Anisotropic 2-form Dark Energy

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Some motivations

1. Anisotropic inflation.
2. Parity breaking inflationary signatures.
3. The imprinted signatures in CMB are different from those by vector fields.
4. Less explored than vector models.
5. Complementary approach to higher spin fields.
6. Anisotropic expansion in Bianchi backgrounds and dark energy models.

The Model

DE field

Perfect fluid

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V_{eff}(\phi) - \frac{1}{12}f(\phi)H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} + P_f(Z)\right]$$

Coupling function

2-form contribution

$$f(\phi) = f_0 e^{-\mu\phi/M_{\rm pl}}$$

$$\mu \geq \mathcal{O}(1)$$

The Model

The potential

$$V(\phi) = V_0 e^{-\lambda\phi/M_{\rm pl}}, \qquad \lambda \geq \mathcal{O}(1)$$

Field configuration

$$B_{\alpha\beta}\mathrm{d}x^\alpha \wedge \mathrm{d}x^\beta = 2v_B(t)\mathrm{d}y \wedge \mathrm{d}z$$

The background

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}\mathrm{d}x^2 + e^{2\sigma(t)}(\mathrm{d}y^2 + \mathrm{d}z^2)\right]$$

Equations of Motion (EoM)

$$3M_{\rm pl}^2 H^2 \left(1 - \Sigma^2\right) = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_B + \rho_f,$$

$$M_{\rm pl}^2 \left(\dot{H} + 3H^2\Sigma^2\right) = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{3}\rho_B - \frac{1}{2}\left(\rho_f + P_f\right),$$

$$M_{\rm pl}^2 \left[H\dot{\Sigma} + \left(\dot{H} + 3H^2\right)\Sigma\right] = -\frac{2}{3}\rho_B,$$

$$\dot{\rho}_f + 3H\left(\rho_f + P_f\right) = 0,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - \frac{f_{,\phi}}{f}\rho_B = 0.$$

$$H \equiv \dot{\alpha}$$

$$\Sigma \equiv \frac{\dot{\sigma}}{H}$$

$$\rho_f = \rho_m + \rho_r$$

$$P_m = 0$$

$$P_r = \rho_r/3$$

$$\rho_B = \frac{f(\phi)}{2} e^{-4\alpha - 4\sigma} \dot{v}_B^2$$

Dynamical system

$$x_1 = \frac{\dot{\phi}}{\sqrt{6}HM_{\rm pl}}\,, \quad x_2 = \frac{\sqrt{V}}{\sqrt{3}HM_{\rm pl}}\,, \quad \Omega_B = \frac{\rho_B}{3H^2M_{\rm pl}^2}\,,$$

$$\Omega_r = \frac{\rho_r}{3H^2M_{\rm pl}^2}\,, \quad \Omega_m = \frac{\rho_m}{3H^2M_{\rm pl}^2}\,.$$

Friedmann constraint $\longrightarrow$ $3M_{\rm pl}^2H^2\left(1-\Sigma^2\right) = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_B + \rho_f,$

$\downarrow$

$$\Omega_m = 1 - x_1^2 - x_2^2 - \Sigma^2 - \Omega_B - \Omega_r\,.$$

Dynamical system

$$x_1' = \frac{3}{2}x_1\left(x_1^2 - x_2^2 + \Sigma^2 - 1 - \frac{1}{3}\Omega_B + \frac{1}{3}\Omega_r\right) + \frac{\sqrt{6}}{2}\left(\lambda x_2^2 - \mu\Omega_B\right) ,$$

$$x_2' = \frac{1}{2}x_2(3x_1^2 - 3x_2^2 + 3\Sigma^2 + 3 - \sqrt{6}\lambda x_1 - \Omega_B + \Omega_r) ,$$

$$\Sigma' = \frac{1}{2}\Sigma\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 - 3 - \Omega_B + \Omega_r\right) - 2\Omega_B ,$$

$$\Omega_B' = \Omega_B(3x_1^2 - 3x_2^2 + 3\Sigma^2 + 4\Sigma + 1 + \sqrt{6}\mu x_1 - \Omega_B + \Omega_r) ,$$

$$\Omega_r' = \Omega_r\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 - 1 - \Omega_B + \Omega_r\right) ,$$

Dynamical system

$$w_{\text{eff}} = x_1^2 - x_2^2 + \Sigma^2 - \frac{1}{3}\Omega_B + \frac{1}{3}\Omega_r \longrightarrow$$

