Cosmology From Gauge Fields

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What is our Universe made of?

EYMH Theory in $SU(2)$

Action →

$$S = \int \left[\frac{m_p^2}{2}R - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - (D_\mu\Phi^a)^\dagger (D^\mu\Phi_a) - V\left(\Phi_a^\dagger\Phi^a\right) + L_{mr}\right]\sqrt{-g}d^4x$$

Covariant derivative

$$D_\mu\Phi^a = \partial_\mu\Phi^a - \frac{i}{2}\gamma A^b_\mu\sigma_b\Phi^a$$

Mexican Hat Potential

$$V = \frac{\lambda}{4}(\Phi^2 - \Phi_0^2)^2$$

FLRW Universe → Homogeneous and Isotropic

$$ds^2 = -dt^2 + a(t)^2 dx_i dx_j \delta^{ij}$$

Cosmic Triad

$$A^b_\mu = \left(0, f(t)\delta^b_i\right)$$

- M. Rinaldi, Class. Quantum Grav. **32**, 045002 (2015).

Energy Tensor →

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}\right)}{\delta g^{\mu\nu}}$$

Contribution of the Scalar Field →

$$T^{\Phi}_{\mu\nu} = 2\left(D_{(\mu}\Phi^{a}\right)^{\dagger}\left(D_{\nu)}\Phi_{a}\right) - g_{\mu\nu}\left[\left(D^{\mu}\Phi^{a}\right)^{\dagger}\left(D_{\mu}\Phi_{a}\right) + V\right]$$

$$T^{\Phi}_{0i} = 2\left(D_{(0}\Phi^{a}\right)^{\dagger}\left(D_{i)}\Phi_{a}\right) = \gamma f \mathrm{Im}\left(\dot{\Phi}^{\dagger}_{a}\sigma_{i}\Phi^{a}\right) \neq 0$$

Solution? → Endless

$$\dot{\phi}_1\psi_2 - \phi_1\dot{\psi}_2 + \dot{\phi}_2\psi_1 - \phi_2\dot{\psi}_1 = 0$$
$$\dot{\phi}_1\psi_2 + \phi_1\dot{\psi}_2 - \dot{\phi}_2\psi_1 - \phi_2\dot{\psi}_1 = 0$$
$$\dot{\phi}_1\psi_1 - \phi_1\dot{\psi}_1 - \dot{\phi}_2\psi_2 + \phi_2\dot{\psi}_2 = 0$$

Excellent!!! ✓

$$T^{\Phi}_{00} = \dot{\Phi}^2 + \frac{3\gamma^2 f^2 \Phi^2}{4a^2} + V$$

$$T^{\Phi}_{ij} = \left[\dot{\Phi}^2 - \frac{\gamma^2 f^2 \Phi^2}{4a^2} - V\right] a^2 \delta_{ij}$$

Unitary Gauge

$$\Phi^{a} = (\Phi, 0)$$

Field Equations

Friedmann Equations

$$3H^2 = \frac{3\dot{f}^2}{2a^2} + \frac{3\gamma^2 f^4}{2a^4} + \dot{\Phi}^2 + \frac{3\gamma^2 f^2 \Phi^2}{4a^2} + V + \rho_m + \rho_r$$

$$\dot{H} = -\left[\frac{\dot{f}^2}{a^2} + \frac{\gamma^2 f^4}{a^4} + \dot{\Phi}^2 + \frac{\gamma^2 f^2 \Phi^2}{4a^2} + \frac{1}{2}\rho_m + \frac{2}{3}\rho_r\right]$$

Equation for the Higgs Field →

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{3\gamma^2 f^2 \Phi}{4a^2} + V_\Phi = 0$$

Equation for the Gauge Field →

$$\ddot{f} + H\dot{f} + \frac{2\gamma^2 f^3}{a^2} + \frac{1}{2}\gamma^2\Phi^2 f = 0$$

Dynamical System

Normalized Expansion Variables:

$$x = \frac{1}{\sqrt{2}}\frac{\dot{f}}{aH} \qquad y = \frac{1}{\sqrt{2}}\frac{\gamma f^2}{a^2 H} \qquad w = \frac{\gamma f \Phi}{2aH} \qquad z = \frac{1}{\sqrt{3}}\frac{\dot{\Phi}}{H} \qquad v = \frac{1}{H}\sqrt{\frac{V}{3}} \qquad r = \frac{1}{H}\sqrt{\frac{\rho_r}{3}} \qquad l = \sqrt{2}a/f$$

Fixed Points

Kination ⟶ $x=0,\quad y=0,\quad w=0,\quad z=1,\quad v=0,\quad r=0,\quad l=0,\quad q=2$

Radiation ⟶ $x^2+y^2+r^2=1,\quad w=0,\quad z=0,\quad v=0,\quad l=0,\quad q=1$

Matter ⟶ $x=0,\quad y=0,\quad w=0,\quad z=0,\quad v=0,\quad r=0,\quad l=0,\quad q=1/2$

Transition ⟶ $x=0,\quad y=0,\quad w=1,\quad z=0,\quad v=0,\quad r=0,\quad l=0,\quad q=0$

