Light propagation in Horndeski vector-tensor theory

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Work "in progress??" (2016-????) with Cyril Pitrou, Jean-Philippe Uzan and Pierre Fleury.
Based on arXiv: xxxx.xxxx [gr-qc]. Coming "soon"

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Some motivations

1. To conclude the unfinished projects!
2. Understand the basics of light propagation in modified gravity models.
3. Search for observational consequences of modified gravity models.
4. Study Horndeski's model which is a well motivated model involving vector-tensor couplings.

Electromagnetism in flat space (without gravity)

Action and equation of motion

U(1) gauge symmetric action with 1st order derivatives

Maxwell action $\longrightarrow$

$$S_M[A] = -\frac{1}{4}\int \mathrm{d}^4x\, F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge invariance $\rightarrow$ $A_\nu \rightarrow A_\nu + \partial_\nu \xi$

Equation of motion $\longrightarrow$

$$\frac{\delta S_M[A]}{\delta A_\nu} = \partial_\mu F^{\mu\nu} = 0.$$

Lorentz gauge: $\partial^\nu A_\nu = 0$ $\longrightarrow$ $\Box A_\mu = 0.$

Electromagnetism in curved space (minimal coupling)

Action and equation of motion

U(1) gauge symmetric action with 1st order derivatives

Maxwell and Einstein $\longrightarrow$ $$S_M[A] = -\frac{1}{4}\int \mathrm{d}^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \qquad A_\nu \to A_\nu + \partial_\nu \xi$$

Equation of motion $\longrightarrow$ $$\frac{\delta S_M[A]}{\delta A_\nu} = \nabla_\mu F^{\mu\nu} = 0.$$

Lorentz gauge: $\nabla^\nu A_\nu = 0$

$$\nabla_\mu F^{\mu\nu} = \nabla_\mu \nabla^\mu A^\nu - \nabla_\mu \nabla^\nu A^\mu = \nabla_\mu \nabla^\mu A^\nu - \nabla^\nu(\nabla_\mu A^\mu) - R^\mu{}_{\rho\mu}{}^\nu A^\rho\,.$$

$$\Box A_\mu - R^\nu{}_\mu A_\nu = 0.$$

Geometric optics

Electromagnetic waves

Linearly polarized ansatz $\longrightarrow$ $A_\mu = a_\mu e^{i\varphi} + c.c$

a_μ $\longrightarrow$ slowly varying amplitude

φ $\longrightarrow$ rapidly varying phase

$k_\mu \equiv \partial_\mu \varphi$ $\longrightarrow$ wave four-vector

Gauge: $\nabla^\mu A_\mu = (\nabla^\mu a_\mu + i k_\mu a^\mu) e^{i\varphi} + c.c = 0$

$\nabla^\mu a_\mu = 0$ & $k_\mu a^\mu = 0$ $\longrightarrow$ Transverse waves

Geometric optics

Eikonal approximation. Wave effects are irrelevant: No interference, no diffraction, light ray trajectories.

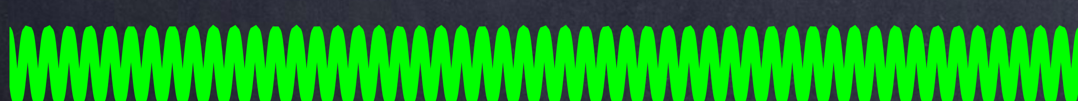
Flat space

$$\lambda \ll r \leftrightarrow \partial\varphi \gg \partial a/a$$

r: Typical length scales of the system

$\partial\varphi \gg \partial a/a$: The phase varies much quicker than the amplitude

r —> a single length scale in flat space.



Small wavelength. Geometric optics limit.

Large wavelength. Wave effects are important.

Geometric optics

Eikonal approximation. Wave effects are irrelevant:
No interference, no diffraction, light ray trajectories.

