Correlation functions of sourced gravitational waves in inflationary scalar vector models. A symmetry based approach

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Outline

Motivations

- CMB observations allow the existence of anomalies.
- Vector fields are natural sources for statistical anisotropies, parity violation, among others.
- Vector fields can enhance the production of inflationary gravitational waves (GW) and leave distinctive signatures in their correlation functions.
- Statistical descriptors in cosmology relates theory and observation.
- Symmetries gives information about physical systems.

Key idea

1. We exploit the isomorphism between the $4D$ de Sitter group and the $3D$ Conformal group.
2. We will assume that correlation functions of primordial perturbations are invariant under conformal transformations.

The Method

1. Solve the equations of motion of the fields.
2. Find the asymptotic fields in the limit when $-k\tau \to 0$.
3. Find the conformal weight of the fields in $3D$ by using the transformation laws for the fields in $4D$.
4. Propose a general form for the correlation function.
5. Find the form of the correlator using the conditions imposed on the fields.
6. Use the Ward identities to constrain the form of the correlation functions.

The Scalar-Vector Coupled System

The action

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}\left[f_1(\phi)F^{\mu\nu}F_{\mu\nu} + f_2(\phi)\tilde{F}^{\mu\nu}F_{\mu\nu}\right] + S_{\mathrm{E-H}} + S_\phi \ .$$

The conformal de Sitter line element

$$ds^2 = \frac{1}{H^2\tau^2}\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right)$$

The scalar conformal weight

$$\Delta_\varphi = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right)$$

- Dilatation imply that the coupling functions must be proportional and of the same order: $f_1 = \alpha f_2$ and $f_1 \propto (-H\tau)^{-2n}$

Vector field conformal weight

$$\Delta_u = n + 2 \ .$$

The scalar and tensor perturbations

The equations for the tensor perturbations

$$\gamma_\lambda'' - \frac{2}{\tau}\gamma_\lambda' - \nabla^2\gamma_\lambda = \frac{2}{M_p^2}\Pi_\lambda^{lm} T_{lm}^{EM}, \quad \gamma_\lambda = \Pi_\lambda^{ij}\gamma_{ij}, \quad \Pi_\lambda^{lm} = \frac{\epsilon^l_{-\lambda}(\vec{k})\epsilon^m_{-\lambda}(\vec{k})}{\sqrt{2}}$$

Scalar perturbation expansion

$$\zeta = \zeta^{(0)} + \zeta^{(1)}$$

Tensor perturbation expansion

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)}$$

The SSP

$$\zeta^{(1)} \propto (\hat{E}_i \delta E^i)\zeta^{(0)}$$

The STP

$$\gamma_{ij}^{(1)} \propto (\hat{E}_i \delta E^i)\gamma_{ij}^{(0)}$$

Momentum projectors

$$\Delta_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j, \quad \hat{\eta}_{ij} = \eta_{ija}\hat{k}^a$$

The Ward Identities

Dilatation Ward identity for a N-point correlation function

$$\left[-3(N-1)+\sum_{a=1}^{N}\Delta_a-\sum_{a=2}^{N}\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\right]\langle V_i^{(1)}(\vec{k}_1)\cdots V_j^{(N)}(\vec{k}_N)\rangle'=0\,.$$

SCT Ward identity for a N-point correlation function

$$b^m\sum_{a=1}^{N}\left[2(\Delta_a-3)\frac{\partial}{\partial k_a^m}+k_{m(a)}\nabla^2_{k_a}-2\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\frac{\partial}{\partial k_a^m}\right]\langle V_{i_1}^1(\vec{k}_1)\cdots V_{i_N}^N(\vec{k}_2)\rangle'-$$

$$-2\sum_{a=1}^{N}\sum_{l=1}^{N}\left[\left(b^{j_l}\frac{\partial}{\partial k_a^{i_l}}-b_{i_l}\frac{\partial}{\partial k_{(a)j_l}}\right)\langle V_{i_1}(\vec{k}_1)\cdots V_{j_l}(\vec{k}_l)\cdots V_{i_N}(\vec{k}_{i_N})\rangle'\right]=0\,.$$

The Spectrum

Expansion of the different Spectrum

$$\langle OO \rangle = \langle O^{(0)} O^{(0)} \rangle + \langle O^{(1)} O^{(0)} \rangle + \langle O^{(0)} O^{(1)} \rangle + \langle O^{(1)} O^{(1)} \rangle$$

The scalar spectrum

$$\langle \zeta(\vec{k}_1)\zeta(-\vec{k}_1)\rangle' = \alpha_0 k_1^{-3+2\Delta_{\zeta 0}} \left[1 + g_\zeta(k_1)\left(1 - (\hat{E}\cdot\hat{k}_1)^2\right)\right], \quad g_\zeta(k_1) \equiv \left(\frac{\alpha_1}{\alpha_0}\right) k_1^{2\Delta_u}$$