Dark Energy ⟶ $x=0,\quad y=0,\quad w=0,\quad z=0,\quad v=1,\quad r=0,\quad l=0,\quad q=-1$

Stability Analysis

Jacobian → $J = J(x, y, w, z, v, r, l)$

Kination → $(3, 3, 2, 1, 1, 1, 1)$ → Repeller

Radiation → $(2, -1, 1, 1, 1, 0, 0)$ → Saddle

Matter → $\left(-\frac{3}{2}, \frac{3}{2}, 1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ → Saddle

Transition → $(-2, -1, -1, -1, -1, 1, 1)$ → Saddle - Inconsistent

Dark Energy → $(-3, -3, -2, -2, -2, -1, 1)$ → Attractor!!!

Numerical Analysis

Physical properties

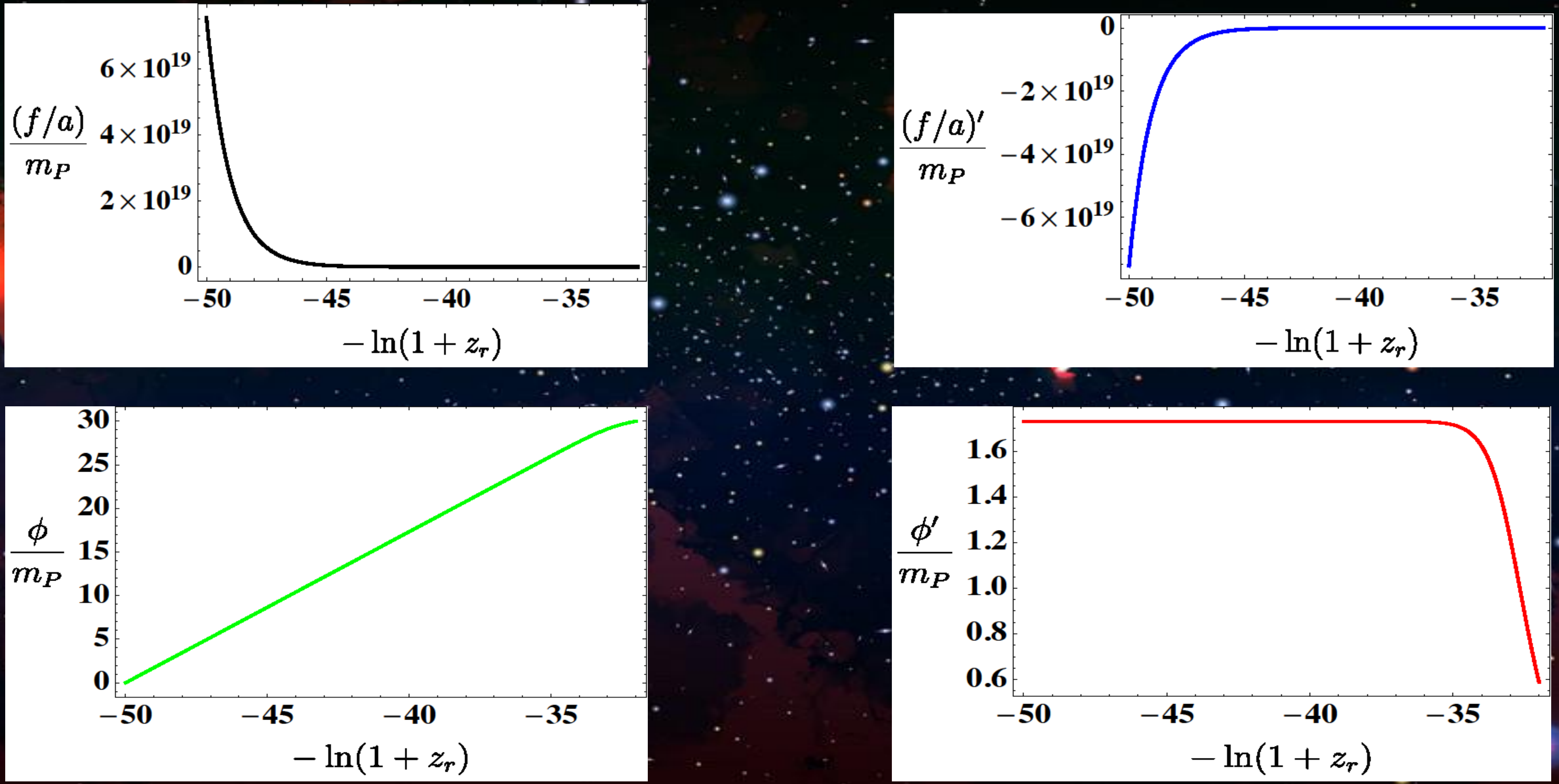
$$\frac{f/a}{m_P} = \frac{\sqrt{2}}{l} \qquad \frac{(f/a)'}{m_P} = \sqrt{2}\left(x - \frac{1}{l}\right)$$

