The Scalar Galileon and its constraints from GW170817 and GRB170817A

Jhan Nicolás Martínez and
Yeinzon Rodríguez.

Universidad
Industrial de
Santander

Introduction

Scalar fields play an important role in cosmology and particle physics; and including the fact that every physical theory of scalar field avoids tachyonic instabilities here we build the action for the scalar Galileon and study the implications of the mentioned detections on this theory.

Introduction

1. 1974, G. W. Horndeski.
2. 1918, A. Einstein.
3. 2016, LIGO.
4. 2017, LIGO-Virgo.
5. 2001, Gravitational Cherenkov Radiation.

B. P. Abbott et. al., *Astrophys. J.*, 2017.

Ostrogradski's instability

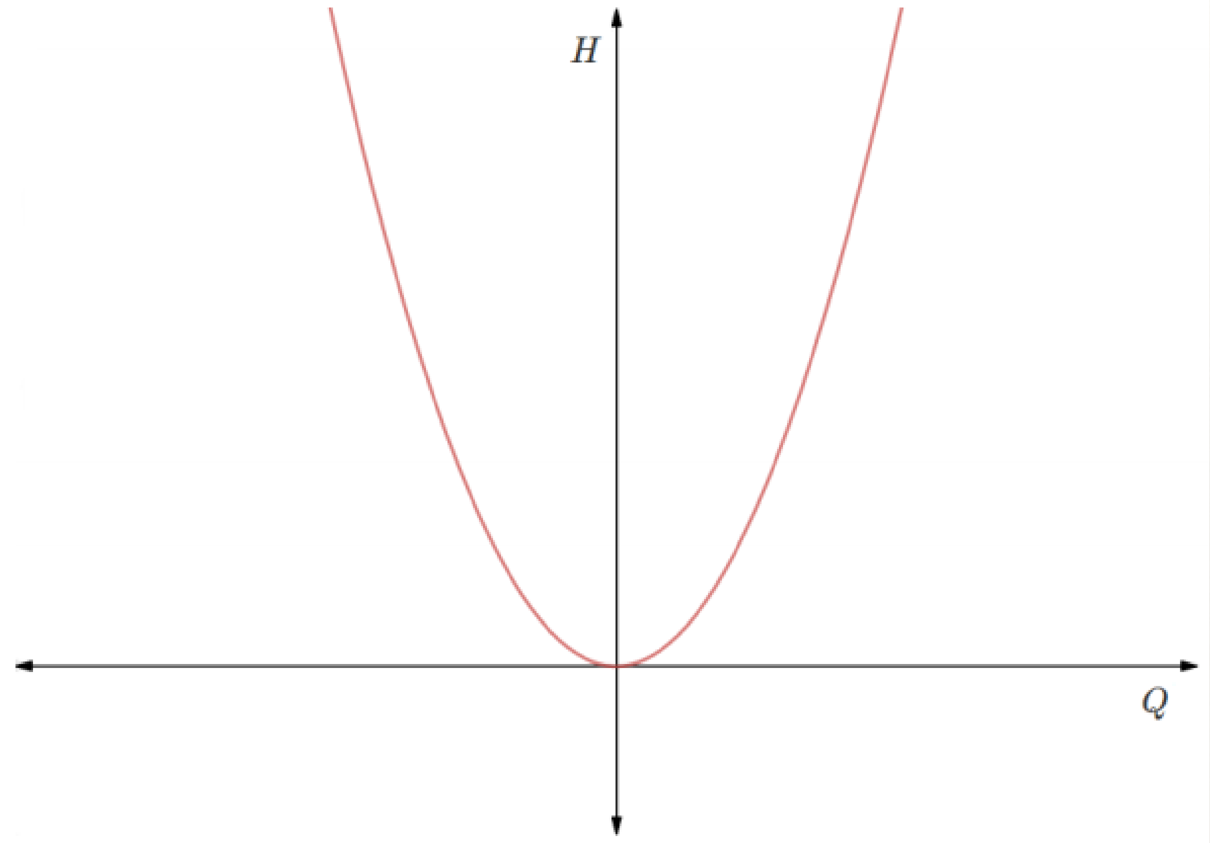

Figure 1: Physical system with Hamiltonian bounded from below.

