Anisotropic Inflation with Coupled $p-$forms

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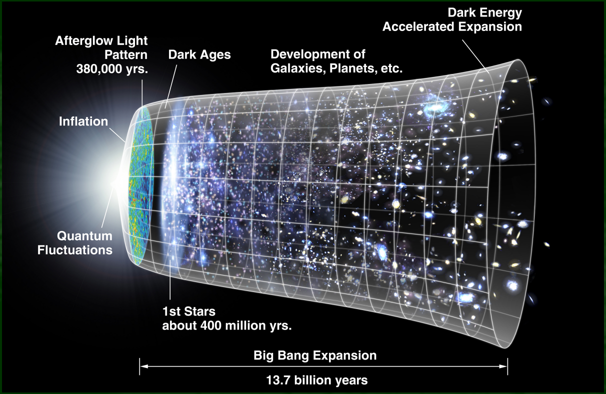
Motivation



- Anomalies in the CMB
- Anisotropic inflation
- Late-time acceleration; anisotropic dark energy
- A complete description of the Universe

The model

A general action allowing couplings between $p-$forms, and couplings with kinetic functions of a scalar field

$$\mathcal{L}_p = -\frac{1}{2}\sum_{p=1}^{3}\frac{f_p(\phi)}{(p+1)!}F_{(p)}^2 - \frac{g_1(\phi)}{4}F_{(1)\mu_1\mu_2}\tilde{F}_{(1)}{}^{\mu_1\mu_2} - \frac{g_2(\phi)}{2}A_{(2)\mu_1\mu_2}\tilde{F}_{(1)}{}^{\mu_1\mu_2} - g_3(\phi)\tilde{F}_{(3)}\,.$$

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \underbrace{V_{\rm eff}(\phi)}_{3-\text{form}} - \underbrace{\frac{f_1(\phi)F^2}{4}}_{1-\text{form}} - \underbrace{\frac{f_2(\phi)H^2}{12}}_{2-\text{form}} - \underbrace{\frac{m_v B\tilde{F}}{2}}_{\text{coupled}}\right]$$

with

$$F_{\mu_1\mu_2} = \partial_{[\mu_1}A_{\mu_2]}\,, \quad H_{\mu_1\mu_2\mu_3} = \partial_{[\mu_1}B_{\mu_2\mu_3]}\,,$$

$$F^2 \equiv F_{\mu_1\mu_2}F^{\mu_1\mu_2}\,, \quad H^2 \equiv H_{\mu_1\mu_2\mu_3}H^{\mu_1\mu_2\mu_3}\,, \quad B\tilde{F} \equiv B_{\mu_1\mu_2}\tilde{F}^{\mu_1\mu_2}\,.$$

Background equations

A_μ in the x direction $A_\mu = (0, v_A(t), 0, 0)$.
$B_{\mu\nu}$ orthogonal to the 1-form field $B_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = 2v_B(t)\mathrm{d}y \wedge \mathrm{d}z$.

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}\mathrm{d}x^2 + e^{2\sigma(t)}(\mathrm{d}y^2 + \mathrm{d}z^2)\right] ,$$

$a \equiv e^{\alpha(t)}$ is an isotropic scale factor, $\sigma(t)$ is a spatial shear.

$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{3M_{\mathrm{pl}}^2 e^{3\alpha}}{N}\left(\dot{\sigma}^2 - \dot{\alpha}^2\right) + e^{3\alpha}\frac{\dot{\phi}^2}{2N} - Ne^{3\alpha}V_{\mathrm{eff}}(\phi) + \frac{f_1(\phi)}{2N}e^{\alpha+4\sigma}\dot{v}_A^2 + \frac{f_2(\phi)}{2N}e^{-\alpha-4\sigma}\dot{v}_B^2 + m_v\dot{v}_A v_B\right]$$

$$\ddot{v}_A + \left(\frac{f_{1,\phi}}{f_1}\dot{\phi} + \dot{\alpha} + 4\dot{\sigma}\right)\dot{v}_A + \frac{m_v^2}{f_1 f_2}\left(v_A + \frac{p_B}{m_v}\right) = 0\,,$$

$$\ddot{v}_B + \left(\frac{f_{2,\phi}}{f_2}\dot{\phi} - \dot{\alpha} - 4\dot{\sigma}\right)\dot{v}_B + \frac{m_v^2}{f_1 f_2}\left(v_B - \frac{p_A}{m_v}\right) = 0\,.$$

