Arbitrarily coupled $p-$forms in cosmological backgrounds

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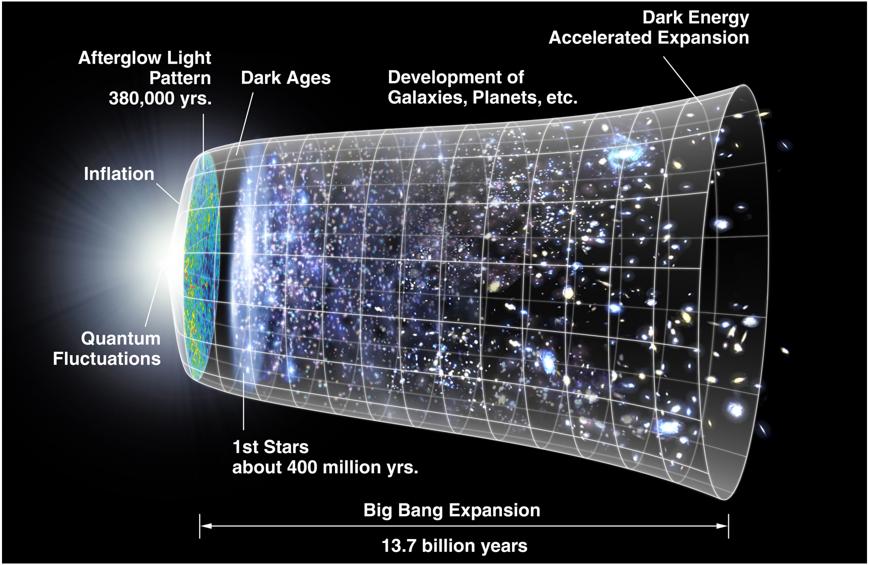
Outline


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Modifying Gravity

Theorem (Lovelock's Theorem)

In a four-dimensional space-time the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\alpha\beta}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term:

$$E^{\alpha\beta} = \alpha\sqrt{-g}\left[R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R\right] + \lambda\sqrt{-g}g^{\alpha\beta},$$

where α and λ (cosmological constant) are constants, and $R_{\alpha\beta}$ and R are the Ricci tensor and scalar curvature, respectively.

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1. Consider extra degrees of freedom more than the metric tensor.

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2. Work in a space with dimensionality different than four.
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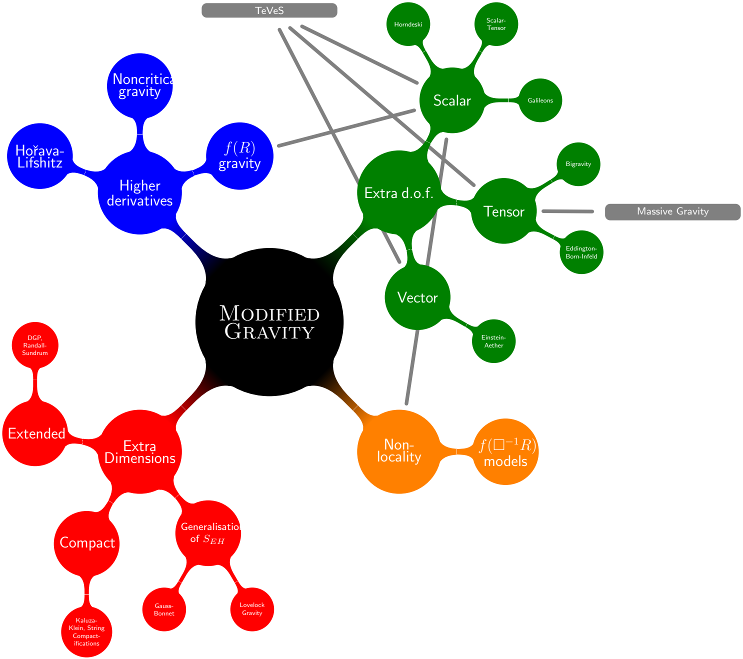
4. Consider non-locality.


