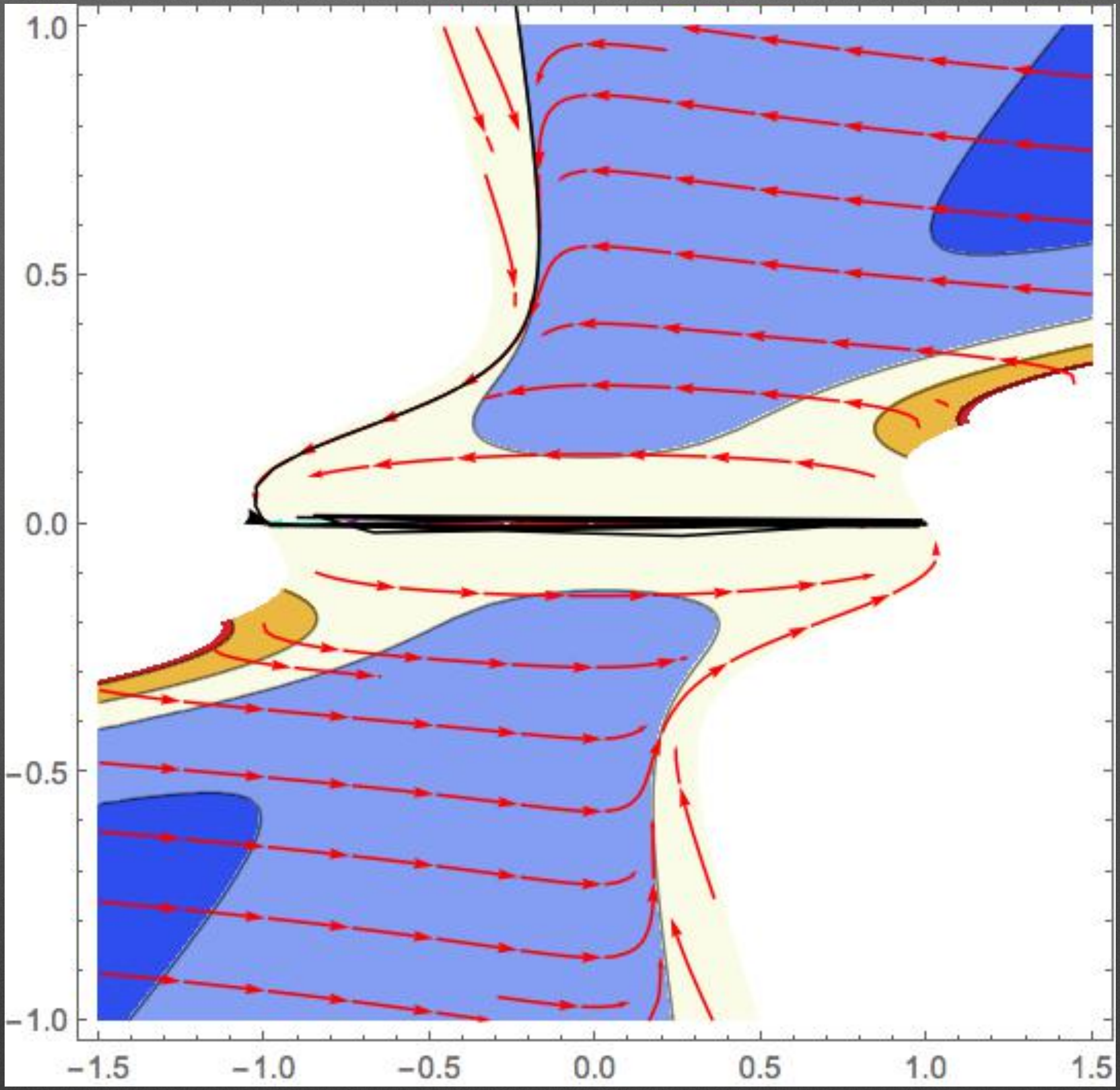

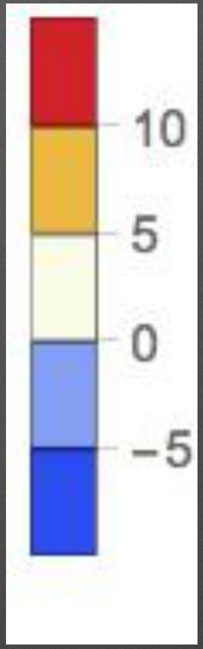

Generalized SU(2) Proca Inflation

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Sequel

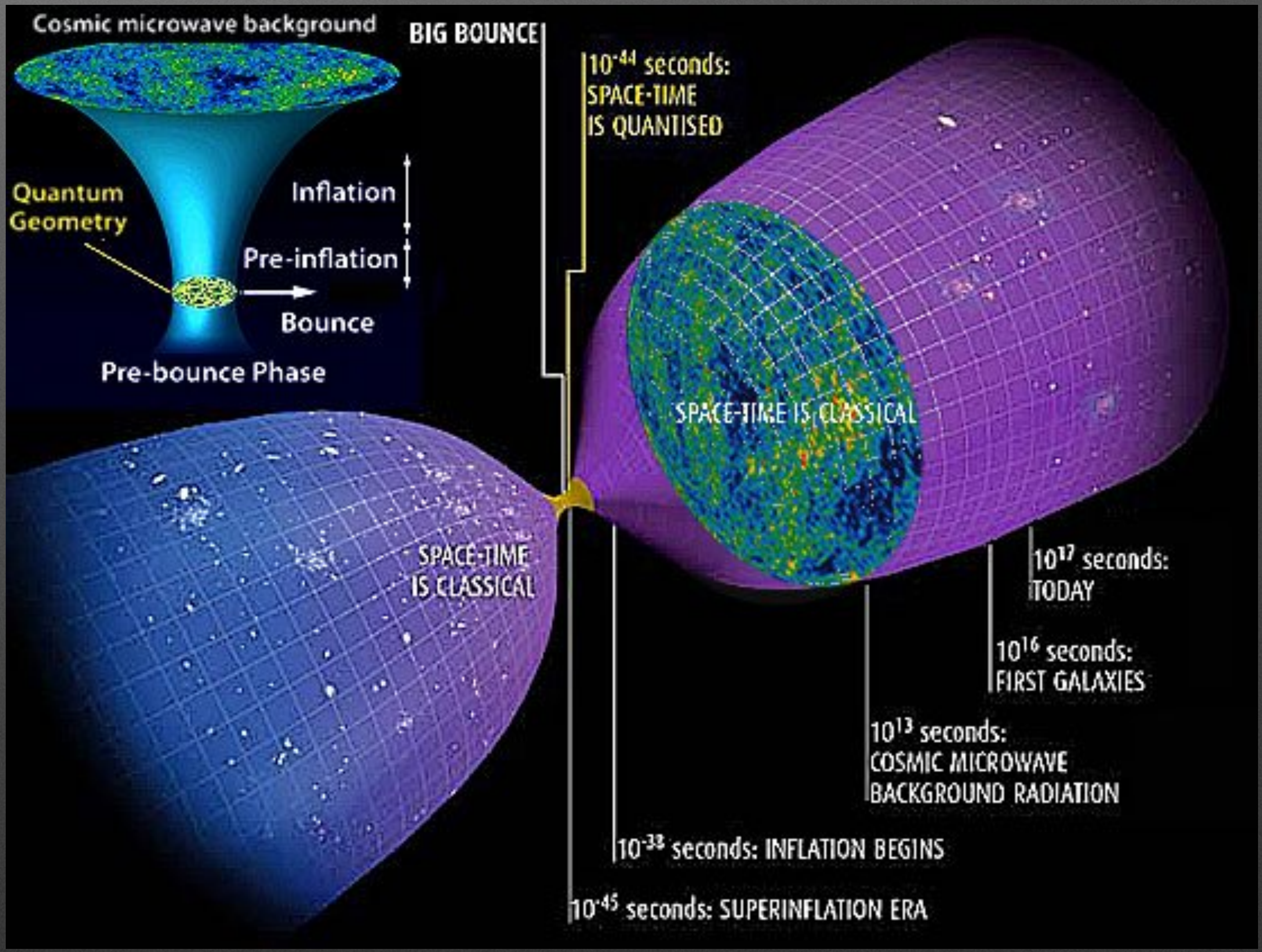
- E. Allys, P. Peter, and Y. Rodríguez, "Generalized Proca action for an Abelian vector field", JCAP 1602 (2016) 004.
- E. Allys, J. P. Beltrán-Almeida, P. Peter, and Y. Rodríguez, "On the 4D generalized Proca action for an Abelian vector field", JCAP 1609 (2016) 026.
- E. Allys, P. Peter, and Y. Rodríguez, "Generalized SU(2) Proca theory", Phys. Rev. D 94 (2016) 084041.
- Y. Rodríguez and A. A. Navarro, "Scalar and vector Galileons", J. Phys. Conf. Ser. 831 (2017) 012004.
- Y. Rodríguez and A. A. Navarro, "Non-Abelian S-term dark energy and inflation", Phys. Dark Univ. 19 (2018) 129.
- A. Gallego Cadavid and Y. Rodríguez, "A Systematic Procedure to Build the Beyond Generalized Proca Field Theory", arXiv:1905.10664 [hep-th].

The problem of initial conditions in cosmology

- Although there is no firm physical principle that underlies it, the reasonable expectation is that models of the universe should not be sensitive to initial conditions.

The problem of initial conditions in cosmology

- Inflation addresses the special initial conditions of the Hot Big Bang theory (underlying the flat and horizon problems) in an interesting and important way.
- Nontheless, inflation remains as a phenomenological scenario that is yet to be rooted in a fundamental theory.