Both 1- and 2-form fields acquire the effective mass term $m_v/\sqrt{f_1 f_2}$ through their interactions. Varying the action with respect to N, α, σ, and ϕ, we obtain

$$3M_{\rm pl}^2\left(\dot{\alpha}^2 - \dot{\sigma}^2\right) = \frac{1}{2}\dot{\phi}^2 + V_{\rm eff}(\phi) + \rho_A + \rho_B, \qquad M_{\rm pl}^2\left(\ddot{\alpha} + 3\dot{\alpha}^2\right) = V_{\rm eff}(\phi) + \frac{1}{3}\rho_A + \frac{2}{3}\rho_B\,,$$

$$M_{\rm pl}^2\left(\ddot{\sigma} + 3\dot{\alpha}\dot{\sigma}\right) = \frac{2}{3}\rho_A - \frac{2}{3}\rho_B, \qquad \ddot{\phi} + 3\dot{\alpha}\dot{\phi} + V_{{\rm eff},\phi} - \frac{f_{1,\phi}}{f_1}\rho_A - \frac{f_{2,\phi}}{f_2}\rho_B = 0\,,$$

where ρ_A and ρ_B are the energy densities of 1- and 2-forms

$$\rho_A = \frac{f_1(\phi)}{2}e^{-2\alpha+4\sigma}\dot{v}_A^2\,, \qquad \rho_B = \frac{f_2(\phi)}{2}e^{-4\alpha-4\sigma}\dot{v}_B^2\,.$$

the energy densities obey

$$\dot{\rho}_A = -4\rho_A\left(\dot{\alpha} + \dot{\sigma} + \frac{\dot{f}_1}{4f_1}\right) - 2m_v\sqrt{\frac{\rho_A\rho_B}{f_1 f_2}}, \quad \dot{\rho}_B = -2\rho_B\left(\dot{\alpha} - 2\dot{\sigma} + \frac{\dot{f}_2}{2f_2}\right) + 2m_v\sqrt{\frac{\rho_A\rho_B}{f_1 f_2}}.$$

Anisotropic inflation: uncoupled case ($m_v = 0$)

In this case

$$\rho_A = \frac{p_A^2}{2f_1(\phi)} e^{-4\alpha-4\sigma}\,, \qquad \rho_B = \frac{p_B^2}{2f_2(\phi)} e^{-2\alpha+4\sigma}\,.$$

Slow-roll approximations $\dot{\phi}^2/2 \ll V_{\rm eff}(\phi)$ and $|\ddot{\phi}| \ll |3\dot{\alpha}\dot{\phi}|$.
$3M_{\rm pl}^2\dot{\alpha}^2 \simeq V_{\rm eff}(\phi)$ and $3\dot{\alpha}\dot{\phi} \simeq -V_{{\rm eff},\phi}$,
Then, the critical couplings $f_1(\phi) \propto e^{-4\alpha}$ and $f_2(\phi) \propto e^{-2\alpha}$ correspond to

$$f_1(\phi) = e^{4\int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi}\,, \qquad f_2(\phi) = e^{2\int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi}\,.$$

these couplings separately give rise to anisotropic inflation with the non-vanishing shear $\Sigma \equiv \dot{\sigma}$

$$\frac{\Sigma}{H} \simeq \frac{\epsilon}{12(\alpha+\alpha_0)}\,, \qquad \text{for} \quad p_A \neq 0\,, \quad p_B = 0\,,$$

$$\frac{\Sigma}{H} \simeq -\frac{\epsilon}{3(\alpha+\alpha_0)}\,, \qquad \text{for} \quad p_A = 0\,, \quad p_B \neq 0\,,$$

with $H \equiv \dot{\alpha}$, $\qquad \epsilon \equiv -\frac{\dot{H}}{H^2}$

$$f_1(\phi) = e^{4c_1 \int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi}, \qquad f_2(\phi) = e^{2c_2 \int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi},$$

