A taste of flavor symmetries from string theory

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In collaboration with Y. Olguín-Trejo & R. Pérez-Martínez: arxiv:1708.01595 & arxiv:1808.06622

P. Vaudrevange: arxiv:1811.00580

(B. Carballo-Pérez, E. Peinado: arxiv:1607.06812)

The need for discrete flavor symmetries

- Need to explain
  - three flavors of SM particles
  - observed mass patterns
  - CKM, PMNS phases
  - neutrino physics
  - suppression of proton decay
  - baryogenesis
  - CP violation
  - (even) dark matter?
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- There is a plethora of non-Abelian discrete symmetries that satisfy basic phenomenological constraints

- Problem: Which one is THE right one ??

Strings as a guiding principle

In Abelian, toroidal heterotic orbifolds

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B: discrete charges related to S $\rightarrow \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \mathbb{Z}_{N_3} \times \cdots$

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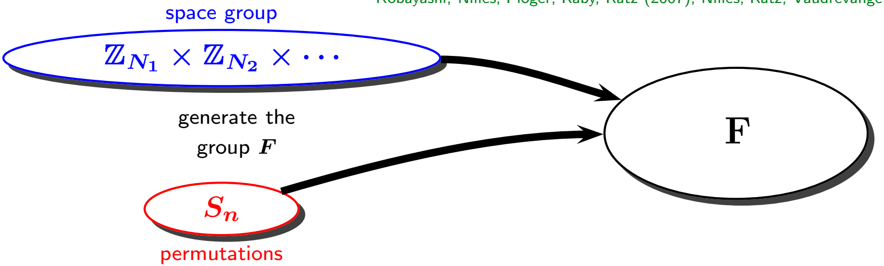

$\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots \cup S_n = F$

Abelian non-Abelian **non-Abelian!**

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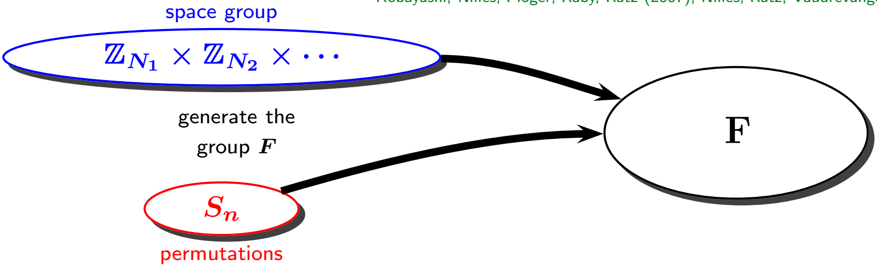


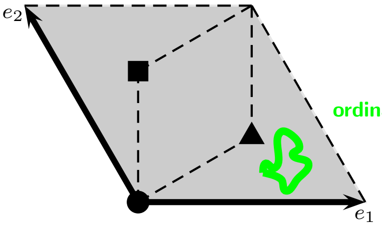
$\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots \rtimes S_n = F$

Abelian — non-Abelian — **non-Abelian!**

Untwisted and twisted closed strings in $\mathbb{T}^2/\mathbb{Z}_3$

Two types of massless *closed strings*:

ordinarily closed and closed under the orbifold



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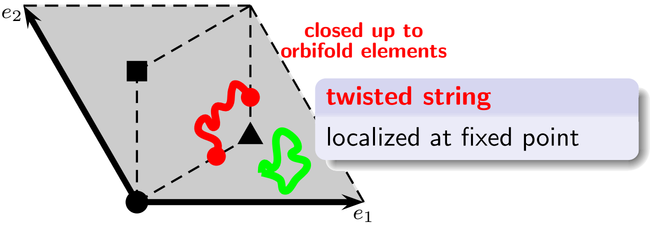

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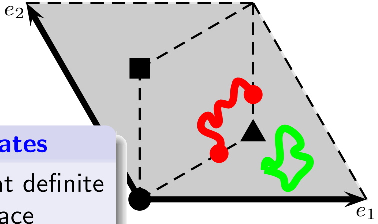

localized twisted states

LEEF states appear at definite points in compact space

Twisted strings located at fixed points $\rightarrow$ LEEF states localized

Identified with constructing elements $g_\ell \in S$ (boundary conditions for string closure)

$$g_\ell = (\vartheta^{k^{(\ell)}}, n_\alpha^{(\ell)} e_\alpha)$$

Invariance under S

- Interacting strings join for the time they interact:

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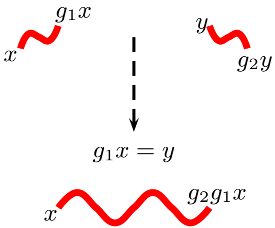

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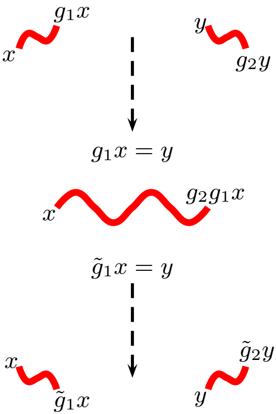

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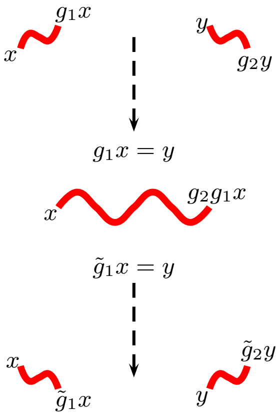

$$g_2\, g_1 = \tilde{g}_2\, \tilde{g}_1 = \mathbb{1}$$

for L interacting strings:

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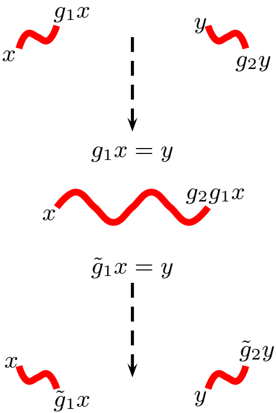

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space group selection rule:

$$\prod_{\ell=1}^{L}(\vartheta^{k^{(\ell)}},\, n_\alpha^{(\ell)} e_\alpha) = (\mathbb{1}, \bigcup_\ell (\mathbb{1} - \vartheta^{k^{(\ell)}})\Lambda)$$

- E.g. $\vartheta^N = \mathbb{1} \quad \longrightarrow \quad \sum k^{(\ell)} = 0 \mod N \quad \longrightarrow \quad \mathbb{Z}_N$

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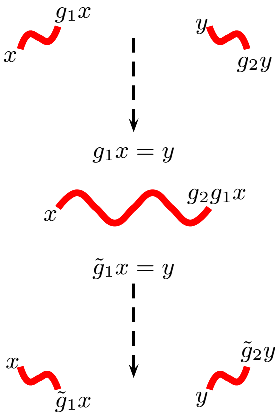

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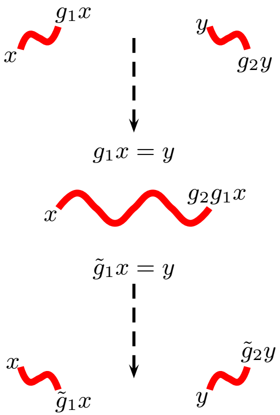

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Can be reformulated as

$$\prod_\ell h_\ell g_\ell h_\ell^{-1} = (\mathbb{1}, 0)$$

We propose a (non-faithful) representation

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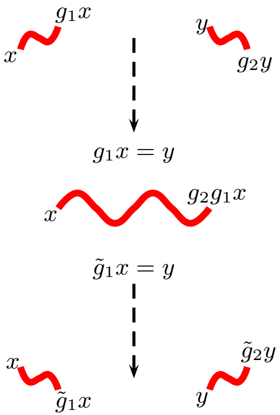

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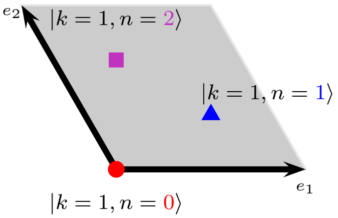
- This Abelianization lead to $\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots$

SRS, Vaudrevange(2018)

Two 2D examples: $\mathbb{T}^2/\mathbb{Z}_3$

space group rule



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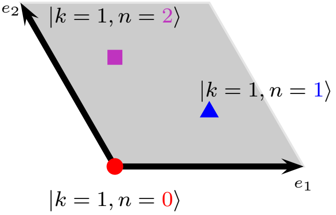

$$\prod_{\ell=1}^{L}(\vartheta^{k^{(\ell)}}, n^{(\ell)}e_1) = (\vartheta^{\sum_\ell k^{(\ell)}}, (\textstyle\sum_\ell n^{(\ell)})e_1)$$
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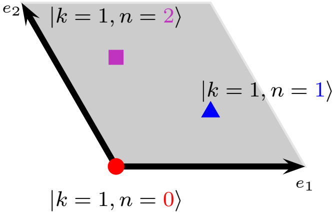

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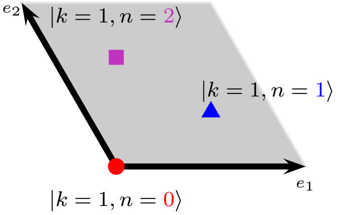

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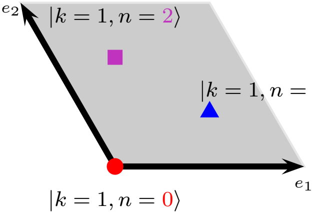

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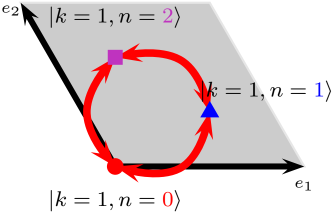

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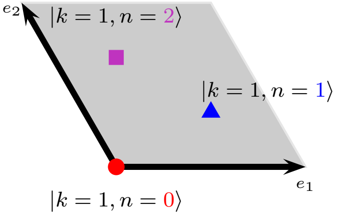

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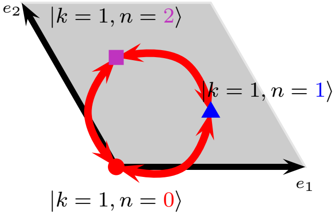

$\Downarrow$

permutation symmetry:

S_3

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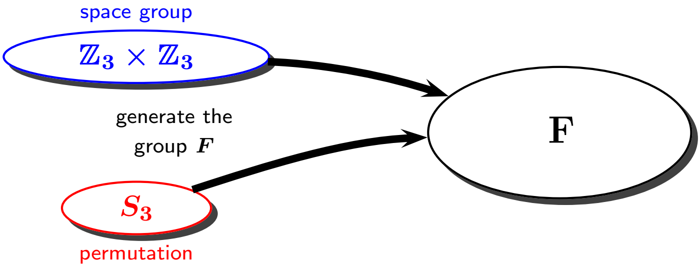
- Invariance of couplings under:



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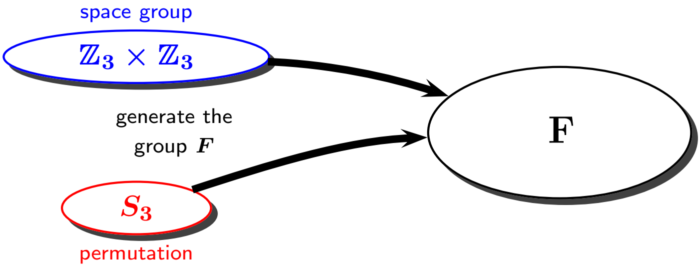

$$\underbrace{\mathbb{Z}_3 \times \mathbb{Z}_3}_{\text{Abelian}} \cup \underbrace{S_3}_{\text{non-Abelian}} \quad = \quad \underbrace{F}_{\text{non-Abelian!}}$$

- Flavor symmetry

$$F = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) = \Delta(54)$$

Examples in 1D, 2D

$$\rightarrow \quad F = D_4 = S_2 \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

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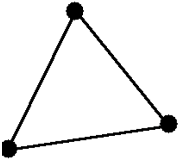

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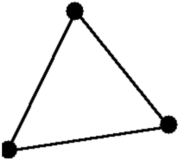

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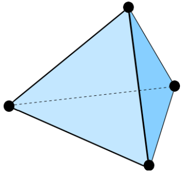

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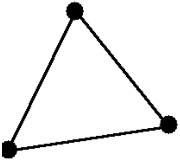

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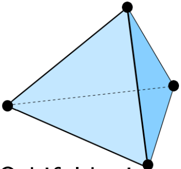
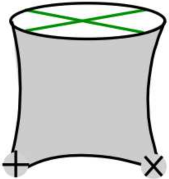


$$\rightarrow \quad F = D_4 \times D_4/\mathbb{Z}_2$$

Orbifold with roto-translations



$$\rightarrow \quad F = D_4$$

Natural question 1:

Aiming at a guiding principle from an UV complete theory,...

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Aiming at a guiding principle from an UV complete theory,...

what flavor symmetries are realized
in heterotic orbifolds?

Explore all 138 Abelian space groups S already known

Repeat previous steps for the all possible space groups

Orbifold label	Twist vector(s)	# of $\mathbb{Z}$ classes	# of affine classes
$\mathbb{Z}_3$	$\frac{1}{3}(1,1,-2)$	1	1
$\mathbb{Z}_4$	$\frac{1}{4}(1,1,-2)$	3	3
$\mathbb{Z}_6$–I	$\frac{1}{6}(1,1,-2)$	2	2
$\mathbb{Z}_6$–II	$\frac{1}{6}(1,2,-3)$	4	4
$\mathbb{Z}_7$	$\frac{1}{7}(1,2,-3)$	1	1
$\mathbb{Z}_8$–I	$\frac{1}{8}(1,2,-3)$	3	3
$\mathbb{Z}_8$–II	$\frac{1}{8}(1,3,-4)$	2	2
$\mathbb{Z}_{12}$–I	$\frac{1}{12}(1,4,-5)$	2	2
$\mathbb{Z}_{12}$–II	$\frac{1}{12}(1,5,-6)$	1	1
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{2}(1,0,-1)$	12	35
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{4}(1,0,-1)$	10	41
$\mathbb{Z}_2 \times \mathbb{Z}_6$–I	$\frac{1}{2}(0,1,-1)$, $\frac{1}{6}(1,0,-1)$	2	4
$\mathbb{Z}_2 \times \mathbb{Z}_6$–II	$\frac{1}{2}(0,1,-1)$, $\frac{1}{6}(1,1,-2)$	4	4
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$\frac{1}{3}(0,1,-1)$, $\frac{1}{3}(1,0,-1)$	5	15
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$\frac{1}{3}(0,1,-1)$, $\frac{1}{6}(1,0,-1)$	2	4
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$\frac{1}{4}(0,1,-1)$, $\frac{1}{4}(1,0,-1)$	5	15
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$\frac{1}{6}(0,1,-1)$, $\frac{1}{6}(1,0,-1)$	1	1

Fischer, Ratz, Torrado, Vaudrevange (2013)

We find all possible flavor symmetries (without enhancement)

Orbifold		Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^4$
	(1,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(1,3)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
	(1,4)	–
	(2,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^3$
	(2,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(2,3)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
	(2,4)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(2,5)	$(D_4 \times D_4)$
	(2,6)	–
	(3,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
	(3,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(3,3)	$(D_4 \times D_4)/\mathbb{Z}_2$
	(3,4)	–
	(4,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
	(4,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(5,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
	(5,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(5,3)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(5,4)	$(D_4 \times D_4)$
	(5,5)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(6,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
	(6,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(6,3)	D_4
	(7,1)	$(D_4 \times D_4)$
	(7,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(8,1)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(9,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
	(9,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(9,3)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(10,1)	$D_4 \times \mathbb{Z}_2$

Orbifold		Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(10,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
	(11,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
	(12,1)	$(D_4 \times D_4)$
	(12,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$
	(1,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(1,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(1,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(1,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(1,6)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
	(2,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$
	(2,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(2,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(2,4)	$D_4 \times \mathbb{Z}_4$
	(2,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(2,6)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(3,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
	(3,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(3,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(3,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(3,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(3,6)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(4,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
	(4,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(4,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(4,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(4,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(5,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2)$
	(5,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(6,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
	(6,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Orbifold		Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(6,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(6,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(6,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(7,1)	$D_4 \times \mathbb{Z}_4$
	(7,2)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
	(7,3)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
	(8,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
	(8,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(8,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
	(9,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
	(9,2)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
	(9,3)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
	(10,1)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
	(10,2)	$\mathbb{Z}_4 \times \mathbb{Z}_2$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	(1,1)	$(D_4 \times D_4 \times \mathbb{Z}_6)/\mathbb{Z}_2$
	(1,2)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
	(2,1)	$(D_4 \times D_4 \times \mathbb{Z}_6)/\mathbb{Z}_2$
	(2,2)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	(1,1)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
	(2,1)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
	(3,1)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
	(4,1)	$\mathbb{Z}_2 \times \mathbb{Z}_6$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	$(\Delta(54) \times \Delta(54) \times \Delta(54))/\mathbb{Z}_3$
	(1,2)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(1,3)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(1,4)	$\Delta(54) \times \Delta(54)$
	(2,1)	$\Delta(54) \times \Delta(54)$
	(2,2)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(2,3)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(2,4)	$\Delta(54) \times \Delta(54)$
	(3,1)	$\Delta(54) \times \mathbb{Z}_3$
	(3,2)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(3,3)	$\Delta(54) \times \Delta(54)$
	(4,1)	$\Delta(54) \times \Delta(54)$
	(4,2)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
	(4,3)	$\Delta(54) \times \Delta(54)$
	(5,1)	$\mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1,1)	$\Delta(54) \times \mathbb{Z}_6$
	(1,2)	$\mathbb{Z}_3 \times \mathbb{Z}_6$