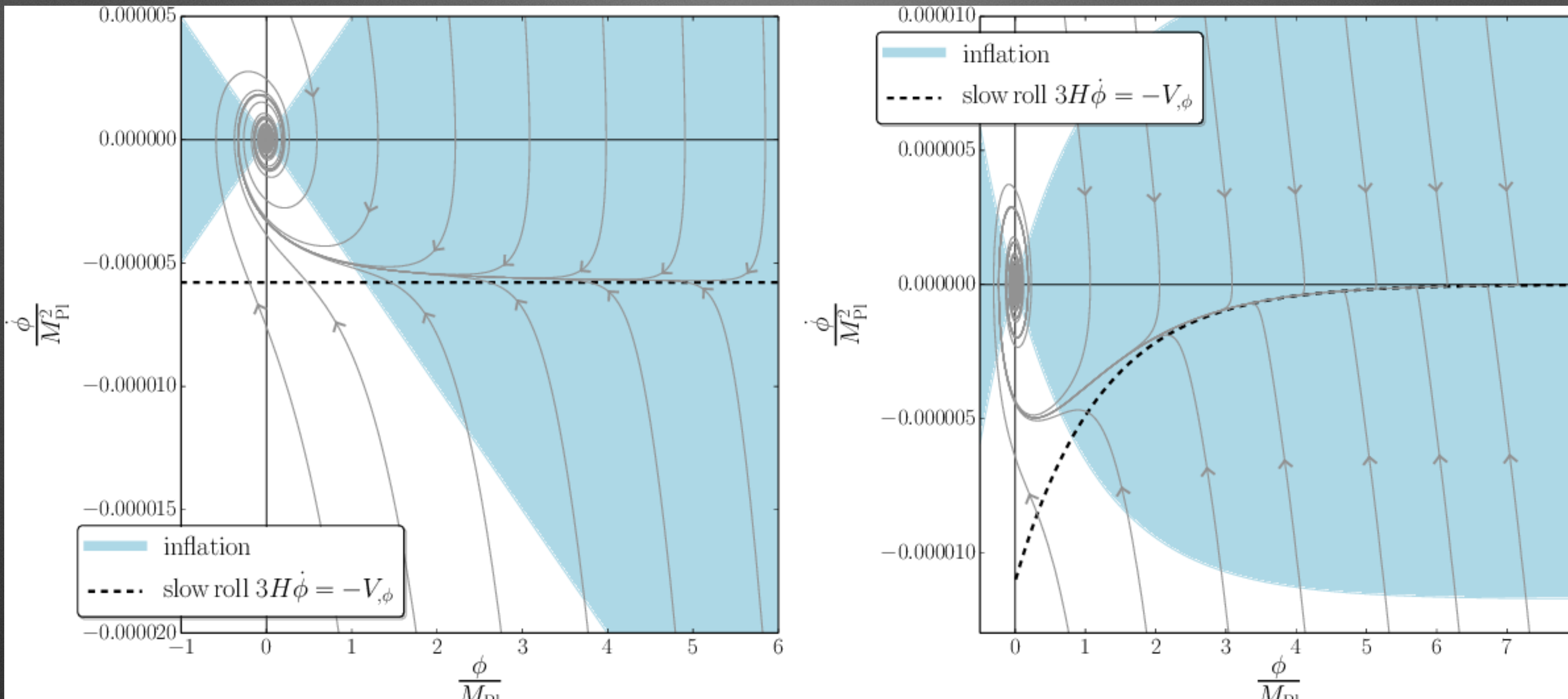


The problem of initial conditions in cosmology

- The era of inflation is believed to set the initial conditions for the Big Bang.
- Nevertheless, inflation itself is not completely free of initial conditions.

For example: the inflaton velocity must be small enough to allow inflation to start.

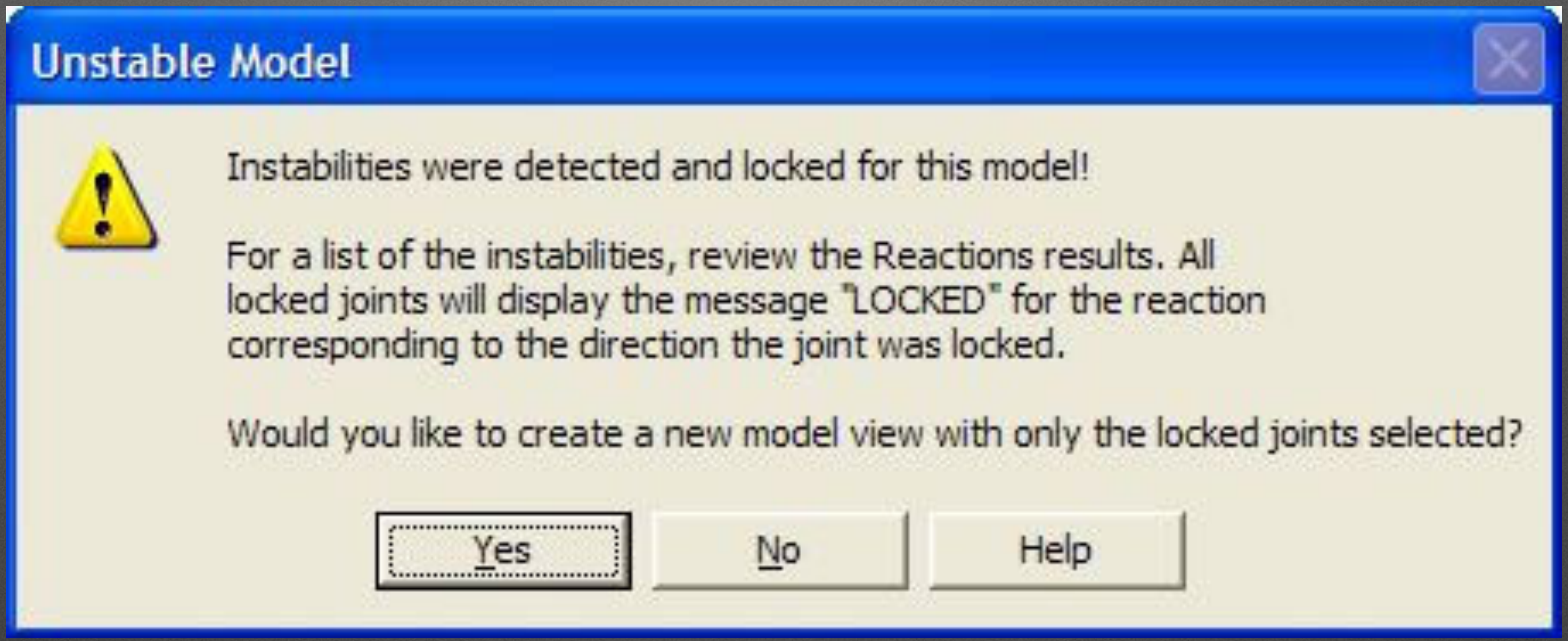
Slow-roll inflation as an attractor in scalar-field inflation



Quadratic potential — Grain et. al., JCAP 2007 — Starobinsky potential

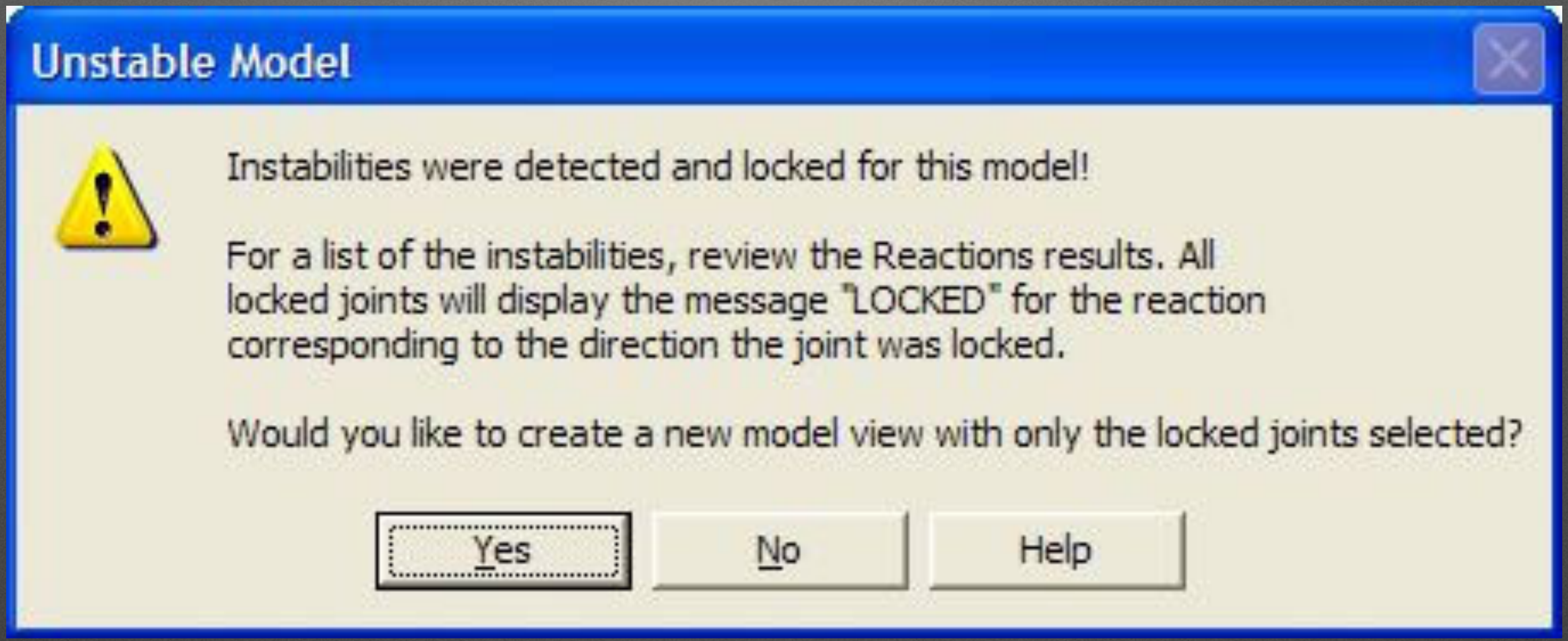
- Slow-roll inflation is an attractor for every single-scalar-field model. However, do all the phase-space trajectories approach the attractor?

Some properties of a good physical theory



- It must avoid any kind of instability:
  - - Tachyonic: a Hamiltonian that is unbounded from below.
  - - Ghost: negative kinetic energy terms.
  - - Laplacian: limitless growing perturbations.