where c_1 and c_2 are constants. Applying the slow-roll approximation $|\ddot{\phi}| \ll |3\dot{\alpha}\dot{\phi}|$

$$\frac{{\rm d}\phi}{{\rm d}\alpha} \simeq -\frac{M_{\rm pl}^2 V_{{\rm eff},\phi}}{V_{\rm eff}} \left(1 - \frac{c_1 p_A^2}{\epsilon_V V_{\rm eff}} e^{-4\alpha - 4\sigma - 4c_1 \int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi} - \frac{c_2 p_B^2}{2\epsilon_V V_{\rm eff}} e^{-2\alpha + 4\sigma - 2c_2 \int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi}\right),$$

where

$$\epsilon_V \equiv \frac{M_{\rm pl}^2}{2} \left(\frac{V_{{\rm eff},\phi}}{V_{\rm eff}}\right)^2 .$$

Introducing the quantity

$$x \equiv e^{2c_1\alpha + 2c_1 \int \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} {\rm d}\phi},$$

$$\frac{{\rm d}x}{{\rm d}\phi} = \frac{c_1}{\epsilon_V V_{\rm eff}} \left[2c_1 p_A^2 e^{4(c_1-1)\alpha - 4\sigma} x^{-1} + c_2 p_B^2 e^{2(c_2-1)\alpha + 4\sigma} x^{1-c_2/c_1}\right] \frac{{\rm d}\alpha}{{\rm d}\phi} .$$

- $c_1 > c_2 > 1$ In this case we can ignore the 2-form. The 1-form energy density reduces to

$$\rho_A = \frac{c_1 - 1}{2c_1^2}\epsilon_V V_{\rm eff}\,, \qquad \frac{{\rm d}\phi}{{\rm d}\alpha} = -\frac{M_{\rm pl}^2 V_{{\rm eff},\phi}}{V_{\rm eff}}\frac{1}{c_1}\,.$$

$$\frac{\Sigma}{H} \simeq \frac{c_1 - 1}{3c_1}\epsilon\,, \qquad \epsilon \simeq \frac{\epsilon_V}{c_1}\,,$$

- $c_2 > c_1 > 1$ The 2-form energy density is given by

$$\rho_B = \frac{c_2 - 1}{c_2^2}\epsilon_V V_{\rm eff}\,, \qquad \frac{{\rm d}\phi}{{\rm d}\alpha} = -\frac{M_{\rm pl}^2 V_{{\rm eff},\phi}}{V_{\rm eff}}\frac{1}{c_2}\,,$$

$$\frac{\Sigma}{H} \simeq -\frac{2(c_2 - 1)}{3c_2}\epsilon\,, \qquad \epsilon \simeq \frac{\epsilon_V}{c_2}\,.$$

- $c_1 = c_2 > 1$. The densities could be written as

$$\frac{{\rm d}\phi}{{\rm d}\alpha} = -\frac{M_{\rm pl}^2 V_{{\rm eff},\phi}}{V_{\rm eff}}\frac{1}{c_1}\,,\ \epsilon \simeq \frac{\dot\phi^2}{2M_{\rm pl}^2 H^2} + \frac{2\rho_A + \rho_B}{3M_{\rm pl}^2 H^2} = \frac{\epsilon_V}{c_1}\,,\ P_A = p_A e^{-2\sigma}, P_B = p_B e^{2\sigma}$$

$$\rho_A = \frac{r_{AB}}{2r_{AB}+1}\frac{c_1 - 1}{c_1}\epsilon V_{\rm eff}\,,\ \ \rho_B = \frac{1}{2r_{AB}+1}\frac{c_1 - 1}{c_1}\epsilon V_{\rm eff}\,,\ \ r_{AB} \equiv \frac{\rho_A}{\rho_B} = \frac{P_A^2}{\mathcal{C}P_B^2}\,,$$

$$\frac{\Sigma}{H} \simeq \frac{2(r_{AB} - 1)}{2r_{AB}+1}\frac{c_1 - 1}{3c_1}\epsilon\,.$$