Curved space

$$\partial\varphi \gg \partial a/a \quad \& \quad \partial\varphi \gg r_{curvature}^{-1}$$

We need an additional parameter of length: spacetime curvature!

$$r_{curvature}^{-1} \sim \sqrt{R^{\mu}{}_{\nu\sigma\rho}}$$

A very relaxed condition for cosmological and astrophysical applications!

$$r_{curvature} \sim c/H \sim (\text{few})\text{Gpc} \;\rightarrow\; \lambda \ll \text{Gpc}$$

$$r_{curvature} \sim \sqrt{r^3/r_S} \sim (\text{few})\text{AU} \;\rightarrow\; \lambda \ll (\text{few})\text{AU}$$

Eikonal approximation of Maxwell equations

$$A_\mu = a_\mu e^{i\varphi} + c.c \quad \text{in} \rightarrow \quad \Box A_\mu - R^\nu{}_\mu A_\nu = 0.$$

$$\downarrow$$

$$\Box a_\mu - R^\nu{}_\mu a_\nu - k^\nu k_\nu a^\mu + i(2k^\nu \nabla_\nu a^\mu + a^\mu \nabla_\nu k^\nu) = 0.$$

$$k^\nu k_\nu \gg \{\Box, R^\nu{}_\mu\}$$

$$k^\nu k_\nu = 0. \qquad k^\nu \nabla_\nu a^\mu + \frac{1}{2} a^\mu (\nabla_\nu k^\nu) = 0.$$

$$F_{\mu\nu} = 2ik_{[\mu}A_{\nu]} + c.c. \qquad E^\mu = u_\nu F^{\mu\nu},\ B^\mu = -u_\nu \tilde{F}^{\mu\nu}$$

Orthogonal basis:

$$(k^\mu, E^\mu, B^\mu) \rightarrow k_\mu E^\mu = k_\mu B^\mu = B_\mu E^\mu = 0$$

Null propagation and photon conservation

Dispersion relation: $k^\nu k_\nu = 0.$

$\nabla_\mu (k^\nu k_\nu) = 2k^\nu \nabla_\mu k_\nu = 0.$ $\nabla_\mu k_\nu = \partial_\mu \partial_\nu \varphi - \Gamma^\sigma_{\mu\nu} \partial_\sigma \varphi = \nabla_\nu k_\mu .$

$k^\nu \nabla_\nu k_\mu = 0$ ⟶ Null geodesic!

Conserved current: $k^\nu \nabla_\nu a^\mu + \frac{1}{2} a^\mu (\nabla_\nu k^\nu) = 0.$

$j^\mu \equiv a^2 k^\mu \to \nabla_\mu j^\mu = 0!$ ⟶ Photon conservation!

Photons follows null geodesics!

Electromagnetism in curved space (non-minimal coupling)

Action and equation of motion

Maxwell
Einstein ⟶ $$S_M[A] = -\frac{1}{4}\int \mathrm{d}^4x\,\sqrt{-g}\,\left(F_{\mu\nu}F^{\mu\nu} + l^2 L^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)$$
Horndeski

$$L^{\mu\nu\rho\sigma} \equiv -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\gamma\delta}R_{\alpha\beta\gamma\delta} = 2R^{\mu\nu\rho\sigma} + 4(R^{\mu[\sigma}g^{\rho]\nu} - R^{\nu[\sigma}g^{\rho]\mu} + 2Rg^{\mu[\rho}g^{\sigma]\nu}$$

1. Derived from and action principle
2. Second order equations.
3. Conserved U(1) charge.
4. Reduce to standard EM in flat space.

Conservation of charge and the Einstein-Maxwell field equations, Gregory Walter Horndeski, Journal of Mathematical Physics 17, 1980 (1976).

Electromagnetism in curved space (non-minimal coupling)

Equations of motion

Maxwell
Einstein
Horndeski

$$S_M[A] = -\frac{1}{4}\int d^4x \sqrt{-g}\,\left(F_{\mu\nu}F^{\mu\nu} + l^2 L^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)$$

Parameter with units of length $\longrightarrow$ l^2

Equation of motion

$$\frac{\delta S_M[A]}{\delta A_\mu} = \nabla_\nu F^{\mu\nu} + l^2 L^{\mu\nu\rho\sigma}\nabla_\nu F_{\rho\sigma} = 0.$$

$$\boxed{\Box A^\mu - R^\mu_\nu A^\nu - 2l^2 L^{\mu\nu\rho\sigma}\nabla_\nu\nabla_\rho A_\sigma = 0.}$$

Geometric optics (non-minimal coupling)

We have a parameter with units of length: l!
A priori we dont have any criteria to neglect contributions proportional to l.