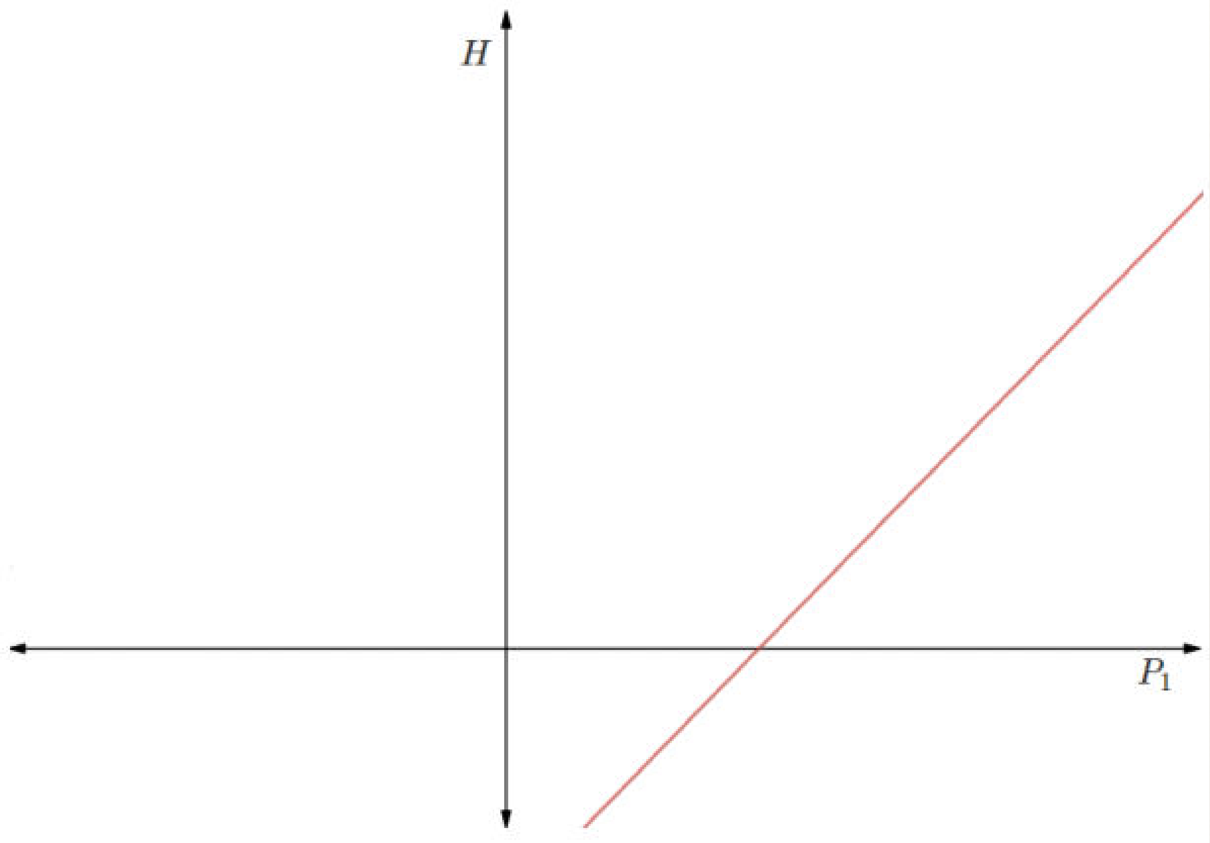

Figure 2: Behavior of H against P_1 for a mechanical system with equations of motion higher than second order.