Numerical solutions

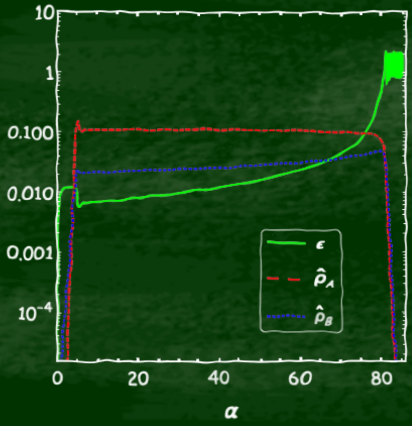
Quadratic potential $\rightarrow V_{\rm eff}(\phi) = \frac{1}{2}\mu^2\phi^2$

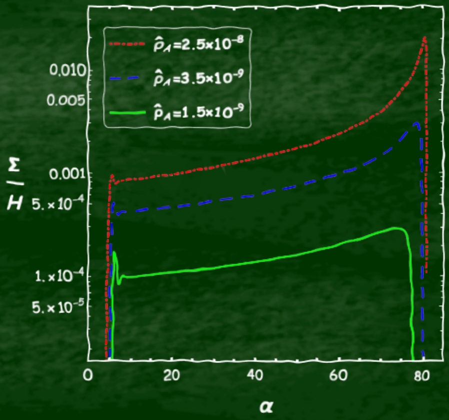
$$\hat{v}_A = \frac{v_A}{M_{\rm pl}}, \quad \hat{v}_B = \frac{v_B}{M_{\rm pl}}, \quad \hat{\rho}_A = \frac{\rho_A}{\mu^2 M_{\rm pl}^2}, \quad \hat{\rho}_B = \frac{\rho_B}{\mu^2 M_{\rm pl}^2}, \quad \hat{\phi} = \frac{\phi}{M_{\rm pl}}, \quad \hat{H} = \frac{\dot{\alpha}}{\mu}, \quad \hat{m}_v = \frac{m_v}{\mu},$$

$$\begin{aligned}
\hat{H} &= \sqrt{\sigma'^2 + \frac{1}{6}\hat{\phi}'^2 + \frac{1}{6}\hat{\phi}^2 + \frac{1}{3}\hat{\rho}_A + \frac{1}{3}\hat{\rho}_B}\,, \\
\hat{H}' &= -3\sigma'^2 - \frac{1}{2}\phi'^2 - \frac{2}{3}\hat{\rho}_A - \frac{1}{3}\hat{\rho}_B\,, \\
\sigma'' &= -3\hat{H}\sigma' + \frac{2}{3}\hat{\rho}_A - \frac{2}{3}\hat{\rho}_B\,, \\
\hat{\phi}'' &= -3\hat{H}\hat{\phi}' - \hat{\phi} + 2c_1\hat{\phi}\hat{\rho}_A + c_2\hat{\phi}\hat{\rho}_B\,.
\end{aligned}$$

$$\begin{aligned}
\hat{\rho}_A' &= -4\hat{\rho}_A\left(\hat{H} + \sigma' + \frac{c_1}{2}\hat{\phi}\hat{\phi}'\right) - 2\hat{m}_v\sqrt{\frac{\hat{\rho}_A\hat{\rho}_B}{f_1 f_2}}\,, \\
\hat{\rho}_B' &= -2\hat{\rho}_B\left(\hat{H} - 2\sigma' + \frac{c_2}{2}\hat{\phi}\hat{\phi}'\right) + 2\hat{m}_v\sqrt{\frac{\hat{\rho}_A\hat{\rho}_B}{f_1 f_2}}\,.
\end{aligned}$$

Solution for the case $m_v = 0$, $c_1 = c_2 = 2$





Anisotropic inflation: coupled case ($m_v \neq 0$)

$$\dot{\rho}_A = -4\rho_A \left(\dot{\alpha} + \dot{\sigma} + c_1 \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} \dot{\phi} \right) - 2\bar{m}_v \sqrt{\rho_A \rho_B}\,,$$

$$\dot{\rho}_B = -2\rho_B \left(\dot{\alpha} - 2\dot{\sigma} - 2c_1 \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} \dot{\phi} \right) + 2\bar{m}_v \sqrt{\rho_A \rho_B}\,.$$

