Slow-Roll Inflation in Scalar-Tensor Models

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1 The model and background equations

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(\phi)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + F_1(\phi)G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - F_2(\phi)\mathcal{G}\right] \quad (1.1)$$

where $G_{\mu\nu}$ is the Einstein's tensor, $\mathcal{G}$ is the GB 4-dimensional invariant given by

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} \quad (1.2)$$

$$F(\phi) = \frac{1}{\kappa^2} + f(\phi), \quad (1.3)$$

and $\kappa^2 = M_p^{-2} = 8\pi G$.

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \quad (1.4)$$

one finds the following equations

$$3H^2F\left(1 - \frac{3F_1\dot{\phi}^2}{F} - \frac{8H\dot{F}_2}{F}\right) = \frac{1}{2}\dot{\phi}^2 + V - 3H\dot{F} \quad (1.5)$$

$$\begin{aligned} 2\dot{H}F\left(1 - \frac{F_1\dot{\phi}^2}{F} - \frac{8H\dot{F}_2}{F}\right) &= -\dot{\phi}^2 - \ddot{F} + H\dot{F} + 8H^2\ddot{F}_2 - 8H^3\dot{F}_2 \\ &\quad - 6H^2F_1\dot{\phi}^2 + 4HF_1\dot{\phi}\ddot{\phi} + 2H\dot{F}_1\dot{\phi}^2 \end{aligned} \quad (1.6)$$

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$$
\begin{aligned}
&\ddot{\phi}+3H\dot{\phi}+V'-3F'\left(2H^2+\dot{H}\right)+24H^2\left(H^2+\dot{H}\right)F_2'+18H^3F_1\dot{\phi}\\
&\quad+12H\dot{H}F_1\dot{\phi}+6H^2F_1\ddot{\phi}+3H^2F_1'\dot{\phi}^2=0
\end{aligned} \tag{1.7}
$$

$$
\epsilon_0=-\frac{\dot{H}}{H^2},\quad \epsilon_1=\frac{\dot{\epsilon}_0}{H\epsilon_0} \tag{1.8}
$$

$$
\ell_0=\frac{\dot{F}}{HF},\quad \ell_1=\frac{\dot{\ell}_0}{H\ell_0} \tag{1.9}
$$

$$
k_0=\frac{3F_1\dot{\phi}^2}{F},\quad k_1=\frac{\dot{k}_0}{Hk_0} \tag{1.10}
$$

$$
\Delta_0=\frac{8H\dot{F}_2}{F},\quad \Delta_1=\frac{\dot{\Delta}_0}{H\Delta_0} \tag{1.11}
$$

$$
\begin{aligned}
V=&H^2F\Big[3-\frac{5}{2}\Delta_0-2k_0-\epsilon_0+\frac{5}{2}\ell_0+\frac{1}{2}\ell_0\left(\ell_1-\epsilon_0+\ell_0\right)\\
&-\frac{1}{2}\Delta_0\left(\Delta_1-\epsilon_0+\ell_0\right)-\frac{1}{3}k_0\left(k_1+\ell_0-\epsilon_0\right)\Big]
\end{aligned} \tag{1.12}
$$

$$
\begin{aligned}
\dot{\phi}^2=&H^2F\Big[2\epsilon_0+\ell_0-\Delta_0-2k_0+\Delta_0\left(\Delta_1-\epsilon_0+\ell_0\right)-\\
&\ell_0\left(\ell_1-\epsilon_0+\ell_0\right)+\frac{2}{3}k_0\left(k_1+\ell_0-\epsilon_0\right)\Big]
\end{aligned} \tag{1.13}
$$

where we used

$$
\ddot{F}=H^2F\ell_0\left(\ell_1-\epsilon_0+\ell_0\right),\quad \ddot{F}_2=\frac{F\Delta_0}{8}\left(\Delta_1+\epsilon_0+\ell_0\right) \tag{1.14}
$$

$$
Y=\frac{\dot{\phi}^2}{H^2F} \tag{1.15}
$$

where it follows that $Y=\mathcal{O}(\varepsilon)$. Under the slow-roll conditions $\ddot{\phi}<<3H\dot{\phi}$ and $\ell_i, k_i, \Delta_i<<1$, it follows from (1.5)-(1.7)

$$
3H^2F\simeq V, \tag{1.16}
$$

$$
2\dot{H}F\simeq-\dot{\phi}^2+H\dot{F}-6H^2F_1\dot{\phi}^2-8H^3\dot{F}_2, \tag{1.17}
$$

$$
3H\dot{\phi}+V'-6H^2F'+18H^3F_1\dot{\phi}+24H^4F_2'\simeq 0 \tag{1.18}
$$

2 Quadratic action for the scalar and tensor perturbations

Scalar Perturbations.