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$S = \int \mathcal{L}(\pi, \partial_\mu \pi, \partial_\mu \partial_\nu \pi) d^4 x,$$

$$\frac{\partial \mathcal{L}}{\partial \pi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} + \partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \pi)} = 0,$$

Scalar Galileon

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$[1] \equiv \Box\pi, \qquad [2] \equiv \pi^{\alpha}_{\beta}\pi^{\beta}_{\alpha},$$

$$[i] \equiv \pi^{\mu_1}_{\mu_2}\pi^{\mu_2}_{\mu_3}\pi^{\mu_3}_{\mu_4}\ldots\pi^{\mu_{i-1}}_{\mu_i}\pi^{\mu_i}_{\mu_1},$$

$$\langle 1\rangle \equiv \pi_{\mu_1}\pi^{\mu_1}_{\mu_2}\pi^{\mu_2}, \qquad \langle 2\rangle \equiv \pi_{\mu_1}\pi^{\mu_1}_{\mu_2}\pi^{\mu_2}_{\mu_3}\pi^{\mu_3},$$

$$\langle i\rangle \equiv \pi_{\mu_1}\pi^{\mu_1}_{\mu_2}\pi^{\mu_2}_{\mu_3}\ldots\pi^{\mu_i}_{\mu_{i+1}}\pi^{\mu_{i+1}},$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\begin{bmatrix} p_1 & p_2 & \dots & p_r \\ 1 & 2 & \dots & r \end{bmatrix} \equiv [1]^{p_1} [2]^{p_2} \dots [r]^{p_r},$$

$$\left\langle \begin{matrix} q_1 & q_2 & \dots & q_s \\ 1 & 2 & \dots & s \end{matrix} \right\rangle \equiv \langle 1 \rangle^{q_1} \langle 2 \rangle^{q_2} \dots \langle s \rangle^{q_s},$$

$$\mathcal{L}^{p_1, p_2, \dots, p_r}_{q_1, q_2, \dots, q_s} = f(\pi, X) \begin{bmatrix} p_1 & p_2 & \dots & p_r \\ 1 & 2 & \dots & r \end{bmatrix} \left\langle \begin{matrix} q_1 & q_2 & \dots & q_s \\ 1 & 2 & \dots & s \end{matrix} \right\rangle,$$

Scalar Galileon

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\mathcal{L} = \sum_{\{p_i\}\{q_i\}} C_{\{p_i\}\{q_i\}} \mathcal{L}^{p_1,p_2,\ldots,p_r}_{q_1,q_2,\ldots,q_s}.$$

$$N = \left(\sum_{i=1}^{r} i p_i\right) + \left(\sum_{j=1}^{s} (j+2) q_j\right),$$

$$n = N - 2\sum_{j=1}^{s} q_j.$$

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$\Box\pi^{\alpha}_{\beta}$$

$$\delta_\pi[i]^{p_i} \supset \frac{2ip_i}{i-1}[\Box(i-1)][i]^{p_i-1}\delta\pi \qquad (i>1),$$

$$\delta_\pi\langle i\rangle^{q_i} \supset 2q_i\langle\Box(i-1)\rangle\langle i\rangle^{q_i-1}\delta\pi \qquad (i>1),$$

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$[\Box(j)] \equiv \sum_{k=1}^{j} \pi^{\mu_1}_{\mu_2} \pi^{\mu_2}_{\mu_3} \ldots \pi^{\mu_{k-1}}_{\mu_k} (\Box \pi^{\mu_k}_{\mu_{k+1}}) \pi^{\mu_{k+1}}_{\mu_{k+2}} \ldots \pi^{\mu_{j-1}}_{\mu_j} \pi^{\mu_j}_{\mu_1},$$

$$= j\ \pi^{\mu_1}_{\mu_2} \pi^{\mu_2}_{\mu_3} \ldots \pi^{\mu_{j-1}}_{\mu_j} (\Box \pi^{\mu_j}_{\mu_1}),$$

$$\langle \Box(j) \rangle \equiv \sum_{k=1}^{j} \pi_{\mu_1} \pi^{\mu_1}_{\mu_2} \pi^{\mu_2}_{\mu_3} \ldots \pi^{\mu_{k-1}}_{\mu_k} (\Box \pi^{\mu_k}_{\mu_{k+1}}) \pi^{\mu_{k+1}}_{\mu_{k+2}} \ldots \pi^{\mu_j}_{\mu_{j+1}} \pi^{\mu_{j+1}}.$$