Some properties of a good physical theory



- It must preserve causality.
- It must preserve unitarity.

The Horndeski theory

- It is a scalar-tensor theory of gravity.
- The action involves, at most, second-order derivatives of the fields involved.
- The field equations are, at most, second order.
- It is a instability-free theory.

Gregory W. Horndeski

Horndeski, Int. J. Theor. Phys. 1974

The Horndeski theory $\left(X \equiv -\frac{1}{2}\nabla_\mu\pi\nabla^\mu\pi\right)$

$$S = \int \left[f(R) + \sum_{N=2}^{5} \mathcal{L}_{N,\pi}^{\text{Gal}} \right] \sqrt{-g}\; d^4x$$

$$\mathcal{L}_{2,\pi}^{\text{Gal}} \equiv G_2(\pi, X)$$

$$\mathcal{L}_{3,\pi}^{\text{Gal}} \equiv G_3(\pi, X)\; \Box\pi$$

$$\mathcal{L}_{4,\pi}^{\text{Gal}} \equiv G_4(\pi, X)\; R \\ +G_{4,X}\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi)\right]$$

$$\mathcal{L}_{5,\pi}^{\text{Gal}} \equiv G_5(\pi, X)\; G_{\mu\nu}(\nabla^\mu\nabla^\nu\pi) \\ -\frac{1}{6}G_{5,X}\Big[(\Box\pi)^3 - 3(\Box\pi)(\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi) \\ +2(\nabla_\mu\nabla^\nu\pi\; \nabla_\nu\nabla^\rho\pi\; \nabla_\rho\nabla^\mu\pi)\Big]$$

"Horndeski scalar theory-or-past, present & future, No. 1"

Deffayet et. al., Phys. Rev. D 2011

Extensions to the Horndeski theory

"Overcoming Gravity"

- Multi Galileons.
- Beyond Horndeski.
- DHOST (Degenerate Higher-Order Scalar-Tensor theories).
- Generalized Proca.
- Beyond generalized Proca.
- Extended vector-tensor (or DHOVT).
- Generalized SU(2) Proca
- Beyond generalized SU(2) Proca

The generalized SU(2) Proca theory

- The standard Proca theory may be seen as the limit (a frozen Higgs) of a valid particle physics model based on a Higgs condensate.

Heisenberg, JCAP 2014
Tasinato, JHEP 2014

- The generalized SU(2) Proca theory may be seen as the limit (invariance under a global SU(2) transformation) of a non-Abelian SU(2) field theory.

Allys et. al., Phys. Rev. D 2016

Building elements

- Theories invariant under continuous local transformations, either Abelian or non Abelian, are built from the gauge field strength tensor $F_{\mu\nu}$, its Hodge dual $\tilde{F}_{\mu\nu}$, and (if the symmetry is spontaneously broken) from the gauge field itself A_μ .

- Theories that don't invoke gauge symmetries are built also from the symmetric version $S_{\mu\nu}$ of the gauge field strength tensor:

$$S_{\mu\nu} \equiv \nabla_\mu A_\nu + \nabla_\nu A_\mu$$

Allys et. al., JCAP 2016

The generalized SU(2) Proca theory

up to six contracted Lorentz indices

$$S = \int \left[f(R) - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \frac{1}{2} m^2 A^\mu_a A^a_\mu + \sum_{N=2}^{4} \alpha_N \mathcal{L}^{\rm Gal}_{N,A} + \sum_{m=1}^{4} f^{\rm Curv}_m \mathcal{L}^{\rm Gal}_{{\rm Curv},m,A} \right] \sqrt{-g}\, d^4x$$

$$\mathcal{L}^{\rm Gal}_{{\rm Curv},1,A} \equiv G_{\mu\nu} A^{\mu a} A^\nu_a$$

$$\mathcal{L}^{\rm Gal}_{{\rm Curv},2,A} \equiv L_{\mu\nu\rho\sigma} F^{\mu\nu a} F^{\rho\sigma}_a$$

$$\mathcal{L}^{\rm Gal}_{{\rm Curv},3,A} \equiv \epsilon_{abc} L_{\mu\nu\rho\sigma} F^{\mu\nu a} A^{\rho b} A^{\sigma c}$$

$$\mathcal{L}^{\rm Gal}_{{\rm Curv},4,A} \equiv L_{\mu\nu\rho\sigma} A^{\mu a} A^\nu_a A^{\rho b} A^\sigma_b$$

Allys et. al., Phys. Rev. D 2016

$$L^{\mu\nu\alpha\beta} \equiv \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}$$

The generalized SU(2) Proca theory

up to six contracted Lorentz indices

$$S = \int \left[f(R) - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \frac{1}{2} m^2 A^\mu_a A^a_\mu + \sum_{N=2}^{4} \alpha_N \mathcal{L}^{\mathrm{Gal}}_{N,A} + \sum_{m=1}^{4} f_m^{\mathrm{Curv}} \mathcal{L}^{\mathrm{Gal}}_{\mathrm{Curv},m,A} \right] \sqrt{-g}\, d^4x$$

$$\mathcal{L}^{\mathrm{Gal}}_{2,A} \equiv f_2(A^a_\mu, F^a_{\mu\nu}, \tilde{F}^a_{\mu\nu}) \qquad \mathcal{L}^{\mathrm{Gal}}_{3,A} \equiv 0 \qquad \alpha_4 \mathcal{L}^{\mathrm{Gal}}_{4,A} \equiv \alpha_{4,1} \mathcal{L}^{\mathrm{Gal},1}_{4,A} + \alpha_{4,2} \mathcal{L}^{\mathrm{Gal},2}_{4,A} + \alpha_{4,3} \mathcal{L}^{\mathrm{Gal},3}_{4,A}$$

$$\mathcal{L}^{\mathrm{Gal},1}_{4,A} \equiv \frac{1}{4}(A_b \cdot A^b) \left[S^{\mu a}_\mu S^\nu_{\nu a} - S^{\mu a}_\nu S^\nu_{\mu a} + A_a \cdot A^a R \right] + \frac{1}{2}(A_a \cdot A_b) \left[S^{\mu a}_\mu S^{\nu b}_\nu - S^{\mu a}_\nu S^{\nu b}_\mu + 2 A^a \cdot A^b R \right]$$

$$\mathcal{L}^{\mathrm{Gal},2}_{4,A} \equiv \frac{1}{4}(A_a \cdot A_b) \left[S^{\mu a}_\mu S^{\nu b}_\nu - S^{\mu a}_\nu S^{\nu b}_\mu + A^a \cdot A^b R \right] + \frac{1}{2}(A^{\mu a} A^{\nu b}) \Big[S^\rho_{\mu a} S_{\nu\rho b} - S^\rho_{\nu a} S_{\mu\rho b} - A^\rho_a A^\sigma_b R_{\mu\nu\rho\sigma} - (\nabla^\rho A_{\mu a})(\nabla_\rho A_{\nu b}) + (\nabla^\rho A_{\nu a})(\nabla_\rho A_{\mu b}) \Big]$$

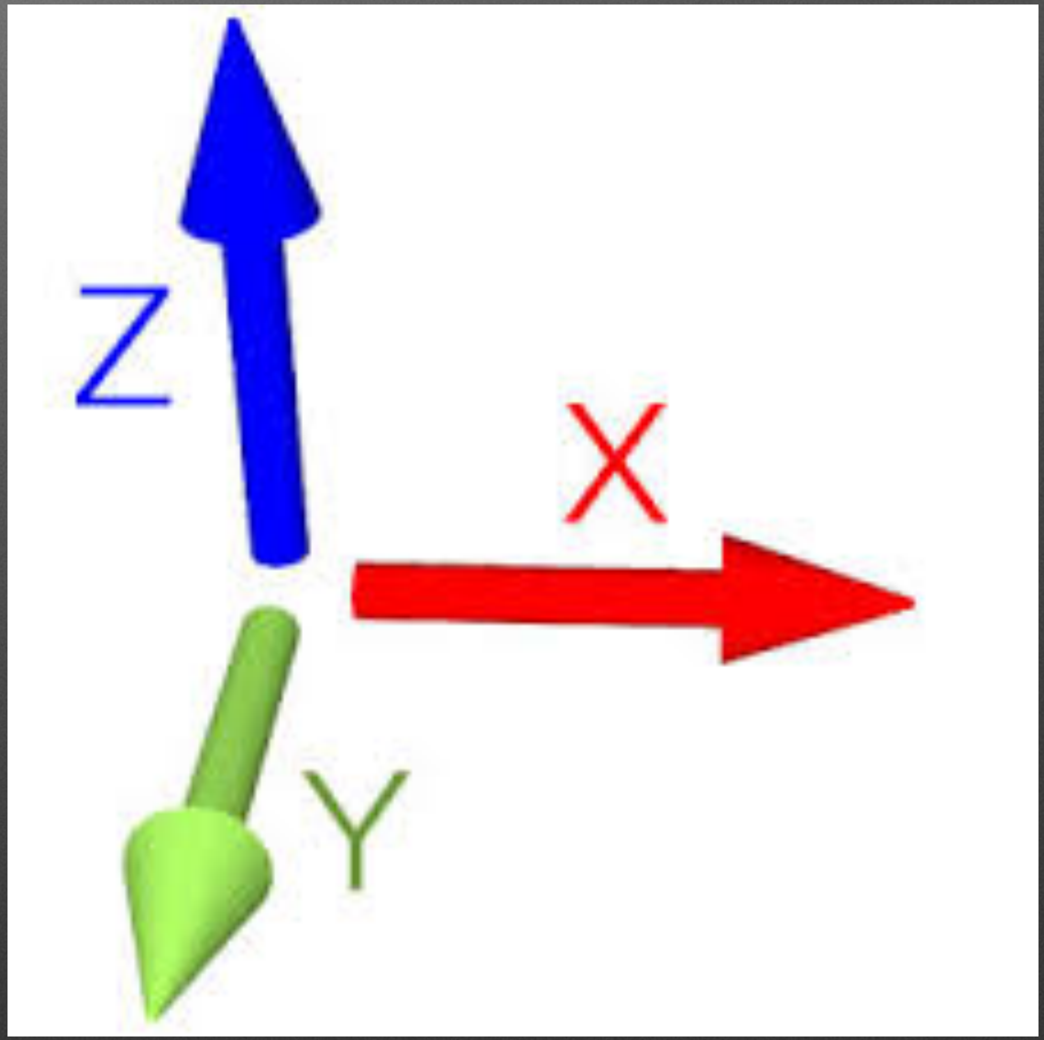
$$\mathcal{L}^{\mathrm{Gal},3}_{4,A} \equiv \tilde{G}^b_{\mu\sigma} A^\mu_a A_{\nu b} S^{\nu\sigma a}$$

$$G^a_{\mu\nu} \equiv \nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu$$

Allys et. al., Phys. Rev. D 2016

The cosmic triad

- We want to describe the observable universe. Therefore, we employ the Friedmann-Lemaitre-Robertson-Walker metric at the background level.
- This is only possible if the field configuration is a "cosmic triad": $A^a_\mu = a\psi\delta^a_\mu$.
- This configuration is invariant both under SU(2), for the field space, and SO(3), for the physical space, in agreement with the homomorphism between the two groups.