$$\frac{\phi}{m_P} = 2\frac{w}{yl} \qquad \frac{\phi'}{m_P} = \sqrt{3}z$$

Initial conditions

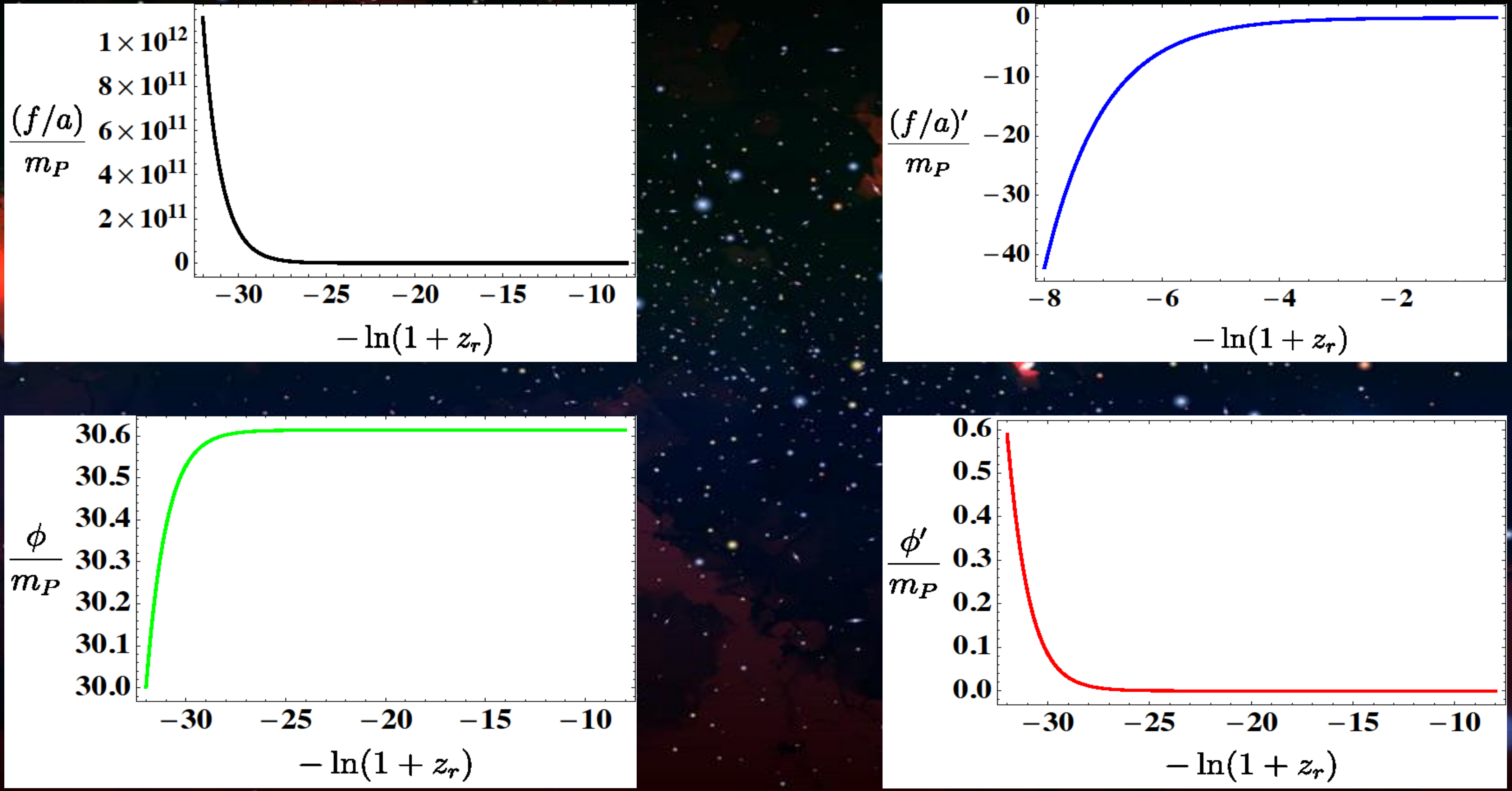
$$r_0 = 10^{-2}\,, \qquad m_0 = 0.557$$
$$x_0 = 10^{-18}\,, \qquad y_0 = 10^{-18}$$
$$w_0 = 10^{-18}\,, \qquad z_0 = 10^{-18}$$
$$v_0 = 0.831\,, \qquad l_0 = 10^2\,.$$