Scalar Galileon

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\frac{\delta \mathcal{L}}{\delta \pi}\delta\pi \supset \frac{2ip_i}{i-1} f(\pi, X)[1]^{p_1}[2]^{p_2}...[i-1]^{p_{i-1}}[\Box(i-1)][i]^{p_{i-1}}...[r]^{p_r} \begin{pmatrix} q_1 & q_2 & \cdots & q_s \\ 1 & 2 & \ldots & s \end{pmatrix} \delta\pi,$$

$$f \begin{bmatrix} p_1+1 & p_2 & \ldots & p_{i-2} & p_{i-1}+1 & p_i-1 & p_{i+1} & \ldots & p_r \\ 1 & 2 & \ldots & i-2 & i-1 & i & i+1 & \ldots & r \end{bmatrix} \left\langle \begin{matrix} q_1 & q_2 & \ldots & q_s \\ 1 & 2 & \ldots & s \end{matrix} \right\rangle,$$

$$\alpha_{[]} = -\frac{ip_i}{(p_1+1)(p_{i-1}+1)(i-1)},$$

Scalar Galileon

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\alpha_{\langle\rangle} f \begin{bmatrix} p_1+1 & p_2 & \dots & p_r \\ 1 & 2 & \dots & r \end{bmatrix} \left\langle \begin{matrix} q_1 & \dots & q_{j-2} & q_{j-1}+1 & q_j-1 & q_{j+1} & \dots & q_s \\ 1 & \dots & j-2 & j-1 & j & j+1 & \dots & s \end{matrix} \right\rangle,$$

$$\alpha_{\langle\rangle} = -\frac{q_j}{(p_1+1)(q_{j-1}+1)}.$$

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$f\begin{bmatrix} p_1 & p_2 & \dots & p_r \\ 1 & 2 & \dots & r \end{bmatrix}\left\langle \begin{matrix} q_1 & q_2 & \dots & q_s \\ 1 & 2 & \dots & s \end{matrix} \right\rangle,$$

$$\Downarrow$$

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$$\Downarrow$$

$$f\begin{bmatrix} p_1+1 & p_2 & \dots & p_{i-2} & p_{i-1}+1 & p_i-1 & p_{i+1} & \dots & p_r \\ 1 & 2 & \dots & i-2 & i-1 & i & i+1 & \dots & r \end{bmatrix}\left\langle \begin{matrix} q_1 & q_2 & \dots & q_s \\ 1 & 2 & \dots & s \end{matrix} \right\rangle.$$

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$\mathcal{L}_q^p = f[1]^p \langle 1 \rangle^q$$
$$= f(\Box\pi)^p(\pi^\mu \pi_{\mu\nu}\pi^\nu)^q,$$

$$p = \sum_{i=1}^{r} i p_i + \sum_{j=1}^{s} (j-1) q_j, \quad q = \sum_{j=1}^{s} q_j,$$

$$p = \frac{1}{2}(3n - N), \qquad q = \frac{1}{2}(N - n).$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$\frac{\delta \mathcal{L}_q^p}{\delta \pi} \delta\pi \supset f(\Box \pi)^p (\pi^\lambda \pi^\rho \pi^\sigma \pi^\tau \pi_{\lambda\rho\sigma\tau})(\pi^\mu \pi_{\mu\nu} \pi^\nu)^{q-2},$$

$$q = 0 \quad \Longleftrightarrow \quad N = n,$$

$$\mathcal{L}_n^{(3)}\{f\} \equiv f(\pi, X) \mathcal{L}_{n+2}^{Gal,3},$$

$$q = 1 \quad \Longleftrightarrow \quad N = n + 2,$$

$$\mathcal{L}_n^{(2)}\{f\} \equiv f(\pi, X) \mathcal{L}_{n+2}^{Gal,2}.$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\mathcal{L}_N^{\mathrm{Gal},1} = \mathcal{A}^{\mu_1 \dots \mu_{n+1} \nu_1 \dots \nu_{n+1}} \pi_{\mu_{n+1}} \pi_{\nu_{n+1}} \pi_{\mu_1 \nu_1} \dots \pi_{\mu_n \nu_n},$$

