

# *Pseudo-scalars non-minimally coupled to gravity during inflation*

*MOCa 2018*

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Based on: arXiv:1803.09743 [astro-ph.CO]*



Bogotá, July 31, 2018.

- 1 Inflation with pseudoscalar-vector interactions
- 2 Slow roll with a steep potential
- 3 CMB signatures. Spectrum, chiral GW and NG
- 4 Non-minimal coupling with gravity
- 5 Inflation with a massive vector
- 6 Final remarks

## Some motivations

### Some motivations/concerns and expectations

- Pseudoscalar fields offer a rich phenomenology in inflationary physics.
- Signatures of statistical anisotropies and parity violation in CMB correlators. Parity violating Non-Gaussianity for instance.
- Enhancement of gravitation waves. Chiral GW.
- Effects on LSS. Non-Gaussian & anisotropic bias.
- Pseudoscalars/axions are suitable dark matter candidates.
- If present during inflation, DM particles should leave some imprint in the CMB.
- Cosmological observations can be used to constraint DM models (couplings): isocurvature, non-gaussianity, scale dependence, anisotropies, etc.
- ...

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## Stable gauge invariant abelian models. Scalar + vector

- General scalar(pseudoscalar) + vector models (allowing for derivative interactions):

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \mathcal{L}(\Phi_I, A_\mu) \right]$$

- Stable and causal gauge invariant ( $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$ ) models:

$$S = \int d^4x \sqrt{-g} \left[ \mathcal{L}_G - \mathcal{L}_{\Phi_I}(\Phi_I, \partial\Phi_I) - \frac{1}{4} f_1(\Phi_I) F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} f_2(\Phi_I) F^{\mu\nu} \tilde{F}_{\mu\nu} \right]$$

A scalar-vector-gravity model:

$$\mathcal{L}_G = \frac{M_P^2}{2} R + h(\Phi_I) R \Rightarrow \text{Non-minimal coupling with gravity}$$

$$\mathcal{L}_{\Phi_I}(\Phi_I, \partial\Phi_I) = \mathcal{L}(\phi, \sigma) \Rightarrow \text{Inflaton plus auxiliary fields}$$

$$f_1(\phi) F^{\mu\nu} F_{\mu\nu} \Rightarrow \text{Non-diluting anisotropic source ("anisotropic hair").}$$

$$f_2(\phi) F^{\mu\nu} \tilde{F}_{\mu\nu} \Rightarrow \text{Parity symmetry breaking.}$$

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# Signatures of vector fields during inflation

Anisotropic and parity violating correlations in models of the form (some type of)  $f_1(\phi)F^2 + f_2(\phi)F\vec{F}$ , Golovnev, Vanchurin, Mukhanov, Dimopoulos, Karčiauskas, Wagstaff, Anber, Sorbo, Ackerman, Carroll, Wise, Bartolo, Dimastrogiovanni, Matarrese, Riotto, Liguori, Ricciardone, Peloso, Valenzuela-Toledo, Rodríguez, Lyth, Gumrukcuoglu, Himmetoglu, Shiraishi, Komatsu, Barnaby, Watanabe, Kanno, Soda, Emami, Firouzjahi...

**Angle dependent correlations. Both scalar and tensor correlations**

$$P_\zeta(k) \equiv \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \rangle' = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta \left( \frac{k}{aH} \right)^{n_\zeta - 1}, \quad \mathcal{P}_\zeta \sim (5 \times 10^{-5})^2$$

$$P_\zeta(k) \Rightarrow P_\zeta(\vec{k}) = P_\zeta(k) \left[ 1 + g_\zeta (\hat{k} \cdot \hat{n})^2 \right], \quad g_\zeta \sim 0.3 \quad (\text{back in 2009}),$$

$$B_\zeta = \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle' = \sum_{L=0} c_L P_L(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) P(k_1) P(k_2) + \text{perm.}$$

$c_L$  = encodes information about the VF couplings,  $P_L$  = Legendre polynomials

Planck 2015  $\rightarrow -0.0225 < g_\zeta < 0.0363, -10.7 < c_0 < 16.7, -89 < c_1 < 324, -57 < c_2 < 47$ .

Optimal estimator for tracking the parity odd features.