Kination



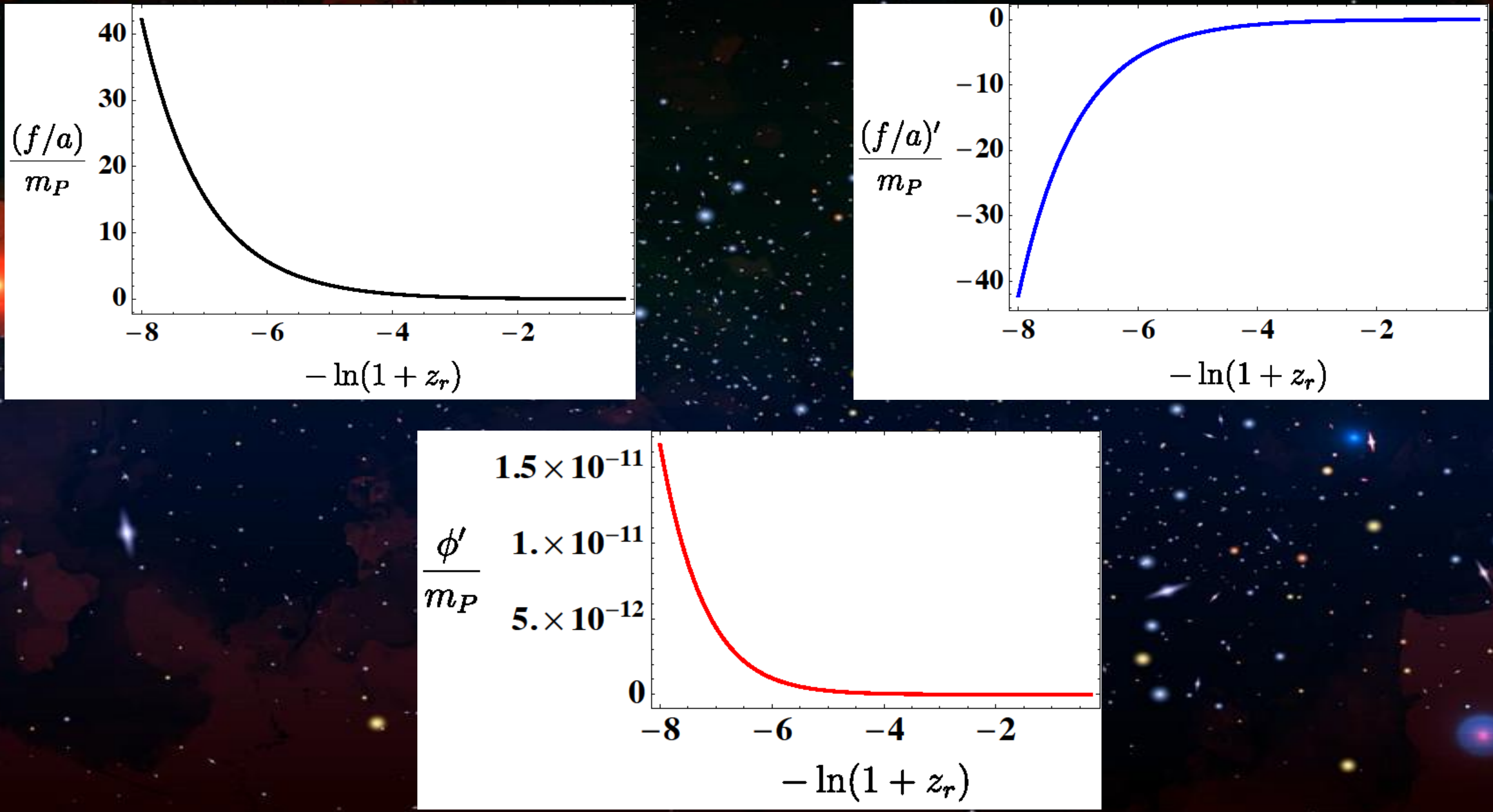
$\frac{(f/a)}{m_P}$
6×10^{19}
4×10^{19}
2×10^{19}
0
−50
−45
−40
−35
$-\ln(1+z_r)$
$\frac{(f/a)'}{m_P}$
0
-2×10^{19}
-4×10^{19}
-6×10^{19}
−50
−45
−40
−35
$-\ln(1+z_r)$
$\frac{\phi}{m_P}$
30
25
20
15
10
5
0
−50
−45
−40
−35
$-\ln(1+z_r)$
$\frac{\phi'}{m_P}$
1.6
1.4
1.2
1.0
0.8
0.6
−50
−45
−40
−35
$-\ln(1+z_r)$