TeVeS
Horndeski
Scalar-Tensor
Scalar
Galileons
Noncritical gravity
Hořava-Lifshitz
$f(R)$ gravity
Higher derivatives
Bigravity
Extra d.o.f.
Tensor
Massive Gravity
Eddington-Born-Infeld
Vector
Modified Gravity
Einstein-Aether
DGP, Randall-Sundrum
Extended
Extra Dimensions
Non-locality
$f(\Box^{-1}R)$ models
Compact
Generalisation of S_{EH}
Gauss-Bonnet
Lovelock Gravity
Kaluza-Klein, String Compactifications

OUTLINE

Formalities

Given a p-form $A_{(p)\,\mu_1,\mu_2\cdots\mu_p}$, its dynamics is introduced by the field strength $F_{(p)} = \mathrm{d}A_p + A_p \wedge A_p$

$$F_{(p)} = \frac{1}{p!}\nabla_{[\mu_1}A_{(p)\mu_2\mu_3\cdots\mu_{p+1}]}\mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} \cdots \wedge \mathrm{d}x^{\mu_{p+1}}.$$

In addition the Hodge dual of the field strength is defined as

$$\tilde{F}_{(p)\nu_1\cdots\nu_{D-p-1}} = \frac{\sqrt{-g}}{(p+1)!}\epsilon_{\mu_1\cdots\mu_{p+1}\nu_1\cdots\nu_{D-p-1}}F_{(p)}^{\;\mu_1\cdots\mu_{p+1}}.$$

The higher rank of a $p-$form in D dimensions is obviously D, this is:

$$A_D \propto \sqrt{-g}\epsilon_{1\ldots D}\mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^D,$$

which is proportional to the $D-$dimensional volume element

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Gauge Invariance

$$A_{(p)\mu_1\cdots\mu_p} \to A_{(p)\mu_1\cdots\mu_p} + \partial_{[\mu_1}\xi_{(p-1)\mu_2\cdots\mu_p]},$$

General Procedure

$$S_{(p)} = -\frac{1}{2}\int F_{(p)} \wedge \star F_{(p)} \equiv -\frac{1}{2(p+1)!}\int \mathrm{d}^D x\sqrt{-g}F_{(p)}^2,$$

where $F_{(p)}^2 \equiv F_{(p)\mu_1\mu_2\ldots\mu_{p+1}}F_{(p)}{}^{\mu_1\mu_2\ldots\mu_{p+1}}$.
In $D-$dimensions we have the following general form to couple $p-$forms of different rank

$$\mathcal{L}_{\text{mixing}} = g_{p_1 p_2\cdots p_r}(\phi)X_{(p_1)} \wedge \cdots \wedge X_{(p_r)},$$

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$$F_{(p)} F^{(n_1)} \ldots F^{(n_i)} \equiv F_{(p)\mu_1\mu_2\ldots\mu_{p+1}} F_{(n_1)}^{\mu_1\mu_2\ldots\mu_{n_1+1}} \ldots F_{(n_i)}^{\mu_s\ldots\mu_{p+1}},$$

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$$\tilde{F}_{(p)} F^{(n)} F^{(m)} \equiv \tilde{F}_{(p)\mu_1\mu_2\ldots\mu_{D-(p+1)}} F_{(n)}^{\;\mu_1\mu_2\ldots\mu_{n+1}} F_{(m)}^{\;\mu_1\mu_2\ldots\mu_{m+1}},$$

Finally, we add gravity to the system.

$$S = \int \mathrm{d}^D x \sqrt{\bar{g}} \left(\frac{\bar{M}_{\mathrm{p}}^{D-2}}{2} \bar{R} - \mathcal{L}_\phi - \mathcal{L}_p \right) ,$$

where

$$\begin{aligned} \mathcal{L}_p &= -\frac{1}{2} \sum_{n=1}^{D-1} f_n(\phi) F_{(n)} \wedge \star F_{(n)} + \sum_{(p_1 p_2 \cdots p_r)} g_{p_1 p_2 \cdots p_r}(\phi) X_{(p_1)} \wedge \cdots \wedge X_{(p_r)}, \\ &= -\frac{1}{2} \sum_{n=1}^{D-1} \frac{f_n(\phi)}{(n+1)!} F_{(n)}^2 + \sum_{(p_1 p_2 \cdots p_r)} g_{p_1 p_2 \cdots p_r}(\phi) X_{(p_1)} \wedge \cdots \wedge X_{(p_r)}, \end{aligned}$$

