solar~7×10⁻⁵eV²
$m_1{}^2$
atmospheric
~2×10⁻³eV²
$m_3{}^2$
?
0



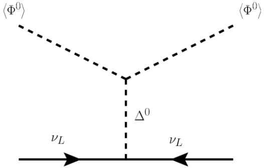
Type I seesaw

LHN $\quad \mathbf{2}\otimes\mathbf{2}\otimes\mathbf{1}$

Minkowski 1977, Gellman, Ramond, Slansky 1980

Glashow, Yanagida 1979, Mohapatra, Senjanovic 1980

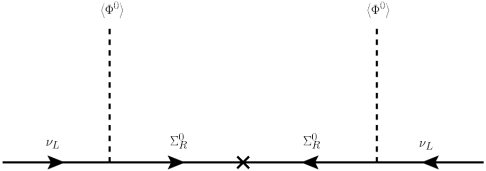
Lazarides Shafi Weterrich 1981, Schechter-Valle 1980 and 1982



Type II seesaw

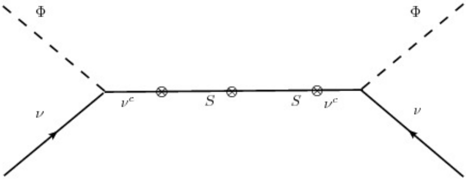
$L\Delta L$ $\quad \mathbf{2}\otimes\mathbf{3}\otimes\mathbf{2}$

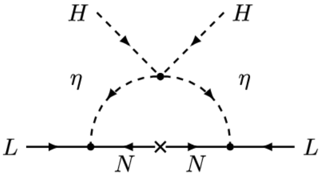
Schechter-Valle 1980 and 1982



Type III seesaw

$LH\Sigma$ $\quad \mathbf{2}\otimes\mathbf{2}\otimes\mathbf{3}$





Inverse seesaw

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2}\left(\begin{array}{ccc} \overline{\nu_L^C} & \overline{N}_R & \overline{S}_R \end{array}\right) \mathbf{M}_\nu \left(\begin{array}{c} \nu_L \\ N_R^C \\ S_R^C \end{array}\right) + H.c$$

$$\mathbf{M}_\nu = \left(\begin{array}{ccc} 0_{3\times3} & \mathbf{M}_1 & 0_{3\times3} \\ \mathbf{M}_1^T & 0_{3\times3} & \mathbf{M}_2 \\ 0_{3\times3} & \mathbf{M}_2^T & \mu \end{array}\right)$$

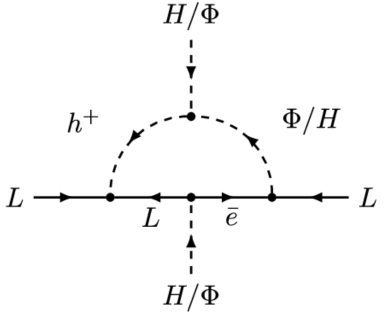
$$\widetilde{\mathbf{M}}_\nu = \mathbf{M}_1 \left(\mathbf{M}_2^T\right)^{-1} \mu \mathbf{M}_2^{-1} \mathbf{M}_1^T$$

$$\mathbf{M}_\nu^{(1)} = -\frac{1}{2}\left(\mathbf{M}_2 + \mathbf{M}_2^T\right) + \frac{1}{2}\mu$$
$$\mathbf{M}_\nu^{(2)} = \frac{1}{2}\left(\mathbf{M}_2 + \mathbf{M}_2^T\right) + \frac{1}{2}\mu$$
$$Q_{\nu_L}^{U(1)_L} = Q_{S_R}^{U(1)_L} = -Q_{N_R}^{U(1)_L} = 1$$

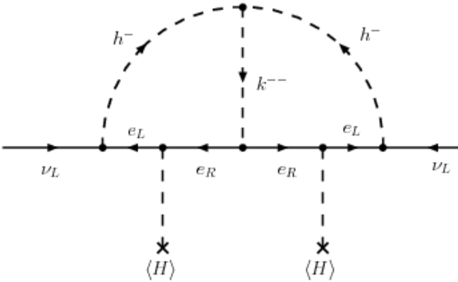
One loop Ma radiative seesaw model

η and N are odd under a preserved Z_2

$$L\widetilde{\eta}N, \frac{\lambda_5}{2}\left(H^\dagger \cdot \eta\right) + h.c$$

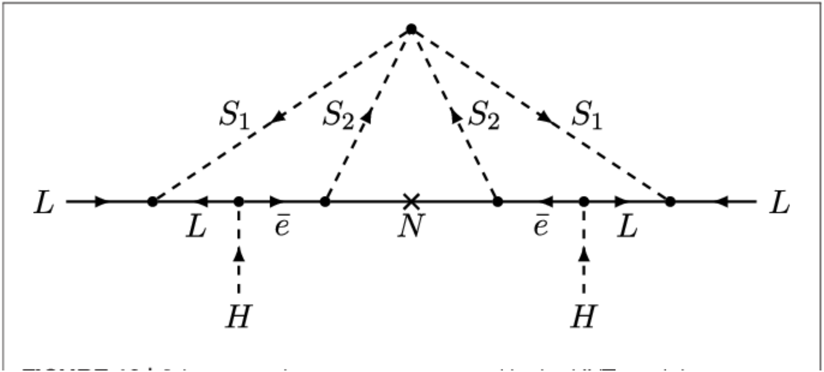


Zee model

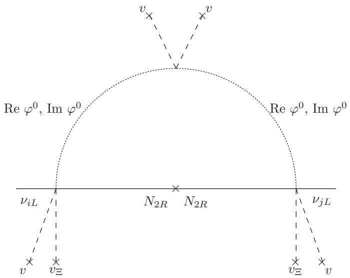


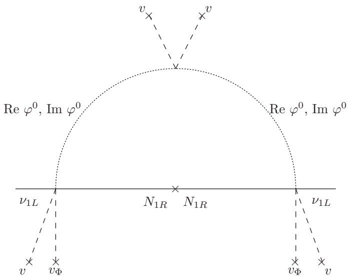
Zee Babu model

Field	Spin	G_{SM}	Z_2
S_1	0	$(1, 1, -1)$	$+$
S_2	0	$(1, 1, -1)$	$-$
N	$\frac{1}{2}$	$(1, 1, 0)$	$-$

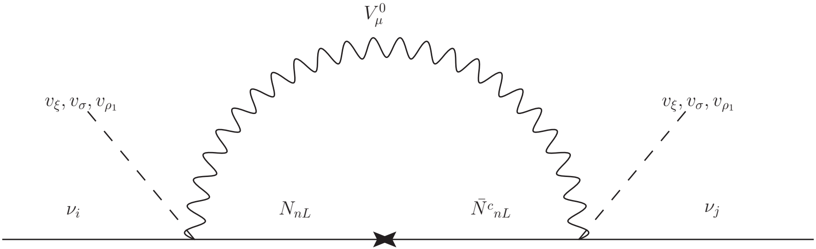


KNT model

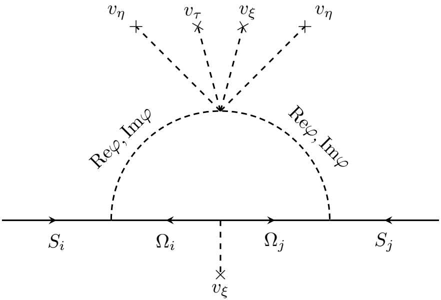




One loop radiative seesaw with non renormalizable Dirac Yukawa terms, N. Bernal, A. E. Cárcamo Hernández, Ivo de Medeiros Varzielas and S. Kovalenko, JHEP **1805**, 053 (2018)



One loop radiative seesaw with non renormalizable interactions of the heavy vectors with Majorana neutrinos and scalar singlets, A. E. Cárcamo Hernández, J. Vignatti and A. Zerwekh, J. Phys. G **46**, no. 11, 115007 (2019)



$$\frac{\kappa}{\Lambda^2}\left(\varphi^*\right)^2\tau^*\xi^*\left(\eta^*\right)^2$$

$$\lambda_3\left(\overline{S}\Omega^C\right)_{\mathbf{1_{0,0}}}\varphi$$

$$\lambda_{1,2}\left(\overline{\Omega}\Omega^C\right)_{\overline{\mathbf{3}}_{S_{1,2}}}\xi$$

One loop radiative inverse seesaw with non renormalizable scalar interactions, A. E. Cárcamo Hernández, S. Kovalenko, J. W. F. Valle and C. A. Vaquera-Araujo, arxiv:hep-ph/1705.06320, JHEP **1902**, 065 (2019)

Some ways of describing the SM charged fermion mass hierarchy are:

1. Spontaneously broken abelian symmetries.
2. Universal Seesaw mechanism.
3. Localization of the profiles of the fermionic zero modes in extradimensions
4. Radiative corrections, for example the sequential loop suppression.
5. Combining spontaneous breaking of discrete symmetries with radiative seesaw processes
6. Combining Universal Seesaw with spontaneous breaking of discrete symmetries.

The Froggatt-Nielsen mechanism has the following features:

- Introduce new gauge singlet scalar, i.e., σ called the flavon, and a global $U(1)_{FN}$ symmetry.
- The $U(1)_{FN}$ charges of the SM fermions (excepting for the top Yukawa term), the Higgs and Flavon fields are such that renormalizable Yukawa terms are forbidden.
- The $U(1)_{FN}$ charge assignments of fermionic and scalar fields generate the following Effective operator:

$$a_{ij}\overline{f}_{iL}Hf_{jR}\left(\frac{\sigma}{\Lambda}\right)^{n_{ij}} \to a_{ij}\left(\frac{v_{\sigma}}{\Lambda}\right)^{n_{ij}}\overline{f}_{iL}Hf_{jR} \quad (1)$$

- The Yukawa hierarchy arises from the $U(1)_{FN}$ charge assignment:

$$n_{ij} = -\frac{1}{q_{\varphi}}\left(q_{\overline{f}_{iL}} + q_{f_{jR}} + q_H\right) \quad (2)$$

The S_3 discrete group

The S_3 is the smallest non-abelian group having a doublet and two singlet irreducible representations. The S_3 group has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010):

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} + \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{2}}, \tag{3}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_{\mathbf{2}}, \qquad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \tag{4}$$

A toy model: Generating $m_b \neq 0$ at one loop level with $m_d = m_s = 0$.

To get massless d, s and b quarks at tree level, we forbidd the operators

$$\overline{q}_{iL}\phi d_{jR}, \qquad i,j = 1,2,3\,, \tag{5}$$

To this end, we consider the following S_3 assignments:

$$q_{iL} \sim \mathbf{1}, \qquad d_{iR} \sim \mathbf{1}', \qquad \phi \sim \mathbf{1} \tag{6}$$

We assume S_3 softly broken and we add gauge singlet scalars η_k $(k = 1,2)$ and vector like down type quarks B_k $(k = 1,2)$ grouped in S_3 doblets as follows:

$$\eta = (\eta_1, \eta_2) \sim \mathbf{2}, \qquad B_{L,R} \sim \mathbf{2} \tag{7}$$

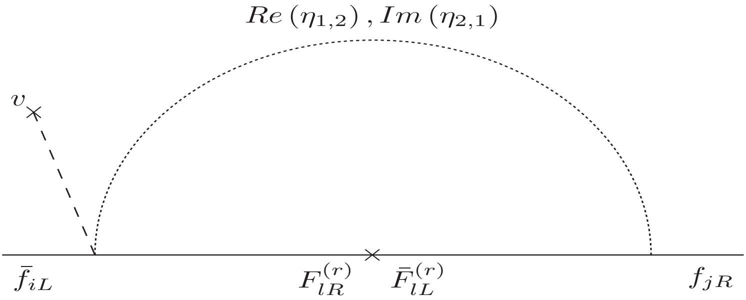
Thus, we are left with the operators:

$$\frac{y_i}{\Lambda}\overline{q}_{iL}\phi\left(B_R\eta\right)_{\mathbf{1}}, \qquad x_j\left(\overline{B}_L\eta\right)_{\mathbf{1}'}d_{jR}, \qquad i,j=1,2,3, \tag{8}$$

which imply:

$$\left(M_D\right)_{ij}\approx\frac{y_ix_j}{\left(16\pi^2\right)^2}f_2\frac{v}{M}\frac{\mu_{12}^3}{\Lambda^3}\mu_{12}, \tag{9}$$

where μ_{12} is a soft breaking mass parameter in $\mu_{12}^2\eta_1\eta_2$. Thus $m_b\neq 0$ at one loop level and $m_d=m_s=0$.



