Beyond the standard model from noncommutative geometry

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Contents

Maths:

Riemannian geometry $\Leftrightarrow$ algebraic picture

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$$(M, g_{\mu\nu}) \quad \Leftrightarrow \quad C^\infty(M)$$

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$$\boxed{\text{The geometry of a finite space}}$$

Physics:

$$\underbrace{M}_{space-time} \times \underbrace{F}_{finite}$$

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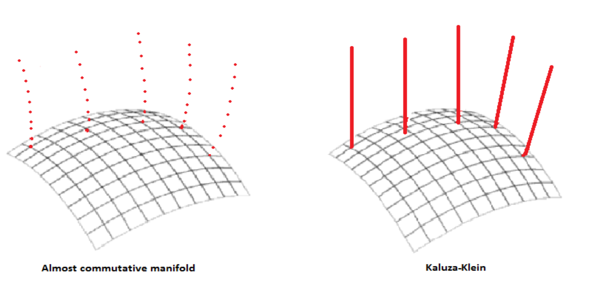

Dirac operator

$$D = \underbrace{\not{\partial} \otimes 1_n}_{continuous} + \underbrace{\gamma_5 \otimes D_F}_{discrete}$$

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Marck Kac (1966): Can one hear the shape of a drum?

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Spectral action

$\wr$

$$s + \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \mathrm{Tr}(\Phi^2) + \mathrm{Tr}(\Phi^4) + \mathrm{Tr}[(D_\mu\Phi)(D^\mu\Phi)]$$

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Unification: $5g_1^2 = 3g_2^2 = 3g_3^2 = f(Y)\lambda$

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Higgs mass :~ 170GeV

Classification of almost-commutative spaces

The classification of $M_A(\mathbb{C}) \oplus M_B(\mathbb{C}) \oplus M_C(\mathbb{C}) \oplus \cdots$ is possible thanks to Krajewski diagrams

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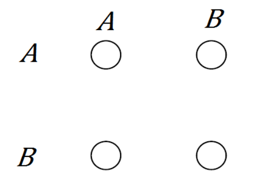

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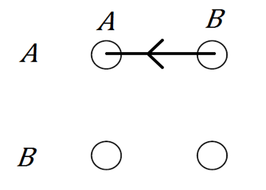

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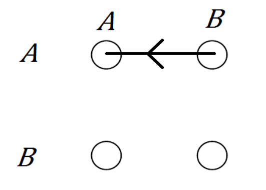

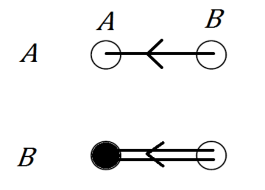

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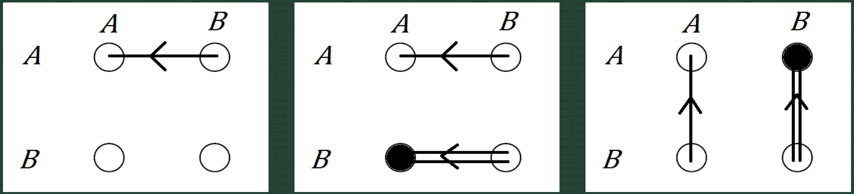

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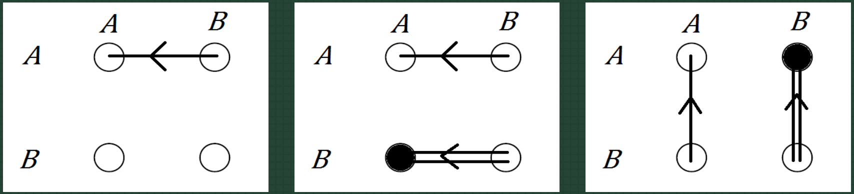
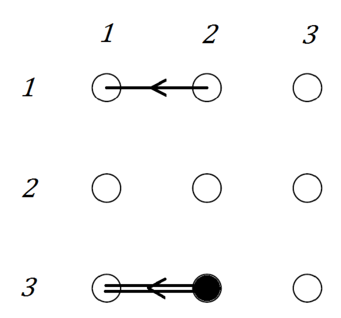