$w_{\text{eff}} \simeq 1/3$ →	Radiation
$w_{\text{eff}} \simeq 0$ →	Matter
$w_{\text{eff}} < -1/3$ →	Cosmic acceleration

Dark sector parameters :

$$\Omega_{\text{DE}} = x_1^2 + x_2^2 + \Sigma^2 + \Omega_B = 1 - \Omega_r - \Omega_m \,,$$

$$w_{\text{DE}} = \frac{3(x_1^2 - x_2^2 + \Sigma^2) - \Omega_B}{3(x_1^2 + x_2^2 + \Sigma^2 + \Omega_B)} \,.$$

Critical points

Isotropic radiation point →

$$x_1 = 0\,, \quad x_2 = 0\,, \quad \Sigma = 0\,,$$
$$\Omega_B = 0\,, \quad \Omega_r = 1\,, \quad \Omega_m = 0\,.$$

Isotropic radiation scaling point →

$$x_1 = \frac{2\sqrt{6}}{3\lambda}\,, \quad x_2 = \frac{2\sqrt{3}}{3\lambda}\,, \quad \Sigma = 0\,,$$
$$\Omega_B = 0\,, \quad \Omega_r = 1 - \frac{4}{\lambda^2}\,, \quad \Omega_m = 0\,.$$

↓

$\lambda > 9.4$ ← $\Omega_{\rm DE} < 0.045$ ← BBN constraint

Critical points

Anisotropic radiation point →

$$x_1 = -\frac{\sqrt{6}\mu}{3\mu^2+8}, \quad x_2 = 0, \quad \Sigma = -\frac{4}{3\mu^2+8},$$

$$\Omega_B = \frac{2}{3\mu^2+8}, \quad \Omega_r = \frac{3\mu^2+4}{3\mu^2+8}, \quad \Omega_m = 0.$$

↓

$\mu > 5.2$ ← $\Omega_{\rm DE} < 0.045$ ← BBN constraint

Critical points

Isotropic matter point →

$$x_1 = 0\,, \quad x_2 = 0\,, \quad \Sigma = 0\,,$$
$$\Omega_B = 0\,, \quad \Omega_r = 0\,, \quad \Omega_m = 1\,.$$

Isotropic matter scaling point →

$$x_1 = \frac{\sqrt{6}}{2\lambda}\,, \quad x_2 = \frac{\sqrt{6}}{2\lambda}\,, \quad \Sigma = 0\,,$$
$$\Omega_B = 0\,, \quad \Omega_r = 0\,, \quad \Omega_m = 1 - \frac{3}{\lambda^2}\,.$$

↓

$\lambda > 12$ ← $\Omega_{\rm DE} < 0.02$ ← CMB constraint

Critical points

Anisotropic matter point

$$x_1 = -\frac{\sqrt{6}\mu}{2(3\mu^2+8)}\,, \quad x_2 = 0\,, \quad \Sigma = -\frac{2}{3\mu^2+8}$$

$$\Omega_B = \frac{3}{2(3\mu^2+8)}\,, \quad \Omega_r = 0\,, \quad \Omega_m = \frac{3\mu^2+6}{3\mu^2+8}$$

CMB constraint → $\Omega_{\rm DE} < 0.02$ → $\mu > 5.5$

Isotropic dark energy point

$$x_1 = \frac{\lambda}{\sqrt{6}}\,, \quad x_2 = \sqrt{1-\frac{\lambda^2}{6}}\,, \quad \Sigma = 0\,,$$

$$\Omega_B = 0\,, \quad \Omega_r = 0\,, \quad \Omega_m = 0\,.$$

$w_{\rm DE} = w_{\rm eff} = -1 + \lambda^2/3$ → $\lambda^2 < 2$

Critical points

Anisotropic dark energy point

$$x_1 = \frac{(2\lambda + \mu)\sqrt{6}}{2\lambda^2 + 5\lambda\mu + 3\mu^2 + 8}, \quad x_2 = \frac{\sqrt{3(\lambda\mu + \mu^2 + 4)(3\mu^2 + 4\lambda\mu + 8)}}{2\lambda^2 + 5\lambda\mu + 3\mu^2 + 8}$$