Combining radiative mechanisms with spontaneously broken symmetries.

The S_3 symmetry is softly broken whereas the Z_8 discrete group is broken.

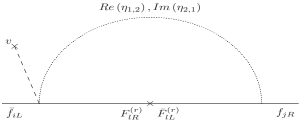
$$\phi\sim\left(\mathbf{1},1\right), \quad \eta=(\eta_1,\eta_2)\sim\left(\mathbf{2},e^{-\frac{\pi i}{4}}\right), \quad \chi\sim\left(\mathbf{1},-i\right),$$
$$v_\chi=\lambda\Lambda, \qquad \lambda=0.225. \tag{10}$$

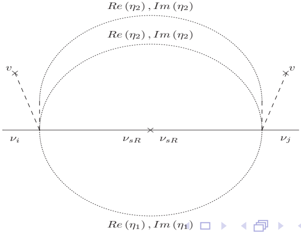
$$\begin{aligned}
q_{jL} &\sim \left(\mathbf{1},e^{-\frac{\pi i(3-j)}{2}}\right), \quad u_{kR}\sim\left(\mathbf{1}',e^{\frac{\pi i(3-k)}{2}}\right), \quad u_{3R}\sim\left(\mathbf{1},1\right),\\
d_{jR} &\sim \left(\mathbf{1}',e^{\frac{\pi i(3-j)}{2}}\right), \quad l_{jL}\sim\left(\mathbf{1},e^{-\frac{\pi i(3-j)}{2}}\right), \quad l_{jR}\sim\left(\mathbf{1}',e^{\frac{\pi i(3-j)}{2}}\right),\\
T_L^{(k)} &\sim \left(\mathbf{2},e^{-\frac{\pi i}{4}}\right), \quad T_R^{(k)}\sim\left(\mathbf{2},e^{\frac{\pi i}{4}}\right), \quad k=1,2,\\
B_L^{(j)} &\sim \left(\mathbf{2},e^{-\frac{\pi i}{4}}\right), \quad B_R^{(j)}\sim\left(\mathbf{2},e^{\frac{\pi i}{4}}\right), \quad j=1,2,3,\\
E_L^{(j)} &\sim \left(\mathbf{2},e^{-\frac{\pi i}{4}}\right), \quad E_R^{(j)}\sim\left(\mathbf{2},e^{\frac{\pi i}{4}}\right),\\
\nu_{kR} &\sim \left(\mathbf{1}',e^{-\frac{\pi i}{4}}\right), \quad k=1,2.
\end{aligned} \tag{11}$$

I use the S_3 discrete group since it is the smallest non-Abelian group.

$$
\begin{aligned}
-\mathcal{L}_Y^{(U)} &= \sum_{j=1}^{3}\sum_{r=1}^{2} y_{jr}^{(u)}\overline{q}_{jL}\widetilde{\phi}\left(T_R^{(r)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{r=1}^{2}\sum_{s=1}^{2} x_{rs}^{(u)}\left(\overline{T}_L^{(r)}\eta\right)_{\mathbf{1}'} u_{sR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{j=1}^{3} y_{j3}^{(u)}\overline{q}_{jL}\widetilde{\phi}u_{3R}\frac{\chi^{3-j}}{\Lambda^{3-j}} + \sum_{r=1}^{2} y_r^{(T)}\left(\overline{T}_L^{(r)}T_R^{(r)}\right)_{\mathbf{1}}\chi + h.c
\end{aligned}
$$

$$
-\mathcal{L}_Y^{(\nu)} = \sum_{j=1}^{3}\sum_{s=1}^{2} y_{js}^{(\nu)}\bar{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{\left[\eta^*\left(\eta\eta^*\right)_{\mathbf{2}}\right]_{\mathbf{1}'}\chi^{3-j}}{\Lambda^{6-j}} + \sum_{s=1}^{2} y_s\overline{\nu}_{sR}\nu_{sR}^{C}\chi + h.c.
$$





Cárcamo Hernández-Kovalenko-Schmidt (CKS) mechanism

In the CKS mechanism the SM fermion mass hierarchy is explained by a sequential loop suppression, so that the masses are generated according to:

$$t\text{-quark} \rightarrow \textit{tree-level mass} \text{ from } \overline{q}_{jL}\widetilde{\phi}u_{3R}, \tag{12}$$

$$b, c, \tau, \mu \rightarrow \textit{1-loop mass;} \text{ tree-level suppressed by a } \textit{symmetry}. \tag{13}$$

$$s, u, d, e \rightarrow \textit{2-loop mass;} \text{ tree-level \& 1-loop suppressed by a } \textit{symmetry}. \tag{14}$$

$$\nu_i \rightarrow \textit{4-loop mass;} \text{ tree-level \& lower loops suppressed by a } \textit{symmetry}. \tag{15}$$

The $S_3 \times Z_2$ particle assignments of the model are:

	ϕ	σ	η
S_3	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
Z_2	1	1	-1

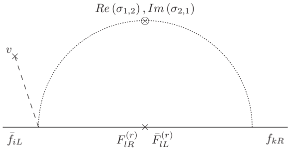
	q_{iL}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	l_{iL}	l_{1R}	l_{2R}	l_{3R}
S_3	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}'$
Z_2	1	-1	1	1	-1	-1	1	1	-1	1	1

	ν_{sR}	T_L	T_R	$\widetilde{T}_L$	$\widetilde{T}_R$	B_L	B_R	$\widetilde{B}_L^{(s)}$	$\widetilde{B}_R^{(s)}$	$E_L^{(s)}$	$E_R^{(s)}$	$\widetilde{E}_L$	$\widetilde{E}_R$
S_3	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$
Z_2	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1

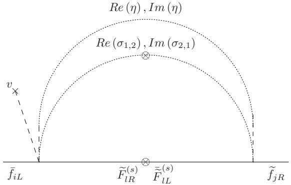
φ is the SM Higgs doublet.

The scalar fields σ and η and all exotic fermions are $SU(2)_L$ singlets.

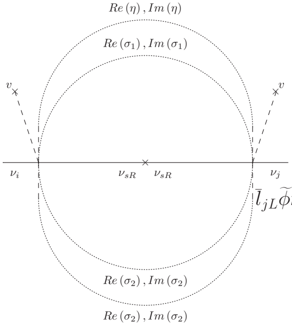
The $S_3 \times Z_2$ discrete group is assumed to be softly broken.



$\overline{f}_{jL}H\left(F_R^{(r)}\sigma\right)_{\mathbf{1}}\frac{1}{\Lambda}$ $\quad$ $\left(\overline{F}_L^{(r)}\sigma\right)_{\mathbf{1}'}f_{kR}$



$\overline{f}_{jL}H\left(\widetilde{F}_R^{(r)}\sigma\right)_{\mathbf{1}}\frac{\eta}{\Lambda^2}$ $\quad$ $\left(\overline{\widetilde{F}}_L^{(r)}\sigma\right)_{\mathbf{1}'}\widetilde{f}_{kR}\frac{\eta}{\Lambda}$



$\bar{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{[\sigma(\sigma\sigma)_{\mathbf{2}}]_{\mathbf{1}'}\eta}{\Lambda^4}$

The mass matrices $M_{U,D}$ of up and down quarks, $M_{l,\nu}$, of charged leptons and light active neutrinos

$$M_U = \begin{pmatrix} \widetilde{\varepsilon}^{(u)}_{11} & \varepsilon^{(u)}_{12} & \kappa^{(u)}_{13} \\ \widetilde{\varepsilon}^{(u)}_{12} & \varepsilon^{(u)}_{22} & \kappa^{(u)}_{23} \\ \widetilde{\varepsilon}^{(u)}_{13} & \varepsilon^{(u)}_{32} & \kappa^{(u)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \widetilde{\varepsilon}^{(d)}_{11} & \widetilde{\varepsilon}^{(d)}_{12} & \varepsilon^{(d)}_{13} \\ \widetilde{\varepsilon}^{(d)}_{21} & \widetilde{\varepsilon}^{(d)}_{22} & \varepsilon^{(d)}_{23} \\ \widetilde{\varepsilon}^{(d)}_{31} & \widetilde{\varepsilon}^{(d)}_{32} & \varepsilon^{(d)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_l = \begin{pmatrix} \widetilde{\varepsilon}^{(l)}_{11} & \varepsilon^{(l)}_{12} & \varepsilon^{(l)}_{13} \\ \widetilde{\varepsilon}^{(l)}_{21} & \varepsilon^{(l)}_{22} & \varepsilon^{(l)}_{23} \\ \widetilde{\varepsilon}^{(l)}_{31} & \varepsilon^{(l)}_{32} & \varepsilon^{(l)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_\nu = \begin{pmatrix} \varepsilon^{(\nu)}_{11} & \varepsilon^{(\nu)}_{12} & \varepsilon^{(\nu)}_{13} \\ \varepsilon^{(\nu)}_{12} & \varepsilon^{(\nu)}_{22} & \varepsilon^{(\nu)}_{23} \\ \varepsilon^{(\nu)}_{13} & \varepsilon^{(\nu)}_{23} & \varepsilon^{(\nu)}_{33} \end{pmatrix} \frac{v^2}{\sqrt{2}\,\Lambda},$$

their entries are generated at different loop-levels:

$$\kappa^{(u)}_{j3} \rightarrow \text{tree-level} \tag{16}$$

$$\varepsilon^{(u)}_{j2}, \varepsilon^{(d)}_{j3}, \varepsilon^{(l)}_{j2}, \varepsilon^{(l)}_{j3} \rightarrow \text{1-loop-level} \tag{17}$$

$$\widetilde{\varepsilon}^{(u)}_{j1}, \widetilde{\varepsilon}^{(d)}_{j1}, \widetilde{\varepsilon}^{(d)}_{j2}, \widetilde{\varepsilon}^{(l)}_{j1} \rightarrow \text{2-loop-level} \tag{18}$$

$$\varepsilon^{(\nu)}_{jk} \rightarrow \text{4-loop-level}, \tag{19}$$

where $j, k = 1, 2, 3$.

$$m_b \sim \frac{y_b^2}{16\pi^2} f_1 \frac{v}{\Lambda} \frac{\mu_{12}}{M} \mu_{12}, \quad (20)$$

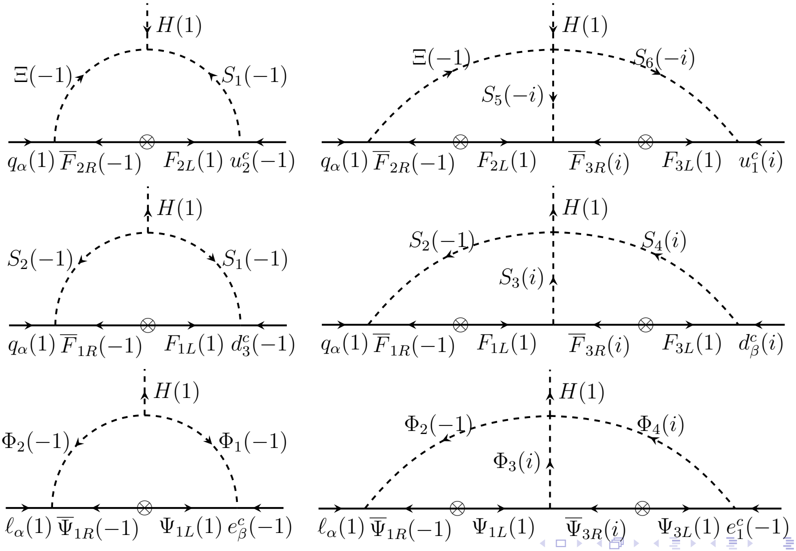
$$m_s \sim \frac{y_s^2}{\left(16\pi^2\right)^2} f_2 \frac{v}{M} \frac{\mu_{12}^3}{\Lambda^3} \mu_{12}, \quad (21)$$

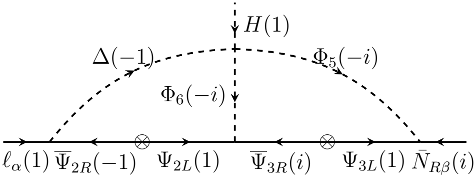
Assuming $y_b^2 f_1 \sim y_s^2 f_2 \sim 1$ and $\mu_{12} \sim M$, we find a rough estimate

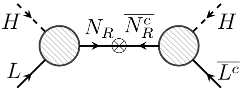
$$\Lambda \sim 10v \sim 2.5\mathrm{TeV} \quad (22)$$

for the correct order of magnitude of m_b and m_s.