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# Natural inflation and electromagnetic dissipation

## Natural inflation pseudoscalar coupled to $U(1)$

$$f_1 = 1/4, f_2 = \alpha\phi/4f, V(\phi) = \Lambda^4(1 + \cos(\phi/f)).$$

$$S_{\phi A} = -\frac{1}{4} \int d^4x \sqrt{-g} \left[ F^{\mu\nu} F_{\mu\nu} + \frac{\alpha}{f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right]$$

↓

$$\nabla_\mu \left( F^{\mu\nu} + \frac{\alpha}{f} \phi \tilde{F}^{\mu\nu} \right) = 0, \quad \& \quad \nabla_\mu \tilde{F}^{\mu\nu} = 0.$$

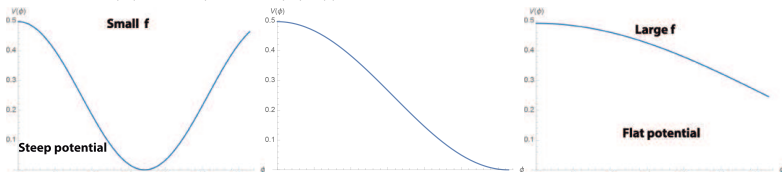
## Background metric

Nearly de Sitter geometry  $a(\tau) \approx -1/H\tau$  with constant Hubble parameter  $H$ ,

$$ds^2 = \frac{1}{H^2 \tau^2} (-d\tau^2 + dx_i dx^i).$$

# Natural inflation and electromagnetic dissipation

**Steep inflation**  $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$ .  $f \sim M_P$



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## Equations of motion

Homogeneous scalar field  $\partial_i \phi = 0 \Rightarrow \partial_i f_1 = \partial_i f_2 = 0$ .

$$\phi'' + 2aH\phi' + a^2 \frac{dV}{d\phi} = \frac{\alpha}{f} a^2 \langle \vec{E} \cdot \vec{B} \rangle, \quad A_i'' - \nabla^2 A_i - \frac{\alpha \phi'}{f} \epsilon_{ijk} \partial_j A_k = 0,$$

$$H^2 = \frac{1}{3M_{\text{P}}^2} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2} \langle \vec{E}^2 + \vec{B}^2 \rangle \right), \quad \dot{H} = -\frac{1}{2M_{\text{P}}^2} \dot{\phi}^2 - \frac{1}{3M_{\text{P}}^2} \langle \vec{E}^2 + \vec{B}^2 \rangle$$

Vector field solution. Polarization decomposition

$$\hat{A}_\lambda(\vec{x}, \tau) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} \vec{\epsilon}_\lambda(\vec{k}) A_\lambda(\tau, \vec{k}) a_\lambda(\vec{k}) e^{i\vec{k} \cdot \vec{x}} + c.c.$$

$$\vec{\epsilon}_\lambda(\vec{k}) \cdot \vec{k} = 0, \quad \vec{k} \times \vec{\epsilon}_\lambda = -i\lambda |\vec{k}| \vec{\epsilon}_\lambda, \quad \vec{\epsilon}_\lambda \cdot \vec{\epsilon}_{\lambda'} = \delta_{\lambda, -\lambda'}, \quad \vec{\epsilon}_\lambda^*(\hat{k}) = \vec{\epsilon}_{-\lambda}(\hat{k}) = \vec{\epsilon}_\lambda(-\hat{k}).$$

$$A''_{\pm} + \left( k^2 \pm \frac{2k\xi}{\tau} \right) A_{\pm} = 0 \quad \text{with} \quad \xi \equiv \frac{\alpha \dot{\phi}_0}{2fH}.$$

$$A_+ \approx \frac{1}{\sqrt{2k}} \left( \frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/aH}} \quad \text{for large } \xi, \quad \xi \gg k\tau/2.$$

Anber & Sorbo JCAP 0610 018 (2006), PRD 81 043534 (2010), PRD 85 123537 (2012).

## Steep potential slow roll

Backreaction effect due to the gauge field coupling. Large  $\xi$

$$\langle \vec{E} \cdot \vec{B} \rangle \approx -\mathcal{I} \frac{H^4}{\xi^4} e^{2\pi\xi}, \quad \frac{1}{2} \langle \vec{E}^2 + \vec{B}^2 \rangle \approx \frac{4\mathcal{I}}{7} \frac{H^4}{\xi^3} e^{2\pi\xi},$$

where  $\mathcal{I} \approx 2.4 \times 10^{-4}$  Anber & Sorbo PRD 85 123537 (2012).