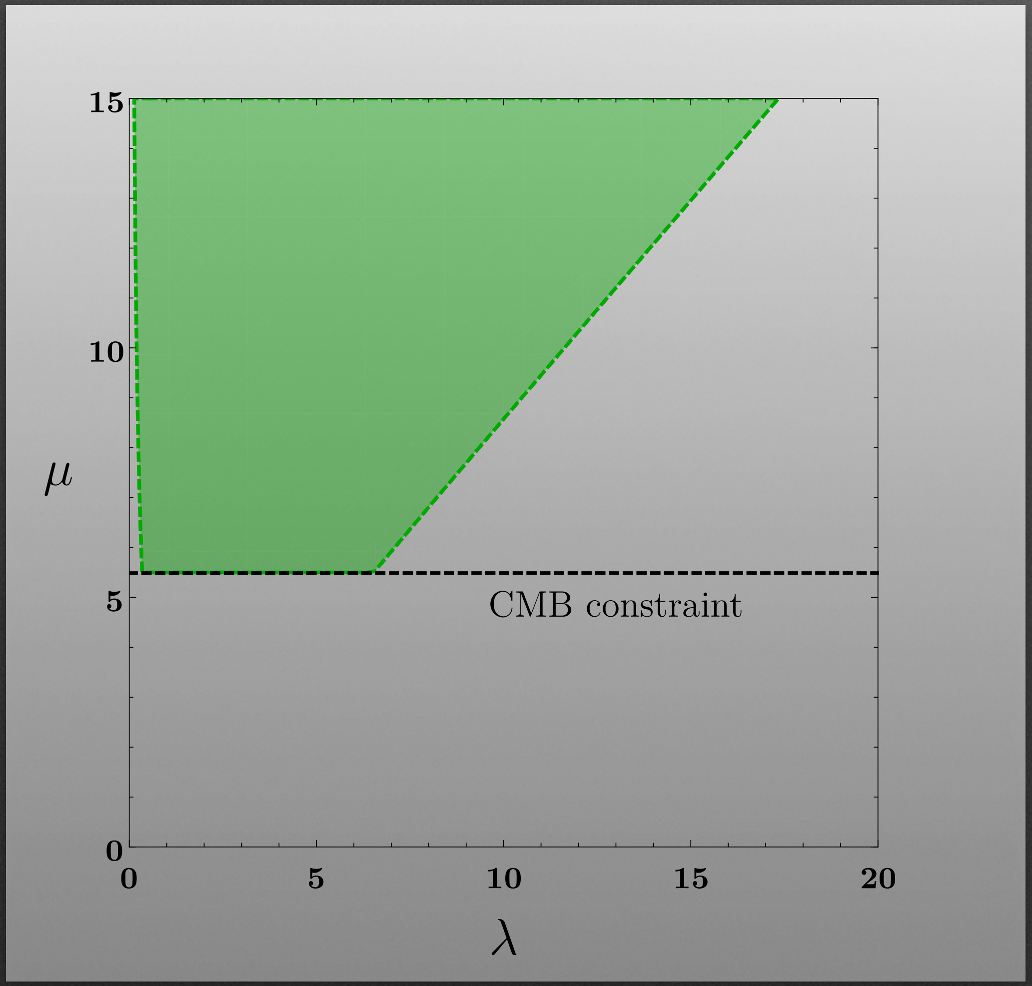
$$\Sigma = -\frac{2(\lambda^2 + \lambda\mu - 2)}{2\lambda^2 + 5\lambda\mu + 3\mu^2 + 8}, \quad \Omega_B = \frac{3(3\mu^2 + 4\lambda\mu + 8)(\lambda^2 + \lambda\mu - 2)}{(2\lambda^2 + 5\lambda\mu + 3\mu^2 + 8)^2}$$

$$\Omega_r = 0\,, \quad \Omega_m = 0$$

$$\lambda^2 + \lambda\mu - 2 > 0\,,$$

$$4\lambda^2 - 2\lambda\mu - 3\mu^2 - 8 < 0\,.$$

$$w_{\rm DE} = w_{\rm eff} = -1 + \frac{2\lambda(2\lambda + \mu)}{2\lambda^2 + 5\lambda\mu + 3\mu^2 + 8}\,.$$