Orbifold		Flavor symmetry
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(2,1)	$\Delta(54) \times \mathbb{Z}_6$
	(2,2)	$\mathbb{Z}_3 \times \mathbb{Z}_6$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$
	(1,2)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(1,3)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(1,4)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(2,1)	$(D_4 \times D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2^2$
	(2,2)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(2,3)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(2,4)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(3,1)	$(D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2$
	(3,2)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(4,1)	$(D_4 \times D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2^2$
	(4,2)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(4,3)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(5,1)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
	(5,2)	$\mathbb{Z}_4 \times \mathbb{Z}_4$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1,1)	$\mathbb{Z}_6 \times \mathbb{Z}_6$

Orbifold		Flavor symmetry
$\mathbb{Z}_3$	(1,1)	$(\Delta(54) \times \Delta(54) \times \Delta(54))/\mathbb{Z}_3^2$
$\mathbb{Z}_4$	(1,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^4$
	(2,1)	$(S_4 \times S_2 \times S_2) \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$
	(3,1)	$(S_4 \times S_4) \ltimes (\mathbb{Z}_4^5 \times \mathbb{Z}_2^2)$
$\mathbb{Z}_6$-I	(1,1)	$\Delta(54)$
	(2,1)	$(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3$
$\mathbb{Z}_6$-II	(1,1)	$\Delta(54) \times [D_4 \times D_4/\mathbb{Z}_2]$
	(2,1)	$[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4 \times D_4]/\mathbb{Z}_2^2$
	(3,1)	$[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4 \times D_4]/\mathbb{Z}_2^2$
	(4,1)	$[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4/\mathbb{Z}_2]$
$\mathbb{Z}_7$	(1,1)	$S_7 \ltimes \mathbb{Z}_7^6$
$\mathbb{Z}_8$-I	(1,1)	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$
	(2,1)	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$
	(3,1)	$S_4 \ltimes (\mathbb{Z}_8 \times \mathbb{Z}_4^2 \times \mathbb{Z}_2)$
$\mathbb{Z}_8$-II	(1,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^3$
	(2,1)	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$
$\mathbb{Z}_{12}$-I	(1,1)	$\Delta(54)$
	(2,1)	$(\Delta(54) \times \mathbb{Z}_{12})/\mathbb{Z}_3$
$\mathbb{Z}_{12}$-II	(1,1)	$(D_4 \times D_4)/\mathbb{Z}_2$

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Natural question 2:

Wilson lines are needed to get MSSM-like models; Ibáñez, Nilles, Quevedo (1987)

They break explicitly (the permutations in) flavor symmmetries...

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They break explicitly (the permutations in) flavor symmmetries...

what flavor symmetries *survive*
in realistic compactifications?

Flavor in realistic orbifolds

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1. Determine all (inequivalent) $\mathrm{E}_8\times\mathrm{E}_8$ gauge embeddings in heterotic string

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2. Select semi-realistic models
   - 4D gauge group $= \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times G_{hidden}$
   - 3 generations of quarks and leptons + a pair H_u, H_d
   - $\sin^2\theta_w(M_{GUT}) = 3/8$
   - only SM-vectorlike extra matter
   - no true anomalies

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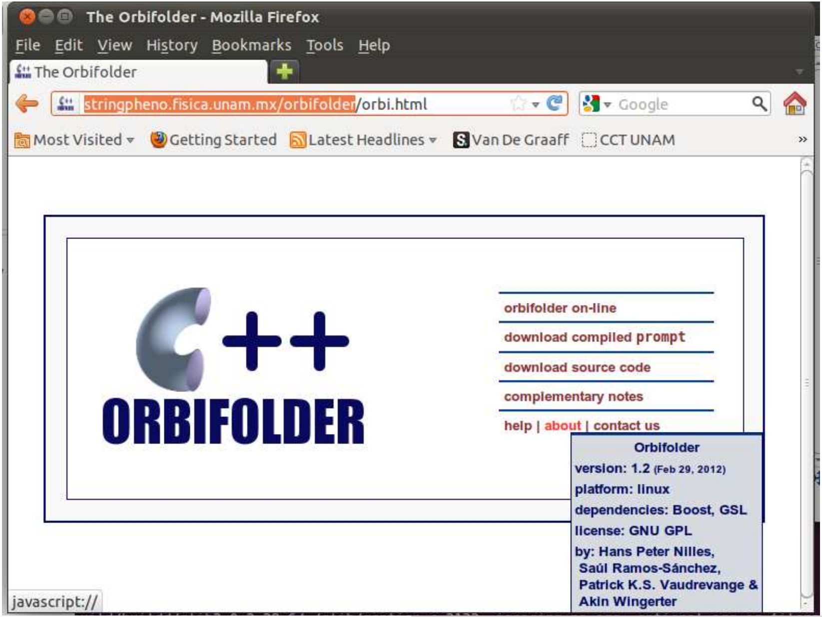
3. Determine the flavor breakdown due to Wilson lines in each case
4. Phenomenology?