Imposing the conditions $\dot{\rho}_A = 0$ and $\dot{\rho}_B = 0$,

$$\frac{\rho_B}{\rho_A} = \frac{9H^2}{4\bar{m}_v^2} \left[\sqrt{1 + \frac{4\bar{m}_v^2}{9H^2}} - 1 \right]^2 .$$

Anisotropic inflation in the regime $\bar{m}_v^2/H^2 \ll 1$.

$$\dot{\rho}_A \simeq -4\rho_A \left(\dot{\alpha} + \dot{\sigma} + c_1 \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} \dot{\phi} + \frac{\bar{m}_v^2}{6H} \right)\,, \quad \dot{\rho}_B \simeq 4\rho_B \left(\dot{\alpha} + \dot{\sigma} + c_1 \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} \dot{\phi} \right) .$$

he condition $\dot{\rho}_B = 0$ holds for

$$\dot{\alpha} + \dot{\sigma} + c_1 \frac{V_{\rm eff}}{M_{\rm pl}^2 V_{{\rm eff},\phi}} \dot{\phi} = 0\,.$$

$$\dot{\rho}_A = -\frac{2\bar{m}_v^2}{3H}\rho_A\,, \quad \Longrightarrow \quad \rho_A = \rho_{A0}\exp\left(-\int_0^\alpha \frac{2\bar{m}_v^2}{3H^2}\,\mathrm{d}\tilde{\alpha}\right)\,,$$

$$\frac{\Sigma}{H} \simeq \frac{2\rho_A}{3V_{\rm eff}}\left(1-\frac{\bar{m}_v^2}{9H^2}\right)\,,$$

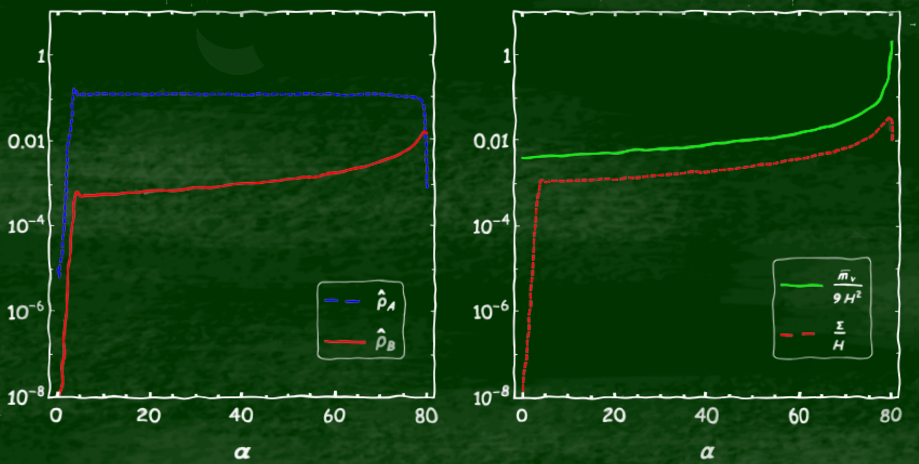
where we employed the slow-roll approximation $3M_{\rm pl}^2H^2 \simeq V_{\rm eff}$ together with the condition $\rho_A \ll V_{\rm eff}$.

$$\frac{\Sigma}{H} \simeq \frac{c_1-1}{3c_1^2}\epsilon_V \simeq \frac{c_1-1}{3c_1}\epsilon\left(1-\frac{c_1-1}{6c_1}\frac{\bar{m}_v^2}{H^2}\right)\,, \quad \epsilon \simeq \frac{\epsilon_V}{c_1}\left(1+\frac{c_1-1}{6c_1}\frac{\bar{m}_v^2}{H^2}\right)\,,$$

Hence the anisotropic shear can survive during inflation for the coupled system of 1- and 2-forms.

Numerical solutions

Solution for the case $\bar{m}_v = \mu$, $c_1 = 2$, $c_2 = -4$



Conclusions

1. $p-$forms could generate anistropic hair during inflationary epoch.
2. General solutions fr the evolution of shear, withput invoking a particular form of the effective potential $V_{\rm eff}$
3. For the coupled case, for the couplings satisfying $c_2 = -2c_1$ and $c_1 > 1$, we found a new class of anisotropic inflationary solutions along which both ρ_A and ρ_B are approximately constant.
4. Inflation is important.