Radiation



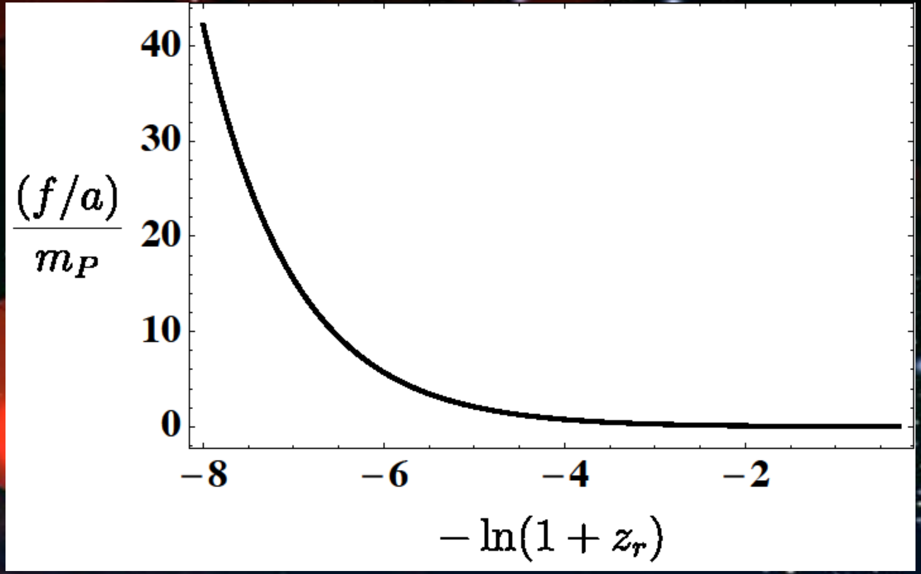
$\frac{(f/a)}{m_P}$
1×10^{12}
8×10^{11}
6×10^{11}
4×10^{11}
2×10^{11}
0
−30 −25 −20 −15 −10
$-\ln(1+z_r)$
$\frac{(f/a)'}{m_P}$
0
−10
−20
−30
−40
−8 −6 −4 −2
$-\ln(1+z_r)$
$\frac{\phi}{m_P}$
30.6
30.5
30.4
30.3
30.2
30.1
30.0
−30 −25 −20 −15 −10
$-\ln(1+z_r)$
$\frac{\phi'}{m_P}$
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−30 −25 −20 −15 −10
$-\ln(1+z_r)$

Matter

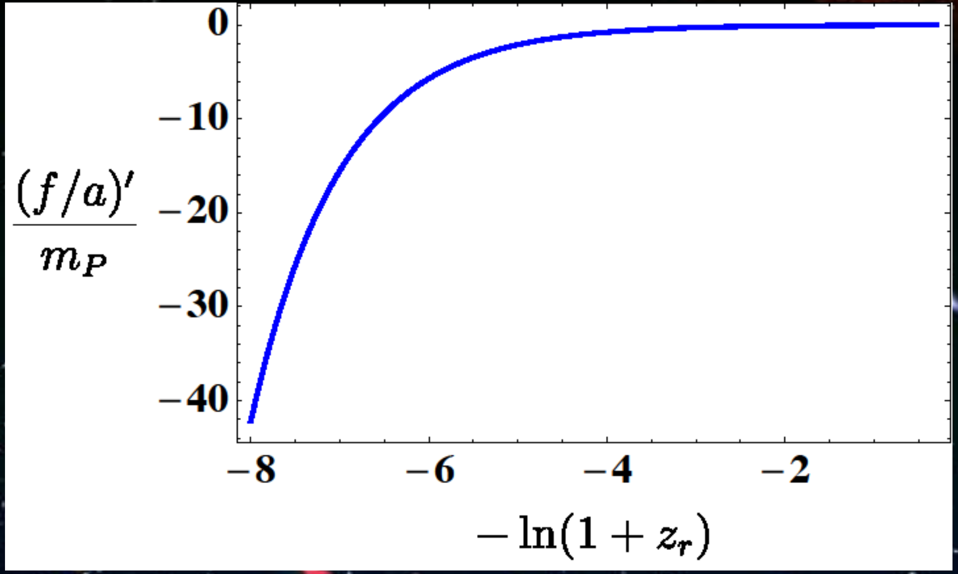


$\frac{(f/a)}{m_P}$
40
30
20
10
0
−8
−6
−4
−2
$-\ln(1+z_r)$
$\frac{(f/a)'}{m_P}$
0
−10
−20
−30
−40
−8
−6
−4
−2
$-\ln(1+z_r)$
$\frac{\phi'}{m_P}$
1.5×10^{-11}
$1.\times10^{-11}$
$5.\times10^{-12}$
0
−8
−6
−4
−2
$-\ln(1+z_r)$

