Constraining the Generalized SU(2) Proca Theory at minimal cost

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Outline

I. Constraints on a theory of gravity:
   - LIGO/VIRGO observations and many other tests
   - Theoretical considerations: ghost and gradient type instabilities, causality

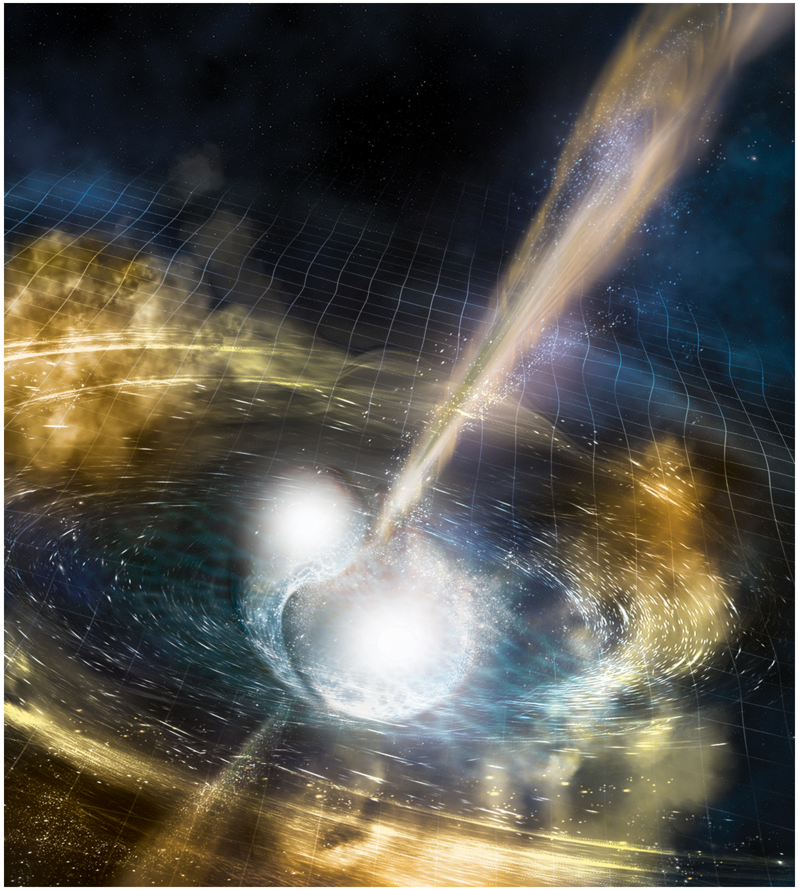
II. Constraining the generalized SU(2) Proca theory

III. Analytical and numerical results

IV. General conclusions and perspectives

LIGO/VIRGO observations

Strong constraints on gravity from GW170817 and GRB 170817A
LIGO Scientific and Virgo and Fermi-GBM and INTEGRAL Collaborations 2017



$$C_T^2 \equiv 1 + \alpha_T,$$

$$|\alpha_T| \lesssim 1 \times 10^{-15}.$$

E. Bellini and I. Sawicki, JCAP 1407, 050 (2014)

At fundamental level

Building up a theory with ghost-free fields demands positive-define kinetic matrix.

Building up a theory with ghadient-free fields demands positive speed propagation

$$\vec{x}^T = \vec{x}_0^T e^{i(wt-kz)}, \; w = c_t k/a.$$

The generalized SU(2) Proca theory

The all possible quartic Lagrangian terms of the theory can be properly rewritten as

$$
\begin{aligned}
\mathcal{L}_4^1 =& \frac{1}{4}(A_b \cdot A^b)[S_\mu^{\mu a} S_{\nu a}^{\nu} - S_\nu^{\mu a} S_{\mu a}^{\nu} + A_a \cdot A^a R] \\
& + \frac{1}{2}(A_a \cdot A_b)[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + 2A^a \cdot A^b R], \\
\mathcal{L}_4^2 =& \frac{1}{4}(A_b \cdot A_b)[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + A^a \cdot A^b R] \\
& + \frac{1}{2}(A^{\mu a} A^{\nu b})[S_{\mu a}^{\rho} S_{\nu \rho b} - S_{\nu a}^{\rho} S_{\mu \rho b} - A_a^\rho A_b^\sigma R_{\mu\nu\rho\sigma} \\
& - (\nabla^\rho A_{\mu a})(\nabla_\rho A_{\nu b}) + (\nabla^\rho A_{\nu a})(\nabla_\rho A_{\mu b})], \\
\mathcal{L}_4^3 =& \tilde{G}_{\mu\sigma}^{b} A_\alpha^\mu A_{\nu b} S^{\nu\sigma a},
\end{aligned}
$$

$$
\mathcal{L}_4^{\mathrm{curv}} = L_{\mu\nu\rho\sigma} A^{\mu a} A^{\nu b} A_a^\rho A_b^\sigma
$$

E. Allys, P. Peter, and Y. Rodríguez, “Generalized SU(2) Proca Theory,” Phys. Rev. D 94, 084041 (2016)

Tensor definitions

The symmetric version and the Abelian version of the gauge field strength tensor

$$S^a_{\mu\nu} \equiv \nabla_\mu A^a_\nu + \nabla_\nu A^a_\mu$$

$$G^a_{\mu\nu} \equiv \nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu$$

$$F^a_{\mu\nu} = \nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu + g\epsilon^a_{bc} A^b_\mu A^c_\nu$$