Sequentially loop suppressed fermion masses at renormalizable level







$$m_D \sim \left(\frac{1}{16\pi^2}\right)^2 \kappa^{(\nu)} v Y^3 \; I_{loop}(m_S/M_R), \tag{23}$$

$$m_\nu \simeq m_D^T M_R^{-1} m_D, \tag{24}$$

$$m_\nu \sim \left(\frac{1}{16\pi^2}\right)^4 (\kappa^{(\nu)})^2 \, Y^6 \frac{v^2}{M_R} \; \left[I_{loop}(m_S/M_R)\right]^2 . \tag{25}$$

The simplified 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$

We consider a $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with (Singer-Valle-Schechter, 1980):

$$Q = T_3 + \beta T_8 + XI, \qquad \beta = -\frac{1}{\sqrt{3}}, \tag{26}$$

331 Models ($SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ Models) are important because:

1. Can explain the origin of fermion generations (Frampton, 1992; Pisano-Pleitez, 1992).
2. These models can have DM candidates (J.K. Mizukoshi, et al, 2011).
3. Can explain the large mass splitting between the heaviest quark family and the two lighter ones (Frampton, 1995).
4. Allow the quantization of electric charge (Pires-Ravinez, 1998; Dong-Long, 2005).
5. Have several sources of CP violation (J.K. Mizukoshi, et al, 1998).
6. Can explain why the Weinberg mixing angle satisfies $\sin^2\theta_W < \frac{1}{4}$.

Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. Possible only with 3 generations!.
Quarks and leptons are unified in the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$Q_L^{1,2} = \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), \qquad Q_L^3 = \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), \tag{27}$$

$$\begin{array}{cc} D_R^{1,2,3} : (3, 1, -1/3), & U_R^{1,2,3} : (3, 1, 2/3), \\ J_R^{1,2} : (3, 1, -1/3), & T_R : (3, 1, 2/3). \end{array} \tag{28}$$

$$L_L^{1,2,3} = \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), \tag{29}$$

$$\begin{array}{ccc} e_R : (1, 1, -1), & \mu_R : (1, 1, -1), & \tau_R : (1, 1, -1), \\ N_R^1 : (1, 1, 0), & N_R^2 : (1, 1, 0), & N_R^3 : (1, 1, 0). \end{array} \tag{30}$$

6 $SU(3)_L$ triplets and 6 $SU(3)_L$ antitriplets $\rightarrow$ Gauge anomalies cancel

Scalars are grouped in the following $[SU(3)_L, U(1)_X]$ representations:

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3),$$
$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3),$$
$$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3, -1/3). \tag{31}$$

The physical scalars are: 4 massive charged Higgs ($H_1^\pm$, $H_2^\pm$), one CP-odd Higgs (A_1^0), 3 neutral CP-even Higgs (h^0, H_1^0, H_3^0) and 2 neutral Higgs ($H_2^0, \overline{H}_2^0$) bosons.

The gauge symmetry in the 3-3-1 model is spontaneously broken in two steps as follows:

$$\mathcal{G} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{v_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v_\eta, v_\rho} SU(3)_C \otimes U(1)_Q , \quad (32)$$

where the hierarchy $v_\eta, v_\rho \ll v_\chi$ among the symmetry breaking scales is fullfilled. Here $v_\eta^2 + v_\rho^2 = v^2$, $v = 246$ GeV.
The quark Yukawa terms are:

$$\begin{aligned} -\mathcal{L}_Y^{(q)} &= \overline{Q}_L^3 \left(\eta h_{\eta 1j}^U + \chi h_{\chi 1j}^U \right) U_R^j + \overline{Q}_L^3 \rho h_{\rho 1j}^D D_R^j \\ &+ \overline{Q}_L^3 \left(\eta h_{\eta 11}^T + \chi h_{\chi 11}^T \right) T_R + \overline{Q}_L^3 \rho h_{\rho 1m}^J J_R^m \\ &+ \overline{Q}_L^n \rho^* h_{\rho nj}^U U_R^j + \overline{Q}_L^n \left(\eta^* h_{\eta nj}^D + \chi^* h_{\chi nj}^D \right) D_R^j \\ &+ \overline{Q}_L^n \rho^* h_{\rho n1}^T T_R^1 + \overline{Q}_L^n \left(\eta^* h_{\eta nm}^J + \chi^* h_{\chi nm}^J \right) J_R^m + h.c, \end{aligned} \quad (33)$$

where $n = 2, 3$ and $i, j = 1, 2, 3$.

The lepton Yukawa terms are:

$$\begin{aligned}-\mathcal{L}_Y^{(l)} &= h_{\rho ij}^{(L)}\overline{L}_L^i \rho e_{jR} + \frac{1}{2}\left(h_\rho\right)_{ij}\varepsilon_{abc}\overline{L}_L^{ia}\left(L_L^{jC}\right)^b\left(\rho^*\right)^c \\ &+ h_{\eta ij}^{(L)}\overline{L}_L^i \eta N_{jR} + h_{\eta ij}^{(L)}\overline{L}_L^i \chi N_{jR} + m_{Nij}\overline{N}_R^i N_R^{jC} + h.c \end{aligned} \tag{34}$$

where $n = 2, 3$ and $i, j = 1, 2, 3$. The neutrino mass terms are:

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2}\begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + H.c, \tag{35}$$

where the neutrino mass matrix is:

$$\begin{aligned} M_\nu &= \begin{pmatrix} 0_{3\times 3} & M_1 & M_2 \\ M_1^T & 0_{3\times 3} & M_3 \\ M_2^T & M_3^T & m_N \end{pmatrix} \\ &= \begin{pmatrix} 0_{3\times 3} & \frac{v_\rho}{2\sqrt{2}}\left(h_\rho^\dagger - h_\rho^*\right) & \frac{v_\eta}{\sqrt{2}} h_\eta^* \\ \frac{v_\rho}{2\sqrt{2}}\left(h_\rho^\dagger - h_\rho^*\right)^T & 0_{3\times 3} & \frac{v_\chi}{\sqrt{2}} h_\chi^* \\ \frac{v_\eta}{\sqrt{2}} h_\eta^\dagger & \frac{v_\chi}{\sqrt{2}} h_\chi^\dagger & m_N \end{pmatrix}, \end{aligned} \tag{36}$$

A 3-3-1 model with sequential loop suppression mechanism

$$\begin{aligned}
\mathcal{G} &= SU(3)_C \times SU(3)_L \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_g} \\
&\xrightarrow{v_\chi, v_\xi} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \times Z_2^{(L_g)} \\
&\xrightarrow{v_\eta} SU(3)_C \times U(1)_{em} \times Z_4 \times Z_2^{(L_g)}, \qquad (37)
\end{aligned}$$

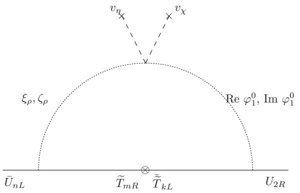
	χ	η	ρ	φ_1^0	φ_2^0	ϕ_1^+	ϕ_2^+	ϕ_3^+	ϕ_4^+	ξ^0
L_g	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	-2	-2	-2
Z_4	1	1	-1	-1	i	i	-1	-1	1	1
Z_2	-1	-1	1	1	1	1	1	-1	-1	1

$$\frac{1}{\Lambda} L_{iL}^C \bar{L}_{jL} \bar{Q}_{3L} d_{kR}$$

Forbidden thanks to $U(1)_{L_g}$.

	Q_{1L}	Q_{2L}	Q_{3L}	U_{1R}	U_{2R}	U_{3R}	T_R	D_{1R}	D_{2R}	D_{3R}	J_{1R}	J_{2R}	$\widetilde{T}_{1L}$	$\widetilde{T}_{1R}$	$\widetilde{T}_{2L}$	$\widetilde{T}_{2R}$	B_L	B_R
L_g	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	0	0	0	2	2	0	0	0	0	0	0
Z_4	-1	-1	1	1	$-i$	1	1	1	1	1	-1	-1	i	1	i	1	-1	-1
Z_2	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	1	1	1

	L_{1L}	L_{2L}	L_{3L}	e_{1R}	e_{2R}	e_{3R}	E_{1L}	E_{2L}	E_{3L}	E_{1R}	E_{2R}	E_{3R}	N_{1R}	N_{2R}	N_{3R}	Ψ_R
L_g	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1	1	1	1	1	1	1	-1	-1	-1	1
Z_4	i	i	i	$-i$	$-i$	$-i$	1	i	i	$-i$	$-i$	$-i$	i	i	i	1
Z_2	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	-1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v_χ
ξ_ρ, ζ_ρ
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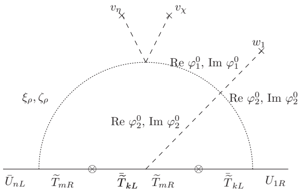
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
U_{2R}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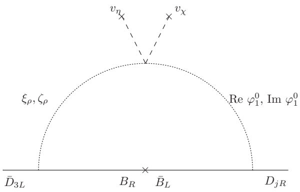
Re φ_2^0, Im φ_2^0
$\bar{U}_{nL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
U_{1R}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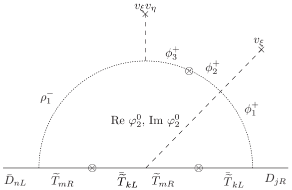
Re φ_1^0, Im φ_1^0
$\bar{D}_{3L}$
B_R
$\bar{B}_L$
D_{jR}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ϕ_3^+
ϕ_2^+
ρ_1^-
ϕ_1^+
Re φ_2^0, Im φ_2^0
$\bar{D}_{nL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
D_{jR}

$$M_U = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} & 0 \\ \widetilde{\varepsilon}_{21}^{(u)} & \varepsilon_{22}^{(u)} & 0 \\ 0 & 0 & y \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_D = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(d)} & \widetilde{\varepsilon}_{12}^{(d)} & \widetilde{\varepsilon}_{13}^{(d)} \\ \widetilde{\varepsilon}_{21}^{(d)} & \widetilde{\varepsilon}_{22}^{(d)} & \widetilde{\varepsilon}_{23}^{(d)} \\ \varepsilon_{31}^{(d)} & \varepsilon_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}} \qquad (38)$$

$$\varepsilon_{n2}^{(u)} = a_{n2}^{(u)}\, l, \qquad \widetilde{\varepsilon}_{n1}^{(u)} = b_{n1}^{(u)}\, l^2,$$

$$\varepsilon_{3j}^{(d)} = a_{3j}^{(d)}\, l, \qquad \widetilde{\varepsilon}_{nj}^{(d)} = b_{nj}^{(d)}\, l^2, \qquad n = 1, 2, \qquad j = 1, 2, 3,$$

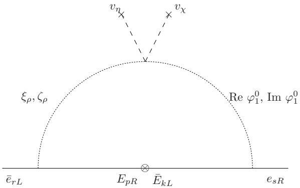
where $l \approx (1/4\pi)^2 \approx 2.0 \times \lambda^4$, $\lambda = 0.225$ is the Wolfenstein parameter.