Topological Terms

The Chern-Pontryagin density or $\theta-$term:

$$S_{\text{CP}} = -\frac{1}{2(p+1)!}\int \mathrm{d}^D x \sqrt{-g}\,\tilde{F}_{(p)\mu_1\cdots\mu_{p+1}} F_{(p)}{}^{\mu_1\cdots\mu_{p+1}}.$$

manifestly independent of the metric. Nevertheless, once it is coupled to a scalar field,

$$S_{\phi\text{CP}} = \int g_1(\phi) F_{(p)} \wedge F_{(p)},$$

it becomes relevant for the dynamics of the scalar and the $p-$form field. For odd dimensions $D = 2p+1$ we have the Chern-Simons invariant

$$S_{\phi\text{CS}} = \int g_2(\phi) A_{(p)} \wedge F_{(p)},$$

For even dimensions, we have the so called $BF-$theories which couples a $p-$form with the field strength of a $(p-1)-$form.

$$S_{\phi\text{BF}} = \int g_3(\phi) A_{(p)} \wedge F_{(D-p-1)},$$

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With the general procedure we arrive to the general Lagrangian coupling $p-$forms in 4 dimensions

$$\mathcal{L}_p(\phi, A_p) = -\frac{1}{2}\sum_{n=1}^{3}\frac{f_n(\phi)}{(n+1)!}F_{(n)}^2 - \frac{f_4(\phi)}{24}\tilde{F}_{(3)} - \frac{g_1(\phi)}{4}F_{(1)\mu_1\mu_2}\tilde{F}_{(1)}{}^{\mu_1\mu_2} - \frac{g_2(\phi)}{2}A_{(2)\mu_1\nu_2}\tilde{F}_{(1)}{}^{\mu_1\mu_2}.$$

Equations of motion and energy-momentum tensor

Energy-momentum tensor

$$
\begin{aligned}
T^{(p)}_{\alpha\beta} &= -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_p)}{\delta g^{\alpha\beta}}, \\
&= f_1(\phi)\left(F_{(1)\,\alpha\mu_2}F_{(1)\beta}{}^{\mu_2} - \frac{1}{4}g_{\alpha\beta}F^2_{(1)}\right) + f_2(\phi)\left(\frac{1}{2}F_{(2)\,\alpha\mu_2\mu_3}F_{(2)\beta}{}^{\mu_2\mu_3} - \frac{1}{12}g_{\alpha\beta}F^2_{(2)}\right) \\
&+ f_3(\phi)\left(\frac{1}{6}F_{(3)\,\alpha\mu_2\mu_3\mu_4}F_{(3)\beta}{}^{\mu_2\mu_3\mu_4} - \frac{1}{48}g_{\alpha\beta}F^2_{(3)}\right) - \frac{f_4(\phi)}{24}\tilde{F}_{(3)}g_{\alpha\beta}.
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Equations of motion for ϕ and $A_{(p)}$

$$\mathcal{E}_\phi = \frac{1}{\sqrt{g}}\frac{\delta(\sqrt{-g}\mathcal{L}_T)}{\delta\phi} = 0, \quad \mathcal{E}_{(p)\mu_1\cdots\mu_p} = \frac{1}{\sqrt{g}}\frac{\delta(\sqrt{-g}\mathcal{L}_p)}{\delta A_{(p)}^{\mu_1\mu_2\cdots\mu_p}} = 0.$$

The coupled 1—form and 2—form system

$$\nabla^{\mu}\left(f_1(\phi)F_{\mu\nu} + g_1(\phi)\tilde{F}_{\mu\nu} + g_2\tilde{B}_{\mu\nu}\right) = 0, \quad \nabla^{\mu}\left(f_2(\phi)H_{\mu\nu\alpha}\right) + \frac{g_2}{2}\tilde{F}_{\nu\alpha} = 0,$$

Solution of the 2—form system

$$F^{\mu\nu} = -\frac{1}{3g_2}\nabla^{[\mu}f_2(\phi)\tilde{H}^{\nu]}, \qquad V^{\mu} \equiv \frac{f_2(\phi)}{3g_2}\tilde{H}^{\mu},$$

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$$S_V = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\left[f_1(\phi)W_{\mu\nu}W^{\mu\nu} + g_1(\phi)W_{\mu\nu}\tilde{W}^{\mu\nu} + 2m^2(\phi)V_{\mu}V^{\mu}\right],$$

with $W_{\mu\nu} = \nabla_{[\mu}V_{\nu]}$. Using the gauge $\nabla_{\nu}\nabla_{\mu}A^{\mu} = -\frac{m^2}{f_1}\partial_{\nu}v$ we obtain

$$\Box A_{\nu} - R_{\nu\mu}A^{\mu} + \frac{\partial^{\mu}f_1}{f_1}(\nabla_{\mu}A_{\nu} - \nabla_{\nu}A_{\mu}) + \frac{\partial^{\mu}g_1}{f_1}\eta_{\mu\nu\alpha\beta}\nabla^{\alpha}A^{\beta} - \frac{m^2}{f_1}A_{\nu} = 0,$$