The Hodge dual and the double dual Riemann tensor

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma a}$$

$$L^{\alpha\beta\gamma\delta} = -\frac{1}{2}\epsilon^{\alpha\beta\mu\nu}\epsilon^{\gamma\delta\rho\sigma} R_{\mu\nu\rho\sigma}$$

General classification of sub-class of SU(2) models

Sub-class of SU(2) Models	Pieces
1	$\alpha L_4^1 + \kappa L_4^2$
2	$\alpha L_4^1 + \kappa L_4^2 + \lambda L_4^3$
3	$\alpha L_4^1 + \kappa L_4^2 + \theta L_4^{cur}$

TABLE I. Sub-class of SU(2) models based on non-Abelian vector fields that may account for early inflation and the late-time accelerated period.

$$\mathcal{S} = \int d^4x \sqrt{-\det(\mathrm{g}_{\mu\nu})}(\mathcal{L}_{\mathrm{E-H}} + \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_4 + \theta\mathcal{L}_4^{\mathrm{curv}}),$$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}, \quad \mathcal{L}_4 \equiv \alpha\mathcal{L}_4^1 + \kappa\mathcal{L}_4^2 + \lambda\mathcal{L}_4^3.$$

Parameterizing: $b \equiv \frac{\kappa}{\alpha}\,; c \equiv \frac{\lambda}{\alpha}\,; d \equiv \frac{\theta}{\alpha}.$

Tensor perturbations

We consider the tensor perturbations in both the metric tensor and the gauge field

$$\delta g_{ij} = a^2(t) h_{ij},$$
$$\delta A_i^a = a(t) t_i^a,$$

where both satisfy the transverse and traceless conditions:

$$\partial^i h_{ij} = h_i^i = 0 \text{ and } \delta_a^i \partial_i t_j^a = \delta_a^i t_i^a = 0.$$

The tensor sector is expressed in terms of 4 dynamical modes

$$\delta g_{11} = -\,\delta g_{22} = a^2 h_+,\ \ \delta g_{12} = a^2 h_\times,$$
$$\delta A^1_\mu = a(0, t_+, t_\times, 0, 0),\ \ \delta A^2_\mu = a(0, t_\times, -t_+, 0, 0).$$

Speed of gravitational waves

We construct the second order action for the metric tensor perturbations

$$S_T^2 = \frac{1}{2}\int d^3x\; dt\; a^3 M_*^2 \left[\dot{h}_{+,\times}^2 - \frac{C_T^2}{a^2}(\partial h_{+,\times})^2\right],$$

where the effective Planck mass and tensor speed excess are read off

$$M_*^2 = \left(1 + \frac{\eta}{M_p^4}\frac{\phi^4}{4}\right) M_p^2, \qquad C_T^2 \equiv 1 + \alpha_T,$$

$$\Rightarrow \quad \alpha_T = \frac{(\phi^4/4M_p^4)(\gamma - \eta)}{1 + (\phi^4\eta/4M_p^4)},$$

$$\eta = 61\alpha + 19\kappa - 16\theta,\; \gamma = 81\alpha - \kappa.$$

Speed of gravitational waves

In general, we assume: $-\infty < \phi < \infty$ and $\alpha \in (0, \infty)$

Model	Constraints
1	$b = 1$
2	$b = 1,\ c =?$
3	$b \leq 81,\ d = \frac{1}{4}(-5 + 5b)$

TABLE II. Type-I sub-class of SU(2) models that must satisfy the observational constraint $\alpha_T = 0$.

Model	Constraints
1	$-\frac{61}{19} \leq b \leq 81$
2	$-\frac{61}{19} \leq b \leq 81,\ c =?$
3	$b \leq 81,\ d \leq \frac{1}{16}(61 + 19b)$

TABLE III. Type-II sub-class of SU(2) models that must avoid exponential growth, i.e. $\alpha_T > -1$.

Ghost-free conditions

We start by writing out the quadratic kinetic action, containing the products of first-order time derivatives for the dynamical modes

$$\dot{\vec{x}}^T = (M_p h_+, t_+, M_p h_\times, t_\times),$$

$$S_K^2 = \int d^3x \; dt \; a^3 \dot{\vec{x}}^T K \; \dot{\vec{x}},$$

where K is a 4X4 symmetric matrix whose dimension is determined by the number of degrees of freedom

$$K_{11} = K_{13} = \frac{1}{4} + \left(\frac{61\alpha + 19\kappa}{8} - 2\theta\right)\phi^4$$

$$K_{22} = K_{44} = 1 + (-5\alpha + \kappa + 2\lambda)\phi^2$$

$$K_{12} = K_{21} = \frac{1}{2}(10\alpha - -3\kappa + 8\theta - 2\lambda)\phi^3,$$

$$K_{34} = K_{43} = K_{12}.$$

Ghost-free conditions

The eigenvalues of the kinetic matrix result in two degenerate solutions

$$\textit{Model 1}:$$

$$\lambda_{\pm} = \frac{10 + 8\alpha(-5+b)\phi^2 + \alpha(61+19b)\phi^4 \pm \sqrt{\Lambda_1}}{16}.$$

$$\textit{Model 2}:$$

$$\lambda_{\pm} = \frac{10 + 8\alpha(-5+b+2c)\phi^2 + \alpha(61+19b)\phi^4 \pm \sqrt{\Lambda_2}}{16}.$$