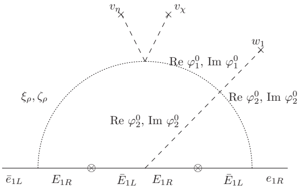
$$a_{12}^{(u)} \simeq a_{22}^{(u)} \approx 0.5, \quad b_{11}^{(u)} \approx 0.25, \quad b_{21}^{(u)} \approx 0.7, \qquad (39)$$

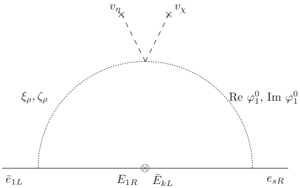
$$a_{31}^{(d)} \simeq -1.6, \quad a_{32}^{(d)} \simeq -2.2, \quad a_{33}^{(d)} \simeq 1.6,$$

$$b_{11}^{(d)} \simeq -12.3 - 0.7i, \quad b_{12}^{(d)} \simeq -7.6 - 1.0i, \quad b_{13}^{(d)} \simeq 11.6 + 0.7i,$$

$$b_{21}^{(d)} \simeq -14.3 + 0.7i, \quad b_{22}^{(d)} \simeq -5.6 + 1.0i, \quad b_{23}^{(d)} \simeq 14.8 - 0.7i.$$







$$a_{11}^{(l)} \approx a_{22}^{(l)} \approx -0.2, \quad a_{12}^{(l)} \approx a_{32}^{(l)} \approx -0.14, \quad a_{13}^{(l)} \approx a_{23}^{(l)} \approx a_{33}^{(l)} \approx 1.1.$$

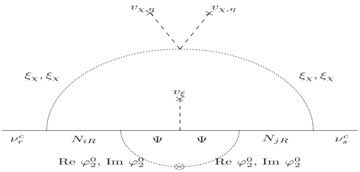
$$M_l = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(l)} & \varepsilon_{12}^{(l)} & \varepsilon_{13}^{(l)} \\ 0 & \varepsilon_{22}^{(l)} & \varepsilon_{23}^{(l)} \\ 0 & \varepsilon_{32}^{(l)} & \varepsilon_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{40}$$

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + y_\Psi \overline{\Psi_R^c} \zeta^0 \Psi_R + h.c,$$

$$M_\nu = \begin{pmatrix} M_1 & 0_{3\times3} & M_3 \\ 0_{3\times3} & M_2 & M_4 \\ M_3 & M_4 & \mathcal{M} \end{pmatrix}, \qquad M_1 = \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}, \qquad M_3 = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_4 = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \\ d_1 & d_2 & d_3 \\ d_4 & d_5 & d_6 \end{pmatrix} \frac{v_\chi}{\sqrt{2}}, \qquad \mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} \\ \mathcal{M}_{12} & \mathcal{M}_{22} & \mathcal{M}_{23} \\ \mathcal{M}_{13} & \mathcal{M}_{23} & \mathcal{M}_{33} \end{pmatrix}$$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$v_{\chi,\eta}$
ξ_χ, ξ_χ
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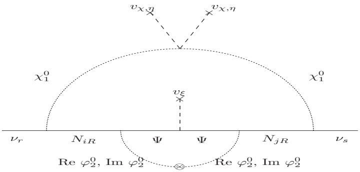
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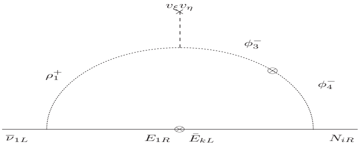
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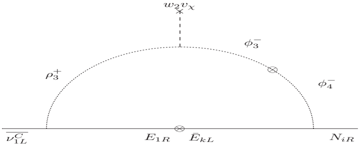
N_{iR}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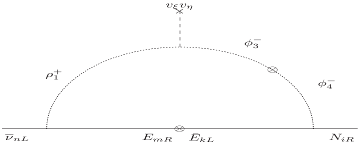
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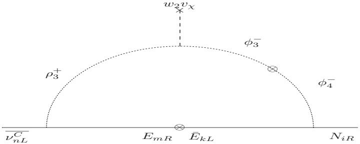
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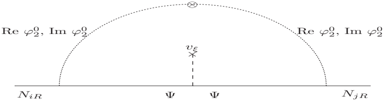
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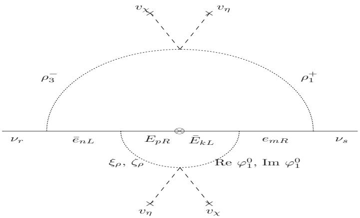
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ξ_ρ, ζ_ρ
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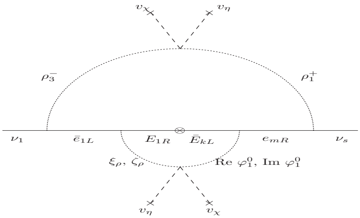
v_η
v_χ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v_η
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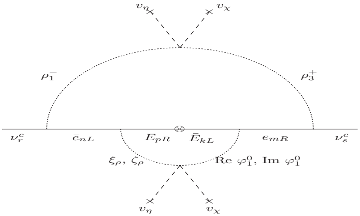
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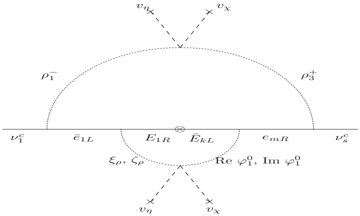
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
v_η
v_χ


v_η
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$\bar{E}_{kL}$
e_{mR}
ν_s^c
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
v_η
v_χ

$$\frac{1}{\Lambda^3}\left(\bar{L}\Psi_R\right)\left(\bar{e}_R L\right)\varphi_2^0 \tag{41}$$

For $m_\Psi < m_2^R$, Ψ_R is a DM candidate.

$$\frac{1}{\Lambda^2}\epsilon_{abc}\left(\eta^\dagger\right)^a\left(\chi^\dagger\right)^b\varphi_1^0\overline{L}_1^c e_{kR} \quad \text{for} \quad k=2,3 \tag{42}$$

$$\Gamma(\varphi^0 \to Z e_1^+ e_{2,3}^-) \simeq \Gamma(\varphi^0 \to \zeta_\eta e_1^+ e_{2,3}^-) \sim \Gamma(\varphi^0 \to h e_1^+ e_{2,3}^-) \sim \frac{m_{\varphi^0}^3 v_\chi^2}{\Lambda^4},$$
$$\Gamma(\varphi^0 \to e_1^+ e_{2,3}^-) \sim m_{\varphi^0}\left(\frac{v_\chi v_\eta}{\Lambda^2}\right)^2. \tag{43}$$

Requiring that the DM candidate φ^0 lifetime be greater than the universe lifetime $\tau_u \approx 13.8$ Gyr, taking into account the limit $v_\chi \gtrsim 90\,\text{TeV}$ and assuming $m_{\varphi^0} \sim 1$ TeV, we estimate the cutoff scale of our model

$$\Lambda > 3 \times 10^{10}\text{GeV} \tag{44}$$

An extended IDM with sequentially loop-generated fermion mass hierarchies.

$$
\begin{aligned}
\mathcal{G} &= SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2^{(1)} \times Z_2^{(2)} \\
&\xrightarrow{v_{\sigma_1}, v_{\rho_3}} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_2^{(2)} \\
&\xrightarrow{v} SU(3)_C \times U(1)_{em} \times Z_2^{(2)},
\end{aligned} \tag{45}
$$

Field	ϕ_1	ϕ_2	σ_1	σ_2	σ_3	ρ_1	ρ_2	ρ_3	η	φ_1^+	φ_2^+	φ_3^+	φ_4^+	φ_5^+
SU_{3c}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
SU_{2L}	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
U_{1Y}	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	1	1	1	1	1
U_{1X}	1	2	-1	-1	-2	0	0	0	1	5	2	3	2	3
$Z_2^{(1)}$	1	1	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1
$Z_2^{(2)}$	1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1

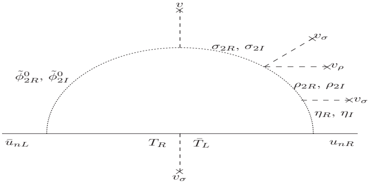
Table: Scalars assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

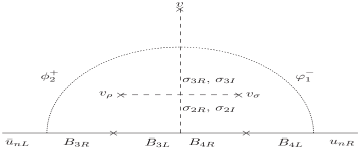
Field	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	T_L	T_R	$\widetilde{T}_L$	$\widetilde{T}_R$	B_{1L}	B_{1R}	B_{2L}	B_{2R}	B_{3L}	B_{3R}	B_{4L}	B_{4R}
SU_{3c}	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
SU_{2L}	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U_{1Y}	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
U_{1X}	0	0	1	2	2	2	-1	-1	-1	1	2	1	1	0	-1	0	-1	-2	-2	-3	-3
$Z_2^{(1)}$	1	1	1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	1	-1	-1

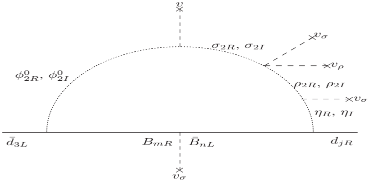
Table: Quark assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

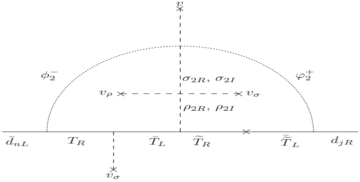
Field	l_{1L}	l_{2L}	l_{3L}	l_{1R}	l_{2R}	l_{3R}	E_{1L}	E_{1R}	E_{2L}	E_{2R}	E_{3L}	E_{3R}	ν_{1R}	ν_{2R}	ν_{3R}	Ω_{1R}	Ω_{2R}	Ψ_R
SU_{3c}	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU_{2L}	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U_{1Y}	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
U_{1X}	0	-3	0	-3	-6	-3	-3	-2	-6	-5	-3	-2	2	-1	2	-1	1	0
$Z_2^{(1)}$	1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	1
$Z_2^{(2)}$	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	1	1

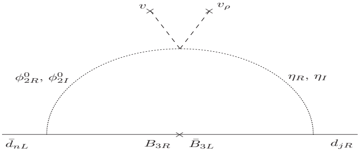
Table: Lepton assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

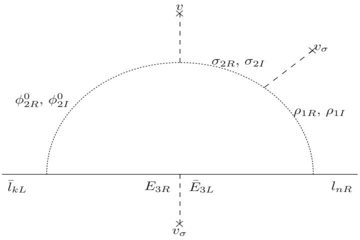

v
$\sigma_{2R},\ \sigma_{2I}$
v_σ
v_ρ
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
$\rho_{2R},\ \rho_{2I}$
v_σ
$\eta_R,\ \eta_I$
$\bar{u}_{nL}$
T_R
$\bar{T}_L$
u_{nR}
v_σ

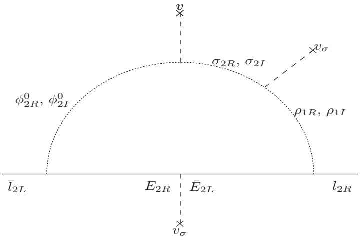

v
ϕ_2^+
$\sigma_{3R},\ \sigma_{3I}$
φ_1^-
v_ρ
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\bar{u}_{nL}$
B_{3R}
$\bar{B}_{3L}$
B_{4R}
$\bar{B}_{4L}$
u_{nR}