3—form system

The 3—form evolves independently from the other antisymmetric tensors.

$$\nabla^{\mu}\left(f_3(\phi)F_{(3)\mu\nu\alpha\beta}+f_4(\phi)\sqrt{-g}\epsilon_{\mu\nu\alpha\beta}\right)=0.$$

Gauge $A_{(3)0ij}=0$ and $\nabla^{k}A_{(3)0ik}=0$.

$$A''_{(3)ijk}+\left(\frac{f'_3}{f_3}-4\frac{a'}{a}\right)A'_{(3)ijk}=-\frac{f'_4}{f_3}\sqrt{-g}\epsilon_{0ijk}.$$

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$$A_{(3)}(\tau)=\int \mathrm{d}\tau'\sqrt{-g(\tau')}\frac{1}{f_3(\phi(\tau'))}\left(c-f_4(\phi(\tau'))\right).$$

This solution is valid for any Friedmann cosmology and for any time dependence of the couplings f_3, f_4.

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$$T^{(3)}_{\alpha\beta}=-\left(\frac{(4!)^2}{2}f_3(\phi)X^2-f_4(\phi)X\right)g_{\alpha\beta},$$

which is a cosmological constant term.

Previous models of inflation *and/or* dark energy:

- Germani & Kehagias (2009)

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2}-\frac{F^2}{12}-\frac{m^2A^2}{4}+\frac{RA_{\mu\nu}A^{\mu\nu}}{6}+\frac{A_{\mu\gamma}R^{\gamma\lambda}A^{\mu}_{\lambda}}{2}\right),$$

- Koivisto, Motta & Pitrou (2009)

$$S=\int \mathrm{d}^Dx\sqrt{-g}\left(\frac{R}{2\kappa^2}-\frac{F^2}{2(n+1)!}-V(A^2)-\frac{\zeta A^2R}{2n!}\right),$$

- Ito & Soda (2015), Ohashi, Soda & Tsujikawa (2013)

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2}-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)-\sum_{p=2}^{4}\frac{1}{4}f_p^2(\phi)F_p^2\right).$$

Outline

Effective energy-momentum tensors

Effect of the 3–form coupled system

The Einstein equations could be written as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G(T^{\phi}_{\mu\nu} + T^{p}_{\mu\nu}),$$

with $T^{\phi}_{\mu\nu}$ the energy-momentum tensor for the scalar field,

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\sigma\phi\partial^\sigma\phi - g_{\mu\nu}V(\phi),$$

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$$T^{(3)}_{\alpha\beta} = f^2\left[\frac{\gamma_3}{6}F_{(3)\beta}{}^{\gamma\delta\rho}F_{(3)\alpha\gamma\delta\rho} - \frac{1}{2}g_{\alpha\beta}\left(\gamma_3\frac{F_3^2}{24} + \gamma_4\frac{\tilde{F}_3}{12}\right)\right],$$

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- Let us asume a FLRW Universe

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2.$$

- Moreover, we fix the form of the 3−form as

$$A_{(3)\mu_1\mu_2\mu_3} = A_3(t)dx \wedge dy \wedge dz,$$

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- Moreover, we fix the form of the 3−form as

$$A_{(3)\mu_1\mu_2\mu_3} = A_3(t)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z,$$

$$\ddot{A}_3 + \left[2\frac{\dot{f}_1}{f_1} - 3H\right]\dot{A}_3 = -\theta a^3 \frac{\dot{f}_1}{f_1}$$

with $\frac{\theta}{2} = \frac{\gamma_4}{\gamma_3}$.

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$$\dot{A}_3 = \left(\frac{\bar{p}_3}{f_1^2} - \frac{\theta}{2}\right) a^3,$$

being $\bar{p}_3$ an integration constant.

Ingredients

- Evolution for the scalar field

$$\ddot{\phi} + V_{,\phi} + 3H\dot{\phi} - \gamma_3 f_1 f_{1,\phi} \left(\frac{\dot{A}_3^2}{a^6} - \theta \frac{\dot{A}_3}{a^6} \right) = 0.$$

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- Including matter

$$T^m_{\mu\nu} = (\rho_m + p_m) u_\mu u_\nu + p_m g_{\mu\nu},$$

being u^μ the 4-velocity of the fluid with $u_\mu u^\mu = -1$, ρ_m the energy density and p_m the preassure.