$$\textit{Model 3}:$$

$$\lambda_{\pm} = \frac{10 + 8\alpha(-5+b)\phi^2 + \alpha(61+19b-16d)\phi^4 \pm \sqrt{\Lambda_3}}{16}.$$

$$\begin{aligned}
\Lambda_1 =& 36 + \alpha\phi^2(96(-5+b) + 4(16\alpha(-5+b)^2 - 3(61+19b))\phi^2 \\
&+ 16\alpha(705 + b(-206+17b))\phi^4 + \alpha(61+19b)^2\phi^6), \\
\Lambda_2 =& (10 + 8\alpha(-5+b+2c)\phi^2 + \alpha(61+19b)\phi^4)^2 \\
&- 32(2 + \alpha\phi^2(2(-5+b+2c) + (61+19b)\phi^2 \\
&+ \alpha(-505 + b^2 + 202c - 8c^2 + 2b(43+7c))\phi^4)), \\
\Lambda_3 =& (10 + 8\alpha(-5+b)\phi^2 + \alpha(61+19b-16d)\phi^4)^2 \\
&- 32(2 + \alpha\phi^2(2(-5+b) + (61+19b-16d)\phi^2 \\
&+ \alpha(-505 + b^2 - 16d(15+8d) + b(86+80d))\phi^4)).
\end{aligned}$$

Ghost-free conditions

The ghost-free conditions for the sub-class of SU(2) models are

Model 1 :

$$\alpha > 0,\, b = 1,\; -\eta_1 < \phi < \eta_1,$$

Model 2 :

$$\alpha > 0,\, b = 1,\; \frac{27}{2} - \sqrt{130} \leq c \leq \frac{27}{2} + \sqrt{130},$$

Model 3 :

$$\alpha > 0,\, b \leq 81,\; d = \frac{1}{4}(-5 + 5b),\; -\eta_3 < \phi < \eta_3,$$

Model 1 :

$$\alpha > 0,\; -43 + \sqrt{2354} \leq b \leq 81.$$

Model 2 :

$$\alpha > 0,\, 1 \leq b \leq 81,\; \frac{1}{8}(101 + 7b) - \frac{1}{8}\sqrt{6161 + 2102b + 57b^2} \leq c \leq \frac{1}{8}(101 + 7b) + \frac{1}{8}\sqrt{6161 + 2102b + 57b^2}.$$

Model 3 :

$$\alpha \gtrsim 10^{-5},\; 5 < b \leq 81,\; \frac{5}{16}(-3 + b) - \frac{1}{16}\sqrt{-785 + 22b + 27b^2} \leq d \leq \frac{5}{16}(-3 + b) + \frac{1}{16}\sqrt{-785 + 22b + 27b^2},$$

Gradient-free Conditions

The action containing first order spatial derivatives is expressed in the form

$$S_L^2 = \int d^3x dt \; (-a \; \partial \vec{x}^T L \; \partial \vec{x}),$$

$$L_{11} = L_{13} = \frac{1}{4} + \left(\frac{81\alpha - \kappa}{8}\right)\phi^4$$

$$L_{22} = L_{44} = 1 + (-5\alpha + \kappa + 2\lambda)\phi^2$$

$$L_{12} = L_{21} = \frac{(10\alpha + \kappa - 4\lambda)}{2}\phi^3,$$

The squared propagation speed follows the dispersion relation

$$\det(\mathrm{c}_s^2 K - L) = 0.$$

Gradient-free Conditions

The squared propagation speeds for all the sub-class of SU(2) models are

Model 1 :

$$c_{s\pm}^2 =(2+\alpha\phi^2(2(-5+b)+(71+9b)\phi^2+3\alpha(-185+b(22+5b))\phi^4)\pm 2(\alpha^2\phi^6(1+\alpha(-5+b)\phi^2)(16b^2+25(-1+b)^2\phi^2 + \alpha(-125+b(-125+b(833+57b)))\phi^4))^{1/2})/(2+\alpha\phi^2(2(-5+b)+(61+19b)\phi^2+\alpha(-505+b(86+b))\phi^4)).$$

Model 2 :

$$c_{s\pm}^2 =(2+\alpha\phi^2(2(-5+b+2c)+(71+9b)\phi^2+\alpha(-555+3b(22+5b)+262c-2bc-16c^2)\phi^4)\pm 2(\alpha^2\phi^6(1+\alpha(-5 + b+2c)\phi^2)(4(-2b+c)^2+25(-1+b)^2\phi^2+\alpha(b(-125+b(833+57b))-2b(384+61b)c+2(41+39b)c^2 + 125(-1+2c))\phi^4))^{1/2})/(2+\alpha\phi^2(2(-5+b+2c)+(61+19b)\phi^2+\alpha(-505+b^2+202c-8c^2+2b(43+7c))\phi^4)).$$

Model 3 :

$$c_{s\pm}^2 =(2+\alpha\phi^2(2(-5+b)+(71+9b-8d)\phi^2+3\alpha(-185+22b+5b^2-8(5+b)d)\phi^4)\pm 2(\alpha^2\phi^6(1+\alpha(-5 + b)\phi^2)(16(b-2d)^2+(5-5b+4d)^2\phi^2+\alpha(b(-125+b(833+57b))-8b(424+15b)d+16(197+3b)d^2+ 25(-5+24d))\phi^4))^{1/2})/(2+\alpha\phi^2(2(-5+b)+(61+19b-16d)\phi^2+\alpha(-505+b^2-16d(15+8d)+b(86+80d))\phi^4)).$$

