DARK ENERGY FROM MODIFIED GRAVITY AND THE HOLOGRAPHIC PRINCIPLE

Germán Darío Rojas Chacón

Luis Norberto Granda

Universidad del Valle

03 December 2018

Contents

Introduction

Many models have been proposed along history in order to describe the origin and evolution of the universe. After the second decade of the XXth century, all of them were constructed by intruducing constants, scalar, vectorial and tensorial fields or any combination between them through a specific coupling in the lagrangian density .

When observations became more and more precise, many of these models were discarded because they did not fit the new observational results. Every model must fit the observations of weak lensing [1], LSS formation [2], barionic oscillations [3], CMB temperature and polarization anisotropy [4], SNIa [5], gravitational waves and gamma ray bursts [6,7,8].

The last two constrain the model to have a late time accelerated expansion phase, and a difference between the predicted (by the model) gravitational waves speed and the speed of light less than 10^{-15}, respectively. Also, the models have to meet the theoretical requirements like the stability of the dark energy sector and the weak field limit, among others [9,10].

The $f(R)$ modified gravity models, in which the Einstein-Hilbert lagrangian is replaced by a general function of the Ricci scalar, are an alternative to explain the evolution of the universe since inflation until dark energy era.

When explaining dark energy, the models must mimic the concordance model (ΛCDM) whose results fit the observa-

Universidad del Valle

tions of late time cosmology. Unfortunately, the difficulty to find a successful explanation of the physical origin of the cosmological constant puts on the table the necessity of suggesting alternatives to it.

The holographic principle [11,12,13,14,15] as a possible approach to a quantum theory of gravity allows to introduce a different interpretation of the vacuum energy.

We propose a reconstruction [16,17,18,19,20,21] of the $f(R)$ function by integration of the field equations obtained through the metric formalism and taking into account an IR holographic cut-off similar to the one proposed by Granda and Oliveros [22] as the source of the vaccum energy.

Universidad del Valle

Marco teórico

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}f(R) + \mathcal{L}_m(g_{\mu\nu}, \psi)\right], \qquad (1)$$

where $f(R)$ is a general function of the Ricci scalar that corresponds to the modification of gravity, $\kappa = 8\pi G$, g is the determinant of the metric tensor $g_{\mu\nu}$, and $\mathcal{L}_m$ is the lagrangian density for the matter sector which can be baryonic, dark matter, or any exotic kind of matter/energy that in our case will be associated to the vacuum energy.

The general field equations obtained by varying the action S with respect to the metric $g_{\mu\nu}$ (metric formalism) are :

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \left(\nabla_\nu\nabla_\mu - g_{\mu\nu}\Box\right)f'(R) = \kappa^2 T^{(m)}_{\mu\nu}, \quad (2)$$

where $\Box \equiv \nabla^\sigma\nabla_\sigma = \nabla_\sigma\nabla^\sigma$ is the covariant d'Alembertian operator, and

$$T^{(m)}_{\mu\nu} \equiv -\frac{2\kappa^2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}$$

is the energy-momentum tensor for the matter sector.

Assuming large scale homogenity and isotropy, we will use the *FLRW* metric :

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 sen^2\theta d\phi^2\right], \quad (3)$$

where K is the spatial curvature ($K = 0$ for a spatially flat universe), and $a(t)$ is the scale factor.

Using this metric and after some manipulation, we find the time and spatial components of the field equations :

$$H^2 = \frac{\kappa^2}{3}\left(\tilde{\rho}^{(m)} + \rho^{(DE)}\right) \tag{4}$$

and

$$2\dot{H} + 3H^2 = -\kappa^2\left(\tilde{p}^{(m)} + p^{(DE)}\right) \ , \tag{5}$$