$$\delta S_s^2 = \int dt d^3x a^3 \left[\mathcal{G}_s\dot{\xi}^2 - \frac{\mathcal{F}_s}{a^2}\left(\nabla\xi\right)^2\right] \tag{2.1}$$

where

$$\mathcal{G}_s = \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T \tag{2.2}$$

$$\mathcal{F}_s = \frac{1}{a}\frac{d}{dt}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T \tag{2.3}$$

with

$$\mathcal{G}_T = F - F_1\dot{\phi}^2 - 8H\dot{F}_2. \tag{2.4}$$

$$\mathcal{F}_T = F + F_1\dot{\phi}^2 - 8\ddot{F}_2 \tag{2.5}$$

$$\Theta = FH + \frac{1}{2}\dot{F} - 3HF_1\dot{\phi}^2 - 12H^2\dot{F}_2 \tag{2.6}$$

$$\Sigma = -3FH^2 - 3H\dot{F} + \frac{1}{2}\dot{\phi}^2 + 18H^2F_1\dot{\phi}^2 + 48H^3\dot{F}_2 \tag{2.7}$$

The sound speed

$$c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} \tag{2.8}$$

$$\mathcal{G}_T = F\left(1 - \frac{1}{3}k_0 - \Delta_0\right) \tag{2.9}$$

$$\mathcal{F}_T = F\left(1 + \frac{1}{3}k_0 - \Delta_0\left(\Delta_1 + \epsilon_0 + \ell_0\right)\right) \tag{2.10}$$

$$\Theta = FH\left(1 + \frac{1}{2}\ell_0 - k_0 - \frac{3}{2}\Delta_0\right) \tag{2.11}$$

$$\begin{aligned}\Sigma = -\,FH^2\Big[&3 - \epsilon_0 + \frac{5}{2}\ell_0 - 5k_0 - \frac{11}{2}\Delta_0 + \frac{1}{2}\ell_0\left(\ell_1 - \epsilon_0 + \ell_0\right)\\ &- \frac{1}{3}k_0\left(k_1 - \epsilon_0 + \ell_0\right) - \frac{1}{2}\Delta_0\left(\Delta_1 - \epsilon_0 + \ell_0\right)\Big]\end{aligned} \tag{2.12}$$

$$\mathcal{G}_S = \frac{F\left(\frac{1}{2}Y + k_0 + \frac{3}{4}W^2(1 - \Delta_0 - \frac{1}{3}k_0)\right)}{\left(1 + \frac{1}{2}W\right)^2} \tag{2.13}$$

$$c_S^2 = 1 + \frac{W^2\left(\frac{1}{2}\Delta_0(\Delta_1+\varepsilon_0+l_0-1)-\frac{1}{3}k_0\right)+W\left(\frac{2}{3}k_0\left(2-k_1-l_0\right)+2\Delta_0\varepsilon_0\right)-\frac{4}{3}k_0\varepsilon_0}{Y+2k_0+\frac{3}{2}W^2(1-\Delta_0-\frac{1}{3}k_0)} \tag{2.14}$$

where

$$W = \frac{\ell_0-\Delta_0-\frac{4}{3}k_0}{1-\Delta_0-\frac{1}{3}k_0} \tag{2.15}$$

First order terms in slow-roll parameters

$$G_S = F\left(\varepsilon_0+\frac{1}{2}l_0-\frac{1}{2}\Delta_0\right) \tag{2.16}$$

$$c_S^2 = 1+\frac{\frac{4}{3}k_0\left(l_0-\Delta_0-\frac{4}{3}k_0\right)-\frac{4}{3}k_0\varepsilon_0}{2\varepsilon_0+l_0-\Delta_0} \tag{2.17}$$

$$\delta S_s^2 = \frac{1}{2}\int d\tau_s d^3x\left[\frac{1}{2}(\tilde{U}')^2-D_i\tilde{U}D^i\tilde{U}+\frac{\tilde{z}''}{\tilde{z}}\tilde{U}^2\right] \tag{2.18}$$

$$\frac{\tilde{z}''}{\tilde{z}} = \frac{a^2H^2}{c_S^2}\Big[2-\epsilon_0+\frac{3}{2}\ell_0+\frac{3}{2}\frac{2\epsilon_0\epsilon_1+\ell_0\ell_1-\Delta_0\Delta_1}{2\epsilon_0+\ell_0-\Delta_0}\Big]. \tag{2.19}$$

$$\tilde{U}_k''+k^2\tilde{U}_k+\frac{1}{\tau_s^2}\left(\mu_s^2-\frac{1}{4}\right)\tilde{U}_k=0 \tag{2.20}$$

where

$$\mu_s^2 = \frac{9}{4}\left[1+\frac{4}{3}\epsilon_0+\frac{2}{3}\ell_0+\frac{2}{3}\frac{2\epsilon_0\epsilon_1+\ell_0\ell_1-\Delta_0\Delta_1}{2\epsilon_0+\ell_0-\Delta_0}\right] \tag{2.21}$$

$$\tilde{U}_k = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{2}(\mu_s - \frac{1}{2})} 2^{\mu_s - \frac{3}{2}} \frac{\Gamma(\mu_s)}{\Gamma(3/2)} \sqrt{-\tau_s} (-k\tau_s)^{-\mu_s}. \tag{2.22}$$