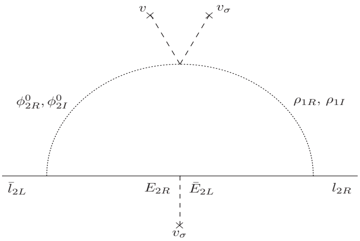

v
$\sigma_{2R},\ \sigma_{2I}$
v_σ
v_ρ
$\phi^0_{2R},\ \phi^0_{2I}$
$\rho_{2R},\ \rho_{2I}$
v_σ
$\eta_R,\ \eta_I$
$\bar{d}_{3L}$
B_{mR}
$\bar{B}_{nL}$
d_{jR}
v_σ

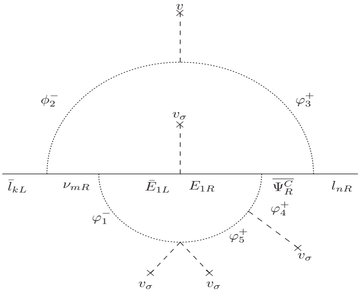

v
ϕ_2^-
$\sigma_{2R},\ \sigma_{2I}$
φ_2^+
v_ρ
v_σ
$\rho_{2R},\ \rho_{2I}$
$\bar{d}_{nL}$
T_R
$\bar{T}_L$
$\widetilde{T}_R$
$\bar{\widetilde{T}}_L$
d_{jR}
v_σ

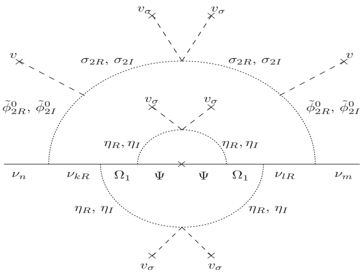

v
v_ρ
$\phi^0_{2R},\ \phi^0_{2I}$
$\eta_R,\ \eta_I$
$\bar{d}_{nL}$
B_{3R}
$\bar{B}_{3L}$
d_{jR}

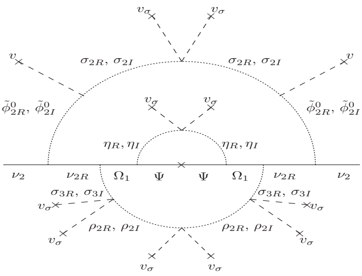

v
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\phi^0_{2R},\ \phi^0_{2I}$
$\rho_{1R},\ \rho_{1I}$
$\bar{l}_{kL}$
E_{3R}
$\bar{E}_{3L}$
l_{nR}
v_σ

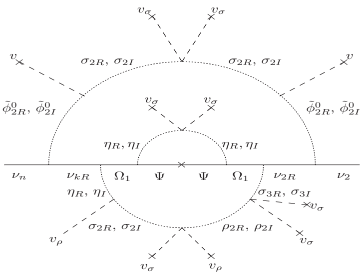

v
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\phi^0_{2R},\ \phi^0_{2I}$
$\rho_{1R},\ \rho_{1I}$
$\bar{l}_{2L}$
E_{2R}
$\bar{E}_{2L}$
l_{2R}
v_σ


v
v_σ
ϕ^0_{2R}, ϕ^0_{2I}
$\rho_{1R},\ \rho_{1I}$
$\bar{l}_{2L}$
E_{2R}
$\bar{E}_{2L}$
l_{2R}
v_σ


v
ϕ_2^-
φ_3^+
v_σ
$\bar{l}_{kL}$
ν_{mR}
$\bar{E}_{1L}$
E_{1R}
$\overline{\Psi_R^C}$
l_{nR}
φ_1^-
φ_4^+
φ_5^+
v_σ
v_σ
v_σ


v_σ
v_σ
v
$\sigma_{2R},\ \sigma_{2I}$
$\sigma_{2R},\ \sigma_{2I}$
v
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_n
ν_{kR}
Ω_1
Ψ
Ψ
Ω_1
ν_{lR}
ν_m
$\eta_R,\ \eta_I$
$\eta_R,\ \eta_I$
v_σ
v_σ


v_σ
v_σ
v
$\sigma_{2R},\ \sigma_{2I}$
$\sigma_{2R},\ \sigma_{2I}$
v
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_2
ν_{2R}
Ω_1
Ψ
Ψ
Ω_1
ν_{2R}
ν_2
$\sigma_{3R},\ \sigma_{3I}$
$\sigma_{3R},\ \sigma_{3I}$
v_σ
v_σ
v_σ
$\rho_{2R},\ \rho_{2I}$
$\rho_{2R},\ \rho_{2I}$
v_σ
v_σ
v_σ

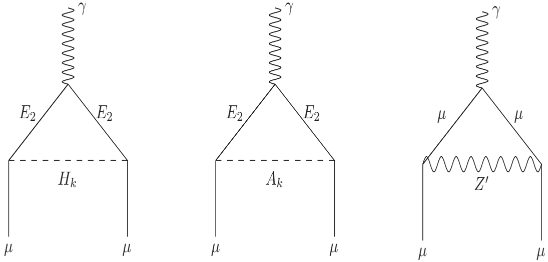

v_σ
v_σ
v
$\sigma_{2R},\ \sigma_{2I}$
$\sigma_{2R},\ \sigma_{2I}$
v
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_n
ν_{kR}
Ω_1
Ψ
Ψ
Ω_1
ν_{2R}
ν_2
$\eta_R,\ \eta_I$
$\sigma_{3R},\ \sigma_{3I}$
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\rho_{2R},\ \rho_{2I}$
v_ρ
v_σ
v_σ
v_ρ

Fitting $R_K = \frac{Br(B \to K\mu^+\mu^-)}{Br(B \to Ke^+e^-)}$ at 1σ and 2σ yields the constraints:

$$14 \text{ TeV} < \frac{M_{Z'}}{g_X} < 20 \text{ TeV} \text{ at } 1\sigma, \quad 13 \text{ TeV} < \frac{M_{Z'}}{g_X} < 26 \text{ TeV} \text{ at } 2\sigma\,.$$

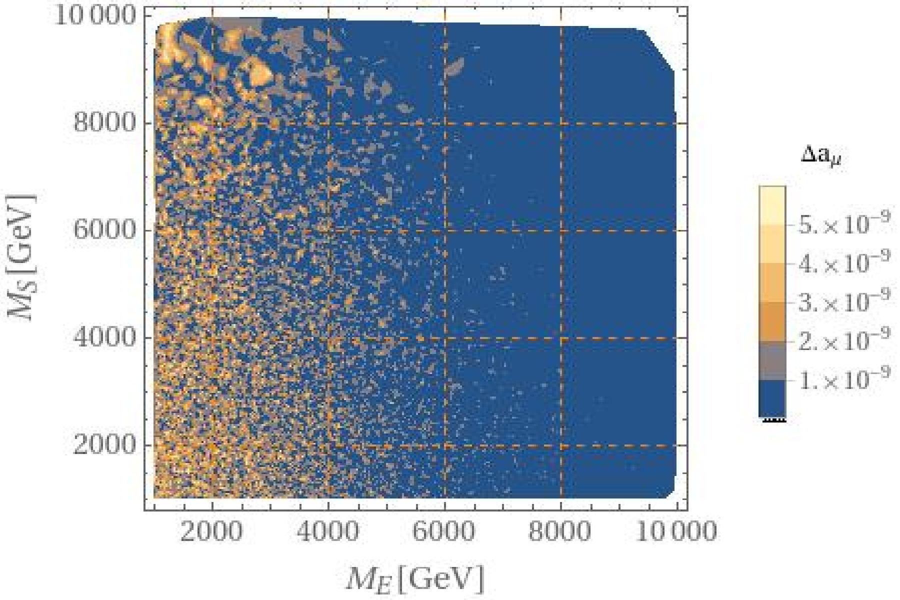
The $e^+e^- \to \mu^+\mu^-$ measurement at LEP imposes the following limit :

$$\frac{M_{Z'}}{g_X} > 12 \text{ TeV}\,. \tag{46}$$

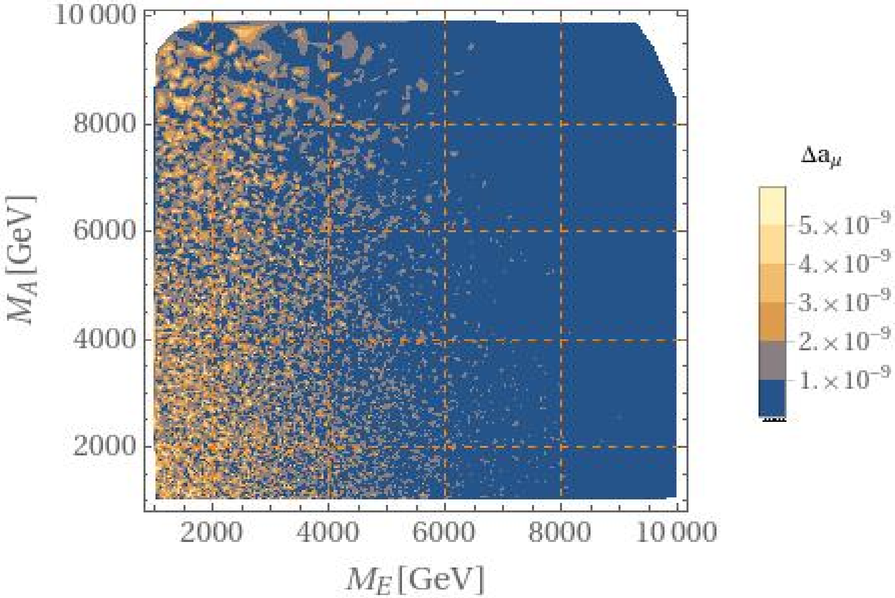


Here H_k and A_k $(k = 1, 2)$ are the physical CP even and CP odd states built from ρ_2 and ϕ_2^0.
We have fixed $\tan\theta = \frac{v}{v_\sigma}$, $M_{Z'} = 1.5$ TeV and $g_X = 0.1$, in consistency with the 2.6σ R_K anomaly. Considering that the muon anomalous magnetic moment is constrained to be in the range:

$$\left(\Delta a_\mu\right)_{\exp} = (26.1 \pm 8) \times 10^{-10}\,, \quad (47)$$



10 000
8000
6000
4000
2000
M_S [GeV]
2000
4000
6000
8000
10 000
M_E [GeV]
Δa_μ
$5.\times10^{-9}$
$4.\times10^{-9}$
$3.\times10^{-9}$
$2.\times10^{-9}$
$1.\times10^{-9}$



10 000
8000
6000
4000
2000
M_A [GeV]
2000
4000
6000
8000
10 000
M_E [GeV]
Δa_μ
$5.\times10^{-9}$
$4.\times10^{-9}$
$3.\times10^{-9}$
$2.\times10^{-9}$
$1.\times10^{-9}$

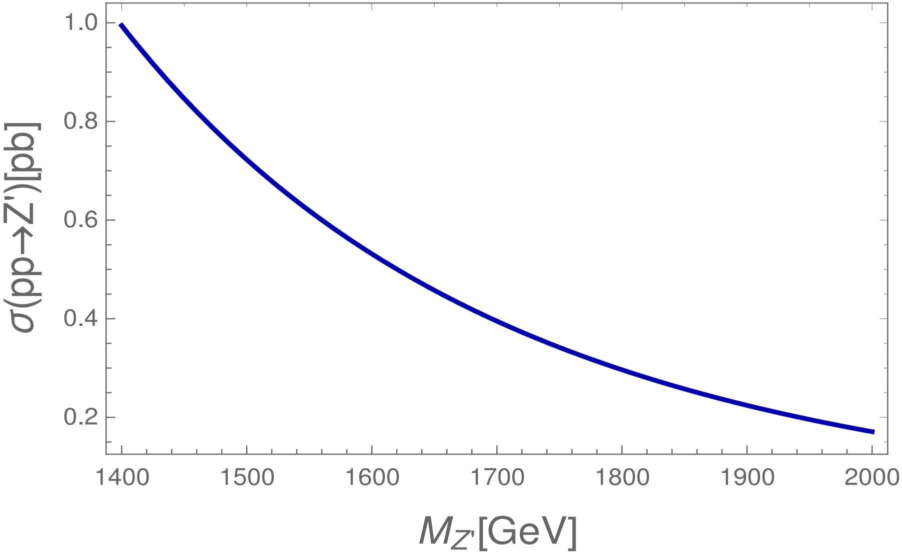


Figure: The total Z' production cross section via the DY mechanism at the LHC for $\sqrt{S} = 13$ TeV and $g_X = 0.1$ as a function of the Z' mass.

