Axions with derivative coupling and inflationary perturbations

Juan P. Beltrán Almeida
Departamento de Física
Facultad de Ciencias
Universidad Antonio Nariño

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Some motivations

1. Anisotropic and parity breaking inflationary signatures.
2. “UV complete” model. Stable under radiative corrections.
3. Testing non minimal couplings with gravity during inflation.
4. Anisotropic and parity breaking signatures in the LSS.

Inflationary models with vectors

A general shift invariant Lagrangian involving scalar, vectors and gravity

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - \frac{1}{2}\nabla_\alpha\phi\,\nabla^\alpha\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]$$

$$+\int \mathrm{d}^4x\sqrt{-g}\left[\frac{\alpha_1\phi}{4f}\tilde{F}^{\mu\nu}F_{\mu\nu} + \frac{\alpha_2\phi}{16}R_{GB} + \frac{\alpha_3\phi}{16}\tilde{R}R\right],$$

Topologic terms

Shift symmetry ⟶ $\phi \to \phi + c$

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$$+\int d^4x\sqrt{-g}\left[\frac{\alpha_1\phi}{4f}\tilde{F}^{\mu\nu}F_{\mu\nu} + \frac{\alpha_2\phi}{16}R_{GB} + \frac{\alpha_3\phi}{16}\tilde{R}R\right],$$

A mass parameter

Topologic terms

Shift symmetry $\longrightarrow$ $\phi \to \phi + c$

Inflationary models with vectors

Broken shift symmetry. A potential is generated

$$\mathscr{L} = \partial_\mu \Phi \partial^\mu \Phi^* - \beta(\Phi\Phi^* - b^2)^2$$

$$\Phi = (b + \delta\Phi)e^{i\phi/f} \longrightarrow \phi \to \phi + c$$

The symmetry is broken by global effects

$$\delta\mathscr{L} \propto e^{-S}(\Phi + \Phi^*) \longrightarrow V(\phi) \propto \cos(\phi/f)$$

$$\boxed{V(\phi) = \Lambda^4(1 + \cos(\phi/f))} \longrightarrow \text{Natural inflation potential}$$

Inflationary models with vectors

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\nabla_\mu\phi\nabla_\nu\phi + V(\phi)\right]$$

$$-\frac{1}{4}\int d^4x\sqrt{-g}\left[F^{\mu\nu}F_{\mu\nu} + \frac{\alpha\phi}{f}\tilde{F}^{\mu\nu}F_{\mu\nu}\right],$$

C. Germani & A. Kehagias, PRL 106 (2011) 161302

M. Anber & L. Sorbo PRD 81 (2010) 043534

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R, \quad \nabla_\mu G^{\mu\nu} = 0.$$ ⟶ 2nd order EOM

Inflationary models with vectors

A general shift invariant Lagrangian involving scalar, vectors and gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(g^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}\chi\phi^2 R\right]$$

$$-\frac{1}{4}\int d^4x\sqrt{-g}\left[F^{\mu\nu}F_{\mu\nu} + \frac{\alpha\phi}{f}\tilde{F}^{\mu\nu}F_{\mu\nu}\right],$$

JB & N. Bernal
PRD 98 (2018) 083519

Motivation: Higgs Inflation like model. Slow roll due to non minimal coupling.

Non minimal coupling with gravity

We add nonminimal coupling with gravity to the previous system

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm P}^2}{2}\left(1+\frac{2h(\phi)}{M_{\rm P}^2}\right)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\frac{\alpha}{f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}\right].$$

where $h(\phi) = \frac{1}{2}\mathcal{X}\phi^2$.

Einstein frame We define the metric $\bar{g}_{\mu\nu} = \Omega(\phi) g_{\mu\nu}$, where $\Omega(\phi) \equiv \left(1+\frac{2h(\phi)}{M_{\rm P}^2}\right)$.