$$\mathcal{L}_N^{\mathrm{Gal},2} = \mathcal{A}^{\mu_1 \dots \mu_n \nu_1 \dots \nu_n} \pi_{\mu_1} \pi_\lambda \pi^\lambda_{\nu_1} \pi_{\mu_2 \nu_2} \dots \pi_{\mu_n \nu_n},$$

$$\mathcal{L}_N^{\mathrm{Gal},3} = X \mathcal{A}^{\mu_1 \dots \mu_n \nu_1 \dots \nu_n} \pi_{\mu_1 \nu_1} \dots \pi_{\mu_n \nu_n},$$

$$\delta^{\mu_1 \dots \mu_n}_{\nu_1 \dots \nu_n} = n! \delta^{[\mu_1}_{\nu_1} \delta^{\mu_2}_{\nu_2} \dots \delta^{\mu_n]}_{\nu_n} = -\mathcal{A}^{\nu_1 \dots \nu_{n+1}}_{\mu_1 \dots \mu_{n+1}},$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$n\mathcal{L}_n^{(2)}\{f\} = \mathcal{L}_n^{(3)}\{f\} - \mathcal{L}_n^{(1)}\{f\},$$

$$\partial_\mu(f(\pi, X)J_n^\mu) = 2\mathcal{L}_n^{(2)}\{f + Xf_X\} + \mathcal{L}_{n-1}^{(1)}\{Xf_\pi\} + \mathcal{L}_n^{(3)}\{f\},$$

$$J_N^\mu = X\mathcal{A}^{\mu\mu_2\ldots\mu_n\nu_1\nu_2\ldots\nu_n}\pi_{\nu_1}\pi_{\mu_2\nu_2}\ldots\pi_{\mu_n\nu_n},$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$\mathcal{L}_n^{(2)}\{f\} = (1-n)\mathcal{L}_{n-1}^{(2)}\left\{\frac{\partial g_1}{\partial \pi}\right\} + \mathcal{L}_n^{(3)}\left\{\frac{g_1}{X}\right\} + \mathcal{L}_{n-1}^{(3)}\{f\}\left\{\frac{\partial g_1}{\partial \pi}\right\} + \text{tot. div.},$$

$$g_1\{f\} = -\frac{1}{2}\int_o^X dY f(\pi, Y),$$

$$\mathcal{L}_n^{(2)}\{f\} = \mathcal{L}_0^{(3)}\left\{\frac{\partial g_{n,1}}{\partial \pi}\right\} + \sum_{i=1}^{n-1}\mathcal{L}_n^{(3)}\left\{\frac{g_{n,i}}{X} + \frac{\partial g_{n,i+1}}{\partial \pi}\right\} + \mathcal{L}_n^{(3)}\left\{\frac{\partial g_{n,n}}{\partial \pi}\right\} + \text{tot. div.},$$

Scalar Galileon

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

C. Deffayet et. al.,*Phys. Rev.*, 2011.

$$\mathcal{L}_0^{(3)} \{f\} = X f,$$

$$g_{n,i} \{f\} \quad \equiv \frac{(n-1)!}{(i-1)!} g_{n-i+1} \{f\},$$

$$g_i \{f\} \equiv -\frac{1}{2^i} \left(\frac{\partial}{\partial \pi} \right)^{i-1} \int_{X_0}^{X} dX_1 \int_{X_0}^{X_1} dX_2 \cdots \int_{X_0}^{X_{i-1}} dX_i f(\pi, X_i),$$

Scalar Galileon

C. Deffayet et. al., *Phys. Rev.*, 2011.

$$D\mathcal{L}_D^{(2)}\{f\} = \mathcal{L}_D^{(3)}\{f\},$$

$$\mathcal{L} = \sum_{n=0}^{D-1} \mathcal{L}_n\{f_n\}.$$

Scalar Galileon in Minkowski spacetime

$(X \equiv \partial_\mu \pi \partial^\mu \pi)$

$$S = \int \sum_{N=2}^{5} \mathcal{L}_{N,\pi}^{\rm Gal} d^4x,$$

$$\mathcal{L}_{2,\pi}^{\rm Gal} \equiv f_2(\pi, X),$$

$$\mathcal{L}_{3,\pi}^{\rm Gal} \equiv f_3(\pi, X)\Box\pi,$$

$$\mathcal{L}_{4,\pi}^{\rm Gal} \equiv f_4(\pi, X)[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)(\partial^\mu\partial^\nu\pi)],$$