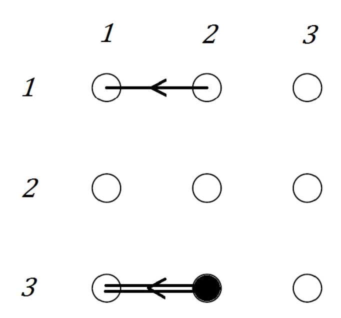
Hilbert space $+$ Dirac operator $\rightarrow$ Higgs field

The matrix $\boxed{\mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C})}$ and the SM



Krajewski diagram for the SM

The matrix $\boxed{\mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C})}$ and the SM



Krajewski diagram for the SM

Representation

$$\underbrace{\begin{pmatrix} 1\otimes 1 & \\ & \begin{pmatrix} 1\otimes 3 & \\ & 1\otimes 3 \end{pmatrix} \end{pmatrix}}_{Right}$$

$$\underbrace{\begin{pmatrix} 2\otimes 1 & \\ & 2\otimes 3 \end{pmatrix}}_{Left}$$

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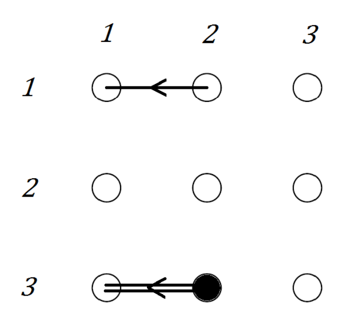

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This is exactly the fermionic content of SM

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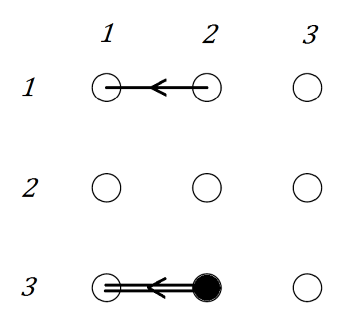

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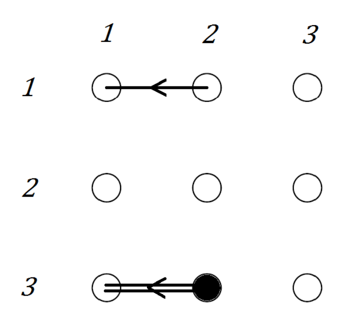

Krajewski diagram for the SM

Dirac operator

$$\underbrace{\begin{array}{c|c|c} m_{(1\times 2)} & & \\ \hline & n_{(1\times 2)} \otimes 1_3 & n'_{(1\times 2)} \otimes 1_3 \end{array}}_{\mathcal{M}}$$

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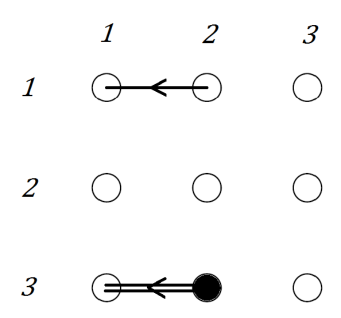

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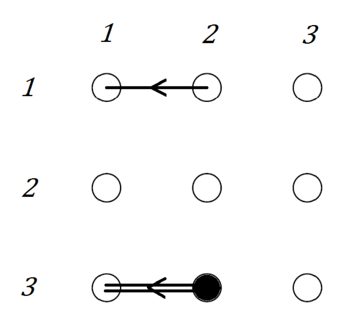

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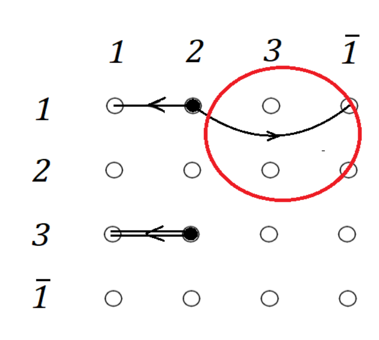
$$\boxed{[D, a] = \underbrace{(z_1, z_2)}_{Higgs} \mathcal{M}}$$

SM + right-handed neutrinos



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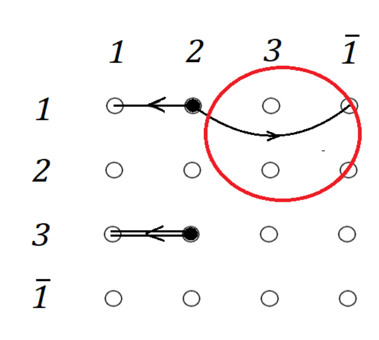