$$\mathcal{L} = \sqrt{-\bar{g}}\left[\frac{M_{\rm P}^2}{2}\bar{R} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{\nabla}_\mu\bar{\phi}\bar{\nabla}_\nu\bar{\phi} - \bar{V}(\bar{\phi}) - \frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} - \frac{1}{4}\frac{\alpha\phi(\bar{\phi})}{f}\bar{F}^{\mu\nu}\bar{\tilde{F}}_{\mu\nu}\right],$$

where $\mathrm{d}\bar{\phi}/\mathrm{d}\phi = K^{1/2}$, $\bar{V}(\phi) \equiv \frac{V(\phi)}{\Omega^2}$, $\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} \equiv \bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}F_{\alpha\beta}F_{\mu\nu}$ and

$$K(\phi) = \frac{1}{\Omega} + \frac{3M_{\rm P}^2}{2\Omega^2}\left(\frac{\partial\Omega}{\partial\phi}\right)^2 = \frac{1+6(\mathcal{X}+\frac{1}{6})\mathcal{X}\left(\frac{\phi}{M_{\rm P}}\right)^2}{\left(1+\mathcal{X}\left(\frac{\phi}{M_{\rm P}}\right)^2\right)^2}.$$

Why this is useful? Correlation functions are invariant under conformal transformation of the metric. Often, it is easier to calculate the correlators in the Einstein frame.

n_s and r

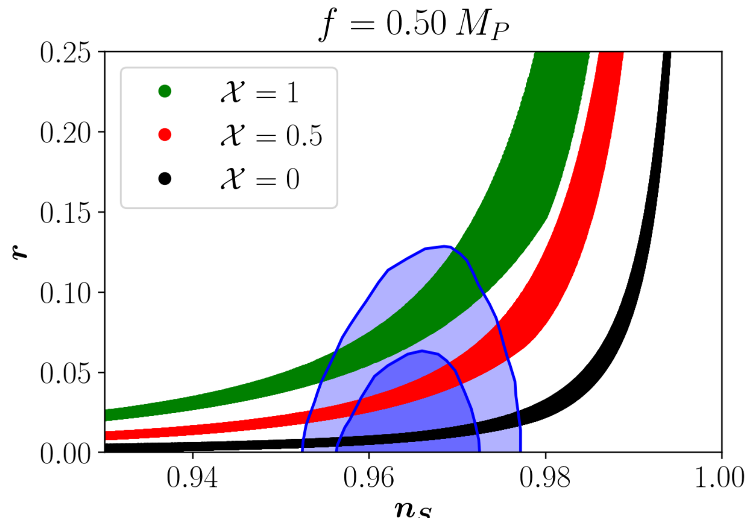
Chiral gravitational waves spectrum

- Spectral index of scalar perturbations

$$\bar{n}_s - 1 \approx 2\bar{\nu}_+ = -K^{1/2}\frac{2f\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}{\pi\alpha\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}\left(1 + \frac{d\ln(\Omega^2 K^{1/2})}{d\bar{\phi}}\frac{\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}\right).$$

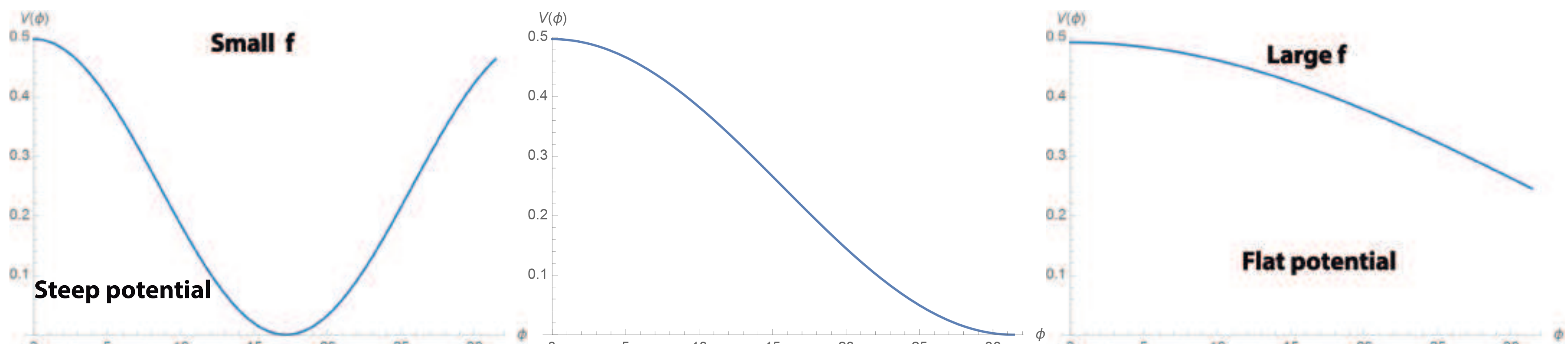
- Tensor to scalar ratio

$$\bar{r} \approx \frac{2V}{3\pi^2 M_P^4 \Omega^2 \mathcal{P}_\zeta} + 2.9\times 10^2 K \frac{\xi^4}{\alpha^2}\left(\frac{f\bar{V}_{\bar{\phi}}}{\bar{V}}\right)^2.$$