Friedmann Equations

$$3H^2 = \frac{1}{M_{\rm Pl}^2}\left(\rho_\phi + \rho_m + \rho_p\right), \qquad 2\dot{H} + 3H^2 = -\frac{1}{M_{\rm Pl}^2}\left(p_\phi + p_m + p_p\right).$$

with

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}.$$

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$$\rho_p \equiv \gamma_3 f^2\left(\frac{\dot{A}_3^2}{2a^6} - \theta\frac{\dot{A}_3}{2a^6}\right), \quad p_p \equiv -\gamma_3 f^2\left(\frac{\dot{A}_3^2}{2a^6} - \theta\frac{\dot{A}_3}{2a^6}\right), \quad w_p = -1.$$

Dynamical System

Let us define the following variables

$$X = \frac{1}{\sqrt{6}M_{\rm pl}}\frac{\dot{\phi}}{H}, \qquad Y = \frac{1}{\sqrt{3}M_{\rm pl}}\frac{\sqrt{V}}{H}, \qquad P_3 = \frac{\sqrt{\gamma_3}f_1\dot{A}_3}{\sqrt{6}M_{\rm pl}Ha^3},$$

using solution for $\dot{A}_3$ with $\theta = 0$. Friedmann equation reads

$$\Omega_m = \frac{\rho_m}{3M_{\rm pl}^2H^2} = 1 - \Omega_{DE}$$

Total dark energy density parameter

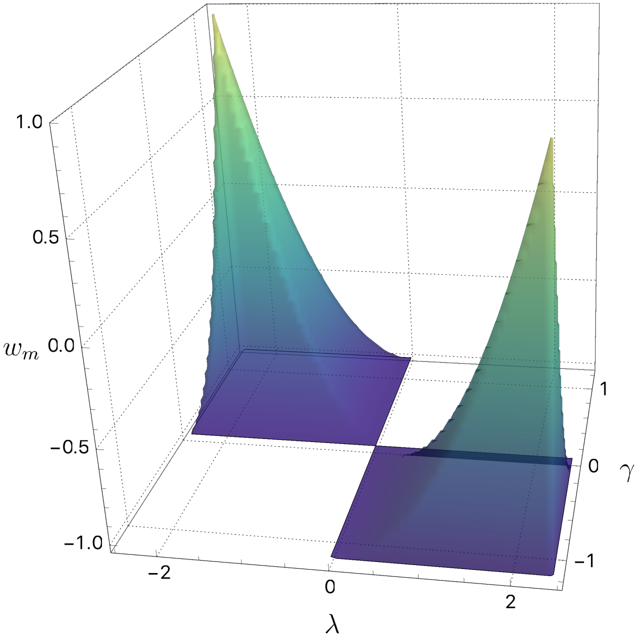
$$\Omega_{DE} = X^2 + Y^2 + P_3^2,$$

Effective E.o.S.

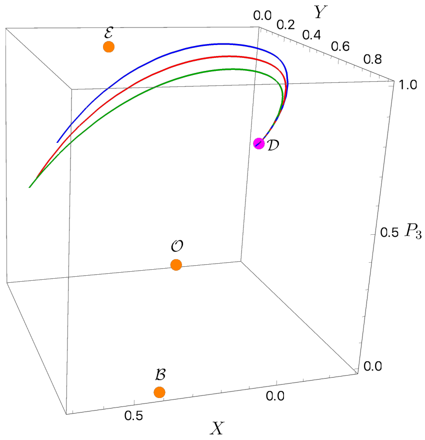
$$w_{eff} \equiv \frac{p_m + p_\phi + p_p}{\rho_m + \rho_\phi + \rho_p} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2},$$