Tthe power spectra

$$\frac{\tilde{z}'}{\tilde{z}} = -\frac{1}{(1-\epsilon_0)\tau_s} \left[1 + \frac{1}{2}\ell_0 + \frac{1}{2} \frac{2\epsilon_0\epsilon_1 + \ell_0\ell_1 - \Delta_0\Delta_1}{2\epsilon_0 + \ell_0 - \Delta_0} \right] = -\frac{1}{\tau_s} \left(\mu_s - \frac{1}{2} \right). \tag{2.23}$$

$$\mu_s = \frac{3}{2} + \epsilon_0 + \frac{1}{2}\ell_0 + \frac{1}{2} \frac{2\epsilon_0\epsilon_1 + \ell_0\ell_1 - \Delta_0\Delta_1}{2\epsilon_0 + \ell_0 - \Delta_0} \tag{2.24}$$

Slowly varying slow-roll parameters

$$\tilde{z} \propto \tau_s^{\frac{1}{2} - \mu_s} \tag{2.25}$$

$$\xi_k = \frac{\tilde{U}_k}{\tilde{z}} \propto k^{-\mu_s} \tag{2.26}$$

$$P_\xi = \frac{k^3}{2\pi^2} |\xi_k|^2 \propto k^{3-2\mu_s} \tag{2.27}$$

$$n_s - 1 = \frac{d \ln P_\xi}{d \ln k} = 3 - 2\mu_s = -2\epsilon_0 - \ell_0 - \frac{2\epsilon_0\epsilon_1 + \ell_0\ell_1 - \Delta_0\Delta_1}{2\epsilon_0 + \ell_0 - \Delta_0} \tag{2.28}$$

Tensor perturbations.

The second order action for the tensor perturbations

$$\delta S_2 = \frac{1}{8} \int d^3x dt \mathcal{G}_T a^2 \left[\left(\dot{h}_{ij} \right)^2 - \frac{c_T^2}{a^2} \left(\nabla h_{ij} \right)^2 \right] \tag{2.29}$$

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \frac{3 + k_0 - 3\Delta_0\left(\Delta_1 + \epsilon_0 + \ell_0\right)}{3 - k_0 - 3\Delta_0}. \tag{2.30}$$

$$v''_{(k)ij} + \left(k^2 - \frac{z''_T}{z_T}\right) v_{(k)ij} = 0, \tag{2.31}$$

Power spectrum for tensor perturbations

$$P_T = \frac{k^3}{2\pi^2} |h_{ij}^{(k)}|^2 \tag{2.32}$$

Tensor spectral index

$$n_T = 3 - 2\mu_T = -2\epsilon_0 - \ell_0 \tag{2.33}$$

r

$$r = \frac{P_T(k)}{P_\xi(k)}. \tag{2.34}$$

$$P_\xi = A_S \frac{H^2}{(2\pi)^2} \frac{\mathcal{G}_S^{1/2}}{\mathcal{F}_S^{3/2}} \tag{2.35}$$

where

$$A_S = \frac{1}{2} 2^{2\mu_s - 3} \left| \frac{\Gamma(\mu_s)}{\Gamma(3/2)} \right|^2$$

$$P_T = 16 A_T \frac{H^2}{(2\pi)^2} \frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}} \tag{2.36}$$

where

$$A_T = \frac{1}{2} 2^{2\mu_T - 3} \left| \frac{\Gamma(\mu_T)}{\Gamma(3/2)} \right|^2.$$

$A_T/A_S \simeq 1$ at the limit $\epsilon_0, \ell_0, \Delta_0, ... << 1$,

$$r = 16\frac{\mathcal{G}_T^{1/2}\mathcal{F}_S^{3/2}}{\mathcal{G}_S^{1/2}\mathcal{F}_T^{3/2}} = 16\frac{c_S^3\mathcal{G}_S}{c_T^3\mathcal{G}_T} \tag{2.37}$$

$$r = 8\left(\frac{2\epsilon_0 + \ell_0 - \Delta_0}{1 - \frac{1}{3}k_0 - \Delta_0}\right) \simeq 8\left(2\epsilon_0 + \ell_0 - \Delta_0\right) \tag{2.38}$$

$$r = -8n_T, \tag{2.39}$$

with $n_T = -2\epsilon_0$. Deviation from the standard consistency relation

$$r = -8n_T + \delta r, \quad \delta r = -8\Delta_0, \tag{2.40}$$

with $n_T = -2\epsilon_0 - \ell_0$.