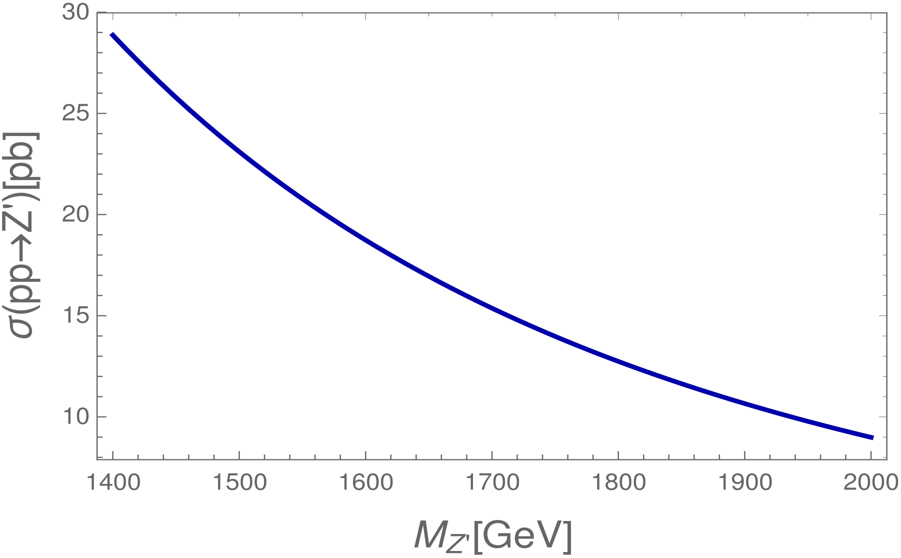


Figure: The total Z' production cross section via the DY mechanism at a future pp collider for $\sqrt{S} = 100$ TeV and $g_X = 0.1$ as a function of the Z' mass.

For $m^2_{\Phi_{DM}} >> v^2$, with $v = 246$ GeV, one has the estimate:

$$< \sigma v > \simeq \frac{\gamma^2}{128\pi m^2_{\Phi_{DM}}}, \tag{48}$$

which results in a DM relic abundance

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1pb}{0.12 < \sigma v >} \simeq \left(\frac{1}{\gamma}\right)^2 \left(\frac{m_{\Phi_{DM}}}{1.1\,TeV}\right)^2, \tag{49}$$

In the scenario with a fermionic DM candidate, when $m^2_{\Omega_{1R}} << m^2_{\eta_R} \sim m^2_{\eta_I} \sim m^2_{\eta}$, one has:

$$< \sigma v > \simeq \frac{9 y_\Omega^4 m_\Omega^2}{16\pi m_\eta^4}. \tag{50}$$

Then, the DM relic abundance is

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1pb}{0.12 < \sigma v >} \simeq \left(\frac{1}{y_\Omega}\right)^4 \left(\frac{400\,GeV}{m_\Omega}\right)^2 \left(\frac{m_\eta}{1.9\,TeV}\right)^4. \tag{51}$$

A minimal 3-3-1 model with radiative mechanisms.

$$\begin{aligned}\mathcal{G} = SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_{L_g} \times Z_4 &\xrightarrow{\Lambda_{int}} \\ SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_2^{(L_g)} \times Z_2 &\xrightarrow{v_\rho} \\ SU(3)_C \times U(1)_Q \times Z_2^{(L_g)} \times Z_2. & \end{aligned} \tag{52}$$

The $SU(3)_L$ scalar triplets of this model are represented as:

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix}, \tag{53}$$

$$\phi = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(\xi_\phi \pm i\zeta_\phi) \\ \phi_3^+ \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix}.$$

whereas the $SU(3)_L$ fermionic triplets and antitriplets take the form:

$$\begin{aligned} Q_{1L} &= (u_1, d_1, J_1,)_L^T, \quad Q_{nL} = (d_n, -u_n, J_n)_L^T, \\ L_{iL} &= (\nu_i, l_i, \nu_i^c)_L^T, \quad n = 2, 3, \quad i = 1, 2, 3. \end{aligned} \tag{54}$$

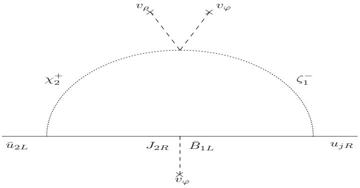
	χ	ρ	ϕ	σ	φ	ϱ	η	S	$\zeta_1^{\pm}$	$\zeta_2^{\pm}$
$SU(3)_C$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1	$\mathbf{1}$	$\mathbf{1}$
$SU(3)_L$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1	$\mathbf{1}$	$\mathbf{1}$
$U(1)_X$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0	0	0	0	±1	±1
$U(1)_{L_g}$	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	2	0	2	4	0	0
Z_4	1	1	i	-1	-1	$-i$	$-i$	-1	-1	$-i$

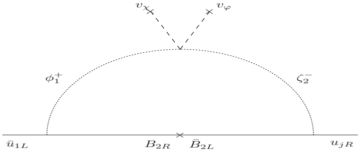
Table: Scalar assignments under $SU(3)_C \times SU(3)_L \times \times U(1)_X \times U(1)_{L_g} \times Z_4$.

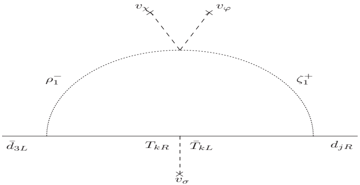
	Q_{1L}	Q_{2L}	Q_{3L}	u_{iR}	d_{iR}	J_{1R}	J_{2R}	J_{3R}	T_{kL}	T_{kR}	T_{2L}	T_{2R}	B_{1L}	B_{1R}	B_{2L}	B_{2R}	L_{iL}	l_{iR}	E_{iL}	E_{iR}	N_{iR}	Ω_{nR}
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(3)_L$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_X$	$\frac{1}{3}$	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	-1	-1	-1	0	0
$U(1)_{L_g}$	$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0	-2	2	2	0	0	0	0	0	0	0	0	$\frac{1}{3}$	1	1	1	-1	-1
Z_4	-1	-1	1	1	1	-1	-1	1	-1	1	$-i$	$-i$	1	-1	i	i	1	-1	i	$-i$	1	i

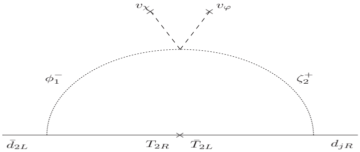
Table: Fermion assignments under $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_{L_g} \times Z_4$. Here $n = 1, 2,$ $k = 1, 3$ and $i = 1, 2, 3$.

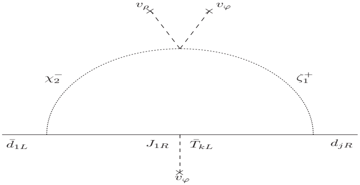
$\frac{1}{\Lambda} L_{iL}^C \bar{L}_{jL} \bar{Q}_{1L} d_{kR}$ is forbidden thanks to $U(1)_{L_g}$ and Z_4.

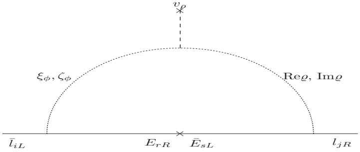

v_ρ
v_φ
χ_2^+
ζ_1^-
$\bar{u}_{2L}$
J_{2R}
$\bar{B}_{1L}$
u_{jR}
v_φ

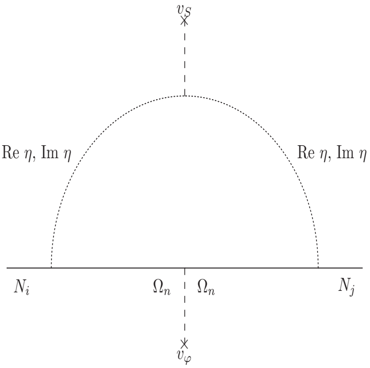

v_χ
v_φ
ϕ_1^+
ζ_2^-
$\bar{u}_{1L}$
B_{2R}
$\bar{B}_{2L}$
u_{jR}

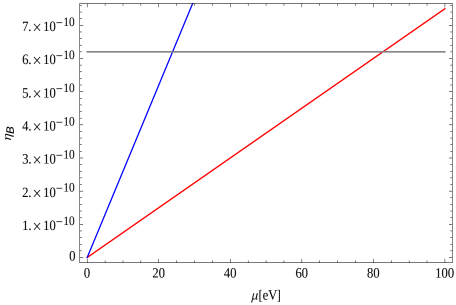

v_χ
v_φ
ρ_1^-
ζ_1^+
$\bar{d}_{3L}$
T_{kR}
$\bar{T}_{kL}$
d_{jR}
v_σ


v_χ
v_φ
ϕ_1^-
ζ_2^+
$\bar{d}_{2L}$
T_{2R}
$\bar{T}_{2L}$
d_{jR}


v_ρ
v_φ
χ_2^-
ζ_1^+
$\bar{d}_{1L}$
J_{1R}
$\bar{T}_{kL}$
d_{jR}
v_φ


v_ϱ
ξ_ϕ, ζ_ϕ
$\mathrm{Re}\varrho$, $\mathrm{Im}\varrho$
$\bar{l}_{iL}$
E_{rR}
$\bar{E}_{sL}$
l_{jR}


v_S
Re η, Im η
Re η, Im η
N_i
Ω_n
Ω_n
N_j
v_φ



The value of VEV is fixed: $v_\chi = 30$ TeV for red line, and $v_\chi = 5$ TeV for blue line. The gray line is the observed baryon asymmetry $\eta_B = 6.2 \times 10^{-10}$

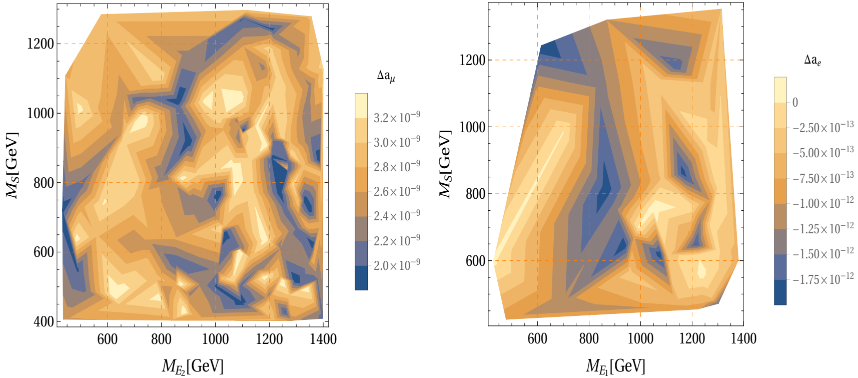


Figure: Allowed parameter space for M_S-M_{E_1} (top panel) and M_S-M_{E_2} (bottom panel) planes with different values of the muon and electron anomalous magnetic moments.

Conclusions

- The SM fermion mass hierarchy can be generated by the loops.
- Implementing the sequential loop suppression mechanism requires to consider vector like exotic fermions and extended scalar sectors.
- Extra symmetries have to be imposed to implement such mechanism.
- Such models have DM particle candidates.
- The mass scale of the non-SM particles are of the order of 1 TeV.
- Fermion masses and mixings, DM, $(g-2)_{e,\mu}$ anomalies, lepton and baryon asymmetry can be accounted for.