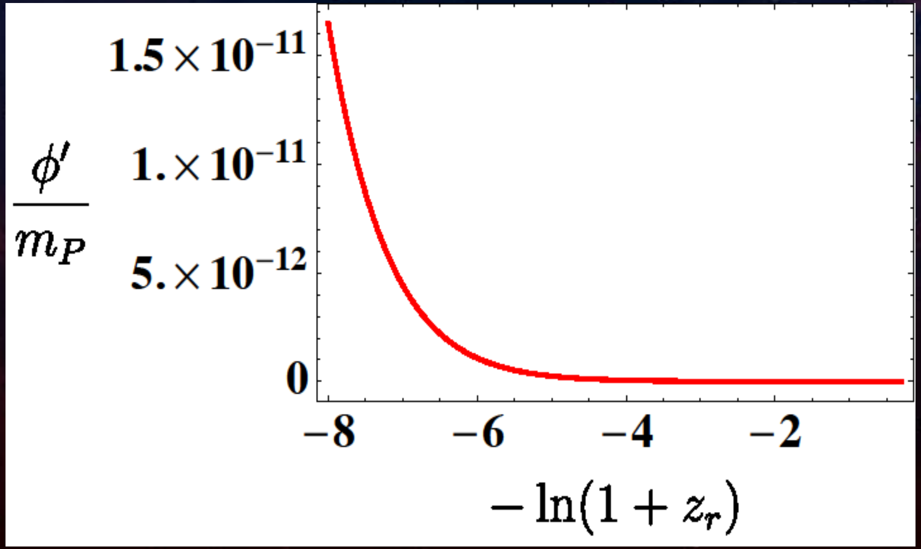
Dark Energy



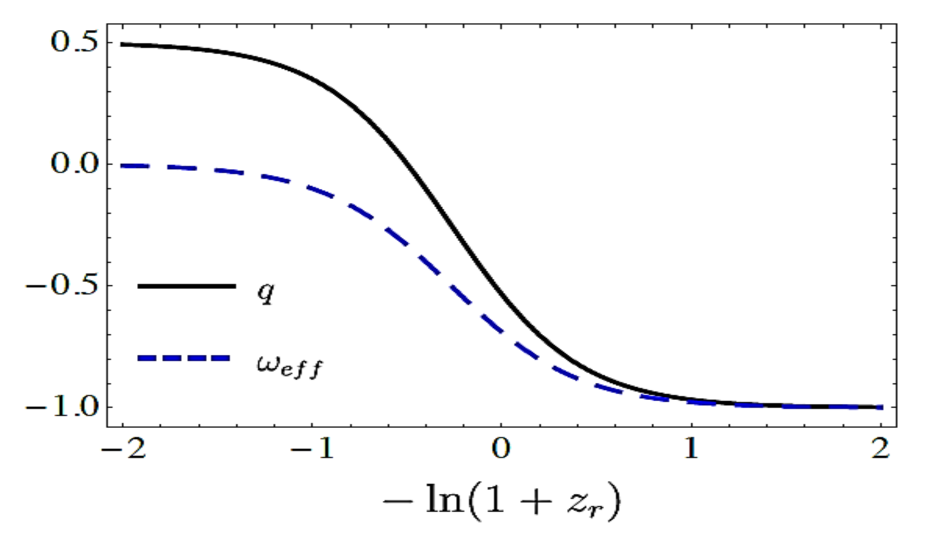
40
30
20
10
0
$\frac{(f/a)}{m_P}$
−8
−6
−4
−2
$-\ln(1+z_r)$