where $\rho = \tilde{\rho}^{(m)} + \rho^{(DE)}$, $p = \tilde{p}^{(m)} + p^{(DE)}$, $\rho^{(DE)} \equiv \rho^{(f)} + \tilde{\rho}_\Lambda$, $p^{(DE)} \equiv p^{(f)} + \tilde{p}_\Lambda$, being $\tilde{\rho}^{(m)} \equiv \frac{\rho^{(m)}}{f'(R)}$, $\tilde{p}^{(m)} \equiv \frac{p^{(m)}}{f'(R)}$, $\tilde{\rho}_\Lambda \equiv \frac{\rho_\Lambda}{f'(R)}$, $\tilde{p}_\Lambda \equiv \frac{p_\Lambda}{f'(R)}$,

$$\rho^{(f)} \equiv \frac{1}{\kappa^2 f'(R)}\left[\frac{Rf'(R) - f(R)}{2} - 3H\dot{R}f''(R)\right] \tag{6}$$

and

$$p^{(f)} \equiv \frac{1}{\kappa^2 f'(R)}\left[2H\dot{R}f''(R) + \frac{f(R) - Rf'(R)}{2} + (\dot{R})^2 f'''(R) + \ddot{R}f''(R)\right]. \tag{7}$$

Autonomous system. From now on, assuming that :

• The content of the universe is matter (ρ_m), and vacuum energy (ρ_Λ),

• the pressure associated to matter at cosmological scales is zero,

• $f(R) \equiv f$ y $f' \equiv F$,

- in a space with *FLRW* metric and $K = 0$, $\Box F = -\ddot{F} - 3H\dot{F}$ y $R = 6\left(2H^2 + \dot{H}\right)$,
- the vacuum energy comes from the Holographic Principle and is given by $\rho_\Lambda \equiv \frac{3}{\kappa^2} F\left(\alpha H^2 + \beta \dot{H}\right)$.

With this, the timelike component of the field equation (2) (eqn. (4)) becomes :

$$1 = \frac{R}{6H^2} - \frac{f}{6H^2 F} - \frac{\dot{F}}{HF} + \frac{\kappa^2 \rho^{(m)}}{3H^2 F} + \alpha + \beta \frac{\dot{H}}{H^2} \ , \qquad (8)$$

which, defining the *dynamical variables* :

$$x \equiv -\frac{\dot{F}}{HF} \quad y \equiv -\frac{f}{6H^2 F} \quad z \equiv \frac{R}{6H^2} = 2 + \frac{\dot{H}}{H^2} \quad \Omega_m \equiv \frac{\kappa^2 \rho^{(m)}}{3H^2 F} \qquad (9)$$

can be written as :

$$1 = x + y + (1+\beta)z + \alpha - 2\beta + \Omega_m \ . \tag{10}$$

As can be seen, the *dynamical variables* x, y and z, with the parameters α and β allow to write the dark energy density parameter coming from $f(R)$ and the vacuum energy ρ_Λ as :

$$\Omega^{(DE)} \equiv (x+y+z) + \left(\alpha + \beta\frac{\dot{H}}{H^2}\right) = x+y+(1+\beta)z+\alpha-2\beta \ , \tag{11}$$

where it is possible to identify $\Omega^{(f)} \equiv x + y + z$ and $\Omega_\Lambda \equiv \alpha + \beta\frac{\dot{H}}{H^2}$. From (10) we also get :

$$\Omega_m = 1 - x - y - (1+\beta)z - \alpha + 2\beta \ . \tag{12}$$

After some manipulation, it is possible to show that :

$$\frac{dx}{dN} = x^2 + \left(\frac{\beta}{m} + \beta - 1\right)xz + (\alpha - 2\beta)x - 3y + (\beta - 2\alpha - 1)z + \alpha - 2\beta - 1 \ , \quad (13)$$

$$\frac{dy}{dN} = xy + \frac{xz}{m} - 2y(z-2) \ , \quad (14)$$

$$\frac{dz}{dN} = -\frac{xz}{m} - 2z(z-2) \ , \quad (15)$$

where $N \equiv Ln\, a$, and $m \equiv \frac{RF'}{F} = \frac{dLnF}{dLnR}$.