Acknowledgements

Thank you very much to all of you for the attention.

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Extra Slides

Combining radiative mechanisms with spontaneously broken symmetries.

The S_3 symmetry is softly broken whereas the Z_8 discrete group is broken.

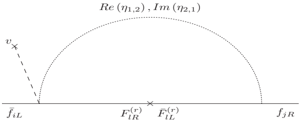
$$\phi \sim (\mathbf{1}, 1), \quad \eta = (\eta_1, \eta_2) \sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad \chi \sim (\mathbf{1}, -i),$$
$$v_\chi = \lambda\Lambda, \qquad \lambda = 0.225. \tag{55}$$

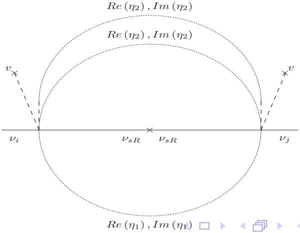
$$\begin{aligned}
q_{jL} &\sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad u_{kR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-k)}{2}}\right), \quad u_{3R} \sim (\mathbf{1}, 1), \\
d_{jR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \quad l_{jL} \sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad l_{jR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \\
T_L^{(k)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad T_R^{(k)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad k = 1, 2, \\
B_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad B_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad j = 1, 2, 3, \\
E_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad E_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \\
\nu_{kR} &\sim \left(\mathbf{1}', e^{-\frac{\pi i}{4}}\right), \quad k = 1, 2.
\end{aligned} \tag{56}$$

I use the S_3 discrete group since it is the smallest non-Abelian group.

$$
\begin{aligned}
-\mathcal{L}_{Y}^{(U)} &= \sum_{j=1}^{3}\sum_{r=1}^{2} y_{jr}^{(u)}\overline{q}_{jL}\widetilde{\phi}\left(T_{R}^{(r)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{r=1}^{2}\sum_{s=1}^{2} x_{rs}^{(u)}\left(\overline{T}_{L}^{(r)}\eta\right)_{\mathbf{1}'} u_{sR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{j=1}^{3} y_{j3}^{(u)}\overline{q}_{jL}\widetilde{\phi}u_{3R}\frac{\chi^{3-j}}{\Lambda^{3-j}} + \sum_{r=1}^{2} y_{r}^{(T)}\left(\overline{T}_{L}^{(r)}T_{R}^{(r)}\right)_{\mathbf{1}}\chi + h.c
\end{aligned}
$$

$$
-\mathcal{L}_{Y}^{(\nu)} = \sum_{j=1}^{3}\sum_{s=1}^{2} y_{js}^{(\nu)}\overline{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{\left[\eta^{*}\left(\eta\eta^{*}\right)_{\mathbf{2}}\right]_{\mathbf{1}'}\chi^{3-j}}{\Lambda^{6-j}} + \sum_{s=1}^{2} y_{s}\overline{\nu}_{sR}\nu_{sR}^{C}\chi + h.c.
$$





The charged fermion mass matrices are:

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)}\lambda^3 & \varepsilon_{12}^{(u)}\lambda^2 & y_{13}^{(u)}\lambda^2 \\ \varepsilon_{21}^{(u)}\lambda^2 & \varepsilon_{22}^{(u)}\lambda & y_{23}^{(u)}\lambda \\ \varepsilon_{31}^{(u)}\lambda & \varepsilon_{32}^{(u)} & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \qquad (57)$$

$$M_{D,l} = \begin{pmatrix} \varepsilon_{11}^{(d,l)}\lambda^4 & \varepsilon_{12}^{(d,l)}\lambda^3 & \varepsilon_{13}^{(d,l)}\lambda^2 \\ \varepsilon_{21}^{(d,l)}\lambda^3 & \varepsilon_{22}^{(d,l)}\lambda^2 & \varepsilon_{23}^{(d,l)}\lambda \\ \varepsilon_{31}^{(d,l)}\lambda^2 & \varepsilon_{32}^{(d,l)}\lambda & \varepsilon_{33}^{(d,l)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

where the dimensionless parameters $\varepsilon_{jk}^{(f)}$ $(j,k=1,2,3)$ with $f=u,d,l$, are generated at one loop level. The invariance of charged exotic fermion Yukawa interactions under the cyclic symmetry requires to consider the Z_8 instead of the Z_4 discrete symmetry.

$$
\begin{aligned}
-L_{gY}^{(q)} &= h_{\chi}^{(T)}\overline{Q}_{3L}\chi T_R + h_{\eta}^{(U)}\overline{Q}_{3L}\eta U_{3R} \\
&+ \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\rho nm}^{(\widetilde{T})}\overline{Q}_{nL}\rho^{*}\widetilde{T}_{mR} + \sum_{n=1}^{2} h_{\varphi_1^0 n2}^{(U)}\overline{\widetilde{T}}_{nL}\varphi_1^0 U_{2R} + \sum_{n=1}^{2} h_{\varphi_2^0 n1}^{(U)}\overline{\widetilde{T}}_{nL}\varphi_2^0 U_{1R} \\
&+ \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\chi nm}^{(J)}\overline{Q}_{nL}\chi^{*}J_{mR} + h_{\rho}^{(B)}\overline{Q}_{3L}\rho B_R + \sum_{j=1}^{3} h_{\varphi_1^0 j}^{(D)}\overline{B}_L\varphi_1^0 D_{jR} \qquad (1) \\
&+ \sum_{n=1}^{2}\sum_{j=1}^{3} h_{\phi_1^+ nj}^{(D)}\overline{\widetilde{T}}_{nL}\phi_1^+ D_{jR} + \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\varphi_2^0 nm}^{(\widetilde{T})}\overline{\widetilde{T}}_{nL}\varphi_2^0\widetilde{T}_{mR} + m_B\bar{B}_L B_R + h.c,
\end{aligned}
$$

$$
\begin{aligned}
-L_{gY}^{(l)} &= h_{\rho}^{(E)}\overline{L}_{1L}\rho E_{1R} + h_{\varphi_2^0}^{(E)}\overline{E}_{1L}\varphi_2^0 E_{1R} + h_{\varphi_2^0}^{(e)}\overline{E}_{1L}\varphi_2^0 e_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(E)}\overline{L}_{nL}\rho E_{mR} \\
&+ h_{\rho}^{(e)}\overline{L}_{1L}\rho e_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(e)}\overline{L}_{nL}\rho e_{mR} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\varphi_1^0 nm}^{(E)}\overline{E}_{nL}\varphi_1^0 E_{mR} \\
&+ \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\varphi_1^0 nm}^{(e)}\overline{E}_{nL}\varphi_1^0 e_{mR} + \sum_{n=2}^{3}\sum_{j=1}^{3} h_{\chi nj}^{(L)}\overline{L}_{nL}\chi N_{jR} \\
&+ \sum_{j=1}^{3}\sum_{n=2}^{3} h_{\phi_4^- nj}^{(e)}\overline{E}_{nL}\phi_4^- N_{jR} + \sum_{j=1}^{3} h_{\varphi_2^0}^{(N)}\overline{\Psi_R^c}\left(\varphi_2^0\right)^{*} N_{jR} + y_{\Psi}\overline{\Psi_R^c}\Psi_R\xi^0 \\
&+ h_{\rho 11}^{(L)}\varepsilon_{abc}\overline{L}_{1L}^{a}\left(L_{1L}^{C}\right)^{b}\left(\rho^{*}\right)^{c} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(L)}\varepsilon_{abc}\overline{L}_{nL}^{a}\left(L_{mL}^{C}\right)^{b}\left(\rho^{*}\right)^{c} + h.c. \qquad (2)
\end{aligned}
$$

$$
V \supset \lambda_1\eta\chi\rho\varphi_1^0 + \lambda_2\eta\chi\rho\left(\varphi_1^0\right)^{*} + \lambda_3\phi_3^-\rho\eta^{\dagger}\xi^0 + \lambda_4\phi_1^-\phi_2^+\left(\varphi_2^0\right)^{*}\left(\xi^0\right)^{*} + w_1\left(\varphi_2^0\right)^2\varphi_1^0 + w_2\phi_3^-\rho\chi^{\dagger} + h.c.. \qquad (3)
$$

$$
\begin{aligned}
L_{gsoft}^{F} &= \sum_{n=1}^{2}\sum_{m=1}^{2}\left(m_{\widetilde{T}}\right)_{nm}\overline{\widetilde{T}}_{nL}\widetilde{T}_{mR} + m_{E_1}\overline{E}_{1L}E_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3}\left(m_E\right)_{nm}\overline{E}_{nL}E_{mR} \\
&+ \sum_{n=2}^{3}\left(m_E\right)_{n1}\overline{E}_{nL}E_{1R} + h.c., \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{F}} = \; & y_{3j}^{(u)}\overline{q}_{3L}\widetilde{\phi}_1 u_{3R} + \sum_{n=1}^{2} x_n^{(u)}\overline{q}_{nL}\widetilde{\phi}_2 T_R + \sum_{n=1}^{2} z_j^{(u)}\overline{T}_L\eta^* u_{nR} + y_T\overline{T}_L\sigma_1 T_R + m_{\widetilde{T}}\overline{\widetilde{T}}_L\widetilde{T}_R + x^{(T)}\overline{T}_L\rho_2\widetilde{T}_R \\
& + \sum_{n=1}^{2} x_n^{(d)}\overline{q}_{3L}\phi_2 B_{nR} + \sum_{n=1}^{2}\sum_{j=1}^{3} y_{nj}^{(d)}\overline{B}_{nL}\eta d_{jR} + \sum_{j=1}^{3} z_j^{(d)}\overline{B}_{3L}\eta^* d_{jR} + \sum_{n=1}^{2} w_n^{(u)}\overline{B}_{4L}\varphi_1^- u_{nR} \\
& + \sum_{k=3}^{4} m_{B_k}\overline{B}_{kL}B_{kR} + \sum_{n=1}^{2} x_n^{(d)}\overline{q}_{nL}\phi_2 B_{3R} + \sum_{n=1}^{2}\sum_{m=1}^{2} y_{nm}^{(B)}\overline{B}_{nL}\sigma_1^* B_{mR} + z^{(B)}\overline{B}_{3L}\sigma_2^* B_{4R} + \sum_{j=1}^{3} w_j^{(d)}\overline{\widetilde{T}}_L\varphi_2^+ \\
& + \sum_{k=1,3} x_{k3}^{(l)}\bar{l}_{kL}\phi_2 E_{3R} + \sum_{k=1,3} y_{3k}^{(l)}\overline{E}_{3L}\rho_1 l_{kR} + x_{22}^{(l)}\bar{l}_{2L}\phi_2 E_{2R} + y_{22}^{(l)}\overline{E}_{2L}\rho_1 l_{2R} \\
& + \sum_{i=1}^{3} y_i^{(E)}\overline{E}_{iL}\sigma_1^* E_{iR} + x_2^{(\nu)}\bar{l}_{2L}\widetilde{\phi}_2\nu_{2R} + \sum_{k=1,3} z_k^{(l)}\overline{\Psi_R^C}\varphi_3^+ l_{kR} + \sum_{k=1,3} z_k^{(\nu)}\overline{E}_{1L}\varphi_1^-\nu_{kR} + z^{(E)}\overline{\Psi_R^C}\varphi_4^+ E_{1R} \\
& + \sum_{k=1,3}\sum_{n=1,3} x_{kn}^{(\nu)}\bar{l}_{kL}\widetilde{\phi}_2\nu_{nR} + \sum_{k=1,3} y_k^{(\Omega)}\overline{\Omega_{1R}^C}\eta^*\nu_{kR} + y^{(\Omega)}\overline{\Omega_{1R}^C}\sigma_3^*\nu_{2R} \\
& + x_1^{(\Psi)}\overline{\Omega_{1R}^C}\eta\Psi_R + x_2^{(\Psi)}\overline{\Omega_{2R}^C}\eta^*\Psi_R + z_\Omega\overline{\Omega_{1R}^C}\sigma_2^*\Omega_{2R} + m_\Psi\overline{\Psi_R^C}\Psi_R + h.c.\,,
\end{aligned}
$$

