Simulating Collisional Dark Matter

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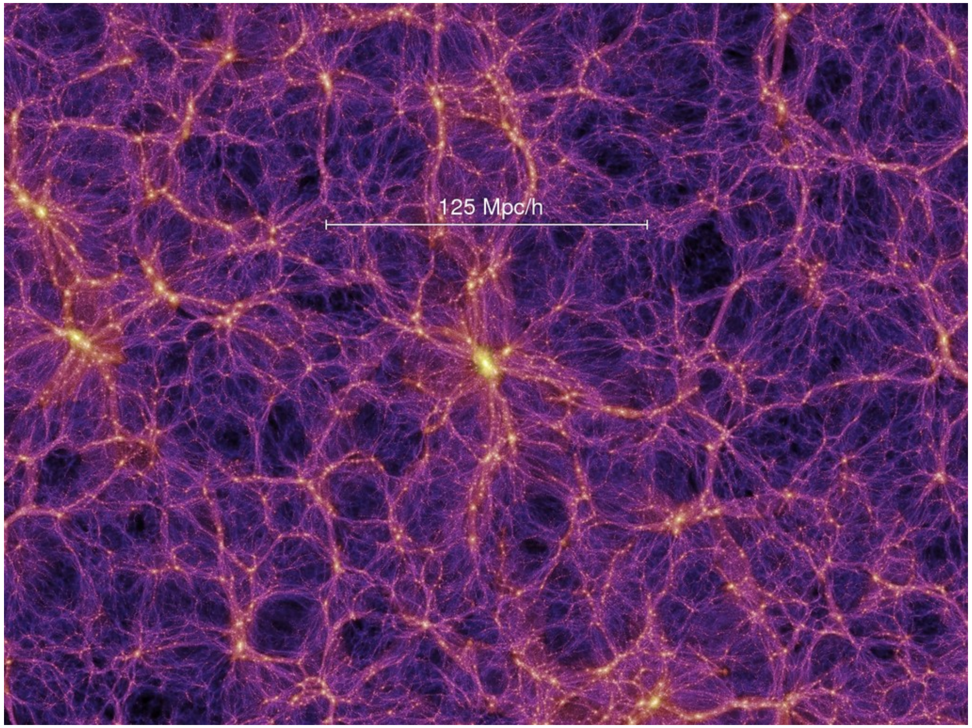
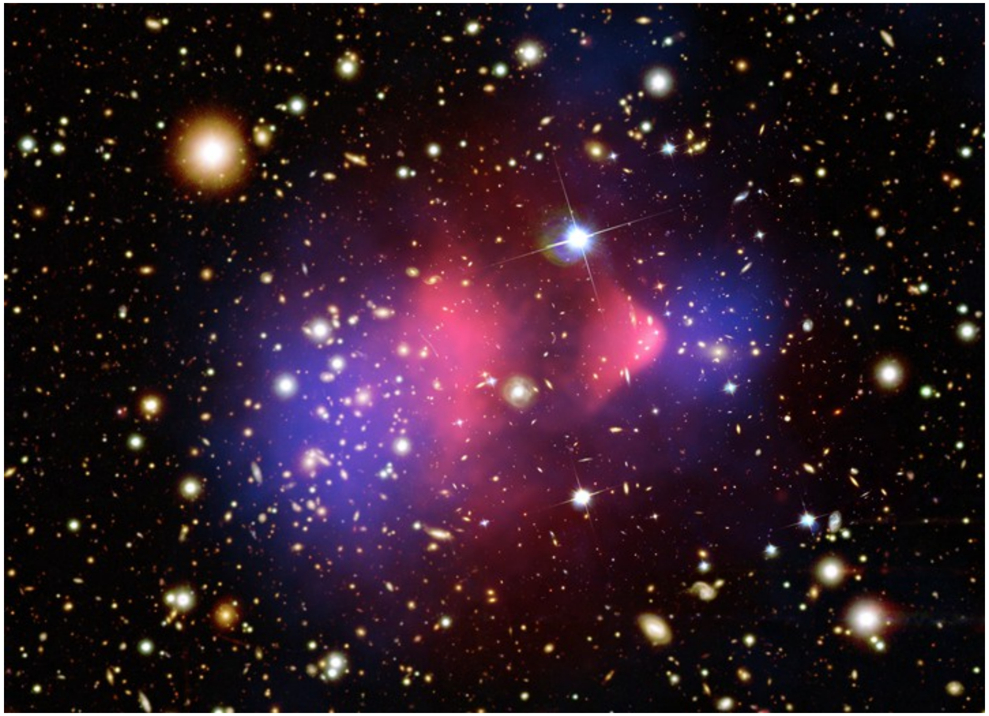
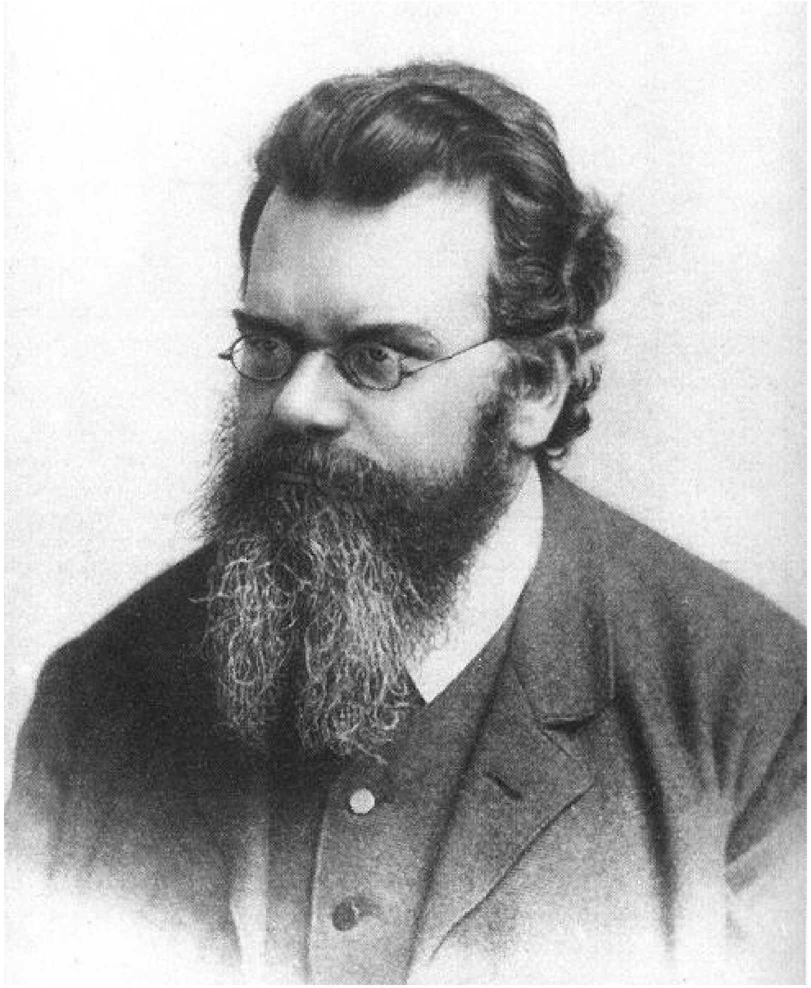
Collisional and Collisionless Dark Matter