Armendáriz-Picón, JCAP 2004

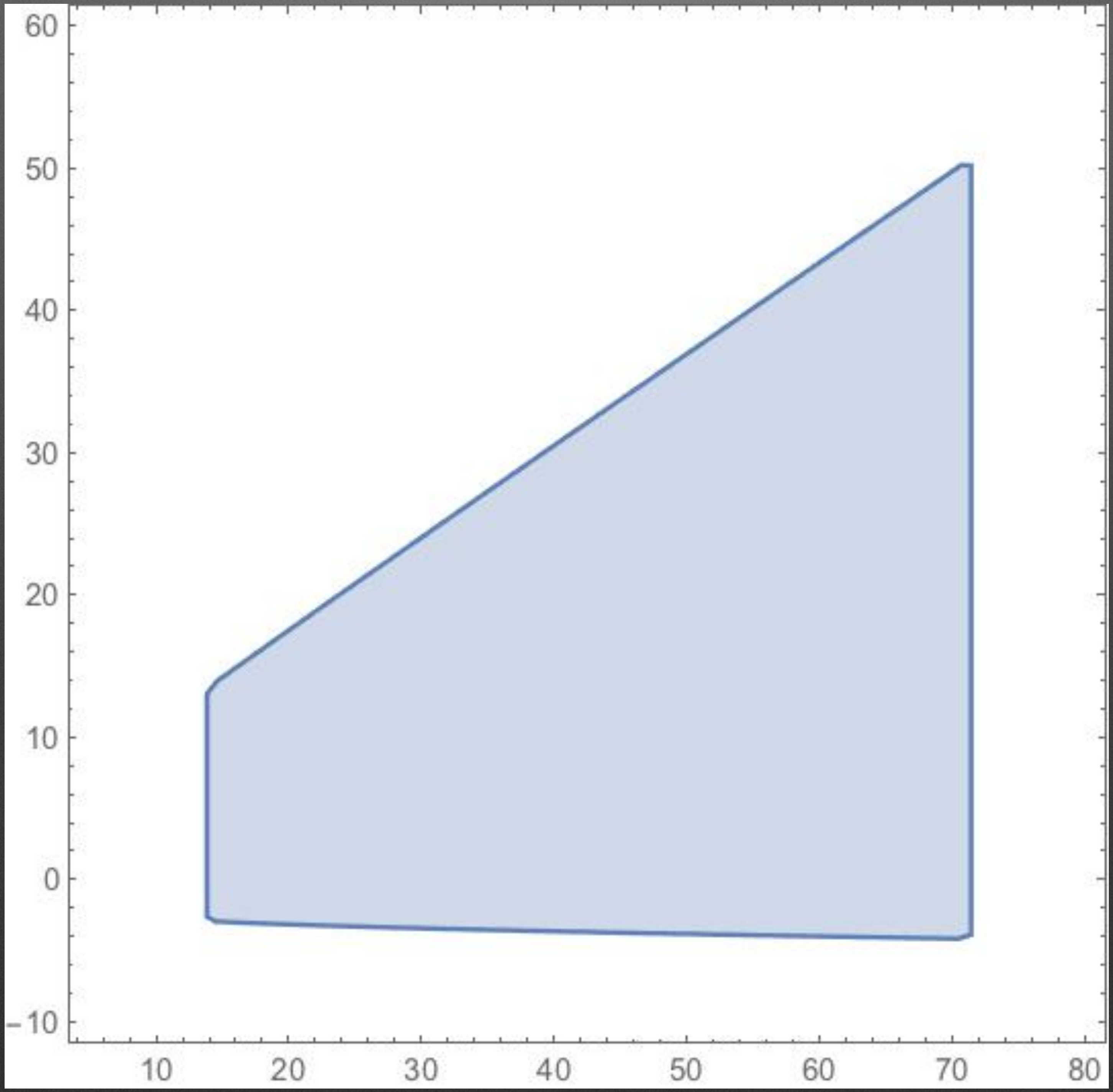
The model

$$S = \int \left[\frac{m_P^2}{2} R - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \alpha_4 \mathcal{L}^{\rm Gal}_{4,A} + f_4^{\rm Curv} \mathcal{L}^{\rm Gal}_{{\rm Curv},4,A} \right] \sqrt{-g}\; d^4x$$

Absence of ghosts and Laplacian instabilities

$\alpha_1 = 0.1 m_P^2$
$\alpha_3/\alpha_1 = 15$
$\psi = 10 m_P$

$f_4^{\mathrm{Curv}}/\alpha_1$



α_2/α_1

A novel mechanism to implement inflation

- Let's split artificially our vector field fluid into two components: one corresponding to the Yang-Mills term and the other corresponding to the new Proca terms.

- What would happen if, due to the dynamics of the vector field, $\rho_{YM} \to \infty$?

- That means a singularity in the energy-momentum tensor which must be avoided. How?: $\rho_{\rm Gal} \to -\infty$ so that $\rho_{\rm Gal}/\rho_{YM} \to -1$. Fine tuning??: yes, but the system could "self tune".

Rodríguez et. al., Phys. Dark Univ. 2018

A novel mechanism to implement inflation

- What about the pressure?: since $P_{YM} = \frac{\rho_{YM}}{3}$, we have again a singularity in the energy-momentum tensor unless $P_{\rm Gal} \to -\infty$, i.e., $P_{\rm Gal}/P_{YM} \to -1$.
- The only possible consistent way to do this is if the new Proca fluid behaves as radiation: $P_{\rm Gal} = \frac{\rho_{\rm Gal}}{3}$.
- The new Proca fluid: a radiation fluid with negative energy density and pressure.

Rodríguez et. al., Phys. Dark Univ. 2018

A novel mechanism to implement inflation

- If the two fluids behave as radiation, could we say that the actual total fluid also behaves as radiation?

- No. 0 divided by 0 could be anything:

$$\omega \equiv \frac{P_{YM} + P_{\rm Gal}}{\rho_{YM} + \rho_{\rm Gal}}$$

- The actual value of the equation of state parameter depends on the characteristics of the model.

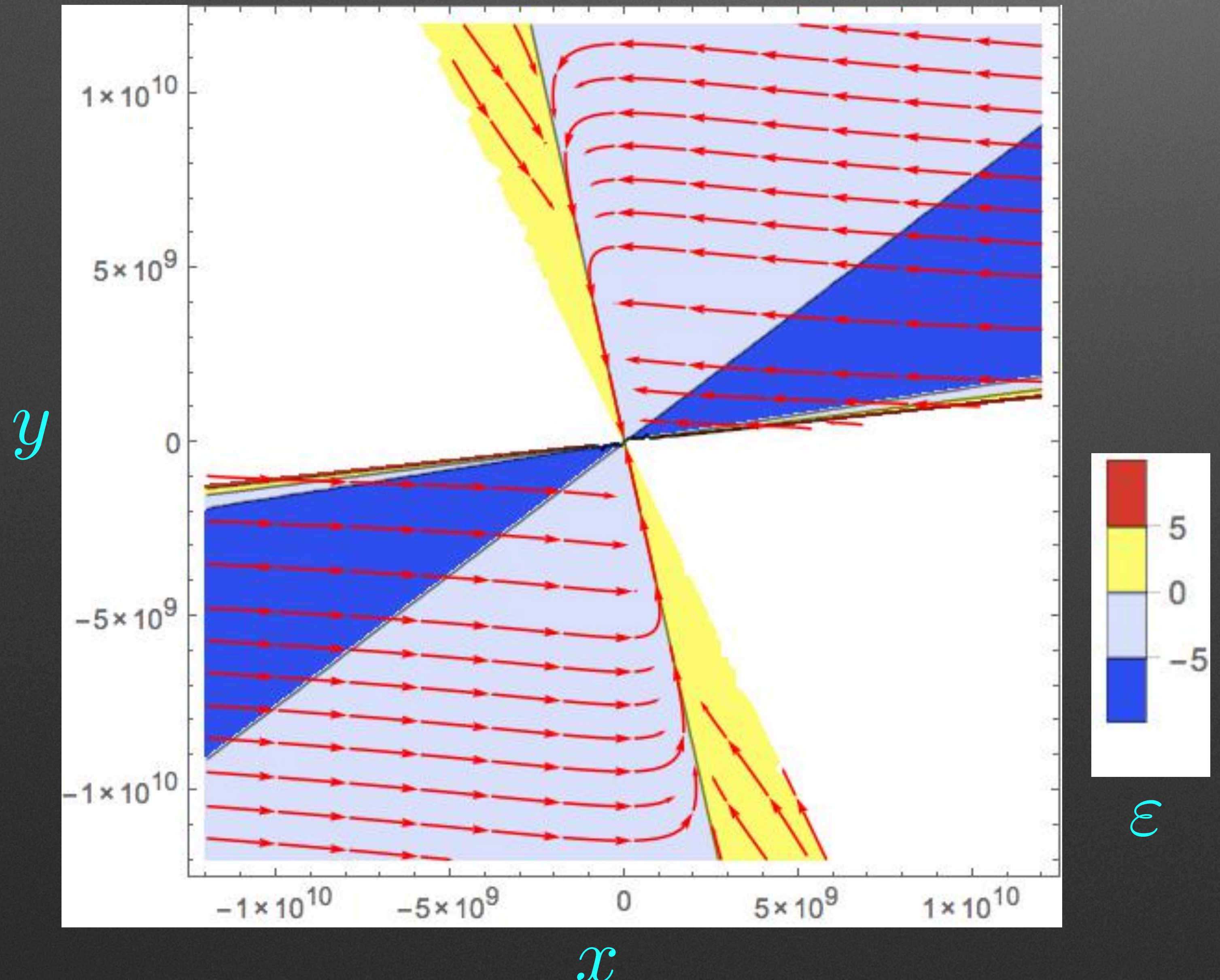
- A realization of this scenario, with $\omega = -1$, i.e., $\varepsilon = 0$ and self tuning will be presented in the following.