New arrow $\rightarrow$ Dirac mass for ν

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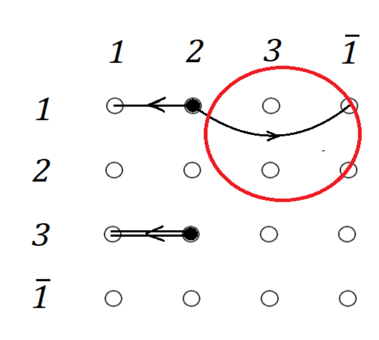

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Majorana $\rightarrow D = \left(\begin{array}{c|c} & T \\ \hline \overline{T} & \end{array}\right)$

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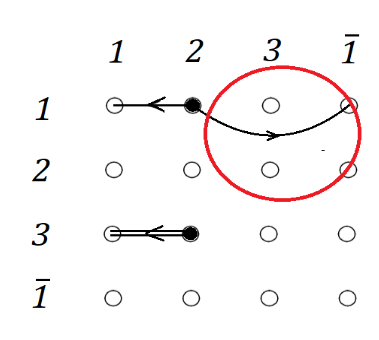

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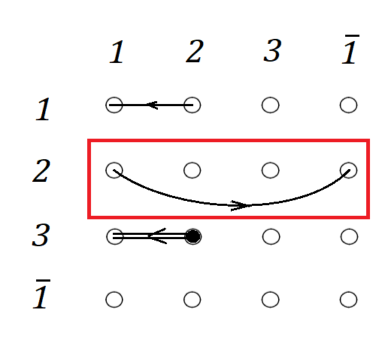
$\rightarrow$ see-saw mechanism

... the axiom of orientability must be forgotten

Vector doublets



Antiparticles: $\underbrace{2}_{rigth} \oplus \underbrace{2}_{left}$

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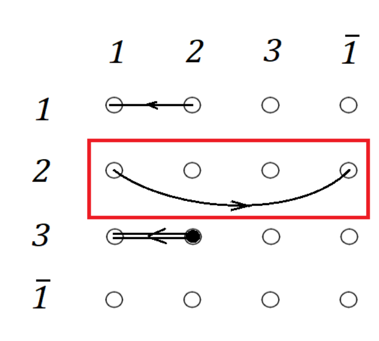

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$$\begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix} \to \Delta m_\psi = 350\text{MeV}$$

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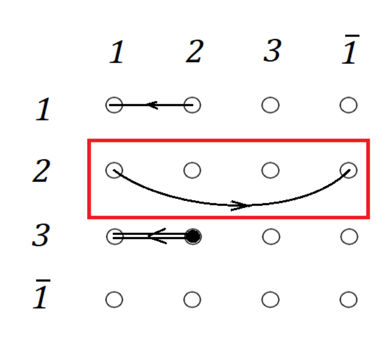

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Life time of ψ^-: $\sim 10^{-9}s$

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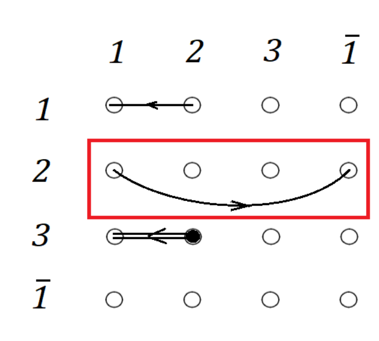
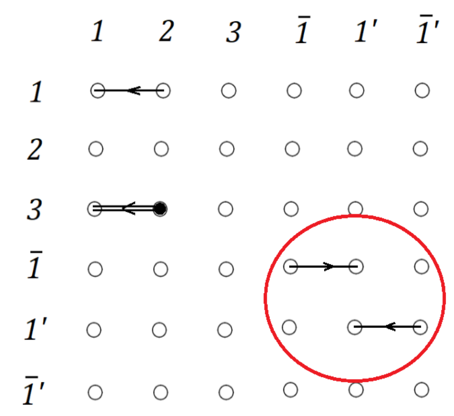


Antiparticles: $\underbrace{2}_{rigth} \oplus \underbrace{2}_{left}$

$$\begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix} \to \Delta m_\psi = 350\text{MeV}$$

Life time of ψ^-: $\sim 10^{-9}s$
dark matter $\to$ 10 & 550 TeV
Higgs mass $\sim$ 178 GeV