Stability of the critical points

Isotropic DE →

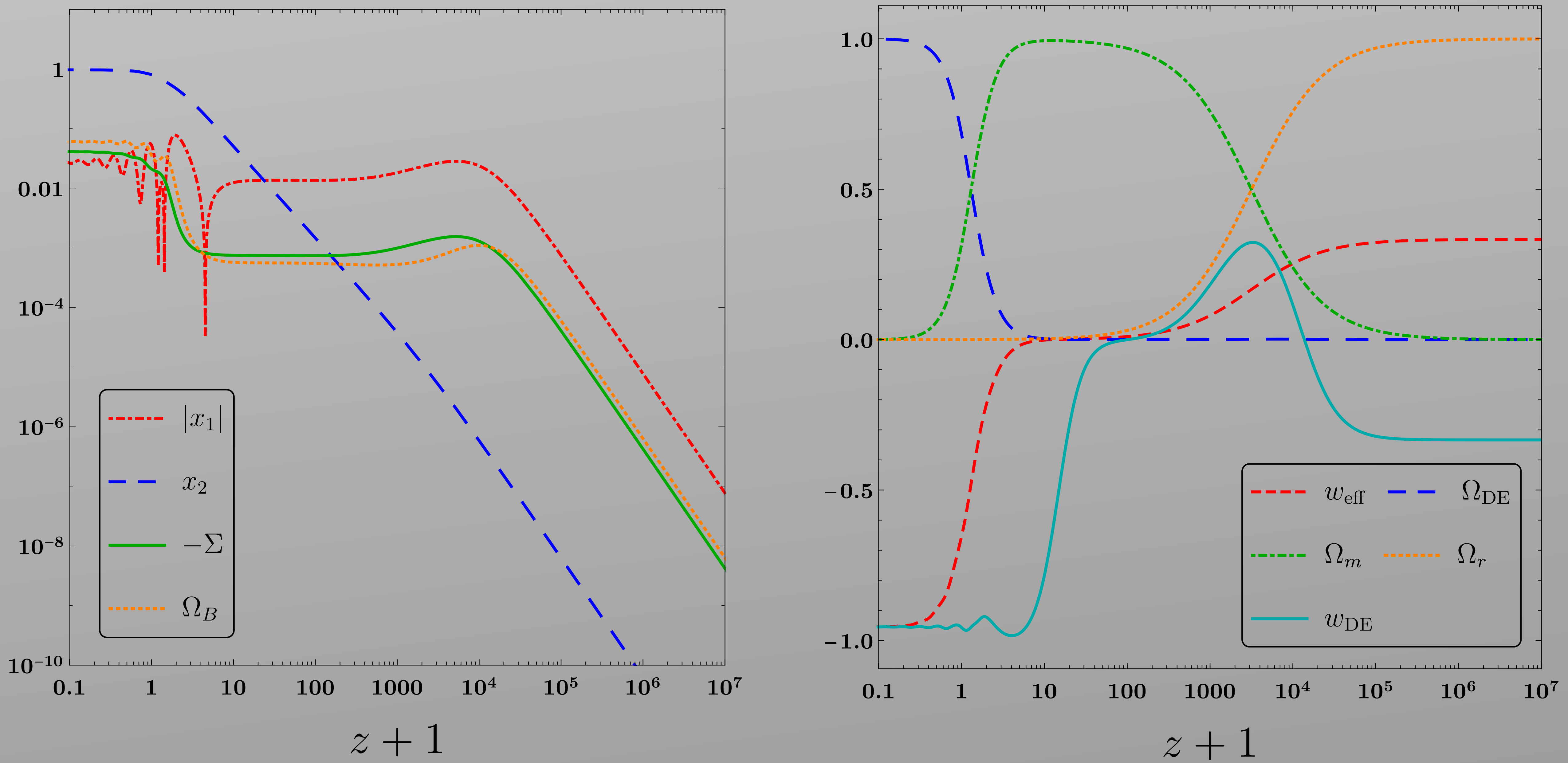
- $\lambda^2 + \lambda\mu - 2 < 0$ → Attractor
- $\lambda^2 + \lambda\mu - 2 > 0$ → Saddle