$$\partial\varphi \gg l^{-1}?$$

$$\nabla_\nu \nabla_\rho A_\sigma \approx [-k_\nu k_\rho a_\sigma + 2ik_{(\nu}\nabla_{\rho)}a_\sigma + i(\nabla_\nu k_\rho)a_\sigma]e^{i\varphi} + c.c$$

$$\downarrow$$

$$(k^\nu k_\nu)a^\mu = -2l^2 L^{\mu\nu\rho\sigma}k_\nu a_\rho k_\sigma$$

$$k^\nu \nabla_\nu a^\mu + \frac{1}{2}(\nabla_\nu k^\nu)a^\mu = +l^2 L^{\mu\nu\rho\sigma}[2k_{(\nu}\nabla_{\rho)}a_\sigma + (\nabla_\nu k_\rho)a_\sigma]$$

Geometric optics (non-minimal coupling)

Consequences

Non null propagation

$$(k^\nu k_\nu)a^\mu = -2l^2 L^{\mu\nu\rho\sigma}k_\nu a_\rho k_\sigma \longrightarrow k^\nu k_\nu = -2l^2 L^{\mu\nu\rho\sigma}\epsilon_\mu k_\nu \epsilon_\rho k_\sigma = -\chi,$$

$$k^2 = -\chi, \longrightarrow \frac{\mathrm{D}k_\mu}{\mathrm{d}v} = -\frac{1}{2}\partial_\mu \chi$$

No photon conservation

$$\nabla_\mu(a^\nu a_\nu k^\mu) = 2\ell^2 L^{\mu\nu\rho\sigma}[2a_\mu k_{(\nu}\nabla_{\rho)}a_\sigma + a_\mu(\nabla_\nu k_\rho)a_\sigma]$$

Geometric optics (non-minimal coupling)

Gravitational birefringence

Gravitational "susceptibility" matrix

Eigenvalues problem

$$\chi^{\mu\nu} \equiv 2l^2 L^{\mu\rho\nu\sigma} k_\rho k_\sigma \longrightarrow \boxed{\chi^{\mu}{}_{\nu} a^{\nu} = -k^2 a^{\mu}}$$

Geometric optics (non-minimal coupling)

Gravitational birefringence

Schwarzschild space

$$ds^2 = [-f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad f(r) \equiv 1 - \frac{2M}{r}$$

↓

$$L^{\mu\nu\alpha\beta} = \frac{2M}{r^3}\left[g^{\mu\beta}g^{\nu\alpha} - g^{\mu\alpha}g^{\nu\beta} + 12e_\theta^{[\mu}e_\phi^{\nu]}e_\theta^{[\alpha}e_\phi^{\beta]} - 12e_t^{[\mu}e_r^{\nu]}e_t^{[\alpha}e_r^{\beta]}\right]$$

$$E \equiv -k_t \qquad L \equiv k_\phi \equiv bE$$

Eigenvalues

$$\chi_1 = -\chi_2 = -\frac{12M\ell^2b^2}{r^5}.$$

→

Eigenvectors

$$\epsilon_1 = \frac{1}{k_\phi}(-k_te_r + k_re_t)$$

$$\epsilon_2 = e_\theta$$

Geometric optics (non-minimal coupling)