Hubble parameter  $H^2 \approx V/3M_P^2$

$$\frac{1}{2} \langle \vec{E}^2 + \vec{B}^2 \rangle \approx \frac{4\mathcal{I}}{7} \frac{H^4}{\xi^3} e^{2\pi\xi} \approx \frac{4}{7} \frac{\xi}{\alpha} fV_\phi \ll 1 \quad \text{for } \alpha \gg \xi.$$

Slow roll parameter  $\epsilon = -\dot{H}/H^2$

$$\epsilon \approx \frac{2\xi^2 f^2}{\alpha^2 M_P^2} + \frac{8}{7} \frac{\xi}{\alpha} \frac{fV_\phi}{V} \ll 1$$

## Perturbations

Equation for the scalar perturbations

$$\delta\phi'' + 2aH\delta\phi' - \nabla^2\delta\phi + a^2V_{\phi\phi}\delta\phi = +a^2\frac{\mathcal{N}\alpha}{f}\delta\left[\vec{E}\cdot\vec{B}\right].$$

$$\delta\phi'' + \frac{1}{\tau}\frac{\alpha\pi V_{\phi}(\phi_0)}{fH^2}\delta\phi' + \frac{1}{\tau^2}\frac{V_{\phi\phi}(\phi_0)}{H^2}\delta\phi \approx a^2\frac{\mathcal{N}\alpha}{f}\delta_{\vec{E}\cdot\vec{B}},$$

$$\delta\phi(\tau, \vec{k}) = \frac{\mathcal{N}\alpha}{f} \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) G(\tau, \tau_1) \int d^3x e^{-i\vec{k}\cdot\vec{x}} \delta_{\vec{E}\cdot\vec{B}}(\tau_1, \vec{x})$$

Green's function

$$G(\tau, \tau') = \frac{\tau'}{\Delta} \left[ \left( \frac{\tau}{\tau'} \right)^{\nu_+} - \left( \frac{\tau}{\tau'} \right)^{\nu_-} \right] \Theta(\tau - \tau') \approx \frac{\tau'}{\Delta} \left( \frac{\tau}{\tau'} \right)^{\nu_+},$$

where

$$\nu_{\pm} \equiv \frac{1}{2} \left( 1 - \frac{\pi\alpha V_{\phi}(\phi_0)}{fH^2} \right) \pm \frac{1}{2} \Delta, \quad \text{and} \quad \Delta \equiv \sqrt{\left( 1 - \frac{\pi\alpha V_{\phi}(\phi_0)}{fH^2} \right)^2 - \frac{4V_{\phi\phi}(\phi_0)}{H^2}}.$$

## Perturbations

### N-point correlation functions

$$\begin{aligned} \langle \delta\phi(\vec{p}_1) \cdots \delta\phi(\vec{p}_n) \rangle &= \delta(\vec{p}_1 + \cdots + \vec{p}_n) \left( \frac{\mathcal{N}\alpha}{f} \right)^n \int d\tau_1 \cdots d\tau_n a_1^2 \cdots a_n^2 G(\tau, \tau_1) \cdots G(\tau, \tau_n) \\ &\times \int d^3x_1 \cdots d^3x_n e^{-i(\vec{p}_1 \cdot \vec{x}_1 + \cdots + \vec{p}_n \cdot \vec{x}_n)} \langle \delta_{\vec{E} \cdot \vec{B}}(\tau_1, \vec{x}_1) \cdots \delta_{\vec{E} \cdot \vec{B}}(\tau_n, \vec{x}_n) \rangle, \end{aligned}$$

where  $a_i \equiv a(\tau_i)$ .

### Spectrum of the perturbations

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &= \delta(\vec{p} + \vec{p}') \left( \frac{\mathcal{N}\alpha}{f} \right)^2 \int d\tau_1 d\tau_2 a_1^2 a_2^2 G(\tau, \tau_1) G(\tau, \tau_2) \\ &\times \int d^3x e^{i\vec{p} \cdot \vec{x}} \langle \delta_{\vec{E} \cdot \vec{B}}(\tau_1, 0) \delta_{\vec{E} \cdot \vec{B}}(\tau_2, \vec{x}) \rangle \\ &\approx \mathcal{F}(\nu_+) \frac{\delta(\vec{p} + \vec{p}')}{p^3} \frac{\mathcal{N}^2 \alpha^2 H^4}{\Delta^2 f^2 \xi^8} e^{4\pi \xi} (-2^5 \xi p \tau)^{2\nu_+}. \end{aligned}$$