Gradient-free Conditions

Model 1:

$$\alpha > 0,\ b = 1,\ -\psi_1 < \phi < \psi_1,$$

$$\psi_1 = \sqrt{\frac{2^{1/3}\alpha^2(3200-6744\alpha)+80\alpha(128000\alpha^3+1727307\alpha^4+843\sqrt{3\Omega_1})^{1/3}+(256000\alpha^3+3454614\alpha^4+1686\sqrt{3\Omega_1})^{2/3}}{1686\alpha^2(128000\alpha^3+1727307\alpha^4+843\sqrt{3}\sqrt{\Omega_1})^{1/3}}}.$$

Model 2:

$$\alpha > 0,\ b = 1,\ \frac{27}{2} - \sqrt{130} \le c \le \frac{27}{2} + \sqrt{130}.$$

Model 3:

$$\alpha > 0,\ \frac{5}{3} < b \le 81,\ -\psi_3 < \phi < \psi_3,$$

$$\psi_3 = \sqrt{\frac{\alpha^2(-81+b)^2+6\alpha^3(-5+b)(405+b(-86+99b))-\alpha(-81+b)(\Omega_3)^{1/3}+(\Omega_3)^{2/3}}{3\alpha^2(405+b(-86+99b))(\Omega_3)^{1/3}}}.$$

Model 1:

$$\alpha > 0,\ -43+\sqrt{2354} \le b \le 81,\ -\Phi_1 < \phi < \Phi_1$$

$$\Phi_1 = \sqrt{\frac{\alpha^2(-81+b)^2+6\alpha^3(-5+b)(605+b(-46+3b))-\alpha(-81+b)(\Delta_1+3\sqrt{6\Theta_1})^{1/3}+(\Delta_1+3\sqrt{6\Theta_1})^{2/3}}{3\alpha^2(605+b(-46+3b))(\Delta_1+3\sqrt{6\Theta_1})^{1/3}}},$$

Model 2:

$$\alpha > 0,\ 1.04795 \lesssim b \le 81,\ \frac{(805001+35000b)}{160000} - \frac{\sqrt{\Theta_2}}{160000} < c < \frac{(805001+35000b)}{160000} + \frac{\sqrt{\Theta_2}}{160000},$$

Model 3:

$$0 < \alpha < 1.40261\times 10^{-6},\ b \gtrsim 40,\ \frac{5}{16}(-3+b) - \frac{1}{16}\sqrt{-785+22b+27b^2} \le d \le \frac{5}{16}(-3+b) + \frac{1}{16}\sqrt{-785+22b+27b^2},$$

Particular setups: type-I sub-class of models

Model 2:

$$b = 1 \Leftrightarrow \alpha_T = 0,$$

$$c_{s\pm}^2 = \frac{-1 \pm 2\sqrt{\chi} + \alpha\phi^2(\omega + \alpha(237 + 2c(-65 + 4c))\phi^4)}{-1 + \alpha\phi^2(\omega + \alpha(209 + 4(-27 + c)c)\phi^4)},$$

$$\chi^{1/2} \equiv \alpha^2(-2 + c)^2\phi^6(1 + 2\alpha(-2 + c)\phi^2)(1 + 40\alpha\phi^4),$$

$$c = 2 \Rightarrow c_{s\pm}^2 = 1.$$

Model 3:

$$d = 1/4(-5 + 5b) \Leftrightarrow \alpha_T = 0,$$

$$c_{s\pm}^2 = \frac{-2 \pm 2\sqrt{2\zeta} + \tau + 3\alpha(135 + b(18 + 5b))\phi^4)}{-2 + \tau + \alpha(405 + b(-86 + 99b))\phi^4)},$$

$$\zeta = -\alpha^2(5 - 3b)^2\phi^6(1 + \alpha(-5 + b)\phi^2)(-2 + \alpha(-81 + b)\phi^4),$$

$$b = 5/3 \Rightarrow c_{s\pm}^2 = 1.$$

Asymptotic limit for Type-II sub-class of models

$$\phi \to \pm\infty \Rightarrow \lambda_\pm = (\eta \pm |\eta|):$$

Model 1 :

$$\lambda_\pm = \alpha(61+19b) \pm \sqrt{\alpha^2(61+19b)^2},\ c_{s\pm}^2 = \frac{3\alpha^2(-185+22b+5b^2) \pm 2\sqrt{\Xi_1}}{\alpha^2(-505+86b+b^2)}.$$

Model 2 :

$$\lambda_\pm = \alpha(61+19b) \pm \sqrt{\alpha^2(61+19b)^2},\ c_{s\pm}^2 = \frac{\alpha^2(-705+31b^2-2b(-23+c)+362c-32c^2) \pm \sqrt{\Xi_2}}{2\alpha^2(-505+b^2+202c-8c^2+2b(43+7c))}.$$

Model 3 :