Rodríguez et. al., Phys. Dark Univ. 2018

Inflation as a non-eternal asymptotic behaviour

$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}$$

$$y \equiv \frac{\psi}{\sqrt{2} m_P}$$



- ***<u>The whole available parameter space</u>*** exhibits a non-eternal asymptotic behaviour that corresponds to an inflationary solution with $\varepsilon = 0$

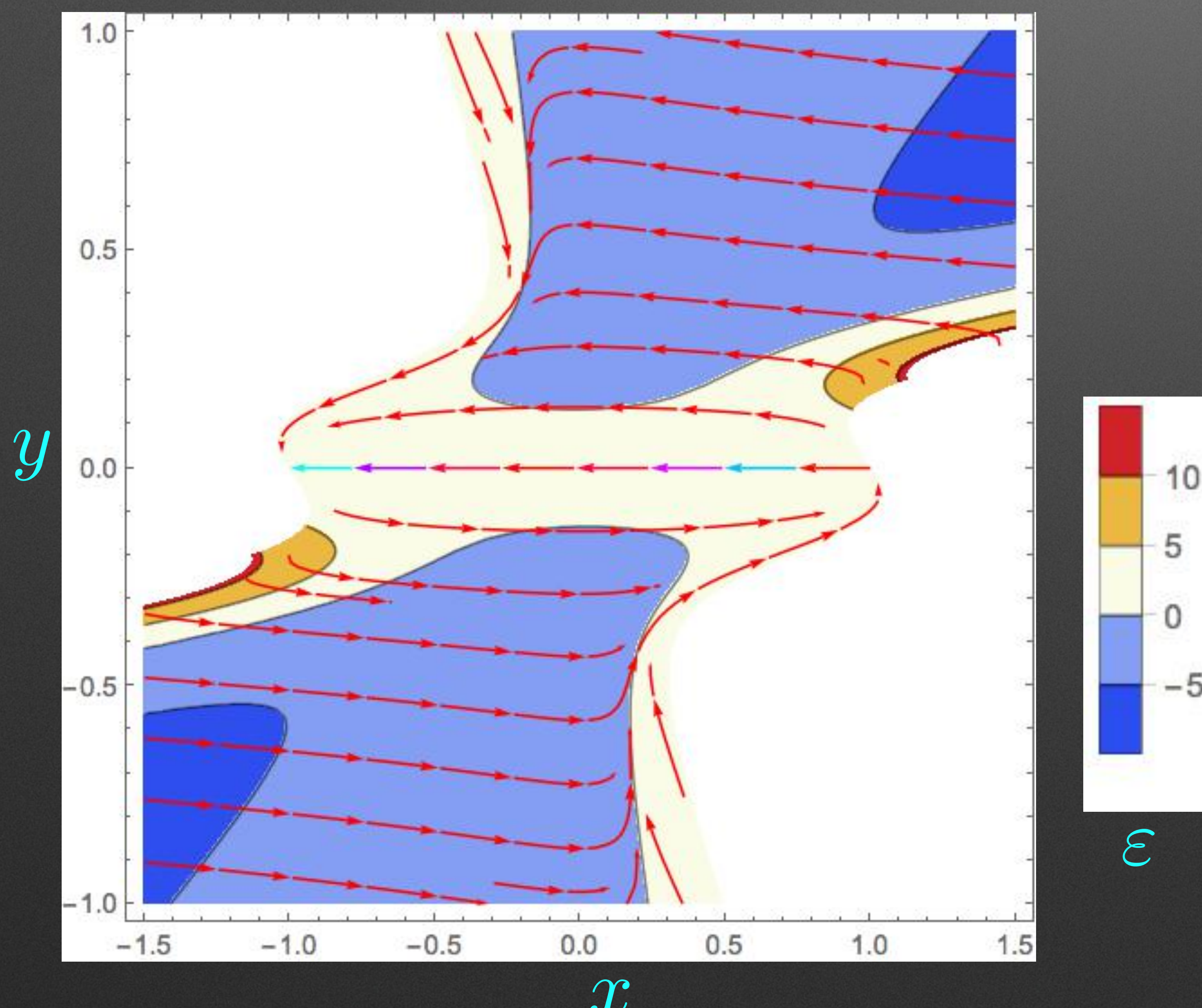
- $y \to \beta x$ where

$$\beta = \frac{203\alpha_{4,1} + 64\alpha_{4,2} + 23 f_4^{\mathrm{Curv}}}{77\alpha_{4,1} - 16\alpha_{4,2} - 9 f_4^{\mathrm{Curv}}}$$

Inflation exit

$$x \equiv \frac{\dot{\psi}}{\sqrt{2} m_P H}$$

$$y \equiv \frac{\psi}{\sqrt{2} m_P}$$



- When x and y approach 0, solutions leave the asymptotic behaviour and inflation ends through oscillations of ψ around 0 and oscillations of ε around 2 (like a scalar inflaton decaying into radiation).

Equations of state parameters

- Analytical results demonstrate that, in the asymptotic limit, when $y \to \beta x$ and $x \to \pm\infty$, the system reveals the following behaviour for energy densities, pressures, and equations of state parameters:

$$\rho_{YM} \to \infty$$

$$P_{YM} \to \infty$$

$$\omega_{YM} \equiv P_{YM}/\rho_{YM} \to \frac{1}{3}$$

$$\rho_{\rm Gal}/\rho_{YM} \to -1$$

$$\rho_{\rm Gal} \to -\infty$$

$$P_{\rm Gal} \to -\infty$$

$$\omega_{\rm Gal} \equiv P_{\rm Gal}/\rho_{\rm Gal} \to \frac{1}{3}$$

$$P_{\rm Gal}/P_{YM} \to -1$$

$$\omega \equiv \frac{P_{YM} + P_{\rm Gal}}{\rho_{YM} + \rho_{\rm Gal}} \to -1$$

- This is consistent with our previous expectations!