Universidad del Valle

Equating each of this equations to zero, we get a closed (autonomous) algebraic system whose roots are critical points of the *dynamical variables*. It is in these points where it is possible to make a stability analysis of the dark energy sector when it is faced to perturbations.

The effective equation of state parameter, which determines the sign of the deceleration parameter, assuming a perfect fluid configuration is given by :

$$\omega_{eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -\frac{1}{3}(2z - 1)$$

Now, the critical points of the autonomous system are :

$P = (x_c, y_c, z_c)$	Ω_m	ω_{eff}
$P_1 = (-4, 5 - \alpha + 2\beta, 0)$	0	$1/3$
$P_2 = (-1, 0, 0)$	$2 - \alpha + 2\beta$	$1/3$
$P_3 = (0, -1 - \alpha, 2)$	0	-1
$P_4 = (1 - \alpha + 2\beta, 0, 0)$	0	$1/3$
$P_5 = \left(\frac{3m}{1+m}, -\frac{1+4m}{2(1+m)^2}, \frac{1+4m}{2+2m}\right)$	$\left[2 - 2\alpha - 2m^2(4+\alpha) +3\beta+m(-3-4\alpha+3\beta)\right] \Big/ \left[2(1+m)^2\right]$	$-\frac{m}{1+m}$
$P_6 = \left(\frac{2m(1-m-\alpha-m\alpha)}{m+2m^2-\beta-m\beta}, \frac{1-4m-\alpha+2\beta}{m+2m^2-\beta-m\beta}, \frac{(1+m)(-1+4m+\alpha-2\beta)}{m+2m^2-\beta-m\beta}\right)$	0	$\frac{1}{3}\left[1 - \left(\left[2(1+m)(-1+4m+\alpha-2\beta)\right] \Big/ \left(m+2m^2-\beta-m\beta\right)\right)\right]$

Table 1. Critical points of the autonomous system.

Universidad del Valle

As can be seen, P_3 is a de Sitter type point.

P_5 can be a matter type point ($\omega_{eff} = 0$, $\Omega_m \approx 1$) or an accelerated one (Quintessence or Phantom type), and its matter content (Ω_m) depends not only on the geometric parameter m but also on the holographic paramenters α and β.

P_6 can have any accelerated scenario (Quintessence, de Sitter or Phantom), and its ω_{eff} not only depends on the geometric parameter m but also on the holographic parameters α and β.

The stability of the system can be determined from the sign of the eigenvalues of the matrix that transforms from the unperturbed system to the perturbed one, evaluated at the critical points. These eigenvalues are :

P_3 : de Sitter type ($\omega_{eff} = -1$) with eigenvalues :

$$j_1 = -3 \quad , \quad j_2, j_3 = \frac{-3m + m\alpha + 2\beta \pm \sqrt{4(-4m + 4m^2 + 4m\alpha + 4m^2\alpha) + (-3m + m\alpha + 2\beta)^2}}{2m} \quad . \tag{16}$$

The conditions on m, α *and* β to get a stable P_3 are :

$$m < -1 \quad \wedge \quad \alpha < \frac{1-m}{1+m} \quad \wedge \quad \beta \geq \frac{1}{2}(3m - m\alpha) + 2\sqrt{m - m^2 - m\alpha - m^2\alpha}$$

$$\vee$$

$$-1 < m < 0 \quad \wedge \quad \alpha > \frac{1-m}{1+m} \quad \wedge \quad \beta \geq \frac{1}{2}(3m - m\alpha) + 2\sqrt{m - m^2 - m\alpha - m^2\alpha}$$

$$\vee$$

$$m > 0 \quad \wedge \quad \alpha < \frac{1-m}{1+m} \quad \wedge \quad \beta \leq \frac{1}{2}(3m - m\alpha) + 2\sqrt{m - m^2 - m\alpha - m^2\alpha}$$