0
−10
−20
−30
−40
$\frac{(f/a)'}{m_P}$
−8
−6
−4
−2
$-\ln(1+z_r)$

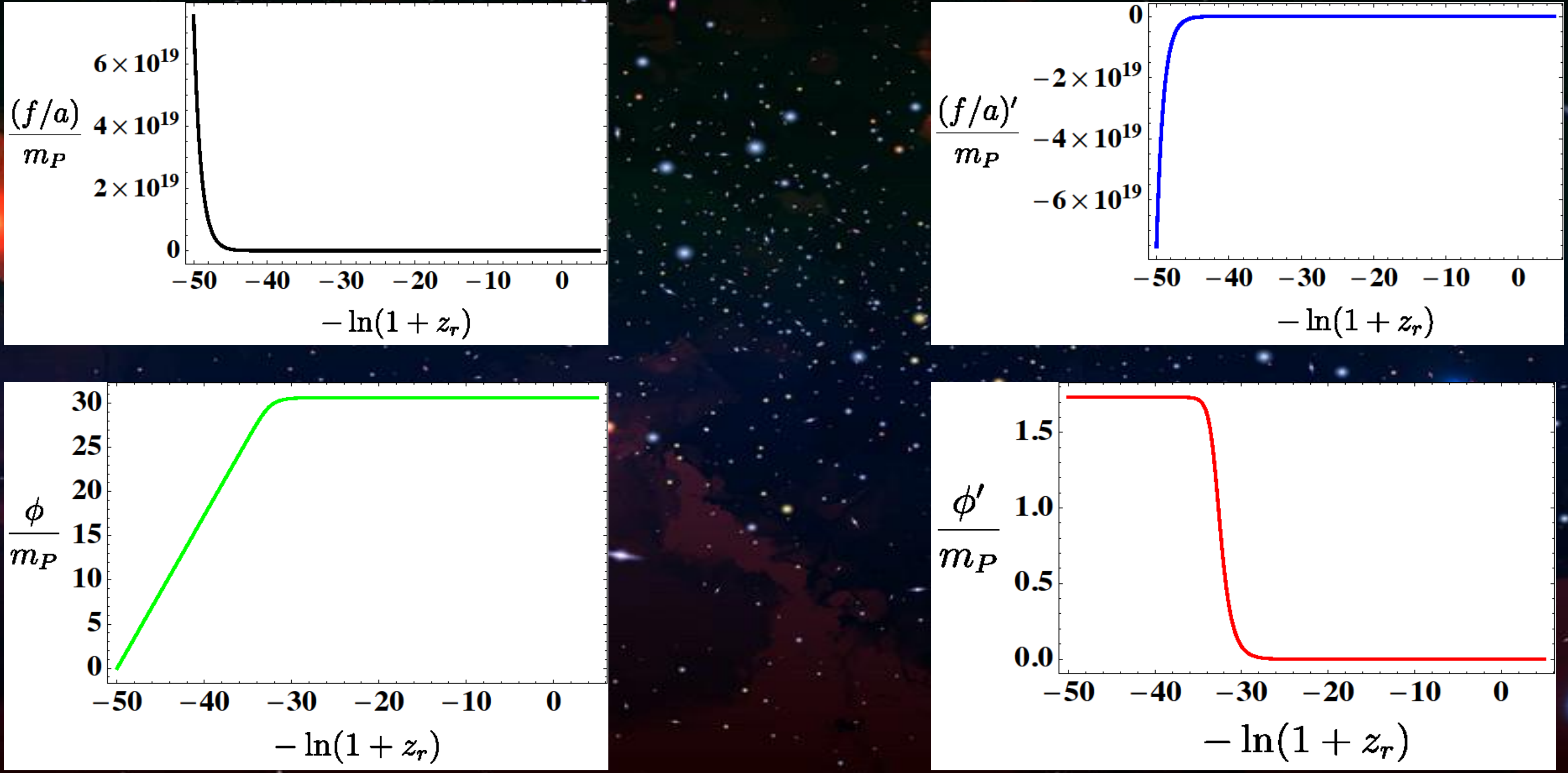


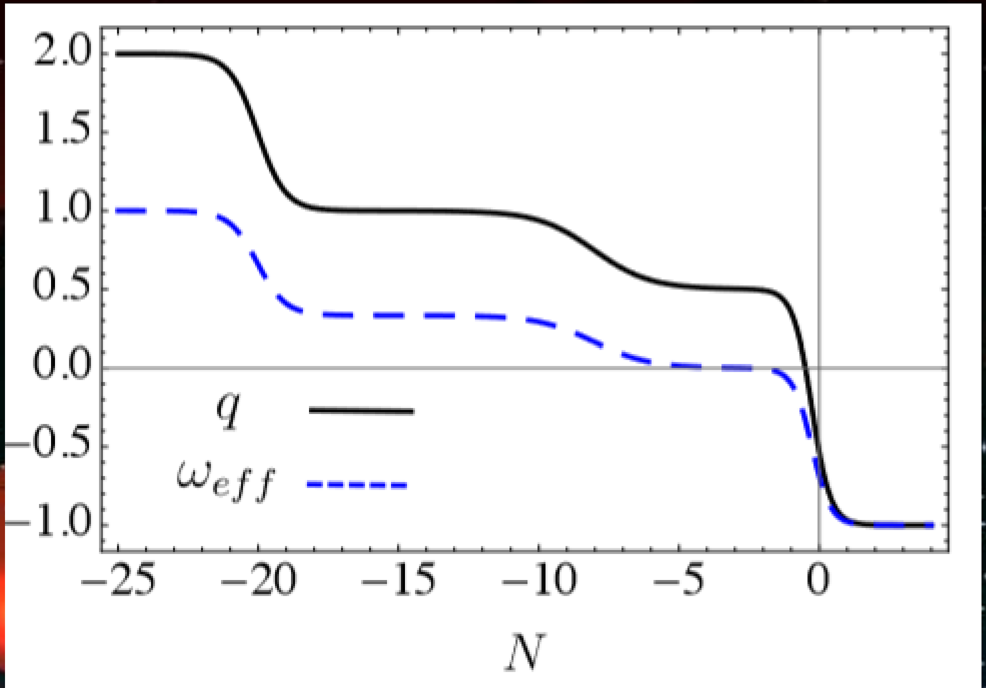
1.5×10^{-11}
$1.\times10^{-11}$
$5.\times10^{-12}$
0
$\frac{\phi'}{m_P}$
−8
−6
−4
−2
$-\ln(1+z_r)$



0.5
0.0
−0.5
−1.0
q
ω_{eff}
−2
−1
0
1
2
$-\ln(1+z_r)$

Global Evolution.