3 Some Models

Model I.

$$F\left(\phi\right) = \frac{1}{\kappa^2} - \xi\phi^2, \qquad V\left(\phi\right) = \frac{1}{2}m^2\phi^2, \qquad F_1\left(\phi\right) = \gamma, \qquad F_2\left(\phi\right) = 0. \tag{3.1}$$

$$\epsilon_0 = \frac{2 + 2\xi\phi^2}{\phi^2 + (m^2\gamma - \xi)\phi^4}, \qquad \epsilon_1 = \frac{4(1 - \xi\phi^2)\left((m^2\gamma - \xi)(\xi\phi^2 + 2)\phi^2 + 1\right)}{\phi^2\left(1 + (m^2\gamma - \xi)\phi^2\right)^2}$$
$$\ell_0 = \frac{4\xi(\xi\phi^2 + 1)}{(m^2\gamma - \xi)\phi^2 + 1}, \qquad \ell_1 = -\frac{4(m^2\gamma - 2\xi)(\xi\phi^2 - 1)}{((m^2\gamma - \xi)\phi^2 + 1)^2}. \tag{3.2}$$

$$\epsilon_0(\phi_E) = 1$$

$$\phi_E^2 = \frac{\sqrt{8m^2\gamma + 4\xi^2 - 12\xi + 1} + 2\xi - 1}{2m^2\gamma - 2\xi} \tag{3.3}$$

$$N = \int_{\phi_I}^{\phi_E} \frac{\phi + (m^2\gamma - \xi)\phi^3}{2\xi^2\phi^4 - 2} d\phi = \frac{1}{8\xi^2}\left[m^2\gamma \ln\left(1 - \xi^2\phi^4\right) - 2\xi \ln\left(1 - \xi\phi^2\right)\right]\Big|_{\phi_I}^{\phi_E} \tag{3.4}$$

Assuming that $\xi\phi^2 << 1$ and $m^2\gamma >> \xi$.

$$\phi_E^2 \approx \left(\frac{2}{m^2\gamma}\right)^{1/2}, \tag{3.5}$$

and from (3.4) we find for ϕ_I

$$\phi_I^2 \approx \left(\frac{8N + 2}{m^2\gamma}\right)^{1/2} \tag{3.6}$$

$$\phi_I \approx (4N + 1)^{1/4}\phi_E$$

$N = 60 \rightarrow \phi_I \approx 3.9\phi_E$.

$$n_s \approx 1 + \frac{2}{(8N + 2)^{1/2}(m^2\gamma)^{1/2}} - \frac{12}{8N + 2} - \frac{8}{(8N + 2)^{3/2}(m^2\gamma)^{1/2}} \tag{3.7}$$

and

$$r \approx \frac{32}{8N + 2} + \frac{64\xi}{(8N + 2)^{1/2}(m^2\gamma)^{1/2}} \tag{3.8}$$

Additional M_p ($\phi \simeq 1$). This can be achieved if $m^2\gamma = 8N + 2$, as follows from (3.6), which gives

$$n_s \approx 1 - \frac{10}{8N + 2} - \frac{8}{(8N + 2)^2}, \quad r \approx \frac{32 + 64\xi}{8N + 2} \tag{3.9}$$

Thus, for 60 e-foldings we find $n_s \approx 0.98$ and $r \approx 0.067$ ($\xi = 10^{-2}$). In this case the inflation begins with $\phi_I = M_p$ and ends with $\phi_E \approx 0.25 M_p$. For the numerical analysis with the exact expressions, we assume $N = 60$, $m = 10^{-6} M_p$. In fact from Eqs. (3.2) follows that the spectral index n_s and the tensor-to-scalar ratio depend on the dimensionless combination $m^2\gamma$. Fig. 1 shows the behavior of n_s and r in the interval $10^2 < m^2\gamma < 5\times 10^2$, for $\xi < 0.1/6, 0.2$.

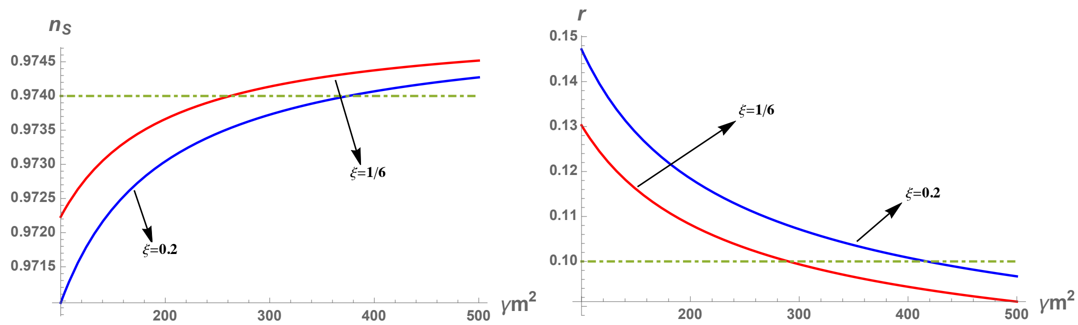


Figure 1: The behavior of the scalar spectral index n_s and r as function of $m^2\gamma$ for some values of ξ. The horizontal lines correspond to the upper limit of the observational quotes from Planck 2015, with values $n_s = 0.968 \pm 0.006$ and $r < 0.1$