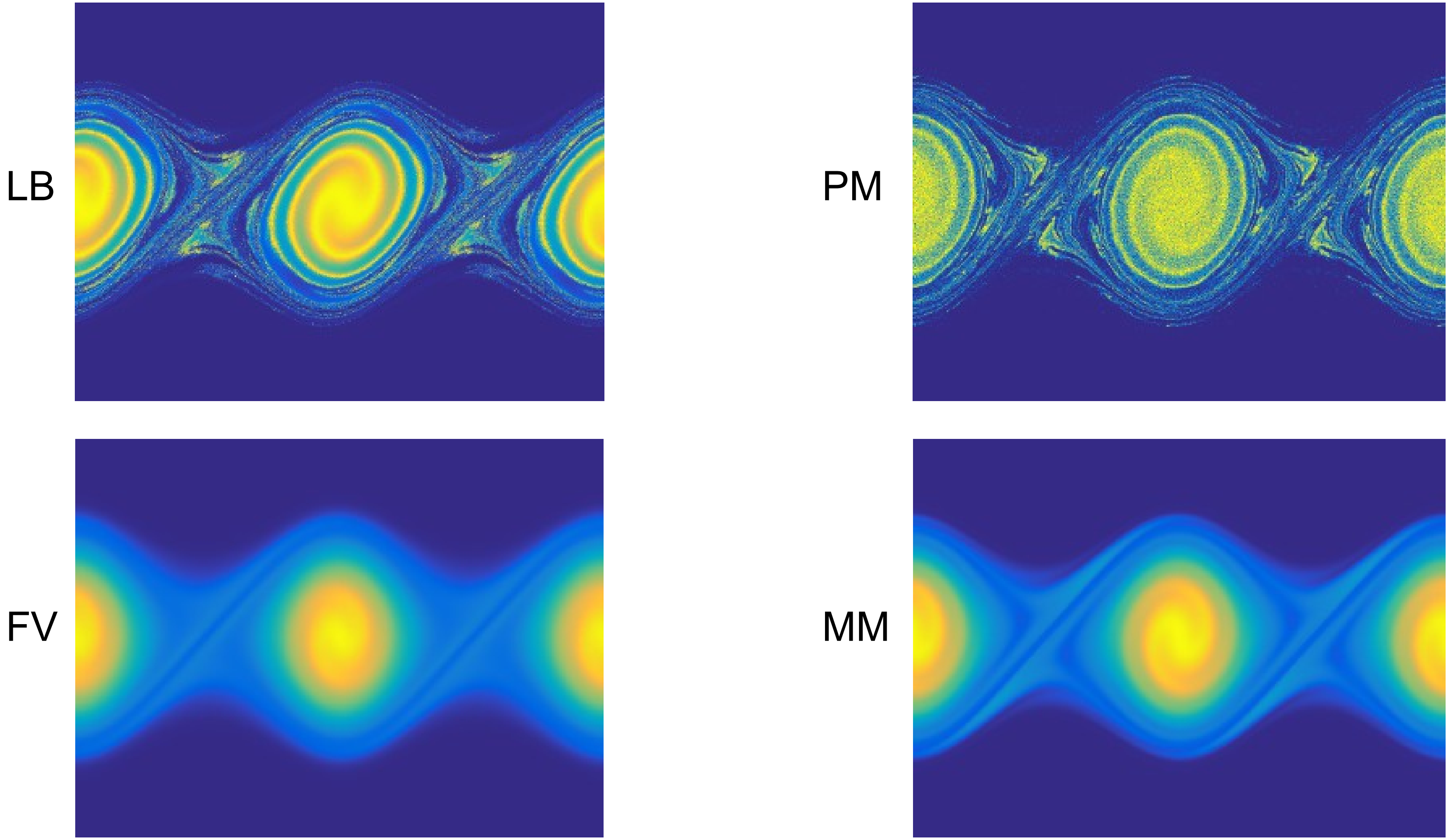
The Boltzmann Equation



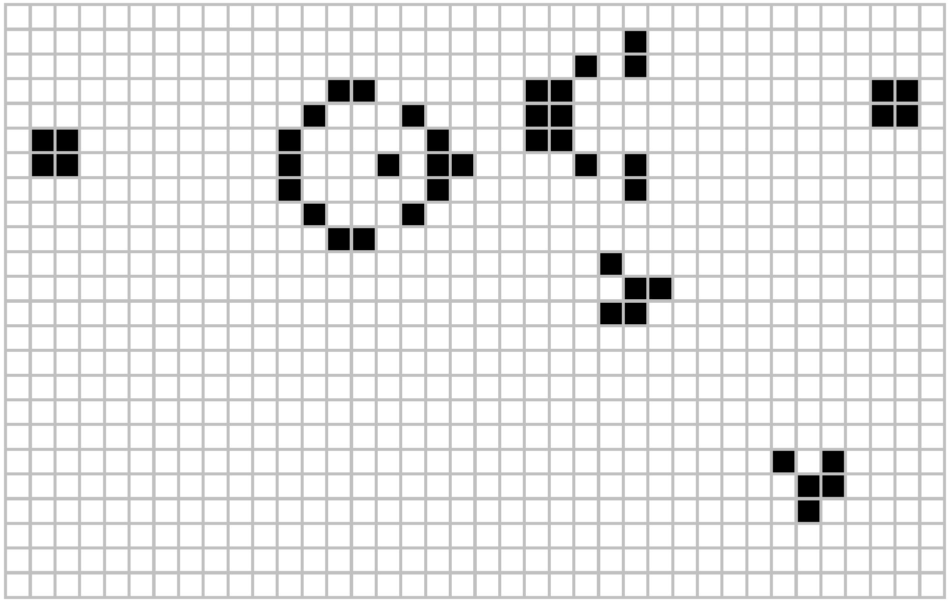
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_{\vec{r}} f + F \cdot \vec{\nabla}_{\vec{p}} = C[f]$$

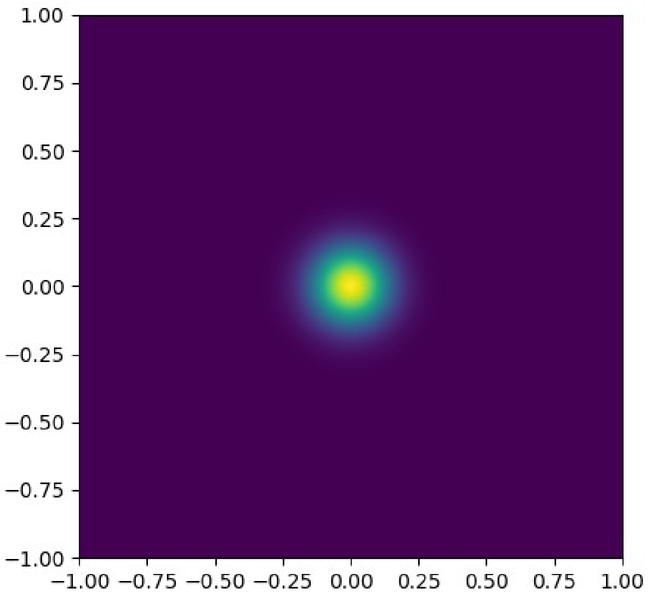
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_{\vec{r}} f + F \cdot \vec{\nabla}_{\vec{p}} = 0$$