$$\mathcal{L}_{5,\pi}^{\rm Gal} \equiv f_5(\pi, X)[(\Box\pi)^3 - 3(\Box\pi)(\partial_\mu\partial_\nu\pi)(\partial^\mu\partial^\nu\pi) + 2(\partial_\mu\partial^\nu\pi\partial_\nu\partial^\rho\pi\partial_\rho\partial^\mu\pi)],$$

Scalar Galileon in curved spacetime

$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$

$$\mathcal{S} = \int \Big[\sum_{N=2}^{5} \mathcal{L}_{N,\pi}^{Gal}\Big] \sqrt{-g}\, \mathrm{d}^4 x,$$

$$\mathcal{L}_{2,\pi}^{\rm Gal} \equiv G_2(\pi, X)\,,$$

$$\mathcal{L}_{3,\pi}^{\rm Gal} \equiv G_3(\pi, X)\,\Box\pi\,,$$

$$\mathcal{L}_{4,\pi}^{\rm Gal} \equiv G_4(\pi, X)[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi)],$$

$$\mathcal{L}_{5,\pi}^{\rm Gal} \equiv G_5(\pi, X)[(\Box\pi)^3 - 3(\Box\pi)(\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi) + 2(\nabla_\mu\nabla^\nu\pi\nabla_\nu\nabla^\rho\pi\nabla_\rho\nabla^\mu\pi)],$$

Scalar Galileon in curved spacetime

$\left(X \equiv -\frac{1}{2}\nabla_\mu\pi\nabla^\mu\pi\right)$

$$\frac{\partial\mathcal{L}}{\partial\pi} - \nabla_\mu\frac{\partial\mathcal{L}}{\partial(\nabla_\mu\pi)} + \nabla_\mu\nabla_\nu\frac{\partial\mathcal{L}}{\partial(\nabla_\mu\nabla_\nu\pi)} = 0,$$

$$-2f\nabla^\alpha\pi\nabla^\mu R_{\mu\alpha}, \qquad \mathcal{L}_4^{'} = G(\pi, X)R,$$

$$6f\nabla^\rho\nabla^\beta\pi\nabla^\gamma\pi\nabla_\gamma G_{\beta\rho}, \qquad \mathcal{L}_5^{'} = G(\pi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\pi),$$

Scalar Galileon in curved spacetime

$\left(X \equiv -\frac{1}{2}\nabla_\mu\pi\nabla^\mu\pi\right)$

$$\mathcal{S} = \int \Big[\sum_{N=2}^{5} \mathcal{L}_{N,\pi}^{Gal} \Big] \sqrt{-g}\, \mathrm{d}^4x,$$

$$\mathcal{L}_{2,\pi}^{\mathrm{Gal}} \equiv G_2(\pi, X),$$

$$\mathcal{L}_{3,\pi}^{\mathrm{Gal}} \equiv G_3(\pi, X)\Box\pi,$$

$$\mathcal{L}_{4,\pi}^{\mathrm{Gal}} \equiv G_4(\pi, X)R + G_{4,X}(\pi, X)[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi)],$$

$$\mathcal{L}_{5,\pi}^{\mathrm{Gal}} \equiv G_5(\pi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\pi) - \frac{1}{6}G_{5,X}\left[(\Box\pi)^3 - 3(\Box\pi)(\nabla_\mu\nabla_\nu\pi)(\nabla^\mu\nabla^\nu\pi) + 2(\nabla_\mu\nabla^\nu\pi\ \nabla_\nu\nabla^\rho\pi\ \nabla_\rho\nabla^\mu\pi)\right],$$