New fermions



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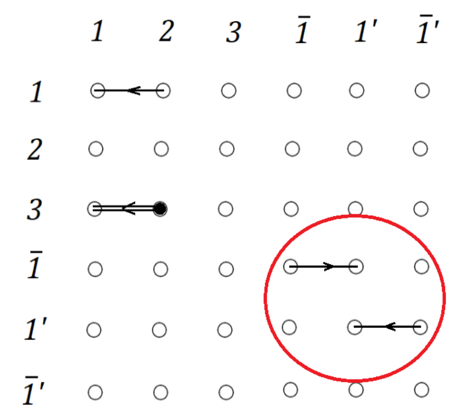
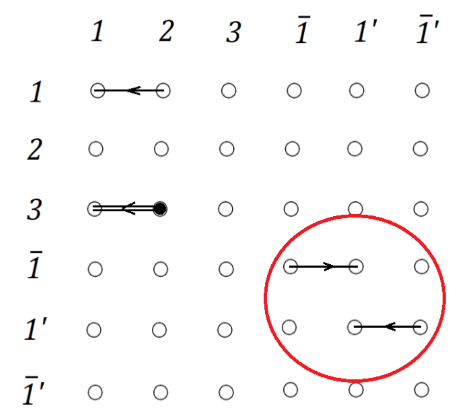


Two arrows

↓

Two new kinds of charged fermions

New fermions



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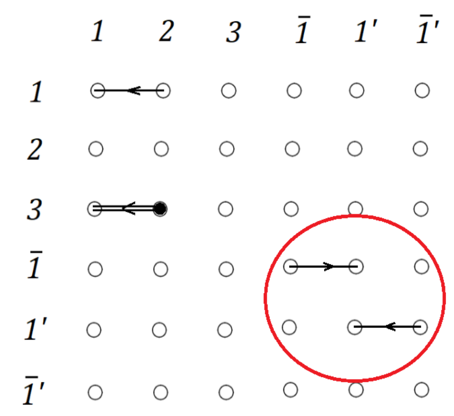
Two gauge invariant Dirac mass terms

New fermions



Two arrows

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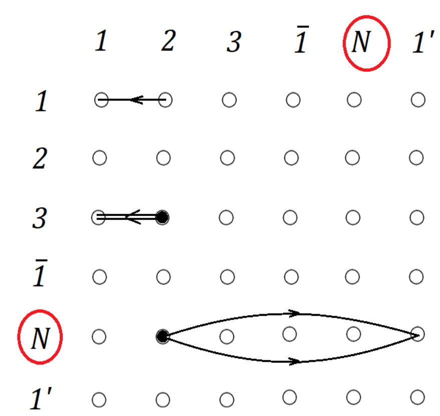
Two gauge invariant Dirac mass terms

↓ Hydrogen-like dark matter

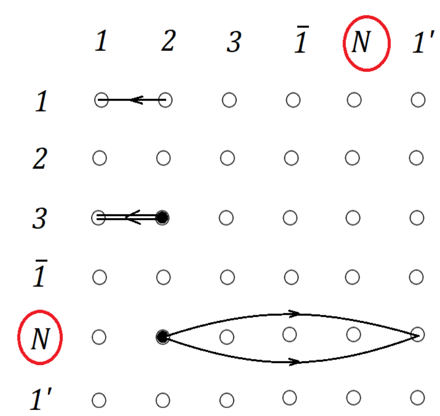
New colours



SM$+SU(N)$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L \oplus (\psi_1)_R \oplus (\psi_2)_R$$

New colours



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Gauge-invariant mass

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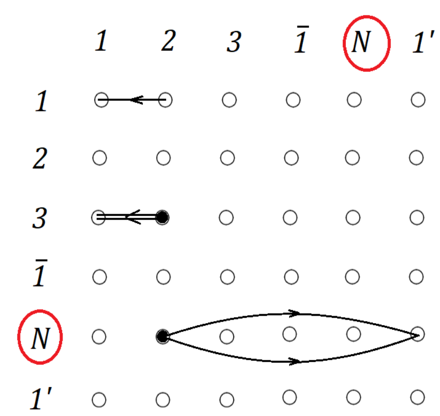