The tensor-scalar spectrum

$$\langle \gamma_\lambda(\vec{k}_1)\zeta(-\vec{k}_1)\rangle' = k_1^{-3+\Delta_{\zeta 0}+\Delta_{\gamma 0}+2\Delta_u} B_\lambda \hat{E}_l \hat{E}_m \Pi_\lambda^{lm} \quad \text{where} \quad B_\lambda = B_1 + \lambda B_2$$

The tensor spectrum

$$\langle \gamma_\lambda \gamma_{\lambda'}\rangle' = T_\lambda \delta_{\lambda,\lambda'} k_1^{-3+2\Delta_{\gamma 0}} \left[1 + g_\gamma(k_1)\left(1 - (\hat{E}\cdot\hat{k}_1)^2\right)\right], \quad g_\gamma(k_1) \equiv \left(\frac{U_\lambda}{T_\lambda}\right) k_1^{2\Delta_u}$$

The Scalar Bispectrum

The expansion of the Scalar Bispectrum

$$\langle\zeta\zeta\zeta\rangle = \langle\zeta^{(0)}\zeta^{(0)}\zeta^{(0)}\rangle + \langle\zeta^{(1)}\zeta^{(0)}\zeta^{(0)}\rangle + (\text{2 perms.}) + \langle\zeta^{(1)}\zeta^{(1)}\zeta^{(0)}\rangle + (\text{2 perms.})$$

$$\langle\zeta^{(0)}(k_1)\zeta^{(0)}(k_2)\zeta^{(0)}(k_3)\rangle = \delta(k_{123})a_0 J_{0\{000\}}$$

$$\langle\zeta^{(1)}(k_1)\zeta^{(1)}(k_2)\zeta^{(0)}(k_3)\rangle = \delta(k_{123})\hat{E}^l\hat{E}^m \left[a_2 J_{2\{000\}}k_2^a k_3^b + \left(a_2 J_{1\{001\}} + a_3 J_{0\{000\}}\right)\delta^{ab}\right]$$
$$\times \left[\Delta_{la}(k_1)\Delta_{mb}(k_2) + i\alpha_2\left(\Delta_{la}(k_1)\hat{\eta}_{mb}(k_2) + \hat{\eta}_{la}(k_1)\Delta_{mb}(k_2)\right) - \alpha_3\hat{\eta}_{la}(k_1)\hat{\eta}_{mb}(k_2)\right]$$

The soft limit

$$\lim_{k_1\to 0}\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \propto k_2^{-\Delta_{\zeta 0}} P_\zeta(k_1)P_\zeta(k_2)\sum_{L=0}^{2} c_L(k_1,k_2)P_L(\hat{k}_1\cdot\hat{k}_2)$$

The Mixed Bispectrum

The expansion of the mixed bispectrum

$$\langle\gamma\zeta\zeta\rangle = \langle\gamma^{(0)}\zeta^{(0)}\zeta^{(0)}\rangle + \langle\gamma^{(0)}\zeta^{(0)}\zeta^{(1)}\rangle + \langle\gamma^{(0)}\zeta^{(1)}\zeta^{(1)}\rangle + \langle\gamma^{(1)}\zeta^{(1)}\zeta^{(0)}\rangle + (\text{ perms.})$$

$$\langle\gamma_\lambda^{(0)}(\vec{k}_1)\zeta^{(0)}(\vec{k}_2)\zeta^{(0)}(\vec{k}_3)\rangle' = B_\lambda \Pi_\lambda^{ab}(\vec{k}_1)k_{2a}k_{2b}J_{2\{000\}}\ , \quad \text{where} \quad B_\lambda = b_0\left[1+\lambda\beta_1\right]$$

$$\langle\gamma_\lambda^{(1)}(\vec{k}_1)\zeta^{(1)}(\vec{k}_2)\zeta^{(0)}(\vec{k}_3)\rangle' = (1+\lambda\beta_7)\hat{E}^l\hat{E}^m\Pi_\lambda^{ab}(\vec{k}_1)\mathcal{B}_{(1)}^{abcd}\times$$
$$\left[\Delta_{lc}(k_1)\Delta_{md}(k_2) + i\beta_5(\Delta_{lc}(k_1)\hat{\eta}_{md}(k_2) + \hat{\eta}_{lc}(k_1)\Delta_{md}(k_2)) + \beta_6\hat{\eta}_{lc}(k_1)\hat{\eta}_{md}(k_2)\right]$$