Model II

$$F = \frac{1}{\kappa^2}, \quad V(\phi) = \frac{1}{2}m^2\phi^2, \quad F_1(\phi) = \gamma, \quad F_2(\phi) = \frac{\eta}{\phi^2} \tag{3.10}$$

η has dimension of $mass^2$ and ϕ is measured in units of M_p.

$$\epsilon_0 = \frac{6 - 8m^2\eta}{3\phi^2(1 + m^2\gamma\phi^2)}, \quad \epsilon_1 = \frac{4(3 - 4m^2\eta)(1 + 2m^2\gamma\phi^2)}{3\phi^2(1 + m^2\gamma\phi^2)^2}$$

$$\Delta_0 = \frac{16m^2\eta(3 - 4m^2\eta)}{9\phi^2(1 + m^2\gamma\phi^2)}, \quad \Delta_1 = \frac{4(3 - 4m^2\eta)(1 + 2m^2\gamma\phi^2)}{3\phi^2(1 + m^2\gamma\phi^2)^2}. \tag{3.11}$$

$$\epsilon(\phi_E) = 1$$

$$\phi_E^2 = \frac{1}{6m^2\gamma}\left[\sqrt{72m^2\gamma - 96m^4\gamma\eta + 9} - 3\right] \tag{3.12}$$

$$N = \frac{3\phi^2(2 + m^2\gamma\phi^2)}{8(4m^2\eta - 3)}\Big|_{\phi_I}^{\phi_E} \tag{3.13}$$

$$n_s = \frac{3m^4\gamma\phi^2\left(\gamma\phi^4 + 16\eta\right) + 6m^2\gamma\phi^2(\phi^2 - 6) + 3\phi^2 + 32m^2\eta - 24}{3\phi^2(1 + m^2\gamma\phi^2)^2}\Big|_{\phi_I} \tag{3.14}$$

$$\frac{3\phi(1 + m^2\gamma\phi^2)}{8m^2\eta - 6}\Big|_{\phi_I} \tag{3.15}$$

For $N = 60$, $m = 10^{-6}M_p$ we can find the behavior of n_s and r in terms of the dimensionless parameter $m^2\gamma$. In Fig. 2 we show the behavior of the scalar field at the beginning and end of inflation for $1 < m^2\gamma < 5$

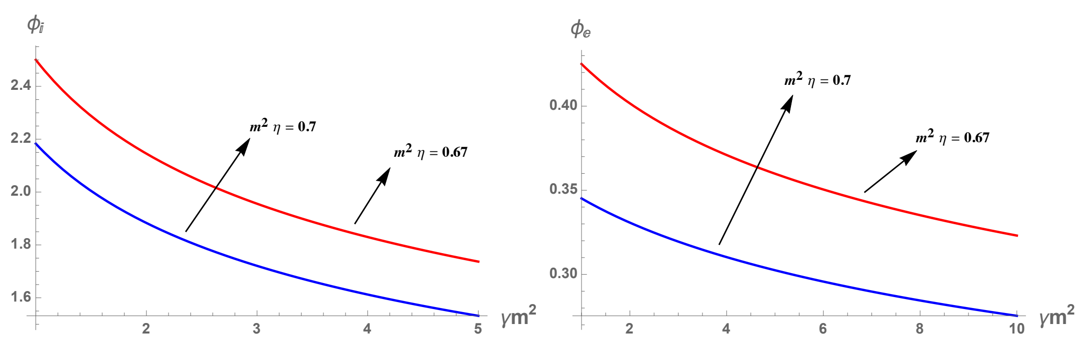


Figure 2: The values of the scalar field at the beginning and end of inflation for $1 < m^2\gamma < 5$ and $m^2\eta = 0, 67, 0.7$ in units of M_p^4.

In Fig. 3 we show the corresponding behavior of n_s and r.