↓

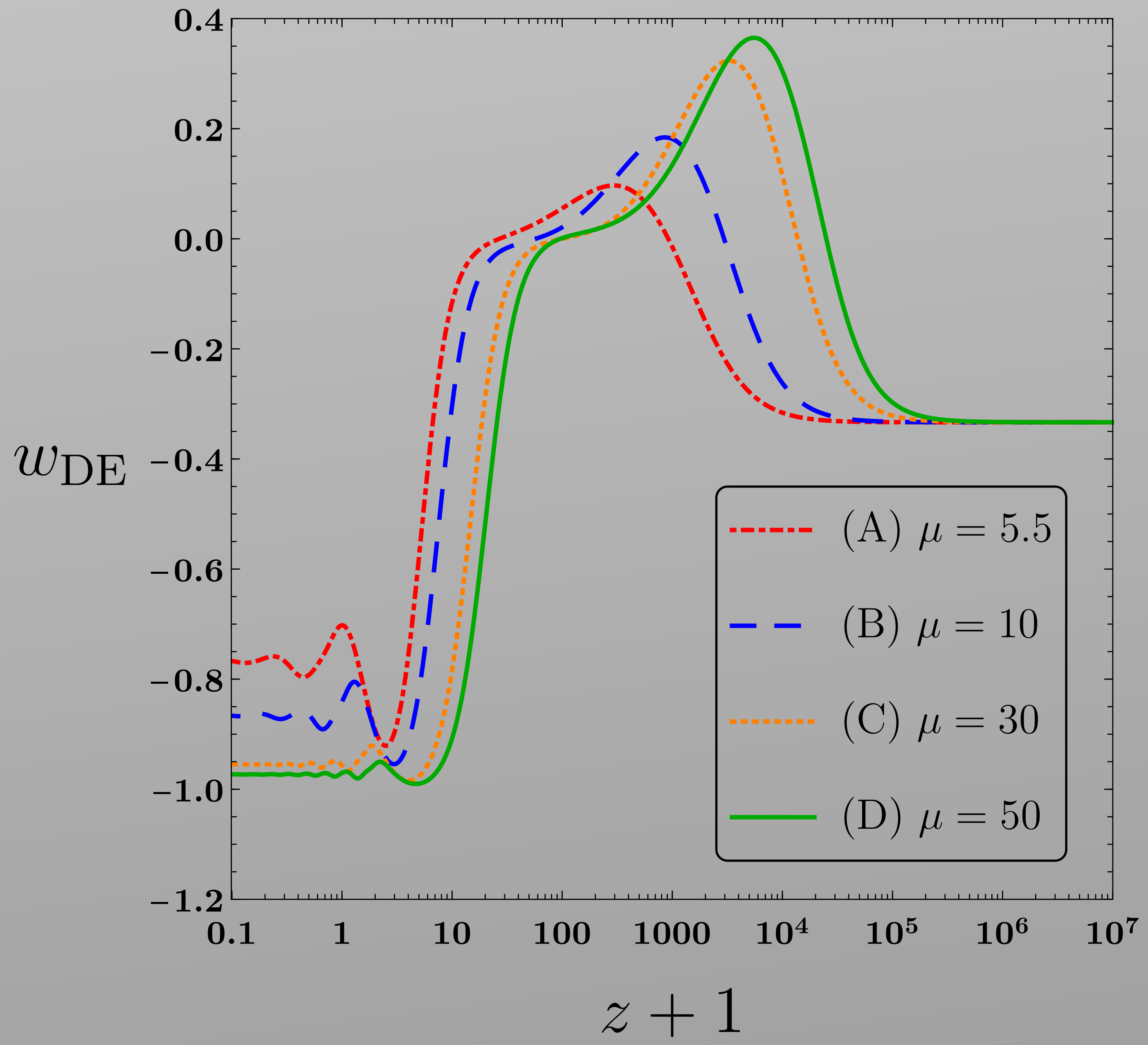
Anisotropic DE

↓

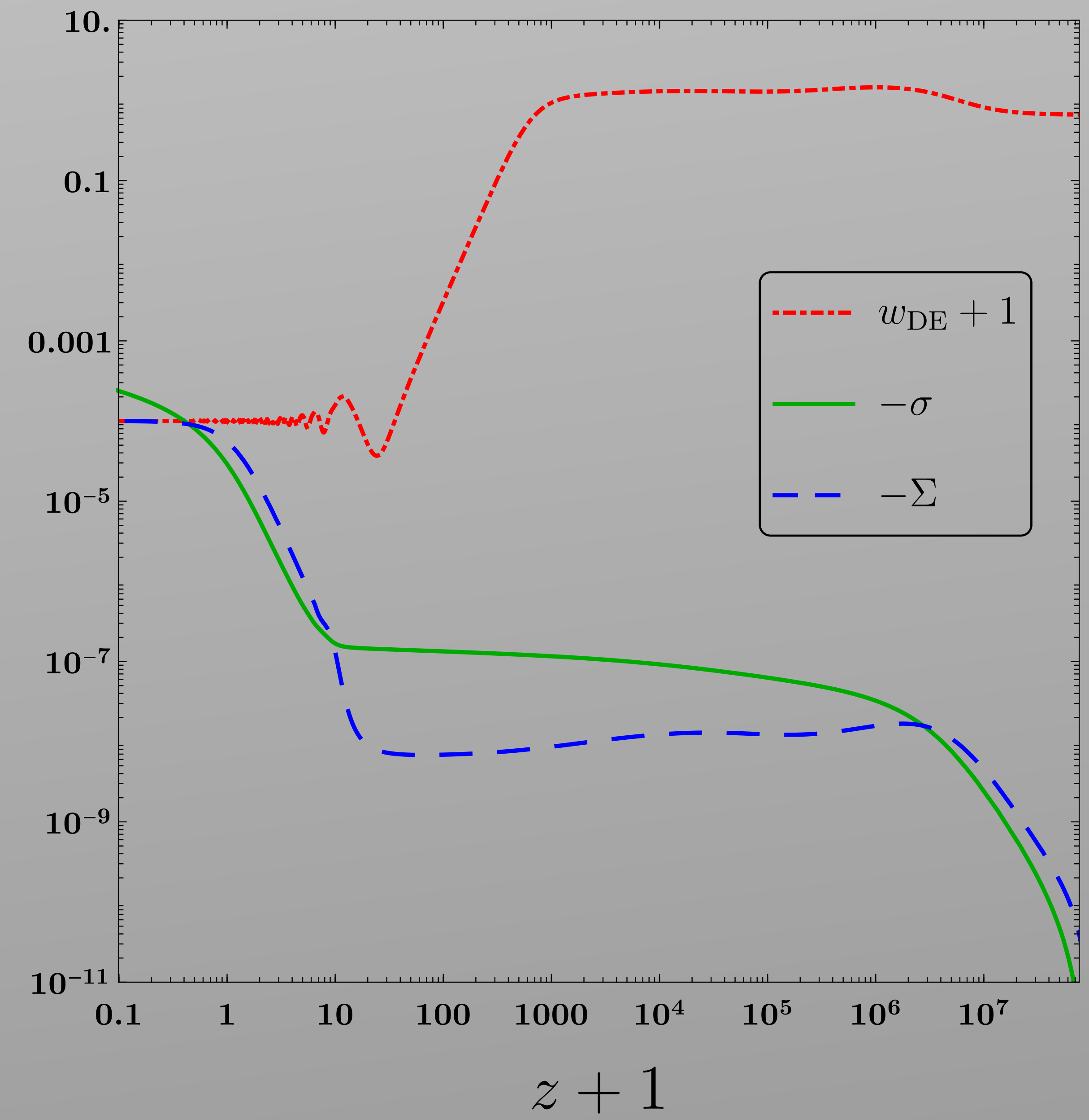
Attractor



1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0.1
1
10
100
1000
10^4
10^5
10^6
10^7
$|x_1|$
x_2
$-\Sigma$
Ω_B
$z+1$
1.0
0.5
0.0
−0.5
−1.0
$w_{\rm eff}$
$\Omega_{\rm DE}$
Ω_m
Ω_r
$w_{\rm DE}$
$z+1$



$w_{\rm DE}$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0.1
1
10
100
1000
10^4
10^5
10^6
10^7
$z+1$
(A) $\mu = 5.5$
(B) $\mu = 10$
(C) $\mu = 30$
(D) $\mu = 50$



10.
0.1
0.001
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0.1
1
10
100
1000
10^4
10^5
10^6
10^7
$z+1$
$w_{\rm DE}+1$
$-\sigma$
$-\Sigma$

Conclusions

- The late time comic acceleration can be realized for coupling constant in the range $\mu \gg \lambda \geq \mathcal{O}(1)$.
- Anisotropic Dark Energy dominated fixed point.
- Oscillating Dark Energy equation of state.
- Isotropic radiation —> Anisotropic radiation —> Anisotropic matter —> Anisotropic accelerated attractor.
- The model leaves imprints on observables (CMB and SN Ia).

Thanks a lot