Speed of gravitational waves

$$\left(X \equiv -\frac{1}{2}\nabla_\mu\pi\nabla^\mu\pi\right)$$

$$g^{\alpha\beta} = a^2\bar{g}^{\alpha\beta}, \qquad \bar{g}^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta},$$

$$\Gamma^\alpha_{\beta\gamma} = \bar{\Gamma}^\alpha_{\beta\gamma} + \delta^\alpha_\beta\partial_\gamma \ln a + \delta^\alpha_\gamma\partial_\beta \ln a - \bar{g}_{\gamma\beta}\partial^\alpha \ln a,$$

$$R_{\alpha\beta} = \bar{R}_{\alpha\beta} - 2\nabla_\alpha\nabla_\beta \ln a - 2\nabla_\alpha \ln a\nabla_\beta \ln a \\ +2\bar{g}_{\alpha\beta}\nabla^\gamma \ln a\nabla_\gamma \ln a - \nabla_\gamma(\bar{g}_{\alpha\beta}\nabla^\gamma \ln a),$$

Speed of gravitational waves

$$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$$

$$^{(0)}\sqrt{-g} = \ a^4,$$

$$^{(1)}\sqrt{-g} = \ 0,$$

$$^{(2)}\sqrt{-g} = -\frac{a^4}{4} h_{\mu\nu} h^{\mu\nu}.$$

Speed of gravitational waves

$$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$$

$$^{(2)}\mathcal{S} = \frac{1}{2}\int d^4x M_*^2\left[(\dot{h}_{ij})^2 - v^2(\nabla h_{ij})^2\right],$$