Natural inflation and electromagnetic dissipation

Steep inflation $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$. $f \sim M_P$



Background metric

Nearly de Sitter geometry $a(\tau) \approx -1/H\tau$ with constant Hubble parameter H,

$$ds^2 = \frac{1}{H^2\tau^2}(-d\tau^2 + dx_i dx^i).$$

Inflationary models with vectors

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$$-\frac{1}{4}\int d^4x\sqrt{-g}\left[F^{\mu\nu}F_{\mu\nu} + \frac{\alpha\phi}{f}\tilde{F}^{\mu\nu}F_{\mu\nu}\right],$$

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Inflationary models with vectors

Equations of motion

$$G_{\mu\nu} = \frac{1}{M_p^2}\left(T^{\phi}_{\mu\nu} + T^{A}_{\mu\nu} - \frac{1}{M^2}\Theta_{\mu\nu}\right) \longrightarrow \text{Gravity}$$

$$\left(g^{\mu\nu} - \frac{1}{M^2}G^{\mu\nu}\right)\nabla_\mu \nabla_\nu \phi - V_\phi - \frac{\alpha}{4f}F^{\mu\nu}\tilde{F}_{\mu\nu} = 0 \longrightarrow \text{Scalar}$$

$$\nabla_\mu\left(F^{\mu\nu} + \frac{\alpha}{f}\phi\,\tilde{F}^{\mu\nu}\right) = 0 \longrightarrow \text{Vector}$$

Inflationary models with vectors

Energy momentum tensor

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V\right) \longrightarrow$$ Scalar

$$T^{A}_{\mu\nu} = F_{\mu\alpha}F_{\nu}{}^{\alpha} - g_{\mu\nu}\frac{1}{4}F^2 \longrightarrow$$ Vector

$$\Theta_{\mu\nu} = -\frac{R}{2}\nabla_\mu\phi\nabla_\nu\phi + 2\nabla_{(\mu}\phi R_{\nu)}{}^{\alpha}\nabla_\alpha\phi - \frac{1}{2}(\nabla\phi)^2 G_{\mu\nu} + \nabla^\alpha\phi\nabla^\beta\phi R_{\mu\alpha\nu\beta} + \nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla_\alpha\phi$$
$$-\nabla_\mu\nabla_\nu\phi\Box\phi + \frac{1}{2}g_{\mu\nu}\left((\Box\phi)^2 - \nabla^\alpha\nabla^\beta\phi\nabla_\alpha\nabla_\beta\phi - 2\nabla_\alpha\phi\nabla_\beta\phi R^{\alpha\beta}\right)$$

↘ Gravity

Inflationary models with vectors

EOM in FLRW

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2\left(1+9\frac{H^2}{M^2}\right)+V(\phi)+\frac{1}{2}(\vec{E}^2+\vec{B}^2)\right) \longrightarrow \text{Gravity}$$

$$\dot{H} = -\frac{1}{2M_P^2}\dot{\phi}^2\left(1-\frac{\dot{H}}{M^2}+3\frac{H^2}{M^2}-2\frac{H\ddot{\phi}}{M^2\dot{\phi}}\right)-\frac{1}{3M_P^2}(\vec{E}^2+\vec{B}^2)$$

$$\ddot{\phi}\left(1+\frac{3H^2}{M^2}\right)+3H\dot{\phi}\left(1+\frac{2\dot{H}}{M^2}+\frac{3H^2}{M^2}\right)=-V_{,\phi}+\frac{\alpha}{f}\langle\vec{E}\cdot\vec{B}\rangle$$

$\longrightarrow$ Scalar

$$A''_\pm+\left(k^2\pm\frac{2\,k\,\xi}{\tau}\right)A_\pm=0 \quad \textbf{with} \quad \xi\equiv\frac{\alpha\,\dot{\phi}_0}{2fH}=\frac{\alpha\,\phi'_0}{2afH}$$