Critical Points

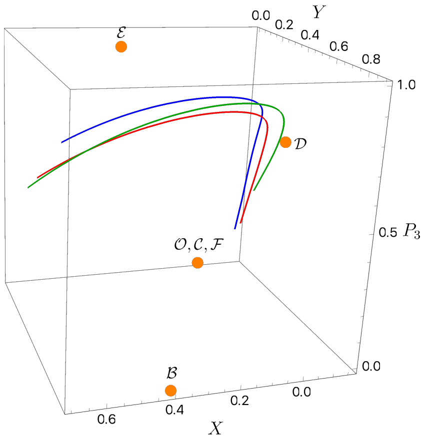
Point	X	Y	P_3	w_{eff}	Ω_{DE}
$\mathcal{O}$	0	0	0	w_m	0
$\mathcal{A}_\pm$	$\pm\, 1$	0	0	1	1
$\mathcal{B}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	0	$-1+\frac{\lambda^2}{3}$	1
$\mathcal{C}$	$\sqrt{\frac{3}{2}\frac{(w_m+1)}{\lambda}}$	$\sqrt{\frac{3}{2}}\sqrt{\frac{1-w_m^2}{\lambda^2}}$	0	w_m	$\frac{3(1+w_m)}{\lambda^2}$
$\mathcal{D}$	0	$\sqrt{\frac{2\gamma}{2\gamma-\lambda}}$	$\sqrt{\frac{\lambda}{\lambda-2\gamma}}$	-1	1
$\mathcal{E}$	$\sqrt{\frac{2}{3}}\gamma$	0	$\sqrt{1-\frac{2\gamma^2}{3}}$	$-1+\frac{4\gamma^2}{3}$	1
$\mathcal{F}$	$\sqrt{\frac{3}{2}\frac{(w_m+1)}{2\gamma}}$	0	$\sqrt{\frac{3}{2}}\sqrt{\frac{1-w_m^2}{4\gamma^2}}$	w_m	$\frac{3(1+w_m)}{4\gamma^2}$


1.0
0.5
w_m 0.0
−0.5
−1.0
−2
0
2
λ
1
0
γ
−1

- *Point* $\mathcal{O}$: Since $\Omega_{DE} = 0$, this point corresponds to a matter dominated universe with $\Omega_m = 1$ with no acceleration $w_{eff} = w_m$.
- *Points* $\mathcal{A}_{\pm}$: In these two points $w_{eff} = 1$ corresponding to stiff matter, with no acceleration.
- *Point* $\mathcal{B}$: Represents acceleration when $w_{eff} < -\frac{1}{3}$, i.e. for $\lambda^2 < 2$. When $\lambda \to 0$, this point represents a de Sitter expansion with $w_{eff} = -1$.
- *Point* $\mathcal{C}$: This point represents a so-called scaling solution where the effective EoS matches the matter EoS. Explicitly $\frac{\Omega_m}{\Omega_{DE}} = \frac{\lambda^2}{3(1+w_m)} - 1$, thus $0 < \Omega_{DE} < 1$ and $0 < \Omega_m = 1 - \Omega_{DE} < 1$. Since, $w_{eff} = w_m$, there is no acceleration.
- *Point* $\mathcal{D}$: Since $\Omega_{DE} = 1$ it describes an accelerated expansion.
- *Point* $\mathcal{E}$: Accelerated expansion for $\gamma = 0$, i.e. a constant coupling between the scalar field and the 3−form. We can also have acceleration providing $\gamma^2 < \frac{1}{2}$.
- *Point* $\mathcal{F}$: New scaling solution. In this case we have $\frac{\Omega_m}{\Omega_{DE}} = \frac{4\gamma^2}{3(1+w_m)} - 1$, and likewise $0 < \Omega_{DE} < 1$, $0 < \Omega_m < 1$. **This point could be of potential interest for the coincidence problem.** Nevertheless, there are no acceleration, due to $w_{eff} = w_m$.



(A)



(B)

Figure: (a) the coupling constants are set to be $\lambda = 1$, $\gamma = -0.4$ in a stiff matter universe $w_m = 1$. The number of e-foldings runs from 0 to 20. (b) same as plot (a) but with a cosmological constant-like term in the matter sector ($w_m = -1$). In this particular case, the points $\mathcal{C}$ and $\mathcal{F}$ correspond exactly to the origin $\mathcal{O}$.

Outline

Conclusions

1. p-forms as interesting approach to dark energy and/or inflation issues.
2. The general constuction of coupled p-forms only allows a $BF-$term; other controbutions are written as Maxwell-like terms for each p-form.
3. The coupled $1-$form and $2-$form system could be written as an action for a massive vector field. That means, a *mass generation* by kinetic couplings of different p-forms.
4. Non-trivial dynamics of the scalar-3-form coupled system. The homogeneous evolution of the 3-form makes this system only interesting at background level.
5. The dynamical system analysis shows some interesting features, in particular the existence of scaling solutions of great interest for the coincidence problem.