Some numerical results

$$\alpha_1 = 0.1 m_P^2$$

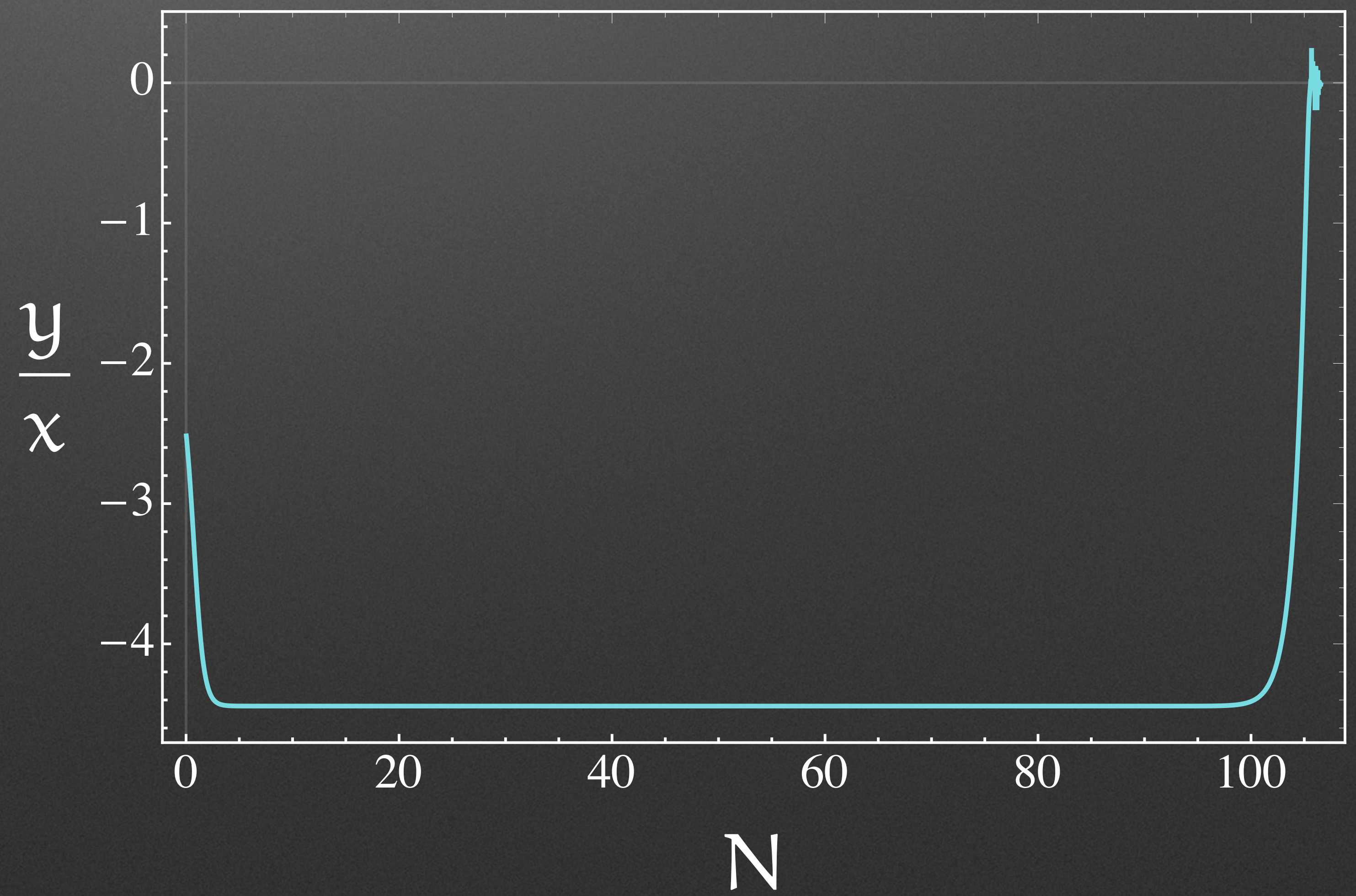
$$\alpha_2/\alpha_1 = 30$$

$$f_4^{\rm Curv}/\alpha_1 = 5$$

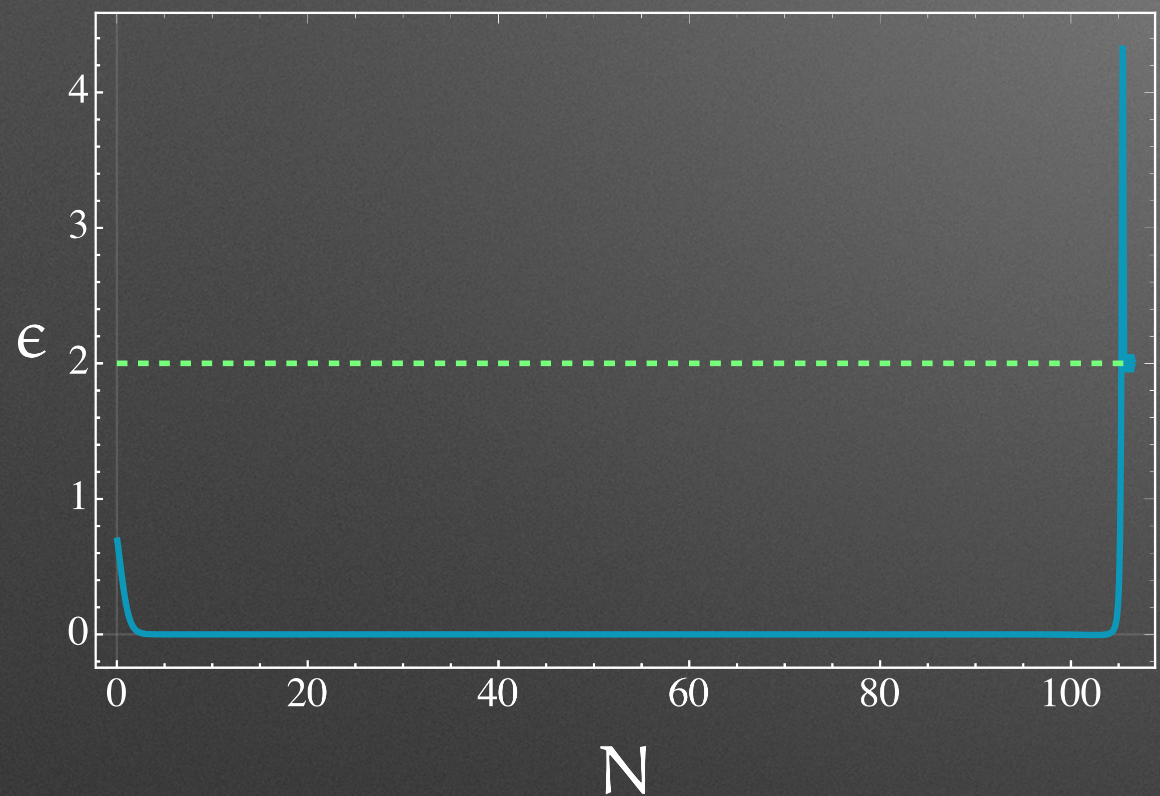
$$x_i = -4 \times 10^9$$

$$y_i = 10^{10}$$

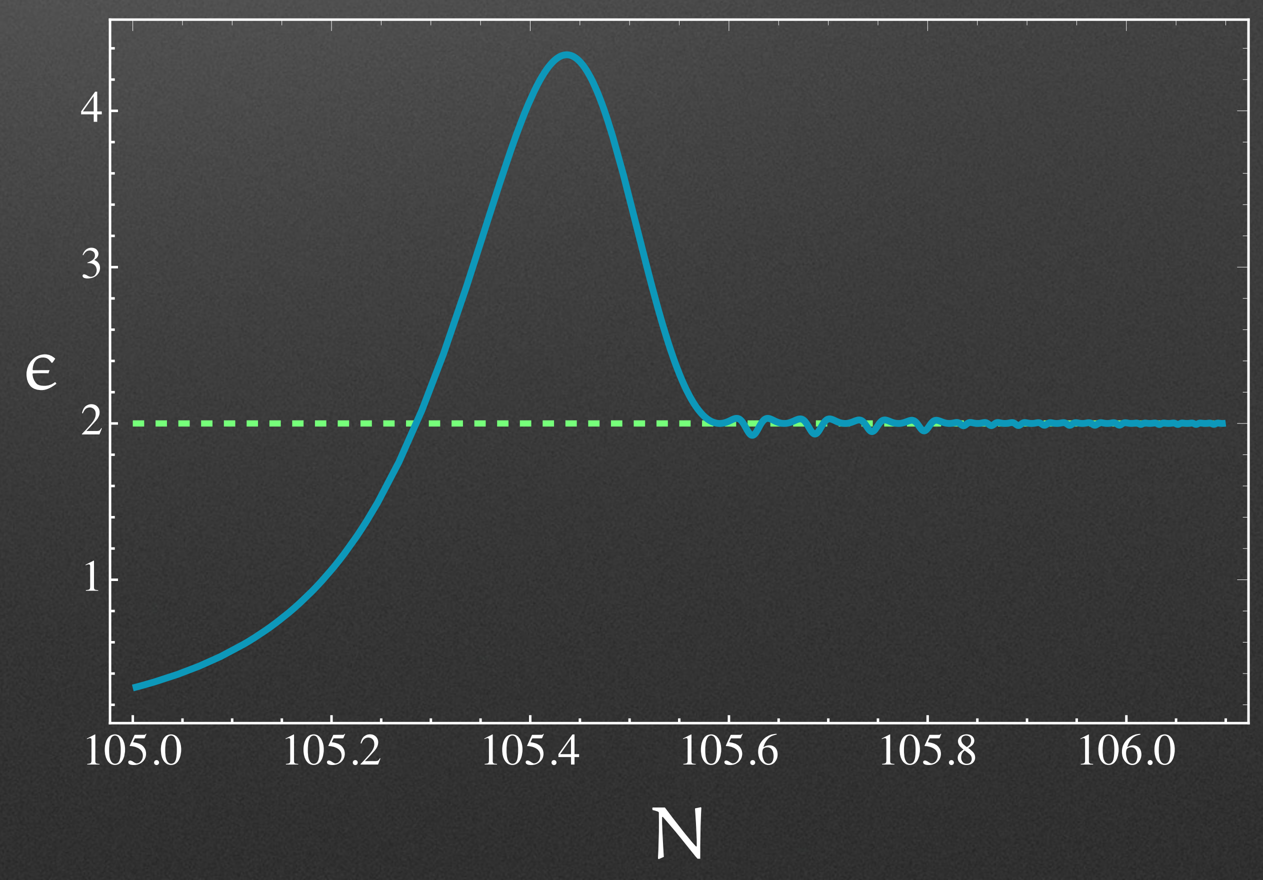
- Asymptotic behaviour $y \to \beta x$



Some numerical results

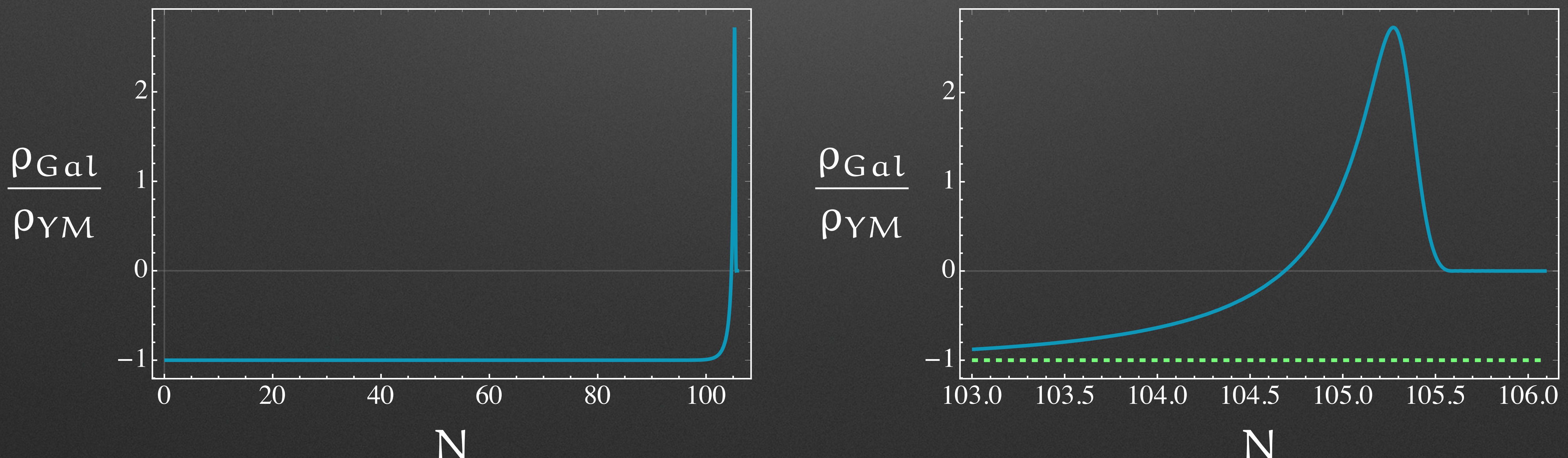


- ε behaviour: first going to 0 and then going to 2. Inflation lasts enough e-folds to solve the classical problems of the standard cosmology.