Approximations used:

$$\Delta^2 \approx \frac{\pi^2 \alpha^2 V_\phi^2}{f^2 H^4}, \quad \dot{\phi}_0^2 = \frac{4f^2 H^2}{\alpha^2} \xi^2, \quad V_\phi \approx \frac{\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle, \quad -\mathcal{I} \frac{H^4}{\xi^4} e^{2\pi \xi}.$$

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# CMB signatures

## Scalar power spectrum

Primordial curvature perturbation  $\zeta = -H\delta\phi/\dot{\phi}_0$

$$P_\zeta(p) \approx \frac{\mathcal{F}(\nu_+)}{8\pi^4 \mathcal{I}^2 \xi^2} (-2^5 \xi p \tau)^{2\nu_+} \approx \frac{5 \times 10^{-2}}{\mathcal{N} \xi^2} (-2^5 \xi p \tau)^{2\nu_+}.$$

$$n_s - 1 \approx 2\nu_+ = -\frac{2fV_{\phi\phi}(\phi_0)}{\pi\alpha V_\phi(\phi_0)}.$$

## Gravitational waves

$$h''_\lambda - \frac{2}{\tau} h'_\lambda + k^2 h_\lambda = \frac{2}{M_P^2} \Pi_\lambda^{lm} T_{lm}, \quad h^{ij}(\vec{k}) = \sqrt{2} \sum_{\lambda=\pm} \epsilon_\lambda^i(\vec{k}) \epsilon_\lambda^j(\vec{k}) h_\lambda(\tau, \vec{k})$$

$$\Pi_\lambda^{lm} = \epsilon_{-\lambda}^l(\vec{k}) \epsilon_{-\lambda}^m(\vec{k}) / \sqrt{2}, \quad T_{ij} = -a^2 (E_i E_j + B_i B_j) + (\dots) \delta_{ij} = -a^{-2} A'_i A'_j + \dots$$

$$h_\lambda(\vec{k}) = -\frac{2H^2}{M_P^2} \int d\tau' G_k(\tau, \tau') (-\tau')^2 \int \frac{d^3q}{(2\pi)^{3/2}} \Pi_\lambda^{ij} A'_i(\vec{q}, \tau') A'_j(\vec{k} - \vec{q}, \tau'),$$

# CMB signatures

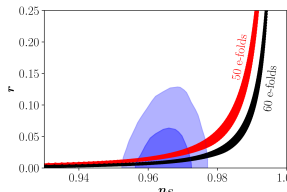
**Chiral gravitational waves spectrum** Sorbo JCAP06(2011)003, Barnaby, Moxon, Namba, Peloso, Shiu, Zhou PhysRevD.86.103508, Cook & Sorbo JCAP11(2013)047, ...

$$\langle h_+ h_+ \rangle \approx \mathcal{A}^+ \frac{H^2}{M_P^2} \frac{e^{4\pi\xi}}{\xi^6} \frac{1}{k^3}, \quad \langle h_- h_- \rangle \approx \mathcal{A}^- \frac{H^2}{M_P^2} \frac{e^{4\pi\xi}}{\xi^6} \frac{1}{k^3}.$$

With  $\mathcal{A}^+ = 8.6 \times 10^{-7}$ ,  $\mathcal{A}^- = 1.8 \times 10^{-9}$ . Testing the statistics of (sourced) GW, Shiraishi, Hikage, Namba, Namikawa, Hazumi Phys.Rev.D94, 043506, arXiv:1606.06082. (Scale dependent and sizeable GW due to source fields like axions)

$\langle B^+ B^+ B^+ \rangle$  at  $3\sigma$  with LiteBIRD

**Tensor to scalar ratio**  $r = \frac{1}{\mathcal{P}_\zeta} \frac{2V(\phi_0)}{3\pi^2 M_P^4} + 2.7 \times 10^2 \frac{\xi^4}{\alpha^2} \left( \frac{f V_\phi(\phi_0)}{V(\phi_0)} \right)^2$ ,



# CMB signatures

**Non-Gaussianities** Anber and Sorbo PhysRevD 85, 123537. Equilateral 3-point correlator of scalar perturbations.

$$\langle \delta\phi(\vec{k}_1)\delta\phi(\vec{k}_2)\delta\phi(\vec{k}_3) \rangle \approx -10^{-9} \mathcal{N} \frac{\alpha^3 H^6}{f^3 \Delta^3 \xi^{12}} e^{6\pi\xi} \frac{\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{k^6}$$

3-point correlator of the primordial curvature perturbation

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = \frac{3(2\pi)^{5/2}}{10} f_{NL} \mathcal{P}_\zeta^2 \frac{\sum_i k_i^3}{\prod_i k_i^3} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

$$f_{NL} \approx 4.4 \times 10^{10} \mathcal{P}^3 \frac{e^{6\pi\xi}}{\xi^9}$$

Large NG for large  $\xi$ , Barnaby and Peloso Phys.Rev.Lett.106, 181301 (2011).