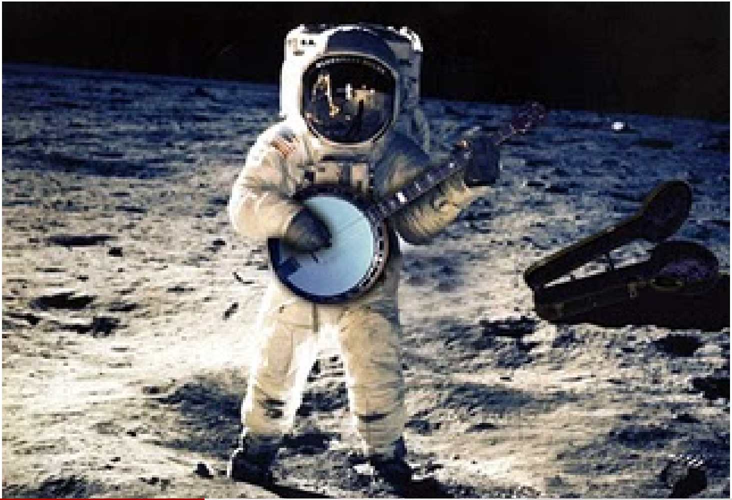
The soft limit

$$\lim_{k_1\to 0}\langle\gamma_\lambda(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle' \propto B_{0\lambda}k_2^{-\Delta_{\gamma 0}}P_\gamma^\lambda(k_1)P_\zeta(k_2)\Pi_{ab}^\lambda(\vec{k}_1)\hat{k}_2^a\hat{k}_2^b\sum_{L=0}^{2}c_L^\lambda(k_1,k_2)P_L(\hat{k}_1\cdot\hat{k}_2)$$

Summary

- Using this method for $f(\phi)(F^2 + \gamma F\tilde{F})$ we found a form for the correlators in agreement with the literature and with some interesting features. With the only down side being that you can't compute the amplitudes of the correlation functions.
- The parity violating term in the action generates an explicit dependence of the polarization in the tensor correlators that could be tested with observation.
- We extrapolated the results of Bzowski, McFadden and Skenderis **JHEP 1403 (2014) 111** to the parity violating case.

THANK YOU!!!



matrices resulting from the conditions over the $\gamma - \zeta$ vector sourced correlation function

$$
\begin{aligned}
B^{(1)}_{ijlm} &= \Delta_{mj}\Delta_{il} + \Delta_{lj}\Delta_{im} - \Delta_{ij}\Delta_{lm},\\
B^{(2)}_{ijlm} &= \Delta_{il}\epsilon_{jma}\hat{k}_a + \Delta_{lj}\epsilon_{ima}\hat{k}_a - \Delta_{ij}\epsilon_{lma}\hat{k}_a,\\
B^{(3)}_{ijlm} &= \epsilon_{ila}\hat{k}_a\epsilon_{jmb}\hat{k}_b + \epsilon_{ima}\hat{k}_a\epsilon_{jlb}\hat{k}_b - \Delta_{ij}\Delta_{lm},\\
B^{(4)}_{ijlm} &= \Delta_{im}\epsilon_{jla}\hat{k}_a + \Delta_{mj}\epsilon_{ila}\hat{k}_a + \Delta_{ij}\epsilon_{lma}\hat{k}_a.
\end{aligned}
$$

Maxwell dual tensor

$$\tilde{F}_{\mu\nu} = \frac{1}{2\sqrt{-g}}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

Conformal Transformations

$$x_i \to x'_i = a_i + M_{ij} x_j \ ,$$
$$x_i \to x'_i = \lambda x_i \ \ \tau \to \tau' = \lambda \tau \ ,$$
$$x_i \to x'_i = \frac{x_i + b_i\left(-\tau^2 + \vec{x}^2\right)}{1 + 2\vec{b}\cdot\vec{x} + b^2\left(-\tau^2 + \vec{x}^2\right)}$$
$$\tau \to \tau' = \frac{\tau}{1 + 2\vec{b}\cdot\vec{x} + b^2\left(-\tau^2 + \vec{x}^2\right)} \ ,$$

The conformal transformation of a r-rank tensor

$$T_{i_1 \dots i_r}(x) \to T'_{i_1 \dots i_r}(x') = \left|\det\left(\frac{\partial x'^l}{\partial x^k}\right)\right|^{\frac{r-\Delta_T}{d}} \frac{\partial x^{j_1}}{\partial x'^{i_1}} \dots \frac{\partial x^{j_r}}{\partial x'^{i_r}} T_{j_1 \dots j_r}(x)$$

The triple K integral

$$J_{N\{p_j\}}(k_1,k_2,k_3) = \int_0^\infty dx x^{1/2+N} \prod_{j=1}^{3} k_j^{\Delta_j-3/2+p_j} K_{\Delta_j-3/2+p_j}(k_j x)$$

The primary SCT Ward identity

$$\mathcal{K}_{ab} = 2(\Delta_i - 2)\frac{1}{k_a}\partial_a - \partial_a^2 - 2(\Delta_b - 2)\frac{1}{k_b}\partial_b + \partial_b^2$$

The triple K integral in the primary SCT Ward identity

$$\mathcal{K}_{ab} J_{N\{p_j\}} = 2p_a J_{N+1\{p_j-\delta_{aj}\}} - 2p_b J_{N+1\{p_j-\delta_{bj}\}}$$