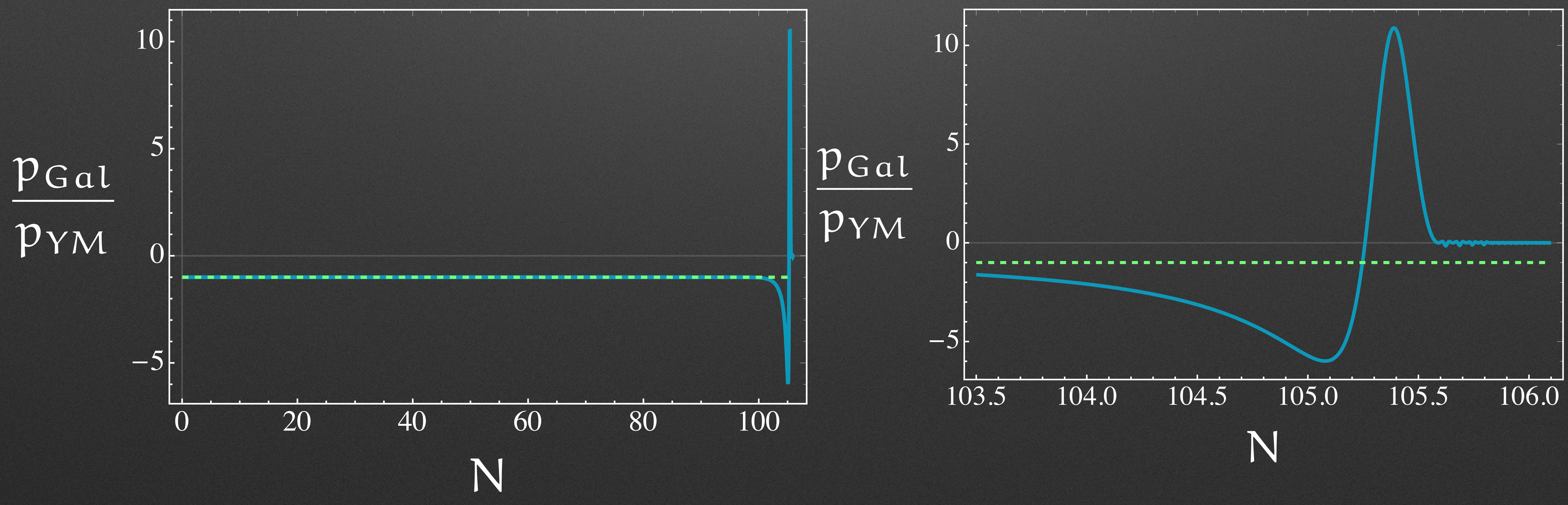
Some numerical results

- $\frac{\rho_{\rm Gal}}{\rho_{YM}}$ behaviour: during inflation, it goes to -1. After the end of inflation, it goes to 0

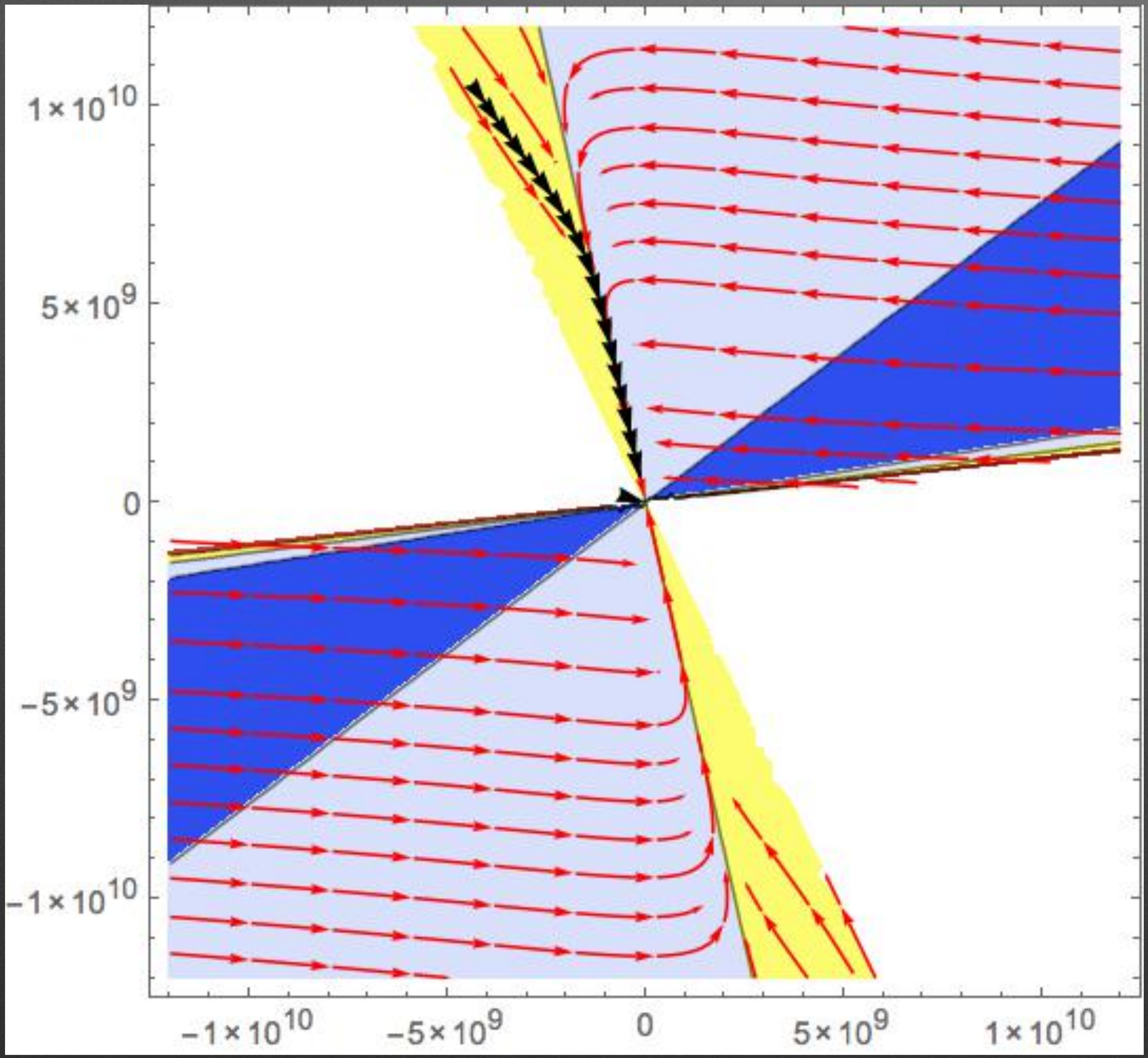


Some numerical results

- $\frac{P_{\rm Gal}}{P_{YM}}$ behaviour: during inflation, it goes to -1. After the end of inflation, it goes to 0



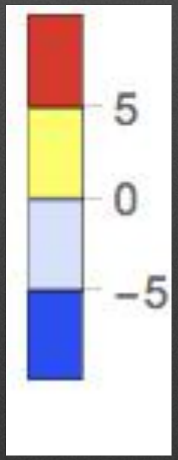
Some numerical results



y

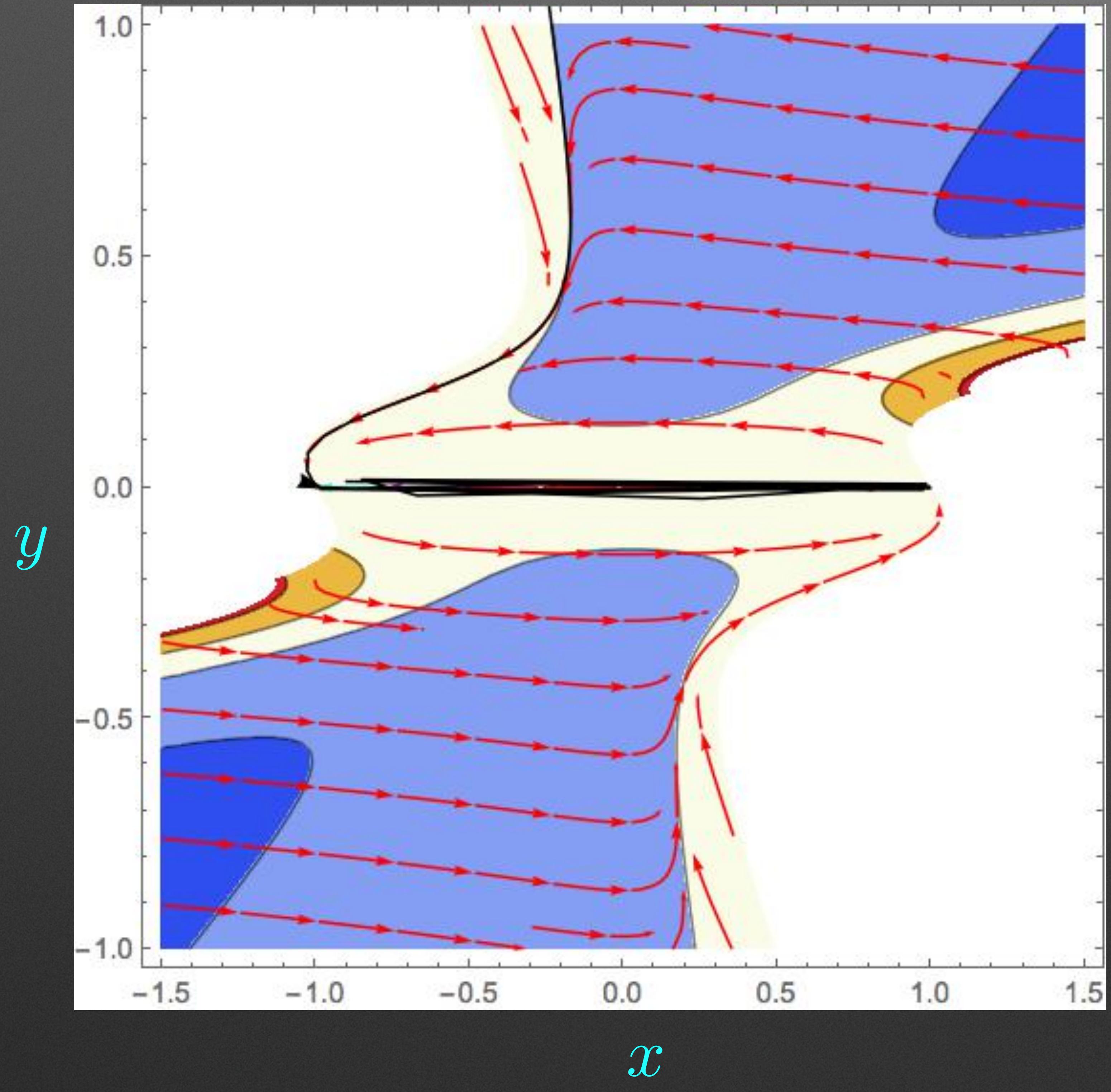
x

- Arbitrary trajectory in the phase in an ε contour plot of the phase space.