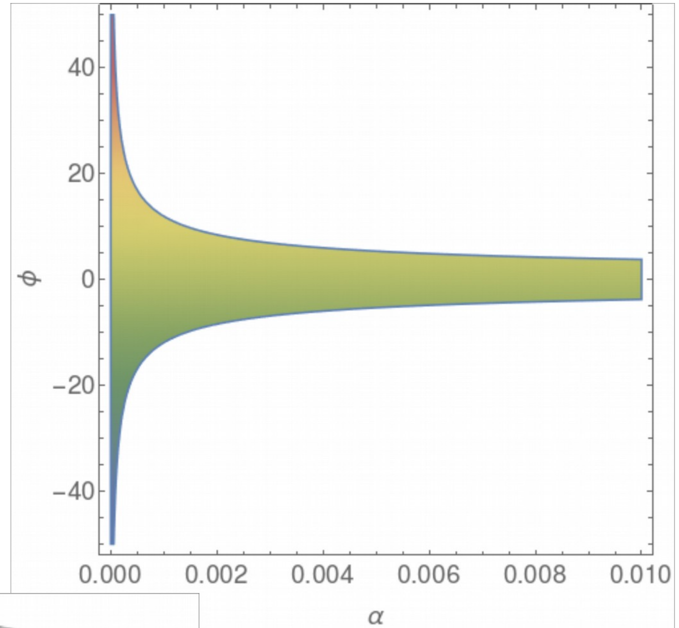
$$\lambda_\pm = \alpha(61+19b-16d) \pm \sqrt{\alpha^2(61+19b-16d)^2},\ c_{s\pm}^2 = \frac{3\alpha^2(5b^2+b(22-8d)-5(37+8d)) \pm 2\sqrt{\Xi_3}}{\alpha^2(-505+b^2-240d-128d^2+b(86+80d))},$$

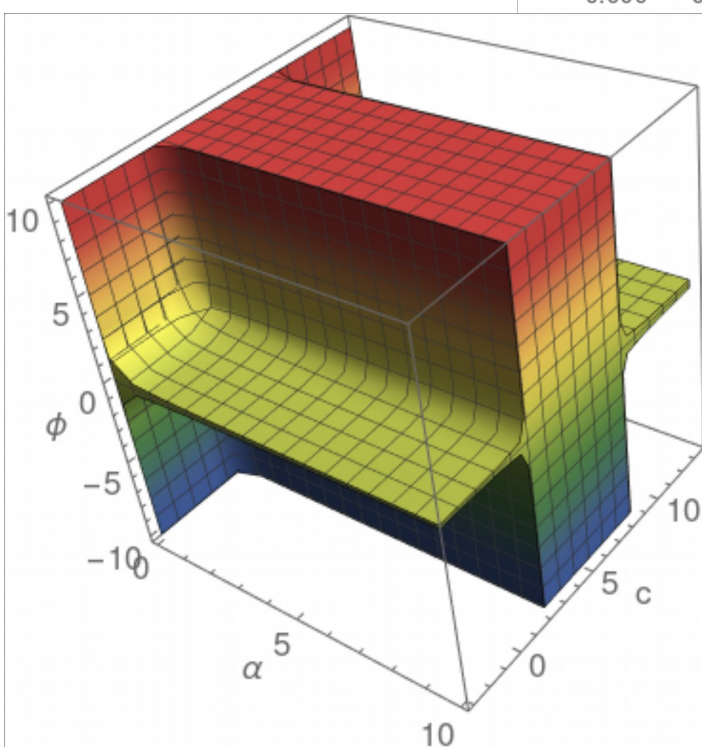
$$\textit{Model 2}: b=1, c=2$$
$$\textit{Model 3}: b=5/2, d=5/6$$