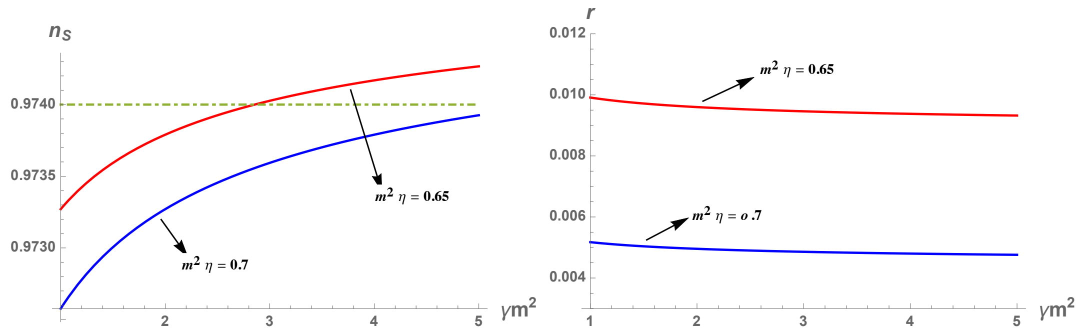


Figure 3: The values n_s and r in the interval $1 < m^2\gamma < 1$ for $m^2\eta = 0, 67, 0.7$ (in units of M_p^4). The horizontal line is the upper limit set by Planck 2015.

Model III.

$$F = \frac{1}{\kappa^2}, \quad V(\phi) = \frac{\lambda}{n}\phi^n, \quad F_1(\phi) = \frac{\gamma}{\phi^n}, \quad F_2(\phi) = \frac{\eta}{\phi^n} \tag{3.16}$$

$$\begin{gathered}\epsilon_0 = \frac{n^2(3n - 8\eta\lambda)}{6(n + 2\gamma\lambda)\phi^2}, \quad \epsilon_1 = \frac{2n(3n - 8\eta\lambda)}{3(n + 2\gamma\lambda)\phi^2}, \quad \Delta_0 = \frac{8n\eta\lambda(3n - 8\eta\lambda)}{9(n + 2\gamma\lambda)\phi^2}, \\ \Delta_1 = \frac{2n(3n - 8\eta\lambda)}{3(n + 2\gamma\lambda)\phi^2}, \quad k_0 = \frac{n\gamma\lambda(3n - 8\eta\lambda)^2}{9(n + 2\gamma\lambda)\phi^2}, \quad k_1 = \frac{2n(3n - 8\eta\lambda)}{3(n + 2\gamma\lambda)\phi^2}\end{gathered} \tag{3.17}$$

$$\phi_E = \frac{n\sqrt{3n - 8\eta\lambda}}{\sqrt{6n + 12\gamma\lambda}} \tag{3.18}$$

$$N = -\frac{3(n + 2\gamma\lambda)}{2n(3n - 8\eta\lambda)}\phi^2\Big|_{\phi_I}^{\phi_E} \tag{3.19}$$

$$\phi_I = \left(\frac{(4N + n)(3n^2 - 8n\eta\lambda)}{6n + 12\gamma\lambda}\right)^{1/2} = \sqrt{\frac{(4N + n)}{n}}\phi_E \tag{3.20}$$

$$n_s = \frac{(6\gamma\lambda + 16n\eta\lambda + 3n)\,\phi^2 - 3n^3 - (6 - 8\eta\lambda)n^2}{3(n + 2\gamma\lambda)\phi^2}\Big|_{\phi_I} = \frac{4N - n - 4}{4N + n} \tag{3.21}$$

$$r = \frac{8n(3n - 8\eta\lambda)^2}{9(n + 2\gamma\lambda)\phi^2}\Big|_{\phi_I} = \frac{16(3n - 8\eta\lambda)}{3(4N + n)} \tag{3.22}$$

Slow-roll parameters N e-folds before the end of inflation

$$\epsilon_0 = \frac{n}{4N + n}, \quad \epsilon_1 = \frac{4}{4N + n}, \quad \Delta_0 = \frac{16\eta\lambda}{3(4N + n)},$$

$$\Delta_1 = \frac{4}{4N + n}, \quad k_0 = \frac{2\gamma\lambda(3n - 8\eta\lambda)}{3(4N + n)(n + 2\gamma\lambda)}, \quad k_1 = \frac{4}{4N + n} \tag{3.23}$$

The tensor-to-scalar ratio depends additionally on the self coupling $\eta\lambda$

The kinetic coupling can lower the values of the scalar field at the end, and therefore at the beginning, of inflation.

Srong coupling regime of the GB coupling spoils the inflation (Δ_0 and k_0 break the slow-roll restrictions)

Strong coupling limit of kinetic coupling all slow-roll parameters and derived quantities are well defined.

$[\eta\lambda] = mass^4$, $[\gamma\lambda] = mass^2$. $\alpha = \eta\lambda = \alpha M_p^4$

The coupling η (and therefore α) can be used to lower the tensor-to-scalar ratio.

$\beta = \gamma\lambda$ leads to consistent inflation in the weak, $\gamma \to 0$, and strong, $\gamma \to \infty$, limits and can take any value between these limits.