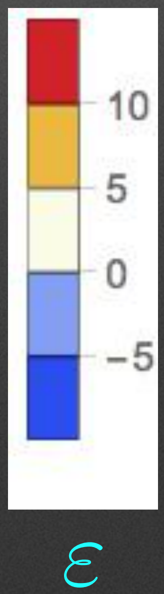


ε

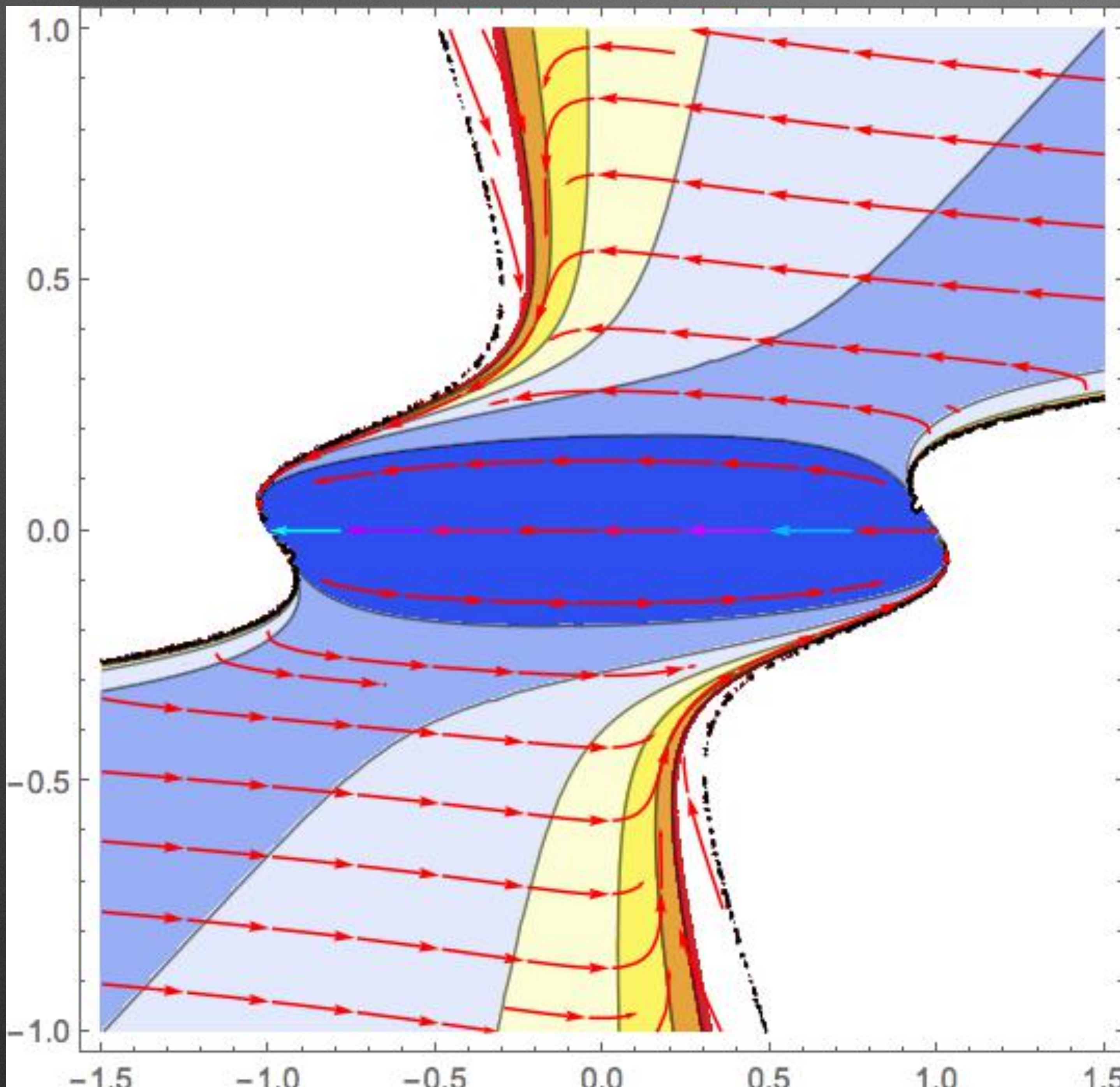
Some numerical results



- Arbitrary trajectory in the phase in an ε contour plot of the phase space (zoom).



Some numerical results



y

x

- H contour plot of the phase space. The inflationary energy scale is always under control: $H/m_P < 1$

H/m_P

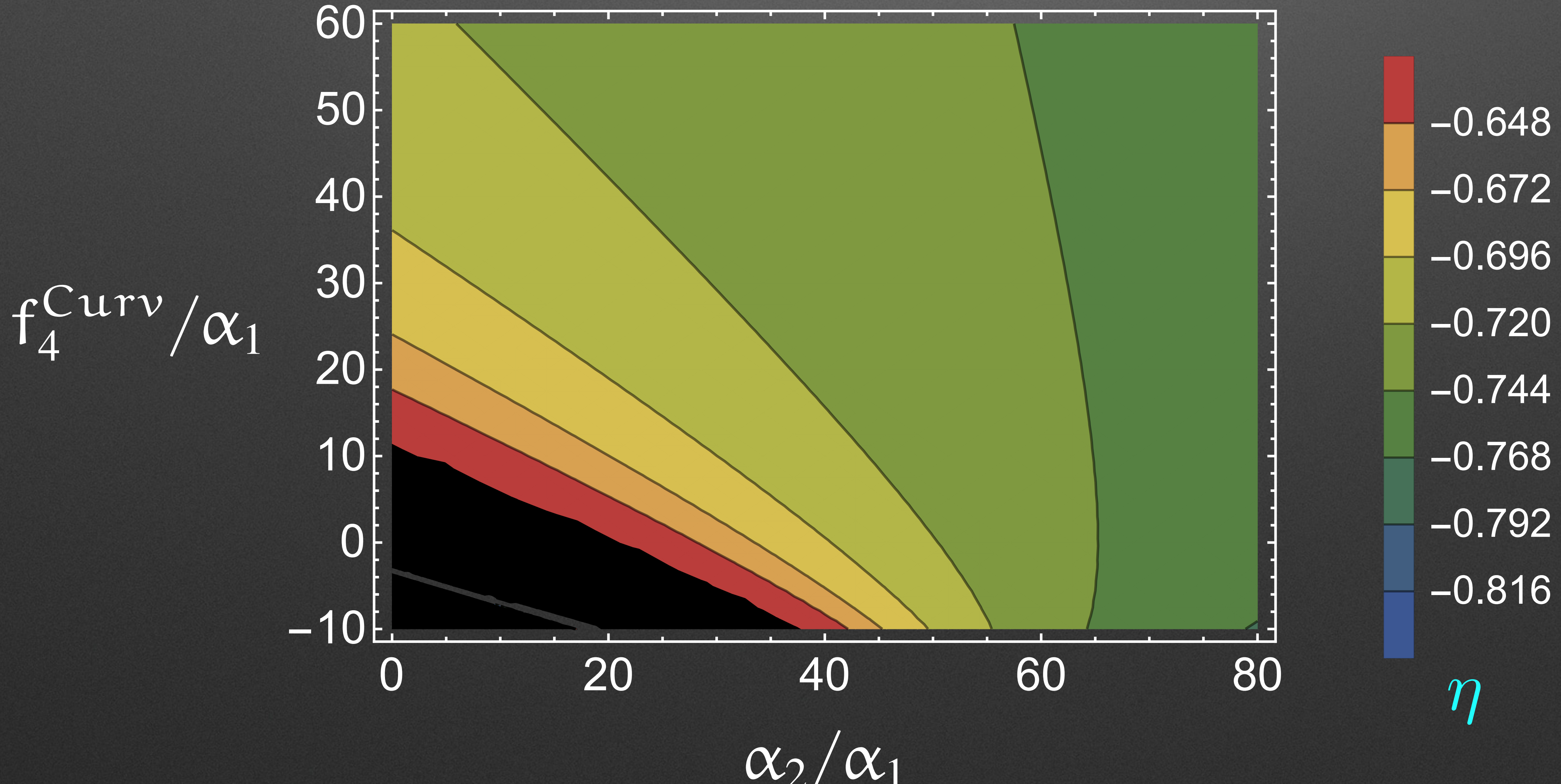
Further exploration of the model

- To investigate the possible attractor nature of the cosmic triad configuration in a more general anisotropic background.
- To investigate the causal structure of the theory.
- To include beyond generalized SU(2) Proca terms and see how they change the model features.
- To obtain the tensor to scalar ratio and the spectral index for the curvature perturbation and see how it fits with observations.

Further exploration of the model

- There is an interesting prediction of the model:

$$\eta = -\frac{287\alpha_1^2 + 384(f_4^{\rm Curv})^2 + 296\alpha_2 f_4^{\rm Curv} + 58\alpha_2^2 + 173\alpha_1(8f_4^{\rm Curv} + 3\alpha_2)}{(26\alpha_1 + 8f_4^{\rm Curv} + 3\alpha_2)(203\alpha_1 + 64f_4^{\rm Curv} + 23\alpha_2)}$$



Thank you!