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## Non minimal coupling with gravity

We add nonminimal coupling with gravity to the previous system

$$\mathcal{L} = \sqrt{-g} \left[ \frac{M_{\text{P}}^2}{2} \left( 1 + \frac{2h(\phi)}{M_{\text{P}}^2} \right) R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} \frac{\alpha}{f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right].$$

where  $h(\phi) = \frac{1}{2} \mathcal{X} \phi^2$ .

**Einstein frame** We define the metric  $\bar{g}_{\mu\nu} = \Omega(\phi) g_{\mu\nu}$ , where  $\Omega(\phi) \equiv \left( 1 + \frac{2h(\phi)}{M_{\text{P}}^2} \right)$ .

$$\mathcal{L} = \sqrt{-\bar{g}} \left[ \frac{M_{\text{P}}^2}{2} \bar{R} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{\nabla}_{\mu} \bar{\phi} \bar{\nabla}_{\nu} \bar{\phi} - \bar{V}(\bar{\phi}) - \frac{1}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \frac{1}{4} \frac{\alpha \phi(\bar{\phi})}{f} \bar{F}^{\mu\nu} \tilde{\bar{F}}_{\mu\nu} \right],$$

where  $d\bar{\phi}/d\phi = K^{1/2}$ ,  $\bar{V}(\bar{\phi}) \equiv \frac{V(\phi)}{\Omega^2}$ ,  $\bar{F}^{\mu\nu} \bar{F}_{\mu\nu} \equiv \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} F_{\alpha\beta} F_{\mu\nu}$  and

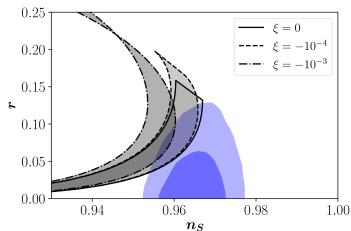
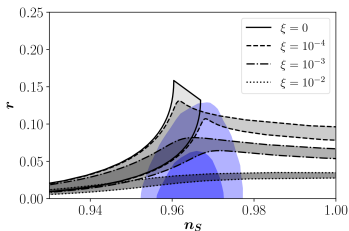
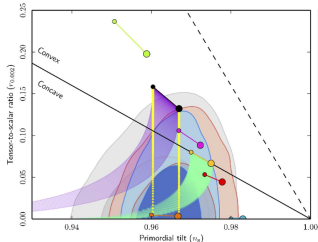
$$K(\phi) = \frac{1}{\Omega} + \frac{3M_{\text{P}}^2}{2\Omega^2} \left( \frac{\partial\Omega}{\partial\phi} \right)^2 = \frac{1 + 6(\mathcal{X} + \frac{1}{6}) \mathcal{X} \left( \frac{\phi}{M_{\text{P}}} \right)^2}{\left( 1 + \mathcal{X} \left( \frac{\phi}{M_{\text{P}}} \right)^2 \right)^2}.$$

**Why this is useful?** Correlation functions are invariant under conformal transformation of the metric. Often, it is easier to calculate the correlators in the Einstein frame.

# Non minimal coupling with gravity

**Applications.** Resurrect models disfavored by Planck.

**Example: Natural inflation.**  $\alpha = 0$  and  $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$



# Non minimal coupling with gravity

**Scalar perturbations.** Back to the pseudoscalar model. Some boring details...

$$\bar{\phi}_0'' + 2\bar{a} \bar{H} \bar{\phi}_0' + \bar{a}^2 \bar{V}_{\bar{\phi}} = \frac{\bar{a}^2}{\Omega^2 K^{1/2}} \frac{\mathcal{N}\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle.$$

$$\delta\bar{\phi}'' + 2\bar{a} \bar{H} \left( 1 - \frac{\pi \alpha \bar{V}_{\bar{\phi}}}{2K^{1/2} f \bar{H}^2} \right) \delta\bar{\phi}' + \bar{a}^2 \left( \bar{V}_{\bar{\phi}\bar{\phi}} + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \bar{V}_{\bar{\phi}} \right) \delta\bar{\phi} = \frac{\bar{a}^2 \mathcal{N} \alpha}{\Omega^2 K^{1/2} f} \delta\vec{E} \cdot \vec{B},$$