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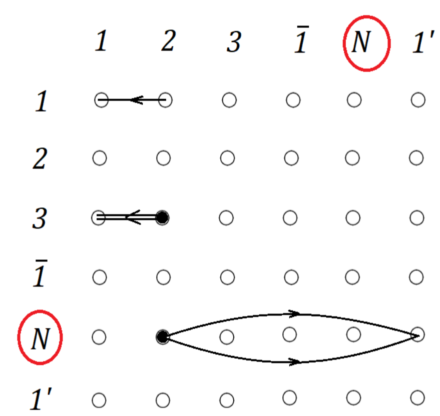
$N > 1 \rightarrow m_H > 170$GeV

$N = 1 \rightarrow m_H \sim 170$GeV

$$\boxed{N = 1 \rightarrow U(1)_X}$$

New colours



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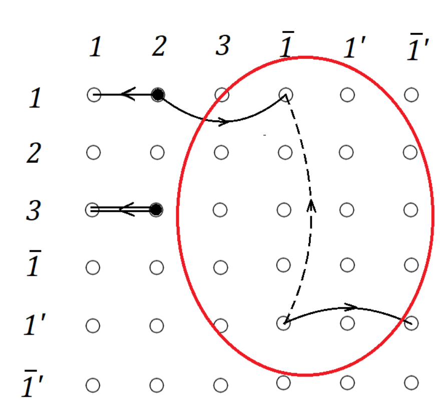
Cold dark matter!

New scalar fields



Right handed ν + Majorana mass

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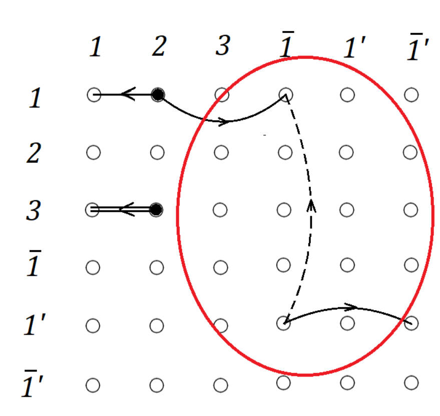

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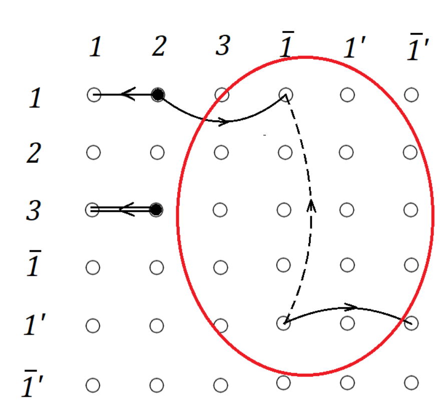

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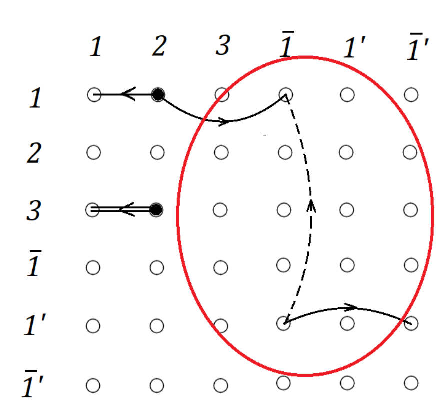
Dashed line: New scalar field

New scalar fields



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Higgs mass ~ 125GeV

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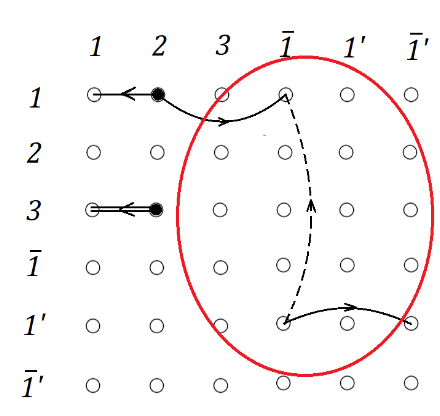

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Breaking: $U(1)_X \to \mathbb{Z}_2$

$\downarrow$

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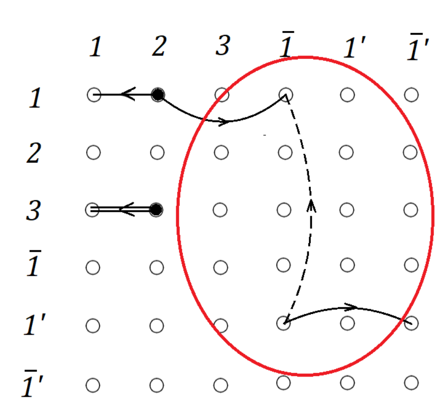