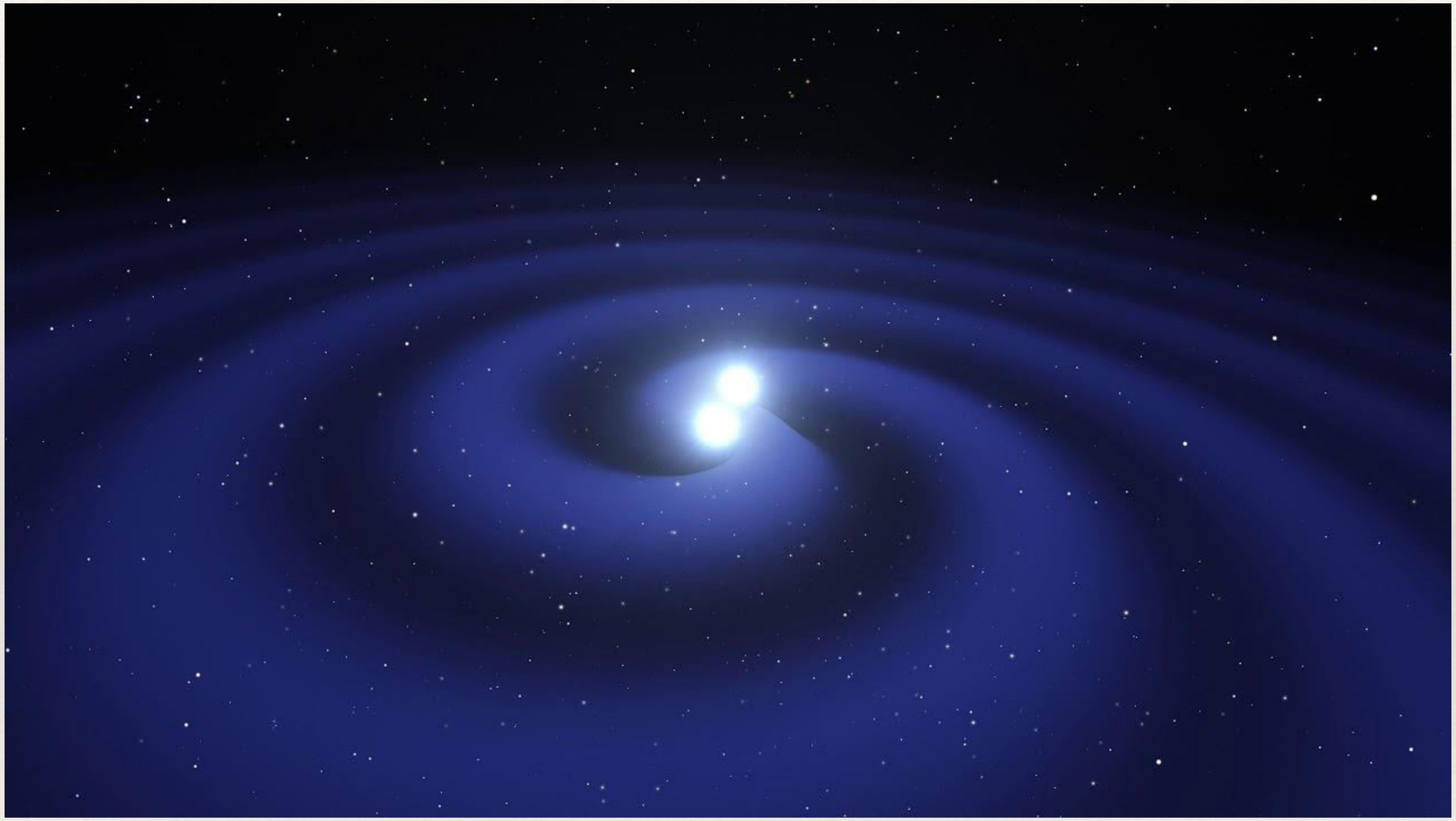
$$v^2 = 1 + \alpha_T,$$

$$M_*^2 \equiv 2(G_4 - 2XG_{4,X} + XG_{5,\pi} - \dot{\pi}HXG_{5,X}),$$

$$M_*^2\alpha_T \equiv 2X[2G_{4,X} - 2G_{5,\pi} - (\ddot{\pi} - \dot{\pi}H)G_{5,X}],$$

Speed of gravitational waves

$$d \approx 40 Mpc, \qquad \Delta t = 1,7 s.$$



G. D. Moore et. al.,*J. High Energy Phys.*, 2001.
B. P. Abbott et al.,*Phys. Rev. Lett.*, 2016.
T. Baker et. al.,*Phys. Rev. Lett.*, 2017.

Speed of gravitational waves

$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$

$$|\alpha_T| \lesssim 1 \times 10^{-15},$$

$$M_*^2 \alpha_T \equiv 2X[2G_{4,X} - 2G_{5,\pi} - (\ddot{\pi} - \dot{\pi}H)G_{5,X}],$$

$$H \;\; \& \;\; \ddot{\pi},$$

G. D. Moore et. al., *J. High Energy Phys.*, 2001.
B. P. Abbott et al., *Phys. Rev. Lett.*, 2016.
T. Baker et. al., *Phys. Rev. Lett.*, 2017.

Speed of gravitational waves

$$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$$

$$G_{4,X} = G_{5,\pi} = G_{5,X} = 0,$$

$$\mathcal{L}_5 \propto G_{\mu\nu} \nabla^\mu \nabla^\nu \pi,$$

G. D. Moore et. al.,*J. High Energy Phys.*, 2001.
B. P. Abbott et al.,*Phys. Rev. Lett.*, 2016.
T. Baker et. al.,*Phys. Rev. Lett.*, 2017.

Speed of gravitational waves

$$\left(X \equiv -\frac{1}{2}\nabla_\mu \pi \nabla^\mu \pi\right)$$

$$S = \int \left[\sum_{N=2}^{4} \mathcal{L}_{N,\pi}^{Gal} \right] \sqrt{-g}\, \mathrm{d}^4 x,$$

$$\mathcal{L}_{2,\pi}^{\mathrm{Gal}} \equiv G_2(\pi, X)\,,$$

$$\mathcal{L}_{3,\pi}^{\mathrm{Gal}} \equiv G_3(\pi, X)\, \Box\pi\,,$$

$$\mathcal{L}_{4,\pi}^{\mathrm{Gal}} \equiv G_4(\pi) R.$$

T. Baker et. al., *Phys. Rev. Lett.*, 2017.
J. Sakstein et. al., *Phys. Rev. Lett.*, 2017.
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Finally...

$$\cdot f(R)$$

- Vector Galileons

- Multifields

J. Wang, *Phys. Rev. Lett.*, 2012.
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E. Allys, *Phys. Rev.*, 2017.

¡Thanks a lot!