Universidad del Valle

P_5 : Depending on m, α and β, ω_{eff} can be greater or less than -1 producing Phantom ($\omega_{eff} < -1$), Quintessence ($-1 < \omega_{eff} < -\frac{1}{3}$) or matter dominance conditions ($\omega_{eff} \approx 0$, $m = 0$ and $\frac{\alpha}{\beta} = \frac{3}{2}$). Its eigenvalues are :

$$
\begin{aligned}
j_1 &= 3(1+m') \quad , \\
j_2, j_3 &= \frac{1}{4m(1+m)}\Big(2m^2\alpha + \beta + m(-3+2\alpha+\beta) \pm \Big[4m^2(8+\alpha)^2 + \beta^2 + \\
& 2m(-8-15\beta+\beta^2+2\alpha(4+\beta)) + 4m^3(40+2\alpha^2-24\beta+\alpha(33+\beta)) \\
& + m^2(-31+4\alpha^2-126\beta+\beta^2+\alpha(84+8\beta))\Big]^{1/2}\Big) \quad ,
\end{aligned} \tag{17}
$$

where $m' \equiv \frac{dm}{dr}$, having that $m \equiv m(r)$, and $r \equiv -\frac{RF}{f} = -\frac{dLnf}{dLnR}$. As can be seen, the value of m' ($m' > -1$ or $m' < -1$) defines the point stability. For the case of dark energy dominance, with a transient grow (saddle point) of the scale factor very close to de Sitter with $-1.05 < \omega_{eff} < -0.95$.

There are several possible conditions on m, α and β, among others :

$$\begin{aligned}
& m < -21 \quad \wedge \quad \beta \leq \frac{m+2m^2}{1+m} \quad \wedge \quad \alpha \leq \Bigg[\frac{(-4-17m-16m^2-\beta-m\beta)}{2m(1+m)} - \sqrt{2}\Big(\Big[2+19m \\
& \qquad + 56m^2 + 48m^3 + \beta + 9m\beta + 24m^2\beta \\
& \qquad + 16m^3\beta\Big] \Big/ \Big[m^2(1+m)^2\Big]\Big)^{1/2}\Bigg] \\
& \vee \\
& m < -21 \quad \wedge \quad \frac{m+2m^2}{1+m} < \beta < \frac{-2-3m}{1+m} \quad \wedge \quad \alpha \leq \Bigg[\frac{(-4-17m-16m^2-\beta-m\beta)}{2m(1+m)} - \sqrt{2}\Big(\Big[2+19m \\
& \qquad + 56m^2 + 48m^3 + \beta + 9m\beta + 24m^2\beta \\
& \qquad + 16m^3\beta\Big] \Big/ \Big[m^2(1+m)^2\Big]\Big)^{1/2}\Bigg] \\
& \vee \\
& m < -21 \quad \wedge \quad \frac{m+2m^2}{1+m} < \beta < \frac{-2-3m}{1+m} \quad \wedge \quad \Bigg\{ \Bigg[\frac{(-4-17m-16m^2-\beta-m\beta)}{2m(1+m)} + \sqrt{2}\Big(\Big[2+19m \\
& \qquad + 56m^2 + 48m^3 + \beta + 9m\beta + 24m^2\beta \\
& \qquad + 16m^3\beta\Big] \Big/ \Big[m^2(1+m)^2\Big]\Big)^{1/2}\Bigg] \leq \alpha \\
& \qquad < \frac{2-3m-8m^2+3\beta+3m\beta}{2+4m+2m^2} \Bigg\} .
\end{aligned} \tag{18}$$