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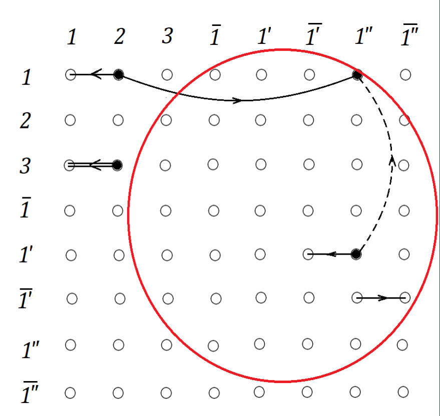
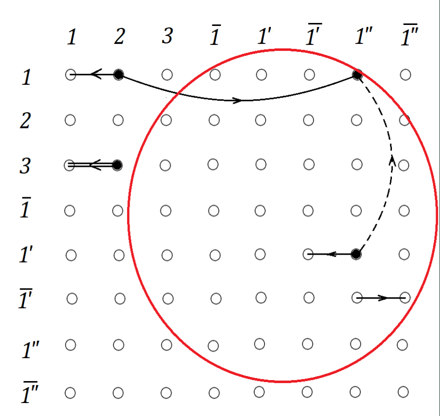
$\downarrow$

Dark photons

Inverse see-saw mechanism

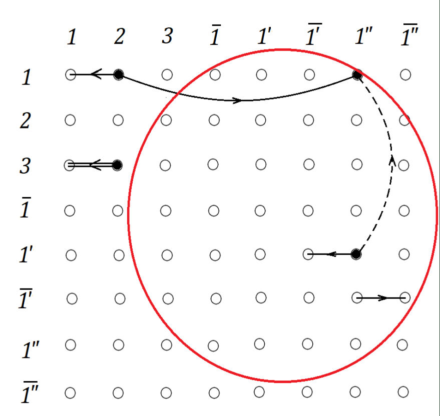


Inverse see-saw mechanism



Right handed ν + Majorana mass

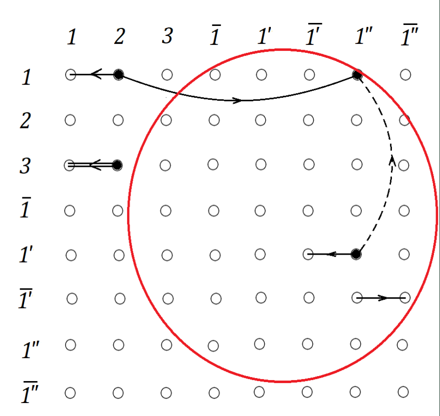
Inverse see-saw mechanism



Right handed ν + Majorana mass

Dashed line: gauge inv. mass

Inverse see-saw mechanism

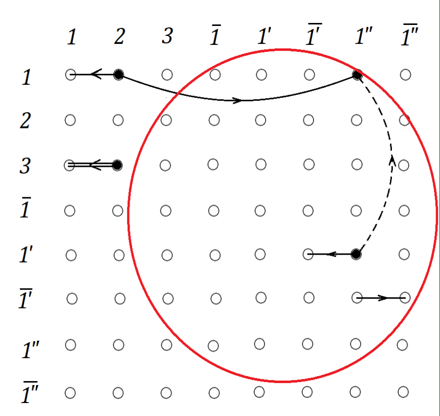


Right handed ν + Majorana mass

Dashed line: gauge inv. mass

New fermion's mass: ~ 1 GeV & 10^6 GeV

Inverse see-saw mechanism



Right handed ν + Majorana mass

Dashed line: gauge inv. mass

New fermion's mass: ~ 1 GeV & 10^6 GeV

$\downarrow$

Fermionic dark matter

+

Dark photons

Outcomes and outlooks

- Krajewski diagrams provides a pictorical way to search for SM extensions.
- Noncommutative geometry allows dark matter and neutrinos mass models but in a more restrictive way than usual efective field theories.
- Until now, all the neutrino masses obtained are at tree-level.
- Are possible neutrino masses at loop level from noncommutative geometry?
- Can one obtain two scalar doublets (at least) that allow us to introduce loop realizations.
- What about non-associative algebras?

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