Power n	n_s	Parameter α range	r in α range
4	0.9508	$1 \leq \alpha \leq 1.3$	$0.0874 \geq r \geq 0.0349$
3	0.959	$0.7 \leq \alpha \leq 1.1$	$0.0746 \geq r \geq 0.0044$
2	0.9669	$0.3 \leq \alpha \leq 0.7$	$0.0746 \geq r \geq 0.0088$
4/3	0.9724	$0.01 \leq \alpha \leq 0.4$	$0.0866 \geq r \geq 0.0177$
1	0.9751	$10^{-3} \leq \alpha \leq 0.3$	$0.0664 \geq r \geq 0.0133$
2/3	0.9778	$10^{-4} \leq \alpha \leq 0.2$	$0.0443 \geq r \geq 0.0089$

Table I. Some values of n_s and r in an appropriate range for α in each case.

Model IV.

$$F = \frac{1}{\kappa^2}, \quad V(\phi) = \frac{\lambda}{n}\phi^n, \quad F_1(\phi) = \frac{\beta}{\phi^{n+2}}, \quad F_2(\phi) = 0 \tag{3.24}$$

$$\begin{aligned} \epsilon_0 &= \frac{n^3}{2n\phi^2 + 4\beta\lambda\phi^4}, \quad \epsilon_1 = \frac{2n^2(n + 4\beta\lambda\phi^2)}{\phi^2(n + 2\beta\lambda\phi^2)^2} \\ k_0 &= \frac{\beta\lambda n^3}{(n + 2\beta\lambda\phi^2)^2}, \quad k_1 = \frac{8\beta\lambda n^2}{(n + 2\beta\lambda\phi^2)^2}. \end{aligned} \tag{3.25}$$

$$\phi_E^2 = \frac{\sqrt{n^2(1 + 4n\beta\lambda)} - n}{4\beta\lambda}. \tag{3.26}$$

$$N = \frac{\beta\lambda\phi^4}{2n^2} - \frac{\phi^2}{2n}\Big|_{\phi_I}^{\phi_E} \tag{3.27}$$

$$\phi_I^2 = \frac{n}{4\beta\lambda}\left(\sqrt{2}n\sqrt{\frac{n+2n\beta\lambda(n+8N)+n\sqrt{1+4n\beta\lambda}}{n^3}}-2\right) \tag{3.28}$$

$$n_s = 1 - \frac{4n\beta\lambda\left[\sqrt{2}n(n+4)f(n,N,\beta,\lambda)-4\right]}{n^3 f^2(n,N,\beta,\lambda)\left[\sqrt{2}nf(n,N,\beta,\lambda)-2\right]}, \tag{3.29}$$

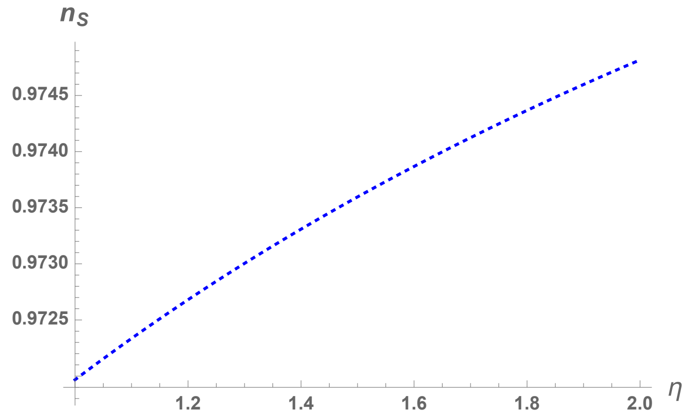
$$f(n,N,\beta,\lambda) = \sqrt{\frac{n+2n\beta\lambda(n+8N)+n\sqrt{1+4n\beta\lambda}}{n^3}}.$$

$$r = \frac{32\sqrt{2}\beta\lambda}{f(n,N,\beta,\lambda)\left[\sqrt{2}nf(n,N,\beta,\lambda)-2\right]} \tag{3.30}$$

$$\alpha = \beta\lambda \tag{3.31}$$

Power n	n_s	Parameter α range	r in α range
4	$0.9666 \leq n_s \leq 0.9667$	$10 \leq \alpha \leq 20$	$0.1335 \geq r \geq 0.1339$
3	$0.9709 \leq n_s \leq 0.971$	$10^2 \leq \alpha \leq 10^3$	$0.0998 \geq r \geq 0.0995$
2	$0.973 \leq n_s \leq 0.9744$	$0.1 \leq \alpha \leq 1$	$0.0756 \geq r \geq 0.0692$
4/3	$0.9721 \leq n_s \leq 0.9736$	$10^{-3} \leq \alpha \leq 10^{-2}$	$0.080 \geq r \geq 0.062$
1	$0.9746 \leq n_s \leq 0.977$	$10^{-3} \leq \alpha \leq 0.05$	$0.06 \geq r \geq 0.04$
2/3	$0.9777 \geq n_s \geq 0.9774$	$10^{-4} \leq \alpha \leq 10^{-2}$	$0.0438 \geq r \geq 0.0313$