$$\delta\bar{\phi}(\bar{\tau}, \vec{k}) = \frac{\mathcal{N}\alpha}{f} \int_{-\infty}^{\bar{\tau}} d\bar{\tau}_1 \frac{\bar{a}^2(\bar{\tau}_1) \bar{G}(\bar{\tau}, \bar{\tau}_1)}{\Omega^2 K^{1/2}} \int d^3x e^{-i\vec{k} \cdot \vec{x}} \delta\vec{E} \cdot \vec{B}(\bar{\tau}_1, \vec{x}),$$

Where:

$$\bar{G}(\bar{\tau}, \bar{\tau}') = \frac{\bar{\tau}'}{\bar{\Delta}} \left[ \left( \frac{\bar{\tau}}{\bar{\tau}'} \right)^{\bar{\nu}+} - \left( \frac{\bar{\tau}}{\bar{\tau}'} \right)^{\bar{\nu}-} \right] \Theta(\bar{\tau} - \bar{\tau}'), \quad \text{with}$$

$$\nu_{\pm} \equiv \frac{1}{2} \left( 1 - \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right) \pm \frac{1}{2} \bar{\Delta} \approx -\frac{1}{2} \left( \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right) \pm \frac{1}{2} \bar{\Delta},$$

$$\bar{\Delta} \equiv \sqrt{\left( 1 - \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right)^2 - \frac{4}{\bar{H}^2} \left( \bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0) + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \bar{V}_{\bar{\phi}}(\bar{\phi}_0) \right)}.$$

## Non minimal coupling with gravity

**Pseudoscalar coupled to gauge fields.** Results for the scalar and tensor spectrum.

- Scalar perturbations spectrum

$$\bar{\mathcal{P}}_{\zeta}(p) \approx \frac{\mathcal{F}(\bar{\nu}_+)}{8\pi^4 \mathcal{I}^2 \xi^2} (-2^5 \xi p \tau)^{2\nu_+} \approx \frac{5 \times 10^{-2}}{\mathcal{N} \xi^2} (-2^5 \xi p \tau)^{2\nu_+}.$$

$$\bar{n}_s - 1 \approx 2\bar{\nu}_+ = -K^{1/2} \frac{2f\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}{\pi\alpha\bar{V}_{\bar{\phi}}(\bar{\phi}_0)} \left( 1 + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \frac{\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)} \right).$$

- Tensor perturbations

$$d\bar{s}^2 = -\bar{a}^2(\bar{\tau}) [-d\bar{\tau}^2 + (\delta_{ij} + \bar{h}_{ij})d\bar{x}_i d\bar{x}_j] \quad \bar{T}_{ij}^{\text{EM}} = -\bar{a}^2(E_i E_j + B_i B_j) + (\dots)\delta_{ij}.$$

$$\langle h_{\pm} h_{\pm} \rangle \approx \frac{\bar{H}^2}{\pi^2 M_P^2} \left( 1 + \mathcal{A}^{\pm} \frac{\mathcal{N} \bar{H}^2}{M_P^2} \frac{e^{4\pi\xi}}{\xi^6} \right)$$

$n_s$  and  $r$

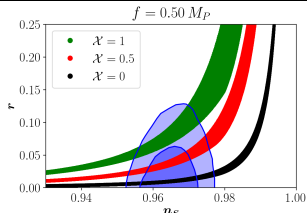
## Chiral gravitational waves spectrum

- Spectral index of scalar perturbations

$$\bar{n}_s - 1 \approx 2\bar{\nu}_+ = -K^{1/2} \frac{2f\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}{\pi\alpha\bar{V}_{\bar{\phi}}(\bar{\phi}_0)} \left( 1 + \frac{d\ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \frac{\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)} \right).$$

- Tensor to scalar ratio

$$\bar{r} \approx \frac{2V}{3\pi^2 M_P^4 \Omega^2 \mathcal{P}_\zeta} + 2.9 \times 10^2 K \frac{\xi^4}{\alpha^2} \left( \frac{f\bar{V}_{\bar{\phi}}}{\bar{V}} \right)^2.$$