Parameter	$\frac{\Delta C_9^{\mu\mu}}{C_9^{SM}}$
Best fit	-0.21
1σ range	-0.27 up to -0.13
2σ range	-0.32 up to -0.08

Table: Constraints on the $C_9^{\mu\mu}$ Wilson coefficient from the LHCb data. Taken from Hurth, et al, 2016.

$$\Delta H_{eff} = -\frac{G_F \alpha_{em} V_{tb} V_{ts}^*}{\sqrt{2}\pi} \sum_{\tilde{l}=e,\mu,\tau} C_9^{\tilde{l}\tilde{l}} \left(\bar{s}\gamma^{\mu} P_L b\right) \left(\bar{\tilde{l}}\gamma^{\mu}\tilde{l}\right) . \quad (58)$$

$$\Delta C_9^{\mu\mu} = -\frac{9g_X^2}{2M_{Z'}^2} \left(V_{DL}^*\right)_{32} \left(V_{DL}\right)_{33} \frac{\sqrt{2}\pi}{G_F \alpha_{em} V_{tb} V_{ts}^*} \simeq -\frac{9g_X^2}{2M_{Z'}^2} \frac{\sqrt{2}\pi}{G_F \alpha_{em}} . \quad (59)$$

The group $\mathcal{G}$ has the following spontaneous breaking pattern:

$$\mathcal{G} = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \Delta(27) \otimes Z_4 \otimes Z_6 \otimes Z_{12}$$
$$\Downarrow \Lambda_{int}$$
$$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes Z_4 \otimes Z_2$$
$$\Downarrow v_{kR}, v_\xi$$
$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2$$
$$\Downarrow v_1, v_{kL}$$
$$SU(3)_C \otimes U(1)_Q \otimes Z_2.$$

We set $v_2 = 0$, for simplicity. φ is the only scalar which do not get VEV.

Field	Φ	χ_{1L}	χ_{1R}	χ_{2L}	χ_{2R}	σ	η	φ	ρ	ϕ	τ	ξ
$\Delta(27)$	$\mathbf{1}_{0,2}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{2,1}$	$\mathbf{1}_{2,2}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{3}$
Z_4	1	1	1	1	1	1	1	i	1	1	1	-1
Z_6	1	1	1	1	1	$\omega^{-\frac{1}{2}}$	ω	$\omega^{-\frac{1}{2}}$	ω	-1	1	ω^2
Z_{12}	1	1	1	1	1	1	$-i$	-1	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	ω^2

Table: Transformation properties of the scalars under the flavor symmetry $\Delta(27) \otimes Z_4 \otimes Z_6 \otimes Z_{12}$.

Field	Q_{iL}	Q_{iR}	L_{iL}	L_{iR}	T_{kL}	T_{kR}	B_{iL}	B_{iR}	E_{iL}	E_{iR}	S_i	Ω_i	Φ	χ_{kL}	χ_{kR}	σ	η	φ	ρ_i	ϕ_i	τ_i	ξ_i
$SU(3)_c$	**3**	**3**	**1**	**1**	**3**	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_R$	**1**	**2**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	-2	-2	0	0	0	1	1	0	0	0	0	0	0	0

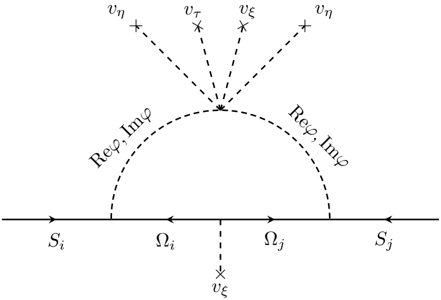
Table: Particle content and transformation properties under $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. Here $i = 1, 2, 3$ and $k = 1, 2$.

Field	Q_{1L}	Q_{1R}	Q_{2L}	Q_{2R}	Q_{3L}	Q_{3R}	T_{1L}	T_{1R}	T_{2L}	T_{2R}	B_{1L}	B_{1R}	B_{2L}	B_{2R}	B_{3L}	B_{3R}
$\Delta(27)$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{2,1}$	$\mathbf{1}_{2,2}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$
Z_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Z_6	-1	-1	ω^2	ω^2	1	1	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	ω^2	ω^2	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	ω	ω	1	1
Z_{12}	1	1	1	1	1	1	-1	-1	1	1	-1	-1	i	i	1	1

Table: Transformation properties of the quarks under the flavor symmetry $\Delta(27) \otimes Z_4 \otimes Z_6 \otimes Z_{12}$.

Field	L_{1L}	L_{1R}	L_{2L}	L_{2R}	L_{3L}	L_{3R}	E_{1L}	E_{1R}	E_{2L}	E_{2R}	E_{3L}	E_{3R}	S	Ω
$\Delta(27)$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{2,0}$	$\mathbf{3}$	$\overline{\mathbf{3}}$
Z_4	1	1	1	1	1	1	1	1	1	1	1	1	1	i
Z_6	-1	-1	ω^2	ω^2	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	ω	ω	$\omega^{-\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$	1	$\omega^{-\frac{1}{2}}$
Z_{12}	-1	-1	-1	-1	-1	-1	1	1	$-i$	$-i$	-1	-1	$-\omega^{\frac{1}{2}}$	$\omega^{-\frac{1}{2}}$

Table: Transformation properties of the leptons under the flavor symmetry $\Delta(27) \otimes Z_4 \otimes Z_6 \otimes Z_{12}$.



$$\frac{\kappa}{\Lambda^2}\left(\varphi^*\right)^2\tau^*\xi^*\left(\eta^*\right)^2$$

$$\lambda_3\left(\overline{S}\Omega^C\right)_{\mathbf{1_{0,0}}}\varphi$$

$$\lambda_{1,2}\left(\overline{\Omega}\Omega^C\right)_{\mathbf{\overline{3}}_{S_{1,2}}}\xi$$

$$\mu \simeq \frac{\lambda_3^2\left(m_R^2-m_I^2\right)v_\xi}{8\pi^2\left(m_R^2+m_I^2\right)\sqrt{2+r^2}}\begin{pmatrix} r\lambda_1 & \lambda_2 e^{i\psi} & \lambda_2 e^{-i\psi} \\ \lambda_2 e^{i\psi} & \lambda_1 e^{-i\psi} & r\lambda_2 \\ \lambda_2 e^{-i\psi} & r\lambda_2 & \lambda_1 e^{i\psi} \end{pmatrix}$$

$$M_\nu^{(1)} = \left(\frac{v v_\xi^2}{v_\rho v_\sigma v_R}\right)^2\mu - \frac{v_L v_\xi^2}{v_R\Lambda^2}\left(M_1+M_1^T\right), \quad \textit{we take } v_L << v_R$$

$$M_\nu^{(2)} = -\frac{1}{2}\left(M_2+M_2^T\right)+\frac{1}{2}\mu, \qquad M_\nu^{(3)} = \frac{1}{2}\left(M_2+M_2^T\right)+\frac{1}{2}\mu.$$

$$v_1 \sim v_{kL} \sim v \ll v_{kR} \sim v_\xi \ll v_\rho \sim v_\phi \sim v_\tau \sim v_\eta \sim v_\sigma \sim \lambda\Lambda, \qquad k = 1, 2.$$

Here $v = 246$ GeV, $v_{kR} \gtrsim \mathcal{O}(10)$ TeV $(k = 1, 2)$ the scale of breaking of the left-right symmetry

$$\langle\rho\rangle = v_\rho\,(1,0,0)\,, \qquad \langle\phi\rangle = v_\phi\,(0,1,0)\,, \qquad \langle\tau\rangle = v_\tau\,(0,0,1)\,, \qquad \langle\xi\rangle = \frac{v_\xi}{\sqrt{2+r^2}}\left(r, e^{-i\psi}, e^{i\psi}\right).$$

$$M_E = \begin{pmatrix} 0_{3\times3} & z\frac{v_L}{\sqrt{2}} \\ z^T\frac{v_R}{\sqrt{2}} & m_E \end{pmatrix}, \qquad z = \begin{pmatrix} z_{11}\lambda^2 & 0 & z_{13}\lambda^2 \\ 0 & z_{22}\lambda & z_{23}\lambda \\ 0 & 0 & z_{33} \end{pmatrix}, \qquad m_E = \begin{pmatrix} m_{E_1} & 0 & 0 \\ 0 & m_{E_2} & 0 \\ 0 & 0 & m_{E_3} \end{pmatrix}$$

$$\widetilde{M}_E = \frac{v_L v_R}{2} z m_E^{-1} z^T = \begin{pmatrix} e_{11}\lambda^7 & e_{12}\lambda^6 & e_{13}\lambda^5 \\ e_{12}\lambda^6 & e_{22}\lambda^5 & e_{23}\lambda^4 \\ e_{13}\lambda^5 & e_{23}\lambda^4 & e_{33}\lambda^3 \end{pmatrix}\frac{v}{\sqrt{2}},$$

$$M_U = \begin{pmatrix} 0_{2\times2} & 0_{2\times1} & x\frac{v_L}{\sqrt{2}} \\ 0_{1\times2} & \alpha_{33}\frac{v}{\sqrt{2}} & 0_{1\times2} \\ x^T\frac{v_R}{\sqrt{2}} & 0_{2\times1} & M_T \end{pmatrix}, \qquad x = \begin{pmatrix} x_{11}\lambda^2 & x_{12}\lambda \\ 0 & x_{22} \end{pmatrix}, \qquad M_T = \begin{pmatrix} m_{T_1} & 0 \\ 0 & m_{T_2} \end{pmatrix}, \qquad m_t = \alpha\frac{v}{\sqrt{2}},$$

$$M_D = \begin{pmatrix} 0_{3\times3} & y\frac{v_L}{\sqrt{2}} \\ y^T\frac{v_R}{\sqrt{2}} & M_B \end{pmatrix}, \qquad y = \begin{pmatrix} y_{11}\lambda^2 & 0 & y_{13}\lambda^3 \\ 0 & y_{22}\lambda & y_{23}\lambda^2 \\ 0 & 0 & y_{33} \end{pmatrix}, \qquad M_B = \begin{pmatrix} m_{B_1} & 0 & 0 \\ 0 & m_{B_2} & 0 \\ 0 & 0 & m_{B_3} \end{pmatrix},$$

$$\widetilde{M}_U = \begin{pmatrix} \frac{v_L v_R}{2} x M_T^{-1} x^T & 0_{2\times1} \\ 0_{1\times2} & m_t \end{pmatrix} = \begin{pmatrix} \left(\frac{a_{12}^2}{a_{22}} + \kappa\lambda^2\right)\lambda^6 & a_{12}\lambda^5 & 0 \\ a_{12}\lambda^5 & a_{22}\lambda^4 & 0 \\ 0 & 0 & \alpha \end{pmatrix}\frac{v}{\sqrt{2}},$$

$$\widetilde{M}_D = \frac{v_L v_R}{2} y M_B^{-1} y^T = \begin{pmatrix} b_{11}\lambda^7 & b_{12}\lambda^8 & b_{13}\lambda^6 \\ b_{12}\lambda^8 & b_{22}\lambda^5 & b_{23}\lambda^5 \\ b_{13}\lambda^6 & b_{23}\lambda^5 & b_{33}\lambda^3 \end{pmatrix}\frac{v}{\sqrt{2}},$$

The rates for $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are expected to be larger in the LR model than in PS model, whereas for $\mu \to e\gamma$ the rates are expected to be similar in both models.