Regular black holes with exotic topologies

Gustavo Melgarejo, Ph.D. Pedro Bargueño de Retes.

May 30, 2019

Departamento de Física, Universidad de los Andes.

Table of contents

Introduction

Black holes

Black holes are solutions of Einstein's field equations in which the gravitational attraction is so strong that nothing can escape from it once a region called the *event horizon* has been crossed. The spacetime for black holes solutions have *singularities*.

The presence of singularities can be recognized by the existence of *incomplete geodesics* that cannot be extended to infinite values of the affine parameter[1].

In these regions the theory *loses its predictive character* and it is no longer possible to define a classical notion of spacetime.

Regular black holes

In 1968 *J.M Bardeen* obtained the first family of regular solutions for Einstein's equations with an event horizon, these solutions satisfy the *weak energy condition* [2, 3, 4].

The Bardeen family of solutions are solutions of Einstein's equations coupled to *non-linear electrodynamics*, which are parameterized by the mass **m** and the charge **e**. The line element for these solutions is as follows [2]:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}r^2 + r^2 d\Omega^2, \tag{1}$$

wherein, $d\Omega^2 = d\theta^2 + \sin^2\theta \; d\phi^2$, and

$$f(r) = 1 - \left(\frac{m}{e}\right)\frac{2(r/e)^2}{((r/e)^2+1)^{3/2}}. \tag{2}$$

Regular black holes

In others works, *Iryna Dymnikova* showed that solutions of regular black holes could be obtained in **spherical symmetry** by imposing the following conditions[5]:

- The metric be asymptotically flat, and finiteness of the mass.
- The metric and the energy density are regular for $r \to 0$.
- That the *dominant energy condition* is fulfilled.

Obtaining thus a metric that is asymptotically Schwarzschild for $r \to \infty$ *and asymptotically de Sitter for* $r \to 0$.

Regular black holes

Also, *Leonardo Balart* and *Elias C. Vagenas* were able to build a family of solutions for static and spherically symmetric charged regular black holes, using *non-linear electrodynamics* coupled to general relativity. To obtain these solutions, they imposed the following three conditions on the metric [4]:

- The weak energy condition must be met.
- The energy-momentum tensor must have the symmetry $T_0^0 = T_1^1$.
- These metrics should behave asymptotically like the Reissner-Nordström metric.

Regular black holes

In 1997 *Arvind Borde* proposed a theorem that shows that in the spacetime for regular black holes that satisfy the *null energy condition*, necessarily there is a *change in the topology* of achronal slices taken along the spacetime [3], in these cases, the slices go from being non-compact to be compact in the neighborhood of the regular black hole core.

Singularity theorems

Singularity theorems

Between 1965 and 1970 *Penrose* and *Hawking* worked to demonstrate some singularity theorems.

These theorems are usually formulated in terms of three conditions [1]:

- Energy conditions.
- Global conditions in the structure of the spacetime.
- Gravity strong enough to trap a region.

Penrose singularity theorem: If the spacetime contains a non-compact Cauchy hypersurface Σ, an closed future-trapped surface, and is fulfilled the condition $R_{\mu\nu}n^{\mu}n^{\nu} \geq 0$ for all null vector n^{μ}, then the spacetime is *null-geodesically incomplete* to the future [6].

Borde's theorem

Borde's theorem

Borde's theorem: Suppose that there is a spacetime $\mathcal{M}$, that a) contains an *eventually future-trapped* surface $\mathcal{T}$, b) the Ricci tensor, $R_{\mu\nu}$, obeys $R_{\mu\nu} n^\mu n^\nu \geq 0$ for all null vectors n^μ, c) is *null-geodesically complete* to the future, and d) is *future causally simple*, i.e., $E^+(X) = i^+(X)$, where X is any achronal compact subset of $\mathcal{M}$. Then there is a compact slice to the causal future of $\mathcal{T}$ [3].

Applications

Penrose diagram for Reissner-Nordström spacetime

For this work we will study regular black holes that have two horizons. So the global structure of spacetime can be described using Penrose diagrams similar to the maximal extension of Reissner-Nordström spacetime, which can be seen in 1.

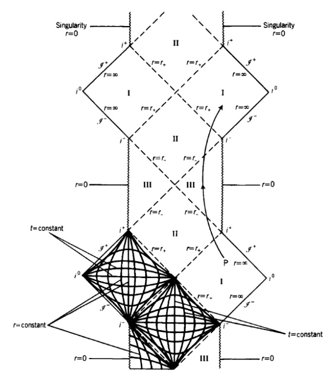

Figure 1: Penrose diagram for Reissner-Nordström black hole [7].

Penrose diagram for space-like identification

To avoid the singularity, the spatial sections corresponding to $r = 0$ have been identified, in this way spacetime is null-geodesically complete to the future.

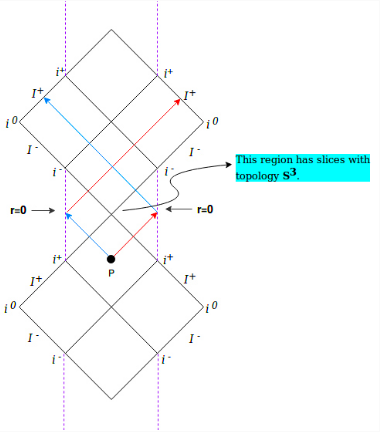

Figure 2: Penrose diagram for space-like identification [8].

Penrose diagram for null identification

To avoid the singularity, We have identified along the r_- horizon, in this way spacetime is null-geodesically complete to the future.

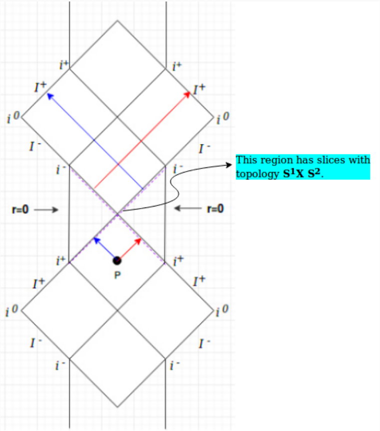

Figure 3: Penrose diagram for null identification [8].

Future work

Future work

1. Determine quantities on hypersurfaces in spacetime for identifications that satisfy the Borde theorem and do not present problems, and relate these with physical quantities that describe our space.
2. Study the analogy that exists between the change of topology of hypersurfaces in spacetime and phase transitions.

References

Referencias i

[1] Stephen Hawking and Roger Penrose.
The nature of space and time.
Princeton University Press, 2010.

[2] Stefano Ansoldi.
Spherical black holes with regular center: a review of existing models including a recent realization with Gaussian sources.
In Conference on Black Holes and Naked Singularities Milan, 2008.

[3] Arvind Borde.
Regular black holes and topology change.
Physical Review D - Particles, Fields, Gravitation and Cosmology, 1997.

Referencias ii

[4] Leonardo Balart and Elias C. Vagenas.
Regular black hole metrics and the weak energy condition.
Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics, 2014.

[5] Irina Dymnikova.
The cosmological term as a source of mass.
Classical and Quantum Gravity, 2002.

[6] Jos M.M. Senovilla and David Garfinkle.
The 1965 Penrose singularity theorem.
Classical and Quantum Gravity, 32(12), 6 2015.

[7] Ray d'Inverno.
Introducing Einstein's Relativity.
Oxford University Press Inc, 1992.

[8] Gustavo Melgarejo and Pedro Bargueño.
Regular black holes with exotic topologies.
in progress.