Vector

Inflationary models with vectors

Solutions for the vector modes

The helicity model + is enhanced. Parity breaking feature.

$$A_+ \approx \frac{1}{\sqrt{2k}} \left(\frac{k}{2\,\xi\, a\, H} \right)^{1/4} e^{\pi\,\xi - 2\sqrt{2\xi\, k/(aH)}}, \quad |\,k\,\tau\,| \ll 2\,\xi$$

$$\langle \vec{E} \cdot \vec{B} \rangle \approx -\,\mathcal{I}\,\frac{H^4}{\xi^4}\, e^{2\pi\xi}, \quad \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle \approx \frac{4\mathcal{I}}{7}\frac{H^4}{\xi^3}\, e^{2\pi\xi},$$

$$\mathcal{I} \approx 2.4 \times 10^{-4}\,.$$

Inflationary models with vectors

Perturbations equations

Friction terms combine. They affect the evolution of the scalar and the perturbations.

$$\delta\phi'' - \frac{2}{\tau}\left(1 - \frac{\pi\alpha V_\phi}{2KfH^2}\right)\delta\phi' + \frac{a^2\, V_{\phi\phi}}{K}\,\delta\phi = \frac{a^2\,\alpha}{Kf}\delta_{\overrightarrow{E}\cdot\overrightarrow{B}}\,.$$

$$K = 1 + 3\frac{H^2}{M^2}$$

$$H \gg M$$

$$\delta\phi(\tau,\overrightarrow{k}) = \frac{\alpha}{Kf}\int_{-\infty}^{\tau} d\tau_1\, a^2(\tau_1)\, G(\tau,\tau_1)\int d^3x\, e^{-i\overrightarrow{k}\cdot\overrightarrow{x}}\,\delta_{\overrightarrow{E}\cdot\overrightarrow{B}}(\tau_1,\overrightarrow{x})\,.$$

$$G(\tau,\tau') = \frac{\tau'}{\Delta}\left[\left(\frac{\tau}{\tau'}\right)^{\nu_+} - \left(\frac{\tau}{\tau'}\right)^{\nu_-}\right]\Theta(\tau-\tau')\,,$$

with

$$\nu_\pm \equiv \frac{1}{2}\left(3 - \frac{\pi\alpha V_\phi}{fH^2K} \pm \Delta\right)$$

$$\Delta \equiv 3\sqrt{\left(1 - \frac{\pi\alpha V_\phi}{3fH^2K}\right)^2 - \frac{4V_{\phi\phi}}{9H^2K}}$$

Inflationary models with vectors

Power specturum of the scalar perturbations

The perturbations are suppressed by the M scale!

$$\mathscr{P}_\zeta(p) \approx \frac{81\pi^2 5\times 10^{-2} M_p^4 \alpha^2}{f^4 K^2 \Delta^2 \xi^2}(-2^5 \xi p\tau)^{2\nu_-} \approx \frac{40 M_p^4 \alpha^2}{f^4 K^2 \Delta^2 \xi^2}(-2^5 \xi p\tau)^{2\nu_-}.$$

$$n_s - 1 \approx 2\nu_- = 3\left(1 - \frac{\pi\alpha V_\phi}{3fH^2K} - \sqrt{\left(1 - \frac{\pi\alpha V_\phi}{3fH^2K}\right)^2 - \frac{4V_{\phi\phi}}{9H^2K}}\right)$$

Non minimal coupling with gravity

Pseudoscalar coupled to gauge fields. Results for the scalar and tensor spectrum.

- Scalar perturbations spectrum

$\mathcal{N} \sim 10^5$

$$\bar{\mathcal{P}}_\zeta(p) \approx \frac{\mathcal{F}(\bar{\nu}_+)}{8\pi^4 \mathcal{I}^2 \xi^2} (-2^5 \xi p \tau)^{2\nu_+} \approx \frac{5 \times 10^{-2}}{\mathcal{N}\xi^2} (-2^5 \xi p \tau)^{2\nu_+} .$$

$$\bar{n}_s - 1 \approx 2\bar{\nu}_+ = -K^{1/2} \frac{2f \bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}{\pi\alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)} \left(1 + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \frac{\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}\right) .$$