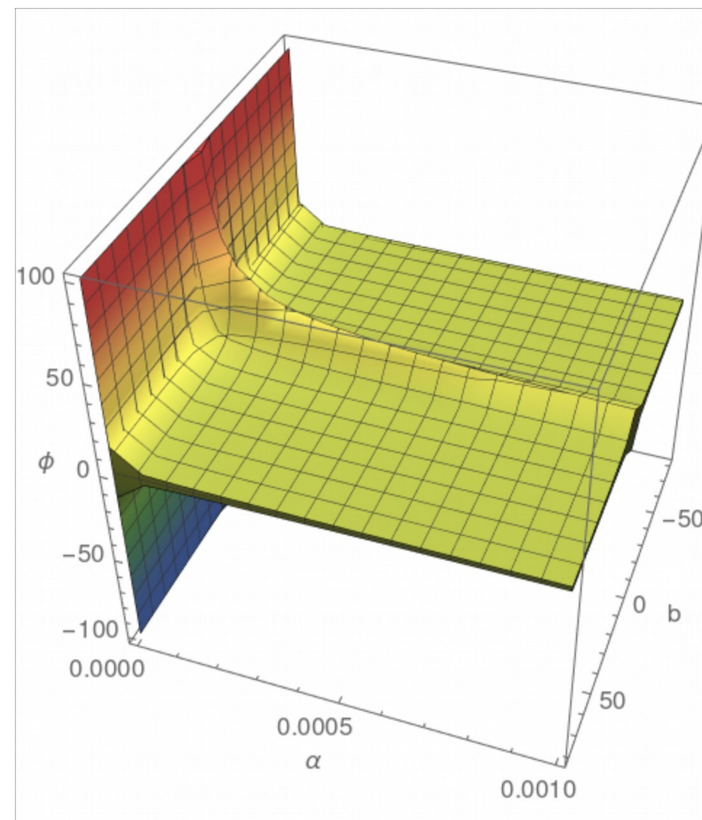
→

$$c_{s\pm}^2 = 1$$

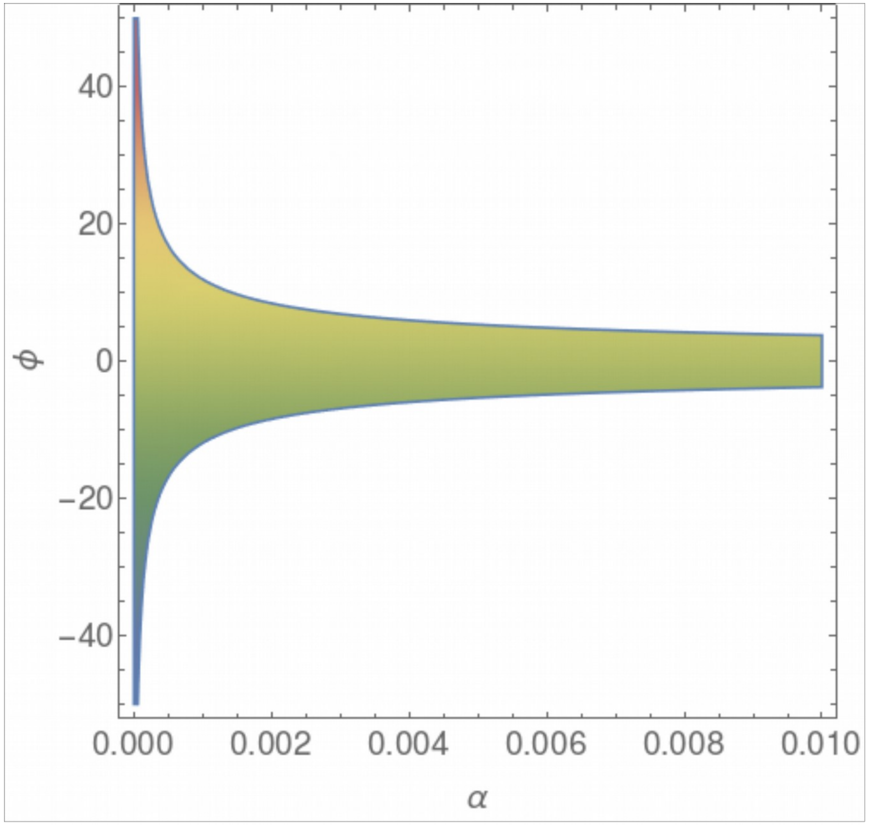
Full Parameter Space for type I models