Orbifolder needed as a tool (Nilles, SRS, Vaudrevange, Wingerter (2011))



Broadest known scan of MSSM-like models

Orbifold		Max # of independent WL	# of MSSM-like models with 0 vanishing WL	1	2	≥ 3	Total
$\mathbb{Z}_3$	(1,1)	3	0	0	0	0	0
$\mathbb{Z}_4$	(1,1)	4	0	0	0	0	0
	(2,1)	3	149	0	0	0	149
	(3,1)	2	27	0	0		27
$\mathbb{Z}_6$-I	(1,1)	1	30	0			30
	(2,1)	1	30	0			30
$\mathbb{Z}_6$-II	(1,1)	3	26	337	0	0	363
	(2,1)	3	14	335	0	0	349
	(3,1)	3	18	335	0	0	353
	(4,1)	2	44	312	0		356
$\mathbb{Z}_7$	(1,1)	1	1	0			1
$\mathbb{Z}_8$-I	(1,1)	2	230	38	0		268
	(2,1)	2	205	41	0		246
	(3,1)	1	389	0			389
$\mathbb{Z}_8$-II	(1,1)	3	1,604	398	21	0	2,023
	(2,1)	2	274	231	0		505
$\mathbb{Z}_{12}$-I	(1,1)	1	556	0			556
	(2,1)	1	555	0			555
$\mathbb{Z}_{12}$-II	(1,1)	2	279	84	0		363

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Orbifold		Max # of independent WL	# of MSSM-like models with					Total
			0	1	2	3	≥ 4	
			vanishing WL					
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	6	1	152	52	0	0	205
	(2,1)	5	13	342	14	0	0	369
	(3,1)	5	4	400	40	0	0	444
	(5,1)	4	2	40	0	0	0	42
	(6,1)	4	344	57	0	0	0	401
	(7,1)	4	21	55	0	0	0	76
	(9,1)	3	25	2	0	0		27
	(10,1)	3	19	2	0	0		21
	(12,1)	2	3	0	0			3
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	4	454	8,637	1,463	26	0	10,580
	(1,6)	2	65	21	0			86
	(2,1)	4	260	4,683	1,131	81	0	6,158
	(2,4)	2	281	47	0			328
	(3,1)	3	13,117	3,637	103	0		16,857
	(4,1)	3	2,911	1,575	33	0		4,519
	(5,1)	3	1,311	742	63	0		2,116
	(6,1)	3	1,814	1,374	58	0		3,246
	(7,1)	3	1,481	1,122	64	0		2,667
	(8,1)	2	839	72	0			911
	(9,1)	2	1,620	522	0			2,142
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	(1,1)	2	467	116	0			583
	(2,1)	2	275	78	0			353
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	3	40	987	81	0		1,108
	(1,4)	1	8	0				8
	(2,1)	2	1,713	239	0			1,952
	(3,1)	2	6	0	0			6
	(4,1)	2	105	110	0			215
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1,1)	1	4,469	24				4,493
	(2,1)	1	495	45				540
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	3	599	12,091	2,258	5		14,953
	(2,1)	2	2,807	3,220	19			6,046
	(3,1)	2	2,039	875	6			2,920
	(4,1)	2	1,876	1,552	6			3,434
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1,1)	0	3,412					3,412

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Flavor symmetry with ℓ non-vanishing WLs $\ell = 1$	 2	 3	 4	Total models
$\mathbb{Z}_3$	(1,1)	3	$\Delta(54)^2$ 0	$\Delta(54) \times \mathbb{Z}_3^2$ 0	$\mathbb{Z}_3^2$ 0		0
$\mathbb{Z}_4$	(1,1)	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 0	$D_4^2 \times \mathbb{Z}_4$ 0	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0	0
	(2,1)	3	$(S_2 \times S_2) \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ 0 $(S_4 \times S_2) \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$S_2 \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ 0 $S_4 \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 149		149
	(3,1)	2	$S_4 \ltimes (\mathbb{Z}_4^4 \times \mathbb{Z}_2)$ 0	$\mathbb{Z}_4^3$ 27			27
$\mathbb{Z}_6$-I	(1,1)	1	$\mathbb{Z}_3^2$ 30				30
	(2,1)	1	$\mathbb{Z}_6 \times \mathbb{Z}_3$ 30				30
$\mathbb{Z}_6$-II	(1,1)	3	$[(D_4 \times D_4)/\mathbb{Z}_2] \times \mathbb{Z}_3^2$ 0 $\Delta(54) \times D_4 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3^2$ 337 $\Delta(54) \times \mathbb{Z}_2^3$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 26		363
	(2,1)	3	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 335 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 14		349
	(3,1)	3	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 333 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 2	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 18		353
	(4,1)	2	$[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2$ 0 $[D_4/\mathbb{Z}_2] \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 312	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2$ 44			356

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Flavor symmetry with ℓ non-vanishing WLs $\ell = 1$	2	3	4	Total models
$\mathbb{Z}_7$	(1,1)	1	$\mathbb{Z}_7^2$ 1				1
$\mathbb{Z}_8$-I	(1,1)	2	$D_4 \times \mathbb{Z}_8$ 38	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 230			268
	(2,1)	2	$D_4 \times \mathbb{Z}_8$ 41	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 204			246
	(3,1)	1	$\mathbb{Z}_8 \times \mathbb{Z}_4$ 389				389
$\mathbb{Z}_8$-II	(1,1)	3	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2$ 21	$D_4 \times \mathbb{Z}_8 \times \mathbb{Z}_2$ 398	$\mathbb{Z}_8 \times \mathbb{Z}_2^3$ 1,604		2,023
	(2,1)	2	$D_4 \times \mathbb{Z}_8$ 231	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 274			505
$\mathbb{Z}_{12}$-I	(1,1)	1	$\mathbb{Z}_3 \times \mathbb{Z}_3$ 556				556
	(2,1)	1	$\mathbb{Z}_{12} \times \mathbb{Z}_3$ 555				555
$\mathbb{Z}_{12}$-II	(1,1)	2	$D_4 \times \mathbb{Z}_2$ 84	$\mathbb{Z}_2^3$ 279			363