Table II. n_s and r in an appropriate range for α in each case.



$$\lim_{\beta\to\infty} n_s = \frac{8N-n-8}{8N+n}, \quad \lim_{\beta\to\infty} r = \frac{16n}{8N+n} \tag{3.32}$$

In the weak coupling limit, $\beta \to 0$, it is found

$$\lim_{\beta\to 0} n_s = \frac{4N-n-4}{4N+n}, \quad \lim_{\beta\to 0} r = \frac{16n}{4N+n}. \tag{3.33}$$

At the strong coupling limit

$$\phi_E \to \left(\frac{n^3}{4\beta\lambda}\right)^{1/4}, \quad \phi_I \to \left(\frac{n^2(8N+n)}{4\beta\lambda}\right)^{1/4} \tag{3.34}$$

At the weak coupling limit

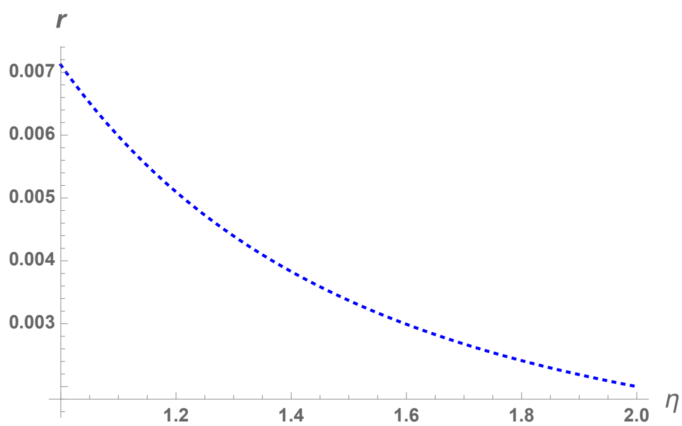
$$\phi_E \to \frac{n}{\sqrt{2}}, \quad \phi_I \to \frac{n\sqrt{4N+n}}{\sqrt{2}} \tag{3.35}$$

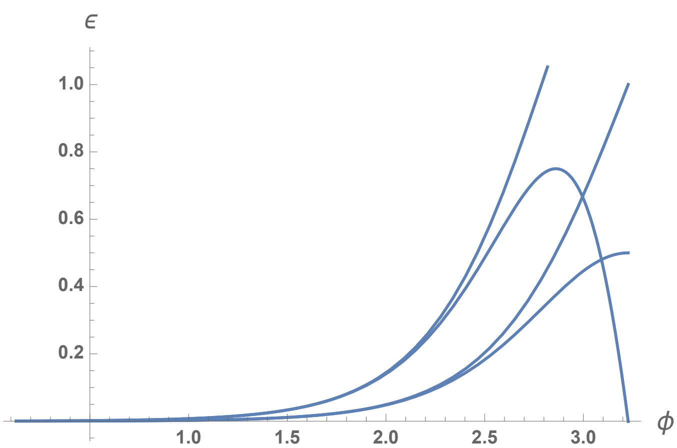
Model V.

$$F = \frac{1}{\kappa^2}, \quad V(\phi) = V_0 e^{-\lambda\phi}, \quad F_1(\phi) = \beta e^{-\eta\phi}, \quad F_2(\phi) = 0$$

4 Discussion

- A consistency relation is found


r
0.007
0.006
0.005
0.004
0.003
1.2
1.4
1.6
1.8
2.0
η


ϵ
1.0
0.8
0.6
0.4
0.2
1.0
1.5
2.0
2.5
3.0
ϕ

- Some models with power-law potential have been analyzed.

- For $V \propto \phi^2$ potential: n_s and r fall in the range allowed by latest observations.

- For $V \propto \phi^2$ potential: n_s falls in the range favored by observations, $r \sim 0.13$

- GB Coupling: not viable in strong coupling limit.

- Kinetic Coupling: viable in strong coupling limit.

- Kinetic coupling: avoids the problem of large fields in chaotic inflation.

- Kinetic Coupling: $F_1 = \beta/\phi^{n+2}$:

- a) n_s and r depend on the kinetic coupling constant .
- b) In the weak coupling $\rightarrow$ standard chaotic.
- c) In the strong coupling $\rightarrow$ n_s increases and r lower.

$$V = \frac{1}{2}\phi^2: \quad n_s^{max} = \frac{4N-5}{4N+1}, \quad r_{min} = \frac{16}{4N+1}$$

$$V = \frac{1}{4}\lambda\phi^4: \quad n_s^{max} = \frac{2N-3}{2N+1}, \quad r_{min} = \frac{16}{2N+1}$$