- Tensor perturbations

$$d\bar{s}^2 = -\bar{a}^2(\bar{\tau}) \left[-d\bar{\tau}^2 + (\delta_{ij} + \bar{h}_{ij}) d\bar{x}_i d\bar{x}_j\right] \qquad \bar{T}_{ij}^{\text{EM}} = -\bar{a}^2 (E_i E_j + B_i B_j) + (\cdots)\delta_{ij} .$$

$$\langle h_\pm h_\pm \rangle \approx \frac{\bar{H}^2}{\pi^2 M_P^2} \left(1 + \mathcal{A}^\pm \frac{\mathcal{N} \bar{H}^2}{M_P^2} \frac{e^{4\pi\xi}}{\xi^6}\right)$$

Inflationary models with vectors

Tensor perturbations

Chiral sourced gravitational waves.

C. Germani & Y. Watanabe
JCAP 1107 (2011) 031

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx_i dx_j\right],$$

$$S_{h^2} = \frac{M_p^2}{8}\int d^3x d\tau\, a^2 \left[\left(1 - \frac{\phi'^2}{2a^2M^2M_p^2}\right)h_{ij}'^2 - \left(1 + \frac{\phi'^2}{2a^2M^2M_p^2}\right)(\nabla h_{ij})^2\right],$$

$$h_{ij}'' + \left(2\frac{a'}{a} + \frac{\beta(\tau)'}{\beta(\tau)}\right)h_{ij}' + k^2 c_t(\tau)^2 h_{ij} = \frac{2}{\beta(\tau)M_P^2}T_{ij}^{EM},$$

$$\beta(\tau) \equiv \left(1 - \frac{\phi'^2}{2a^2M^2M_p^2}\right) < 1 \quad \textbf{and} \quad c_t(\tau)^2 \equiv \frac{1 + \frac{\phi'^2}{2a^2M^2M_p^2}}{1 - \frac{\phi'^2}{2a^2M^2M_p^2}} > 1.$$

Inflationary models with vectors

Tensor perturbations

Chiral sourced gravitational waves.

$$h''_\lambda - 2\left(\frac{1}{\tau} - \frac{\beta'(\tau)}{2\beta(\tau)}\right) h'_\lambda + k^2 c_t(\tau)^2\, h_\lambda = \frac{2}{\beta(\tau) M_P^2} \Pi_\lambda^{lm}\, T_{lm}^{EM}$$

$$\beta(\tau) \equiv \left(1 - \frac{2\xi^2}{3\alpha^2}\frac{f^2}{M_p^2} K\right) < 1 \quad \textbf{and} \quad c_t(\tau)^2 \equiv \frac{1 + \frac{2\xi^2}{3\alpha^2}\frac{f^2}{M_p^2}K}{1 - \frac{2\xi^2}{3\alpha^2}\frac{f^2}{M_p^2}K} > 1$$

Inflationary models with vectors

Spectrum of tensor perturbations

Chiral sourced gravitational waves.

$$\mathscr{P}_h = \frac{k^3}{2\pi^2}\sum_\lambda |h_\lambda|^2 \approx \frac{H^2}{c_t}\pi^2 M_P^2\left(1+\frac{\dot{\phi}^2}{2M_p^2M^2}\right) \longrightarrow \text{Vacuum}$$

$$\mathscr{P}^{(\mathrm{s})\pm} = \mathscr{A}^{\pm}\frac{H^2}{\beta^2 M_P^2}\frac{e^{4\pi\xi}}{\xi^6} \longrightarrow \text{Source}$$

$$\mathscr{P}^{t\pm} = \frac{H^2}{c_t\pi^2 M_P^2\left(1+\frac{\dot{\phi}^2}{2M_p^2M^2}\right)}\left(1+\mathscr{A}^{\pm}\frac{H^2}{\beta^2 M_P^2}\frac{e^{4\pi\xi}}{\xi^6}\right)$$

Enhancement of GW

Inflationary models with vectors

Spectrum of tensor perturbations

Measurable parameter. Tensor to scalar ratio.