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

6,563 $\mathbb{Z}_N$ heterotic orbifolds with MSSM-like properties

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Max. # ℓ of WLs affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLs						Total
				$\ell = 1$	2	3	4	5	6	
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	6	6	$D_4^5/\mathbb{Z}_2^2$ 0	D_4^4 0	$D_4^3 \times \mathbb{Z}_2^2$ 0	$D_4^2 \times \mathbb{Z}_2^4$ 52	$D_4 \times \mathbb{Z}_2^6$ 152	$\mathbb{Z}_2^8$ 1	205
	(1,3)	4	2	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0					0
	(2,1)	5	5	$D_4^4/\mathbb{Z}_2$ 0	$D_4^3 \times \mathbb{Z}_2$ 0	$D_4^2 \times \mathbb{Z}_2^3$ 14	$D_4 \times \mathbb{Z}_2^5$ 342	$\mathbb{Z}_2^7$ 13		369
	(2,3)	3	1	D_4^2 0						0
	(2,5)	3	1	$D_4 \times \mathbb{Z}_2^2$ 0						0
	(3,1)	5	3	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 432	$\mathbb{Z}_2^5$ 12				444
	(3,3)	3	1	$D_4 \times \mathbb{Z}_2$ 0						0
	(4,1)	4	4	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 0	$\mathbb{Z}_2^6$ 0			0
	(5,1)	4	4	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 40	$\mathbb{Z}_2^6$ 2			42
	(5,4)	2	1	$D_4 \times \mathbb{Z}_2^2$ 0						0
	(6,1)	4	4	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 57	$\mathbb{Z}_2^6$ 344			401
	(6,3)	2	0	—						0
	(7,1)	4	2	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^4$ 76					76
	(9,1)	3	3	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 2	$\mathbb{Z}_2^5$ 25				27
	(10,1)	3	1	$\mathbb{Z}_2^3$ 21						21
	(11,1)	3	3	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0	$\mathbb{Z}_2^5$ 0				0

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Max. # ℓ of WLs affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLs $\ell = 1$	2	3	4	5	6	Total
	(12,1)	2	2	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^4$ 3					3
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	4	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 26	$D_4^2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,463	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^3$ 8,637	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 454			10,580
	(1,6)	2	2	$D_4^2 \times \mathbb{Z}_4$ 21	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 65					86
	(2,1)	4	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 81	$D_4^2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,131	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^3$ 4,686	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 260			6,158
	(2,4)	2	1	$\mathbb{Z}_4 \times \mathbb{Z}_2$ 321						328
	(3,1)	3	3	$(D_4^2 \times \mathbb{Z}_4)$ 103	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 3,637	$\mathbb{Z}_4 \times \mathbb{Z}_2^4$ 13,117				16,857
	(4,1)	3	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,308	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 3,187					4,519
	(5,1)	3	3	$D_4^2 \times \mathbb{Z}_4$ 63	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 742	$\mathbb{Z}_4 \times \mathbb{Z}_2^4$ 1,311				2,116
	(6,1)	3	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 884	$\mathbb{Z}_4 \times \mathbb{Z}_2^3$ 2,325					3,246
	(7,1)	3	1	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 2,229						2,667
	(8,1)	2	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 72	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 839					911
	(9,1)	2	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 522	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 1,620					2,142
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	(1,1)	2	2	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_6$ 467	$\mathbb{Z}_2^3 \times \mathbb{Z}_6$ 116					583
	(2,1)	2	2	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_6$ 275	$\mathbb{Z}_2^3 \times \mathbb{Z}_6$ 78					353
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	3	3	$\Delta(54)^2 \times \mathbb{Z}_3$ 81	$\Delta(54) \times \mathbb{Z}_3^3$ 987	$\mathbb{Z}_3^5$ 40				1,108
	(1,4)	1	1	$\mathbb{Z}_3^4$ 8						8
	(2,1)	2	2	$\Delta(54) \times \mathbb{Z}_3^2$ 239	$\mathbb{Z}_3^4$ 1,713					1,952
	(3,1)	2	1	$\mathbb{Z}_3^3$ 6						6

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Max. # ℓ of WLs affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLs: $\ell = 1$	2	3	4	5	6	Total
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(4,1)	2	1	$\mathbb{Z}_3^4$ 127						215
	(1,1)	1	1	$\mathbb{Z}_3^2 \times \mathbb{Z}_6$ 4,469						4,493
	(2,1)	1	1	$\mathbb{Z}_3^2 \times \mathbb{Z}_6$ 495						540
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	3	3	$(D_4^2 \times \mathbb{Z}_4^2)/\mathbb{Z}_2$ 2,258	$D_4 \times \mathbb{Z}_4^2 \times \mathbb{Z}_2$ 12,091	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^3$ 599				14,953
	(2,1)	2	2	$D_4 \times \mathbb{Z}_4^2$ 3,220	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 2,807					6,046
	(3,1)	2	1	$\mathbb{Z}_4^2 \times \mathbb{Z}_2$ 2,914						2,920
	(4,1)	2	1	$D_4 \times \mathbb{Z}_4^2$ 1,552	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 1,876					3,434