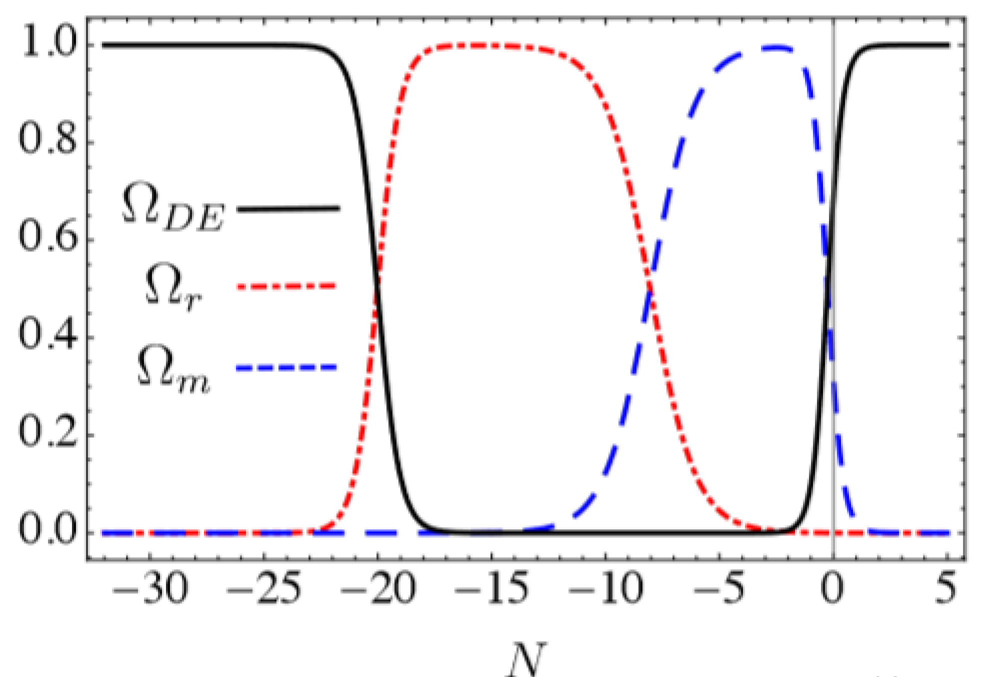


Evolution of the Deceleration Parameter and the Effective State Equation:

The final stage of the Universe is characterized by a negative state equation.

Evolution of the Density Parameters:

The first stage is the domination of the kinetic energy of Φ, passes through radiation and matter eras, eventually it reaches the dark energy stage which is an attractor. The Universe is condemned to suffer accelerated expansion.



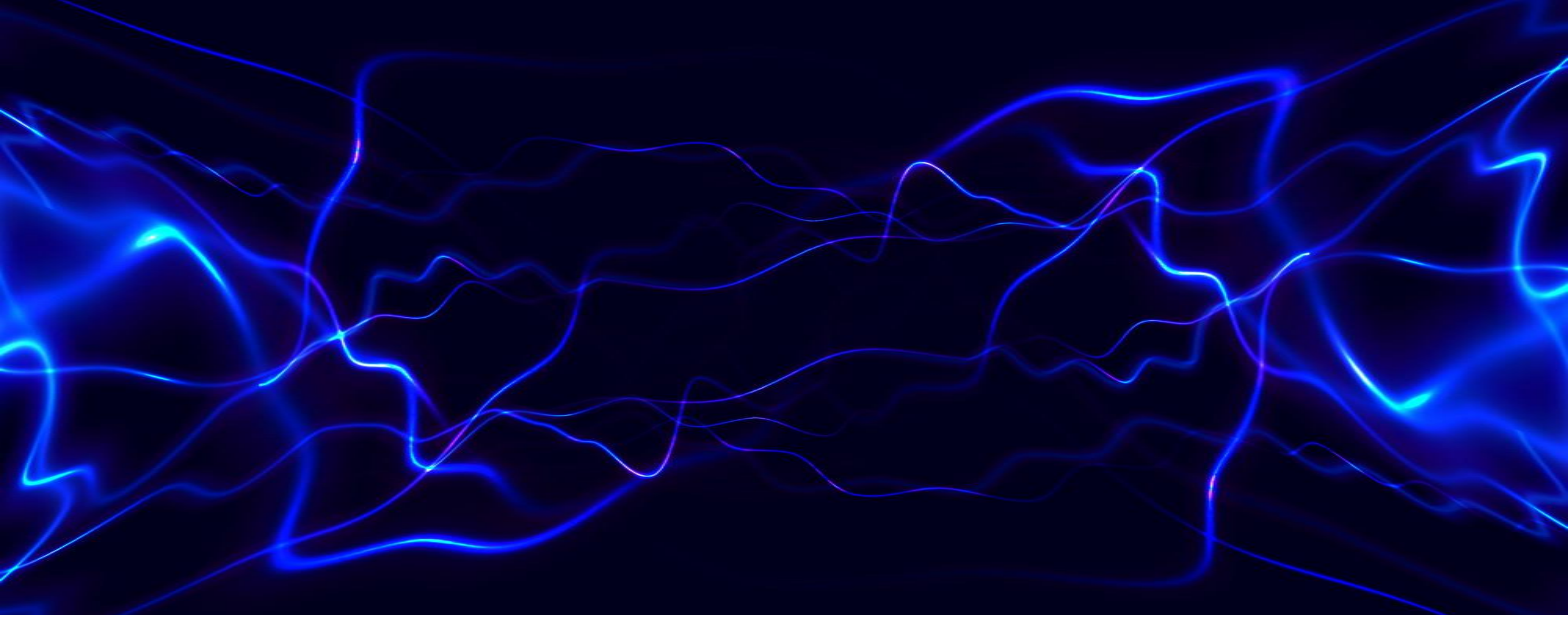
Conclusions.

- The EYMH theory in $SU(2)$ can reproduce the domination epochs of the Universe.

- A specific form for the gauge and Higgs fields is needed in order to get a theory consistent with an isotropic Universe.

- The dynamical system analysis reveals that dark energy domination is the only attractor point.

- The dark energy is kept thanks to the gauge field which is “pushing up” the Higgs field preventing it to fall to its vaccum.



Thank you!!!