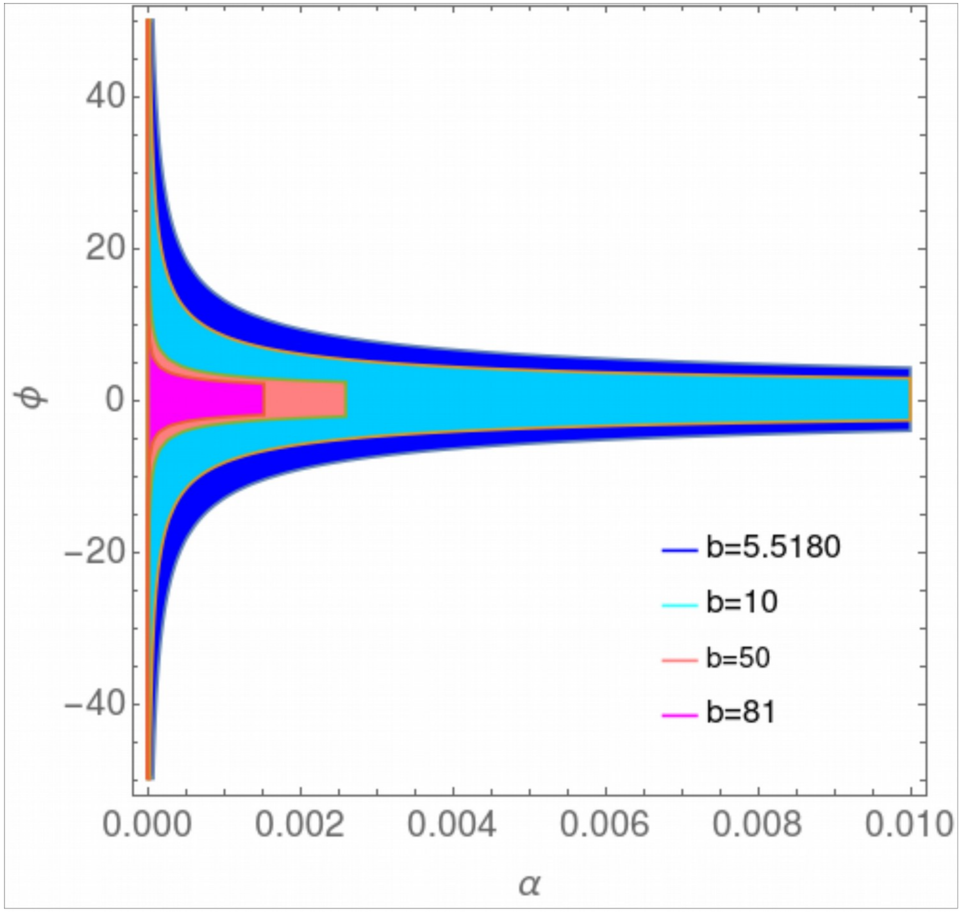




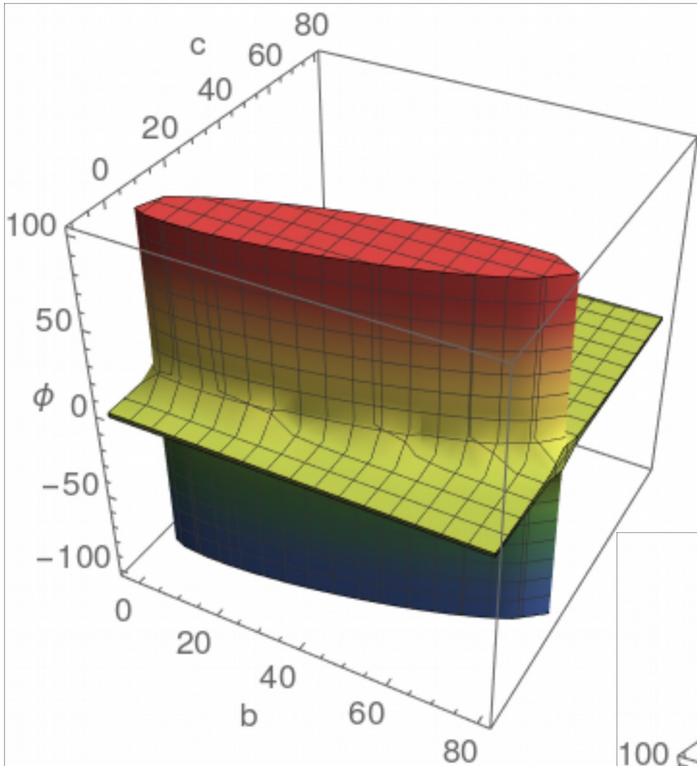


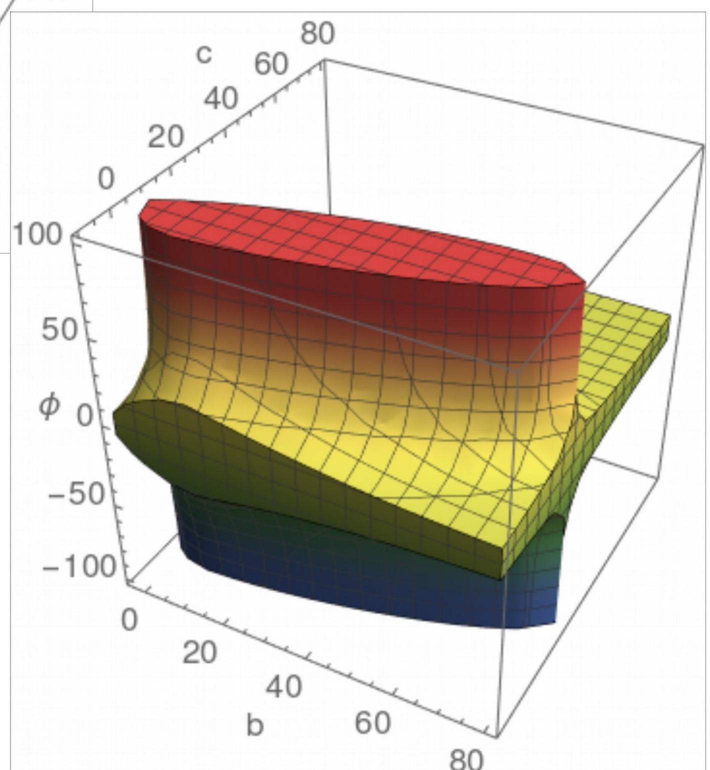
Full Parameter Space for type II: model 1