Angular deflection

$$\delta\phi = \frac{4M}{r_0}\left(1 + \epsilon\frac{4\ell^2}{r_0^2}\right)$$

$$\chi_1 = -\chi_2 = -\frac{12M\ell^2 b^2}{r^5}.$$

$$\epsilon = \pm 1$$

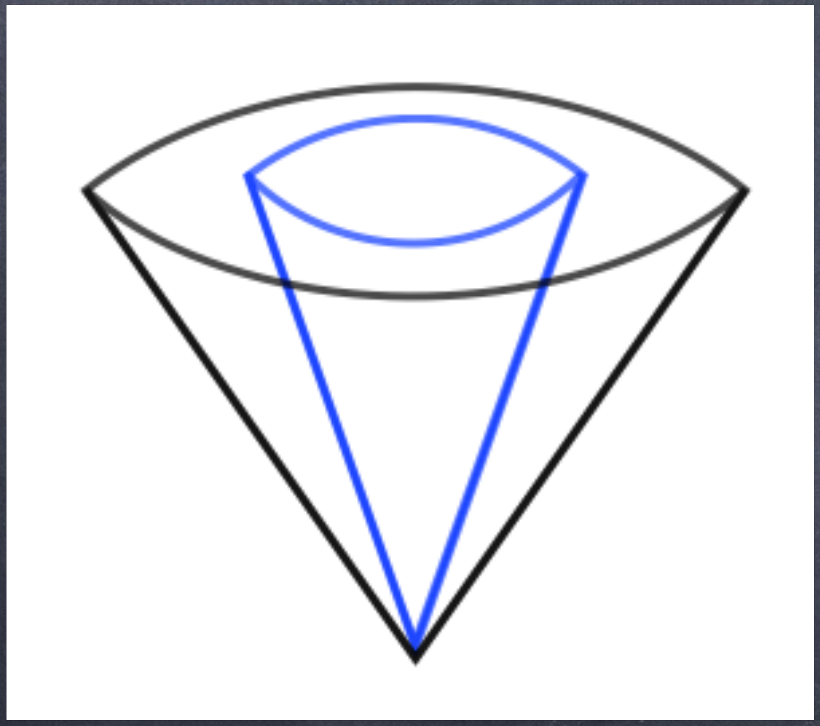
$$r_0^2 \equiv B(r_0)b^2$$

$$B(r) \equiv f(r)\left(1 - \epsilon\frac{12M\ell^2}{r^3}\right)$$

Geometric optics (non-minimal coupling)

Effective or induced metric

$$\tilde{g}_{\mu\nu}k^{\mu}k^{\nu} = (g_{\mu\nu} + \epsilon_{\mu\nu})k^{\mu}k^{\nu} = 0, \quad \epsilon_{\mu\nu} \equiv 2\ell^2 L_{\alpha\mu\beta\nu}\epsilon^{\alpha}\epsilon^{\beta}$$



$$\tilde{\nabla}_{\alpha}\tilde{g}_{\mu\nu} = 0$$

$$\downarrow$$

$$\tilde{\nabla}_{\alpha}(\tilde{g}^{\mu\nu}k_{\mu}k_{\nu}) = 2\tilde{k}^{\nu}\tilde{\nabla}_{\nu}k_{\alpha} = 0$$

$$\tilde{k}^{\nu}\tilde{\nabla}_{\nu}\tilde{k}^{\alpha} = \frac{\mathrm{D}\tilde{k}^{\alpha}}{\mathrm{d}v} = 0$$

Photons follows null geodesics in the induced metric!

Geometric optics (non-minimal coupling)

Effective or induced metric

Photon flux density

$$\tilde{j}^{\nu} \equiv (a^{\alpha}a_{\alpha})\tilde{k}^{\nu} = (a^{\alpha}a_{\alpha})\tilde{g}^{\nu\rho}k_{\rho} = (\tilde{g}_{\alpha\beta}a^{\alpha}a^{\beta})\tilde{k}^{\nu}$$

$$\nabla_{\mu}(a^{\nu}a_{\nu}k^{\mu}) - 2\ell^{2}L^{\mu\nu\rho\sigma}[2a_{\mu}k_{(\nu}\nabla_{\rho)}a_{\sigma} + a_{\mu}(\nabla_{\nu}k_{\rho})a_{\sigma}] = 0$$

$$\nabla_{\nu}\tilde{j}^{\nu} = 0!$$

Photons flux density is conserved!

Energy momentum tensor

A few words about EM tensor

$$T_{\mu\nu} = T_{\mu\nu}^{\text{Maxwell}} - l^2 \left[-L_{\mu\alpha\nu\beta} F^{\alpha\gamma} F^{\beta}{}_{\gamma} + 2\nabla^{\gamma}(\tilde{F}_{\mu\beta})\nabla^{\beta}(\tilde{F}_{\nu\gamma}) \right]$$

Maxwell
physical EM

"Geometrical"
(dielectric) EM

Conclusions and Remarks

1. Light propagation in presence of non minimal couplings to gravity is a subtle issue.
2. Horndeski vector-tensor model is trivially scale invariant, so, no "rainbow" frequency dependent effects are expected.
3. Polarization dependent effects.
4. Induced effective metric is useful to calculate things but is hard to interpret.
5. Energy conditions results crucial to understand/interpret the causal flow of energy in non minimally coupled models.