P_6 : Can be stable or saddle type of dark energy or radiation dominance. The eigenvalues in this point are :

$$j_1 = \frac{2(1+m')(1+m)(1-m-\alpha-m\alpha)}{m+2m^2-\beta-m\beta} \quad ,$$

$$j_2 = \frac{2-2\alpha-2m^2(4+\alpha)+3\beta+m(-3-4\alpha+3\beta)}{m+2m^2-\beta-m\beta} \quad , \qquad (19)$$

$$j_3 = \frac{1-4m-\alpha+2\beta}{m} \quad .$$

Some conditions on m , m' , α and β to find stable Quintessence, de Sitter or Phantom type solutions are :

$$m' > -1 \quad \wedge \quad m < -1 \quad \wedge \quad \beta < \frac{m + 2m^2}{1 + m} \quad \wedge \quad \alpha < 1 - 4m + 2\beta$$

$$\vee$$

$$m' > -1 \quad \wedge \quad m < -1 \quad \wedge \quad \beta > \frac{m + 2m^2}{1 + m} \quad \wedge \quad \frac{1 - m}{1 + m} < \alpha < 1 - 4m + 2\beta$$

$$\vee$$

$$m' > -1 \quad \wedge \quad -1 < m < 0 \quad \wedge \quad \beta > \frac{m + 2m^2}{1 + m} \quad \wedge \quad \alpha < \frac{1 - m}{1 + m}$$

$$\vee$$

$$m' > -1 \quad \wedge \quad m > 0 \quad \wedge \quad \beta < \frac{m + 2m^2}{1 + m} \quad \wedge \quad \alpha > \frac{1 - m}{1 + m}$$

Universidad del Valle

For P_6, ω_{eff} vs m for some values of α and β is :

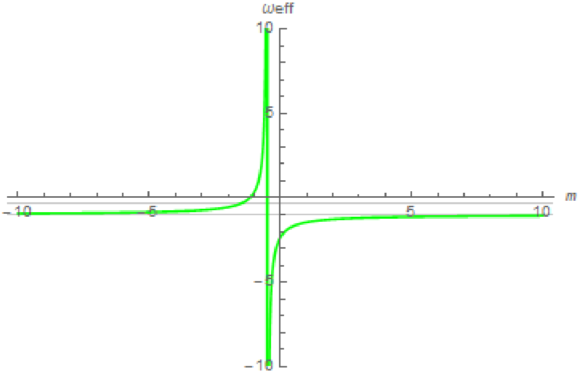

Figure 1. ω_{eff} vs m for $\alpha = 1$ and $\beta = 0$

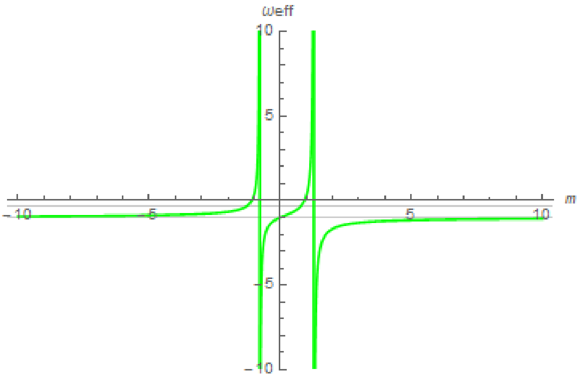

Figure 2. ω_{eff} vs m for $\alpha = 1$ and $\beta = 2$

As can be seen, the parameters associated to the holographic energy density determine the stability and nature of some critical points. Also, in both figures it is possible to see that the interval $-7.6 < \omega_{eff} < -1.07$ is allowed, result that is not present in traditional $f(R)$ models.

Forthcoming research

- Reconstruction (solving the timelike component of the field equation (2)) of $f(R)$ models in the frame of the Holographic Principle.
- Fitting of the parameters associated to some already proposed $f(R)$ models (like the Hu-Sawicki's [23]) taking into account the holographic energy density proposed.
- Study of slow-roll conditions for the scalar field potential found when transforming to the Einstein frame.
- High curvature and weak field limit, solar system tests, and correct cosmological evolution.
- Contrast with the scalar-tensor ratio, spectral index and other parameters that can be calculated from current observations.

Conclusions

Our proposal contains two sources of dark energy, one of them is $f(R)$ and the other one is holographic energy density. Any of them can produce accelerated expansion, but the inclusion of both gives rise to an interplay between them.