# Features of the non-minimally coupled case

- The amplitude of the scalar spectrum is NOT suppressed due to the presence of the non-minimal coupling.
- Non-Gaussianities on scalar correlators are NOT suppressed either.
- Non trivial effect on the higher order statistics of the sourced gravitational waves.
- Modified bound on  $\bar{r}$ :

$$\bar{r} < 1.7 \times 10^{-2} \left( f_{\text{NL}}^{\text{obs}} \right)^{2/3} K^2 \epsilon^2.$$

1 Inflation with pseudoscalar-vector interactions

2 Slow roll with a steep potential

3 CMB signatures. Spectrum, chiral GW and NG

4 Non-minimal coupling with gravity

**5 Inflation with a massive vector**

6 Final remarks



# Inflation with a massive vector

Work in progress with L. Sorbo and M. Anber.

- Retains the nice features of the massless case, chiral GW spectrum, etc.
- A “Higgsed” mechanism to add mass to the vector boson.
- The transverse modes are decoupled from the longitudinal mode.
- The longitudinal mode under control.
- A possible way out of the large amplitude spectrum.
- Hopefully, this mechanism suppress the non-Gaussianities of the perturbations.

# Inflation with a massive vector

**Some details of the massive case** Work in progress with L. Sorbo and M. Anber.

Equations of motion

$$\nabla_\mu \left( F^{\mu\nu} + \frac{\alpha}{f} \phi \tilde{F}^{\mu\nu} \right) - m^2 A^\nu = 0.$$

Longitudinal mode

$$\frac{d A_c}{dx^2} + \left[ 1 + \frac{\mu^2}{x^2} + \frac{2/x^2}{(1 + \mu^2/x^2)^2} \left( 1 - \frac{\mu^2}{x^2} \right) \right] A_c = 0$$

with

$$\Omega(x) \equiv \left[ 1 + \frac{\mu^2}{x^2} + \frac{2/x^2}{(1 + \mu^2/x^2)^2} \left( 1 - \frac{\mu^2}{x^2} \right) \right]^{1/2}.$$

Adiabatic evolution:

$$\frac{\Omega'}{\Omega^2} = \frac{-x^6 (\mu^2 + 2) - 3x^4 (\mu^2 - 2) \mu^2 - 3x^2 \mu^6 - \mu^8}{(x^6 + x^4 (3\mu^2 + 2) + x^2 \mu^2 (3\mu^2 - 2) + \mu^6)^{3/2}} \ll 1.$$

# Inflation with a massive vector

## Some details of the massive case

Transverse modes. They are decoupled from the longitudinal mode evolution!

$$\left( \partial_\tau^2 + k^2 + \frac{m^2}{H^2 \tau^2} \pm \frac{\xi k}{2\tau} \right) A_\pm(k, \tau) = 0, \quad \text{with} \quad \xi = \frac{\alpha \dot{\phi}}{2fH}.$$

Solution

$$A_+ = \frac{e^{\frac{\pi\xi}{2}}}{\sqrt{2k}} W_{i\xi, i\mu}(2ix), \quad \text{with} \quad x = -k\tau, \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{1}{4}}.$$

Analytical approximations for large  $\xi$  and  $\xi \gg \mu$  can be done and the estimate for the spectrum is suppressed by the mass parameter:

$$\mathcal{P}_\zeta(p_1) \approx \frac{1225\mathcal{D}(\mu)}{64\xi^2\mu^{14}} (-8\xi p_1 \tau)^{2\nu_+}.$$

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## Final remarks and hopes

- Interesting possibilities for the GW production mechanism with a pseudoscalar coupling term.
- Statistical anisotropy, parity violating and scale dependent effect are interesting effects that naturally arise in vector field models.
- DM models can be constrained by cosmological observations. Selfcouplings and couplings with the inflaton of the DM sector can be constrained by cosmological parameters such as isocurvature, non-gaussianity, GW, etc.
- Non minimal coupling with gravity is an interesting avenue to explore. This can bring some interesting models such as natural and chaotic inflation inside the allowed  $n_s, r$  region.
- Non trivial modification of the tensor correlators due to non-minimal coupling with gravity.
- Suppression of the correlation functions for massive vector field case.