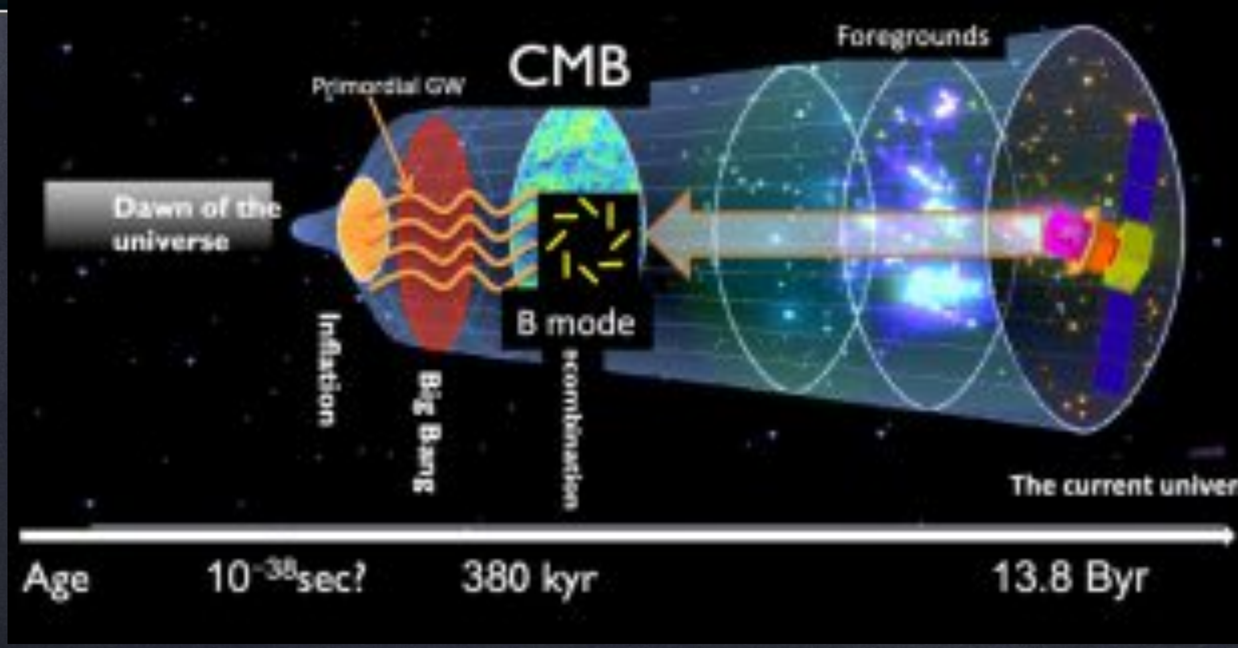
$$r = \frac{\mathscr{P}^{t+} + \mathscr{P}^{t-}}{\mathscr{P}_\zeta} = \frac{H^2}{c_t \pi^2 M_P^2 \left(1 + \frac{\dot{\phi}^2}{2M_p^2 M^2}\right)} \frac{2 + (\mathscr{A}^+ + \mathscr{A}^-)\frac{H^2}{\beta^2 M_P^2}\frac{e^{4\pi\xi}}{\xi^6}}{\mathscr{P}_\zeta}$$

Statistics of the sourced GW. Non gaussianities.

$\langle B^+ B^+ B^+ \rangle$ at 3σ with LiteBIRD

Observations

Lite BIRD: Light satellite for the studies of B-mode polarization and Inflation from cosmic background Radiation Detection (2020).



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Science

LiteBIRD is a satellite that will search for primordial gravitational waves emitted during the cosmic inflation era (around 10^{-38} sec after the beginning of the Universe). It goal is to test representative inflationary models (single-field slow-role models with large field variation) by performing an all-sky CMB polarization survey.

Primordial gravitational waves are expected to be imprinted in the CMB polarization map as special patterns, called the "B-mode". If we succeed to detect them, it will provide entirely new and profound knowledge on how our Universe began.

From the viewpoint of high-energy physics or elementary particle physics, the observation of the CMB B-mode is very important because it will allow us to search for physics in ultra high-energy scales, which are not accessible with man-made accelerators. Measurements of CMB polarization will open a new era of testing theoretical predictions of quantum gravity, including those by the superstring theory.

Conclusions and Remarks

1. Gravitational waves can be a good quantity to "detect" parity breaking signatures.
2. Topologic terms like F~F acquire non trivial dynamics when coupled to a scalar field.
3. Kinetic couplings are useful to reduce the velocity of the inflaton. At the same time, it is useful to suppress the amplitude of the scalar perturbations.
4. Kinetic couplings with the Einstein term maintain 2nd order derivatives in the EOM.