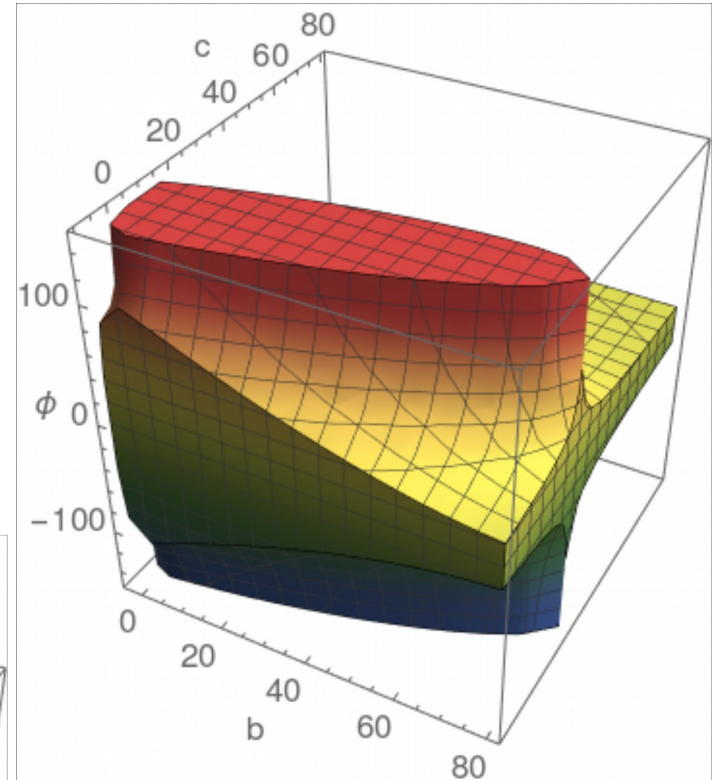




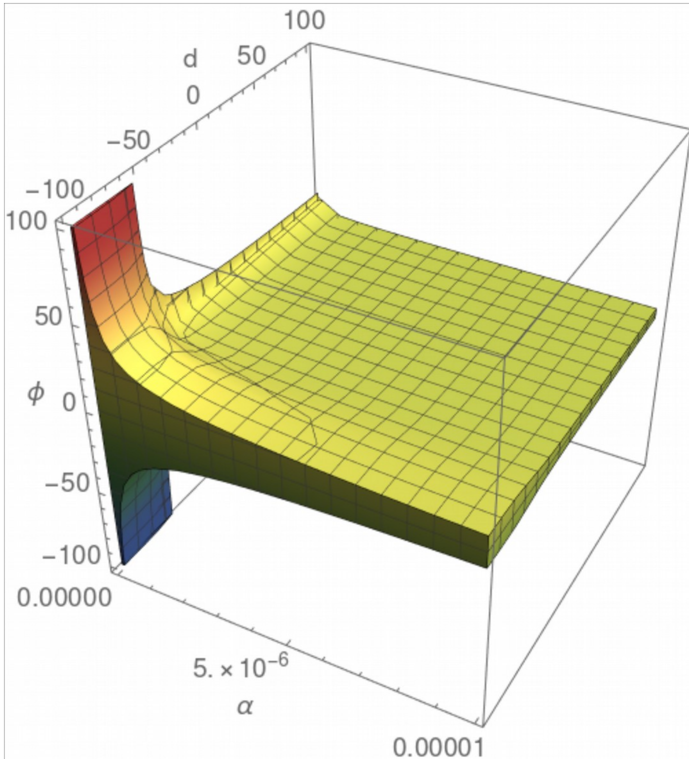
Full Parameter Space for type II: model 2

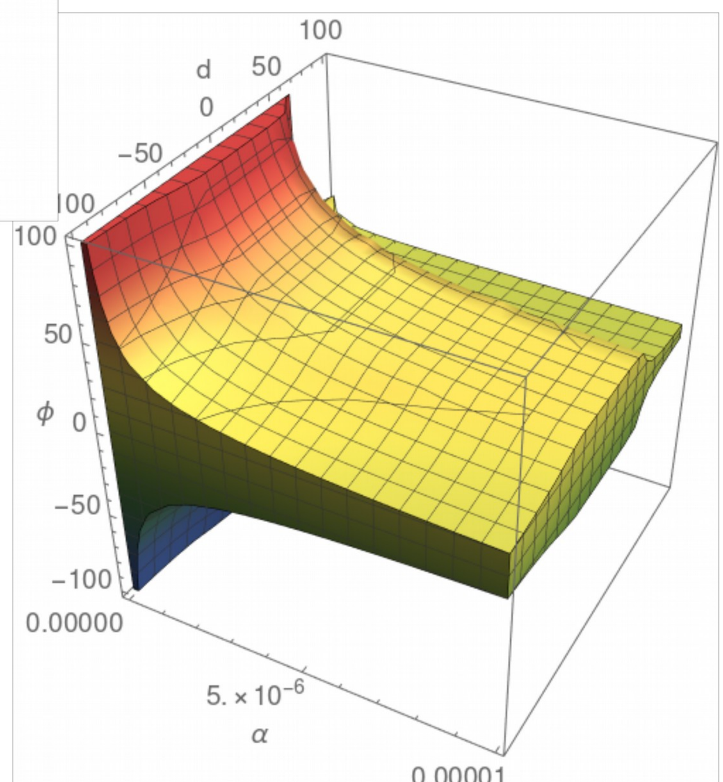


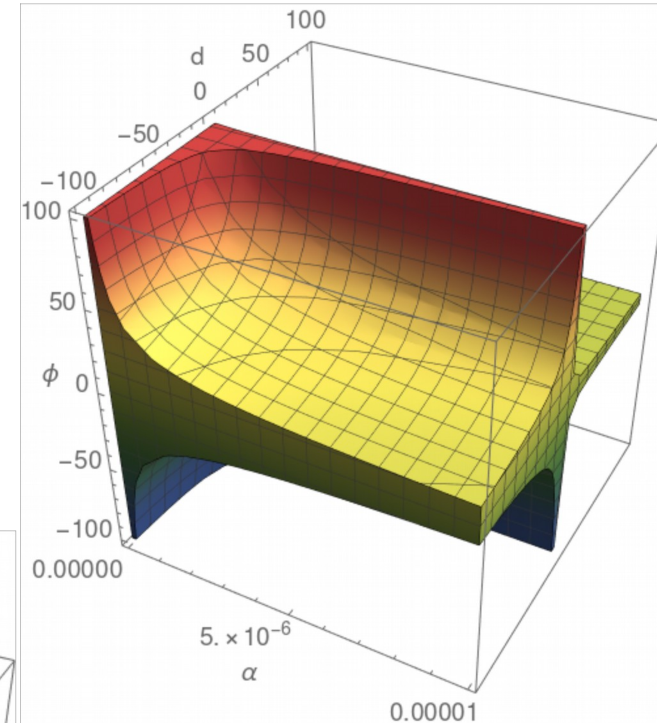




Full Parameter Space for type II: model 3







Conclusions and perpectives

- we evaluated the conditions under which the speed of gravitational waves is consistent with recent LIGO/Virgo observation

- we have sought for a suitable parameter space for a broad sub-class of SU(2) Proca models built by different combinations of pieces of the SU(2) Lagrangian.

- There exists a suitable parameter space independent of the background dynamics.

- It remains for: study the stabilitiy for the other sectors. Ckeck the causality arguments. The former issue will be done however for a particular model.

- GR makes things easier and quite but still with some missing puzzles.