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

87,809 $\mathbb{Z}_N \times \mathbb{Z}_M$ heterotic orbifolds with MSSM-like properties

Pheno with stringy flavor

Lessons from models with D_4

Lebedev, Nilles, SRS, Ratz, Vaudrevange, Wingerter (2008)

- Most of the models ($\sim 46,000$) endowed with D_4

  Olguín-Trejo, Pérez-Martínez, SRS

- Stringy constraint: $\mathbf{2}+\mathbf{1}_0$ family structure, 3^{rd} generation in the bulk

  $\rightarrow$ top Yukawa is the largest

- All exotics can be decoupled in SUSY vacua with $\mathbb{Z}_2^{\text{matter}}$ (and/or $\mathbb{Z}_4^R$)
- In those vacua, D_4 is spontaneously broken $\rightarrow$ pheno?
- Nice Yukawa matrices (with $\widetilde{s} \sim 0.1$) $\rightarrow$ pheno?

$$Y_u = \begin{pmatrix} \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^5 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^6 \\ \widetilde{s}^6 & \widetilde{s}^6 & 1 \end{pmatrix}, Y_d = \begin{pmatrix} 0 & \widetilde{s}^5 & 0 \\ \widetilde{s}^5 & 0 & \widetilde{s}^7 \\ 0 & \widetilde{s}^6 & \widetilde{s}^7 \end{pmatrix}, Y_e = \begin{pmatrix} \widetilde{s}^7 & \widetilde{s}^5 & \widetilde{s}^6 \\ \widetilde{s}^5 & \widetilde{s}^7 & 0 \\ \widetilde{s}^6 & \widetilde{s}^6 & 0 \end{pmatrix}$$

- See-saw with 100's of r.h. neutrinos $\rightarrow$ nicer $M_{\text{see-saw}} \sim 10^{-2} M_{\text{GUT}}$

  Buchmüller, Hamaguchi, Lebedev, SRS, Ratz

$\Delta(54)$ flavor symmetry

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- Simplest options: $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$
- No promising models in $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$ with $\Delta(54)$

Investigate $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifolds!

Flavor symmetries in $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- If all fixed points are degenerate, flavor symmetry is large:

$$F = S_3 \times S_3 \times S_3 \ltimes \left[\mathbb{Z}_3^\vartheta \times \mathbb{Z}_3^\omega \times (\mathbb{Z}_3^m)^3\right]$$

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  WLs break degeneracy of fixed points
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- Models with 2 WLs lead to flavor

$$F = S_3 \ltimes \left[\mathbb{Z}_3^\vartheta \times \mathbb{Z}_3^\omega \times (\mathbb{Z}_3^m)^3\right] \supset \boldsymbol{\Delta}(54) \ ☺$$

Results of search of models

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explanation for three generations!! ☺

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* Numbers statistically compatible with Nilles, Vaudrevange (2014), but we find many more models ☺

$\Delta(54)$ stringy example

Example

$\Delta(54)$ string spectrum

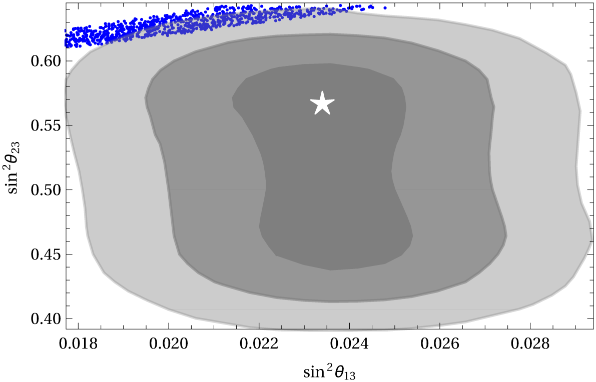
#	irrep	$\Delta(54)$	label	#	anti-irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i				
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\bar{u}_i$				
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\bar{d}_i$				
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i				
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\bar{e}_i$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\bar{\nu}_i$				
1	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{1}_{0}$	H_d	1	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\mathbf{1}_{0}$	H_u
			Flavons				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	ϕ_i^u				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	$\phi_i^{d,e}$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	ϕ_i^{ν}				
2	$(\mathbf{1},\mathbf{1})_{0}$	$2\cdot\mathbf{1}_{0}$	$s^{(d,e)}, s^u$				
128	$(\mathbf{1},\mathbf{1})_{0}$	$77\cdot\mathbf{1}_{0}+16\cdot\mathbf{3}_{12}+\mathbf{3}_{11}$	N_i				
			Exotic states				
16	$(\mathbf{1},\mathbf{2})_{\frac{1}{6}}$	$10\cdot\mathbf{1}_{0}+2\cdot\mathbf{3}_{12}$	v_i	16	$(\mathbf{1},\mathbf{2})_{-\frac{1}{6}}$	$4\cdot\mathbf{1}_{0}+4\cdot\mathbf{3}_{12}$	$\bar{v}_i$
3	$(\mathbf{3},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	y_i	3	$(\overline{\mathbf{3}},\mathbf{1})_{0}$	$3\times\mathbf{1}_{0}$	$\bar{y}_i$
1	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	$\mathbf{1}_{0}$	z_i	1	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{1}_{0}$	$\bar{z}_i$
7	$(\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$4\cdot\mathbf{1}_{0}+\mathbf{3}_{12}$	x_i	7	$(\mathbf{1},\mathbf{1})_{\frac{2}{3}}$	$4\cdot\mathbf{1}_{0}+\mathbf{3}_{11}$	$\bar{x}_i$
51	$(\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	$30\cdot\mathbf{1}_{0}+7\cdot\mathbf{3}_{12}$	w_i	51	$(\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$24\cdot\mathbf{1}_{0}+9\cdot\mathbf{3}_{12}$	$\bar{w}_i$

$$\begin{aligned} W_Y &= y^u_{ijk} Q_i H_u \bar{u}_j \phi^u_k s_u + y^d_{ijk} Q_i H_d \bar{d}_j \phi^{(d,e)}_k s^{(d,e)} + y^e_{ijk} L_i H_d \bar{e}_j \phi^{(d,e)}_k s^{(d,e)} \\ &+ y^{\nu}_{ijkl} L_i H_u \bar{\nu}_j + \lambda_{ijk} \bar{\nu}_i \bar{\nu}_j \bar{\phi}^{\nu}_k \,, \qquad i,j,k = 1,2,3, \end{aligned}$$