The vacuum energy we suggest is an alternative to the Cosmological Constant and because the former does not come from QFT, it could solve the order of magnitude problem that the Cosmological Constant has.

On the other hand, the interplay between modified gravity and holographic energy could improve the fit to observations of modified gravity reconstructed models in this scenario.

References

[1] B. Jain and A. Taylor, Phys. Rev. Lett. 91, 141302 (2003).

[2] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004) ; U. Seljak et al., Phys. Rev. D 71, 103515 (2005).

[3] D. J. Eisenstein et al., Astrophys. J. 633, 560 (2005) ; C. Blake, D. Parkinson, B. Basset, K. Glazebrook, M. Kunz and R. C. Nichol, Mon. Not. Roy. Astron. Soc. 365, 255 (2006).

[4] P. A. R. Ade et al. [Planck Collaboration], arXiv : 1502.01589 [astro-ph.CO] (2015).

[5] S. Perlmutter et al., Astrophys. J. 517, 565 (1999) ; A. G. Riess et al., Astron. J. 116, 1009 (1998) ; A.G. Riess et al., Astron. J. 117, 707 (1999) ; J. L. Tonry et al., Astrophys. J. 594, 1 (2003) ; R.A. Knop et al., Astrophys. J. 598, 102 (2003).

[6] B. Abbott et al. [Virgo, LIGO Scientic], Phys. Rev. Lett. 119, 161101 (2017) [arXiv : 1710.05832 [gr-qc] (2017)].

[7] B. P. Abbott et al., Astrophys. J. Lett. 848, L13 (2017).

[8] B. P. Abbott et al., Astrophys. J. Lett. 848, L12 (2017).

[9] L. Amendola, R. Gannouji, D. Polarski, and S. Tsujikawa, Phys. Rev. D 75, 083504 (2007) [arXiv : 061218 [gr-qc] (2007)].

[10] T. P. Sotiriou and V. Faraoni, arXiv : 0805.1726 [gr-qc] (2008).

Universidad del Valle

[11] G. t'Hooft, arXiv : 9310026 [gr-qc] (1993).

[12] L. Susskind, J. Math. Phys. 36, 6377 (1995) [arXiv : 9409089 [hep-th] (1994)].

[13] W. Fischler and L. Susskind, arXiv : 9806039 [hep-th] (1998).

[14] P. Hořava and D. Minic, Phys. Rev. Lett. 85, 1610 (2000) [arXiv : 0001145 [hep-th] (2000)].

[15] S. D. Thomas, Phys. Rev. Lett 89, 081301 (2002).

[16] S. Nojiri and S. D. Odintsov, Journal of Physics Conference Series 66, 012005 (2007) [arXiv : 0611071 [hep-th] (2006)].

[17] S. Capozziello, V. F. Cardone, A. Troisi, Phys. Rev. D 71, 043503 (2005).

[18] Y-S. Song, W. Hu, I. Sawicki, Phys. Rev. D 75, 044004 (2007).

[19] S. Nojiri, S. D. Odintsov, D. Sáez-Gomez, Phys. Lett. B 681, 74 (2009) [arXiv : 0908.1269 [hep-th] (2009)].

[20] S. Carloni, R. Goswami, and P. K. S. Dunsby, Class. Quant. Grav. 29, 135012 (2012) [arXiv : 1005.1840 [gr-qc] (2010)].

[21] P. K. S. Dunsby, E. Elizalde, R.Goswami, S. D. Odintsov, and D. Sáez-Gomez, Phys. Rev. D 82, 023519 (2010) [arXiv : 1005.2205 [gr-qc] (2010)].

[22] L. N. Granda and A. Oliveros, Phys. Lett. B 669, 275 (2008) ; L. N. Granda and A. Oliveros, Phys. Lett. B 671, 199 (2009).

[23] W. Hu and I. Sawicki, Phys. Rev. D 76, 064004 (2007) [arXiv : 0705.1158 [astro-ph] (2007)].

Universidad del Valle