# II WORKSHOP ON CURRENT CHALLENGES IN COSMOLOGY

OCT 29 - NOV 2  
2018

LSS, DARK ENERGY AND MODIFIED GRAVITY

BOGOTÁ  
COLOMBIA

## INVITED SPEAKERS

Raul Abramo  
Universidade de São Paulo

Luca Amendola  
Universität Heidelberg

Paolo Creminelli  
ICTP Trieste

Cora Dvorkin  
Harvard University

Hendrik Hildebrandt  
Universität Bonn

Jaime E. Forero-Romero  
Universidad de los Andes

Macarena Lagos  
KICP University of Chicago

Jorge Noreña  
PUC Valparaíso

Massimo Pietroni  
Università di Parma and  
INFN Padova

Diego Restrepo  
Universidad de Antioquia

Yeinzon Rodríguez  
Universidad Antonio Nariño  
Universidad Industrial de  
Santander

Björn Malte Schäfer  
Universität Heidelberg

Leonardo Senatore  
Stanford University

Filippo Vernizzi  
IPT Université Paris Saclay

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## VENUE

Centro de convenciones,  
Universidad Antonio Nariño  
Transversal 21 # 96 - 42

Registration deadline:  
September 28, 2018

Abstracts submission deadline:  
September 7, 2018

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## II WORKSHOP ON CURRENT CHALLENGES IN COSMOLOGY

LSS, DARK ENERGY AND MODIFIED GRAVITY

### Overview

The main objective of this second edition of a workshop on current challenges in cosmology is to discuss several topics related with large scale structure (LSS) formation both from a theoretical and an observational point of view. Roughly speaking, the topics of discussion can be classified in the following subjects:

- Cosmological perturbation theory of LSS.
- Effective field theory of LSS.
- Modified gravity models. Galileons, Horndeski and beyond, bigravity, etc.
- Connections between inflationary physics and LSS.
- Weak lensing.
- Connections with observations. Observational ongoing and planned missions.

The workshop is intended to facilitate communication and discussion among researchers, to promote future collaborations and to boost the work of local research groups in theoretical and data-driven cosmology. Along with plenary talks, there will also be discussion sessions aimed to provoke debate and stimulate collaborative work among the participants. The list of invited speakers was designed with the purpose of bringing together a group of researchers with a well known long term trajectory, jointly with a group of young authors with an important progression in their careers during the latest years and with a remarkable academic production in the interest subjects of the workshop.

We will reserve some space for a poster session for the interested participants for Thursday November 1st. Additionally, we will also select six (6) short talks (15' length) for Monday and Tuesday. These sessions are intended for the students and participants to have the opportunity to discuss details about their current research projects on cosmology and interact with the speakers and the other participants.

POSTER

[Download  
Conference Poster](#)

## Programme by speakers.

	Monday 29	Tuesday 30	Wednesday 31	Thursday 1	Friday 2
	Dark energy, modified gravity and LSS	Beyond linear order in LSS	Observations	Connections with inflation and dark matter	Lensing
09:00-10:00	Paolo Creminelli. "Impact of GW observations on Dark Energy models"	Massimo Pietroni. "Time RG Flow methods for CPT, BAO"	Raul Abramo. "Challenges to exploiting large-scale structure data in the next 10 years"	Jorge Noreña. TBA	Cora Dvorkin. "Dark Matter Substructure with Strong Gravitational Lensing"
10:00-11:00	Yeinzon Rodríguez. "Cosmology of the generalized SU(2) Proca theory"	Filippo Vernizzi. "Dark energy and the large-scale structure"	Jaime Forero. "DESI"	Luca Amendola. "Gravitational waves and cosmology"	Hendrik Hildebrandt. "Observational cosmic shear KiDS, weak lensing"
11:00-11:30	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:30-12:30	Discussion session Chair: Jorge Noreña	Discussion session Chair: Yeinzon Rodríguez	Discussion session Chair: Bjorn Malte Schäfer	Discussion session Chair: Filippo Vernizzi	Discussion session Chair: Raul Abramo
12:30-14:30	Lunch	Lunch	Lunch	Lunch	Lunch
14:30-15:30	Macarena Lagos. "Modified gravity models and late universe"	Leonardo Senatore. "EFT of LSS. Applications to data and/or massive neutrinos"	Free time	Diego Restrepo. TBA	Björn Malte Schäfer. "Information content of nonlinear cosmic structures"
15:30-16:00	Coffee break	Coffee break		Coffee break	Coffee break
16:00-17:00	Discussion session Chair: Luca Amendola	Discussion session Chair: Paolo Creminelli		Discussion session Chair: Nicolas Bernal	Summary and Conclusions Chair: XXX
17:00-18:00	Students activity	Students activity		Posters	Finalization
19:00	Welcome cocktail		Dinner	Public talk	

## Listado de charlas