$\Delta(54)$ stringy phenomenology: neutrinos with see-saw

Correlation atmospheric–reactor mixing angles compatible with best fit

Forero, Tortola, Valle (2014)



- atmospheric mixing angle: $51.3^o \lesssim \theta_{23} \lesssim 53.1^o$ (second octant) ☺
- reactor mixing angle: $7.8^o \lesssim \theta_{13} \lesssim 8.9^o$ ☺
- $6meV \lesssim m_{\nu_1} \lesssim 6.8meV$, $65meV \lesssim \sum m_\nu \lesssim 70meV$ ☺

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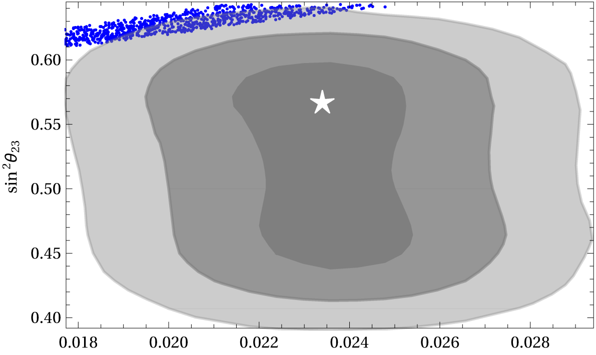

Advantages of model:

Compatible with T2K (2016)

only normal hierarchy admissible

Falsifiable soon

- atmospheric [...] cond octant) ☺
- reactor mixin[...]
- $6meV \lesssim m_{\nu_1}$ [...] eV ☺

$\Delta(54)$ stringy phenomenology: down sector

Down-quark and charged-lepton sectors are symmetric

$$\Rightarrow \quad \tan\theta_C^e \approx \frac{(M_e^D)_{12}}{(M_e^D)_{22}} \approx \sqrt{\frac{m_e}{m_\mu}}$$ leptonic Gatto-Sartori-Tonin ☺

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BUT all other quark and charged-lepton mixing angles are smaller than observed ☹

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leptonic Gatto-Sartori-Tonin ☺

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Additional novel consequence

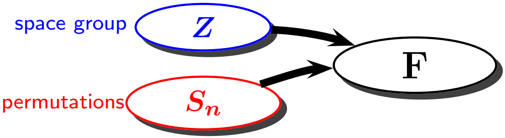
$$\frac{m_s - m_d}{m_b} \overset{!}{=} \frac{m_\mu - m_e}{m_\tau}.$$

more stringent than $b - \tau$ unification ☹

Punch line

- Flavor symmetries from compactification geometry:

Kobayashi,Nilles,Plöger,Raby,Ratz (2007); SRS, Vaudrevange (2018)



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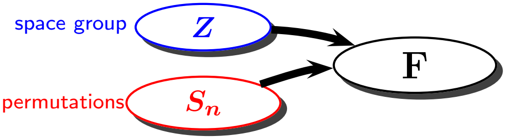

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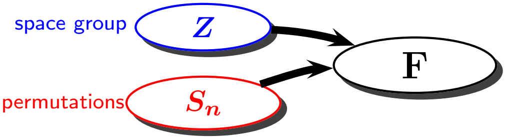

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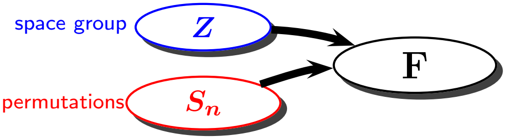

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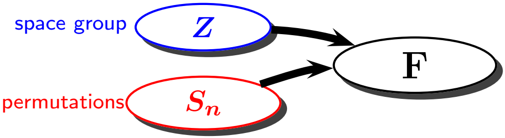

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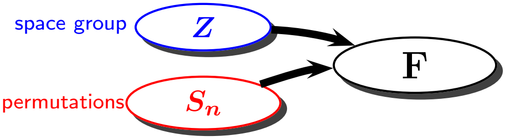

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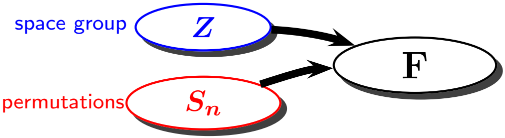

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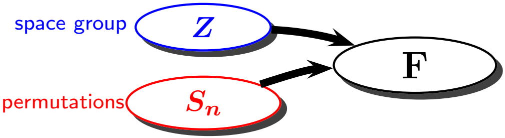

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- Nice pheno, although more challenging in $\Delta(54)$

Possibilities for explanation of CP-violation [Nilles,Ratz,Trautner,Vaudrevange (2018)]

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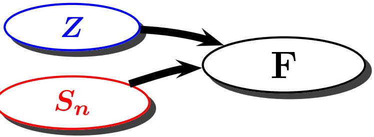

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To do

- compare with other classifying tools
- identify common properties
- study phenomenology
- symmetry enhancement?
- are there predictions?

A shameless ad