Solving Boltzmann Equation

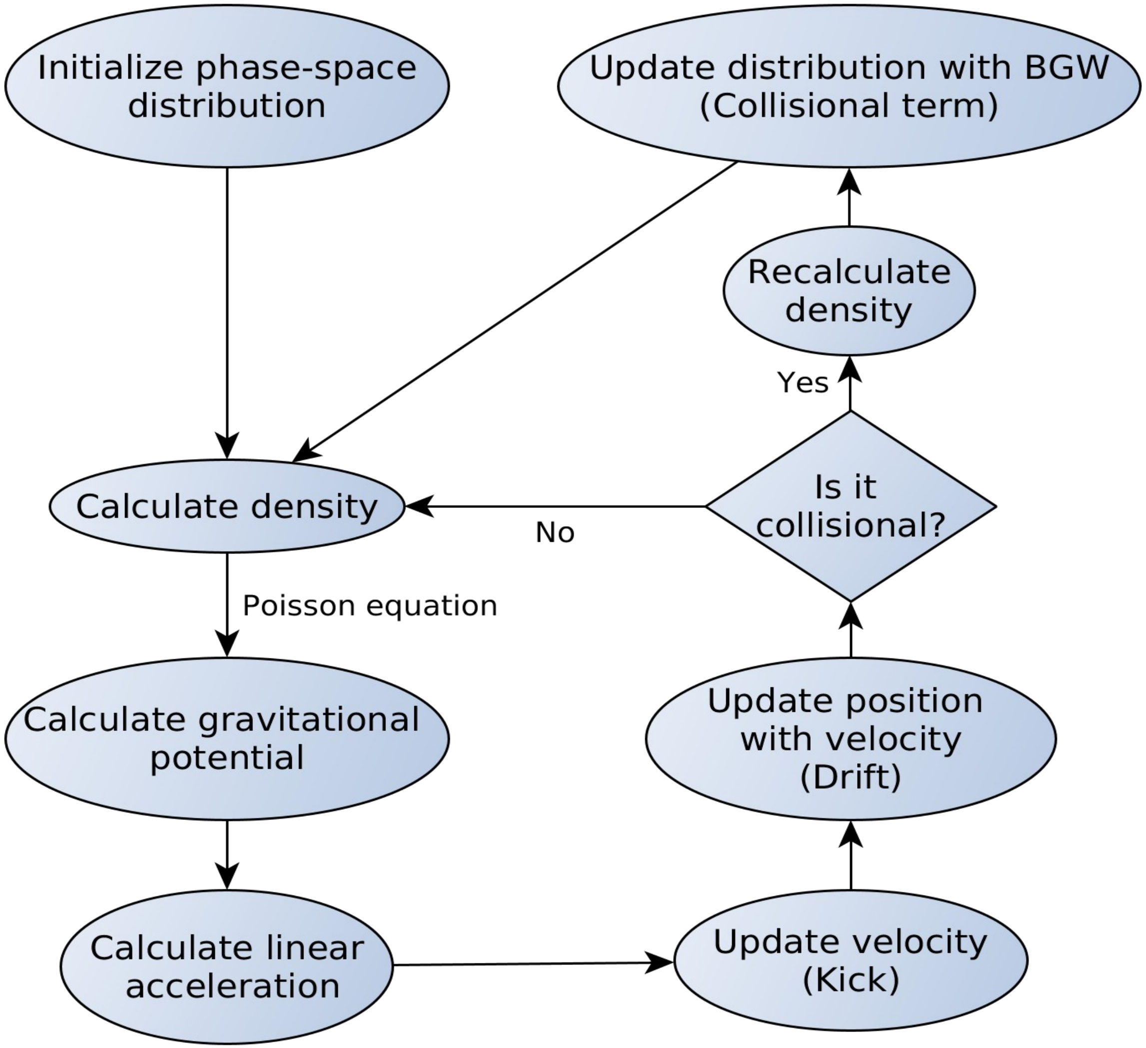


Lattice-Boltzmann and Automatas





Overview of the Algorithm



Some Equations

$$\rho(x, v, t) = \sum_{V_{min}}^{V_{max}} f(x, v, t)\Delta v$$

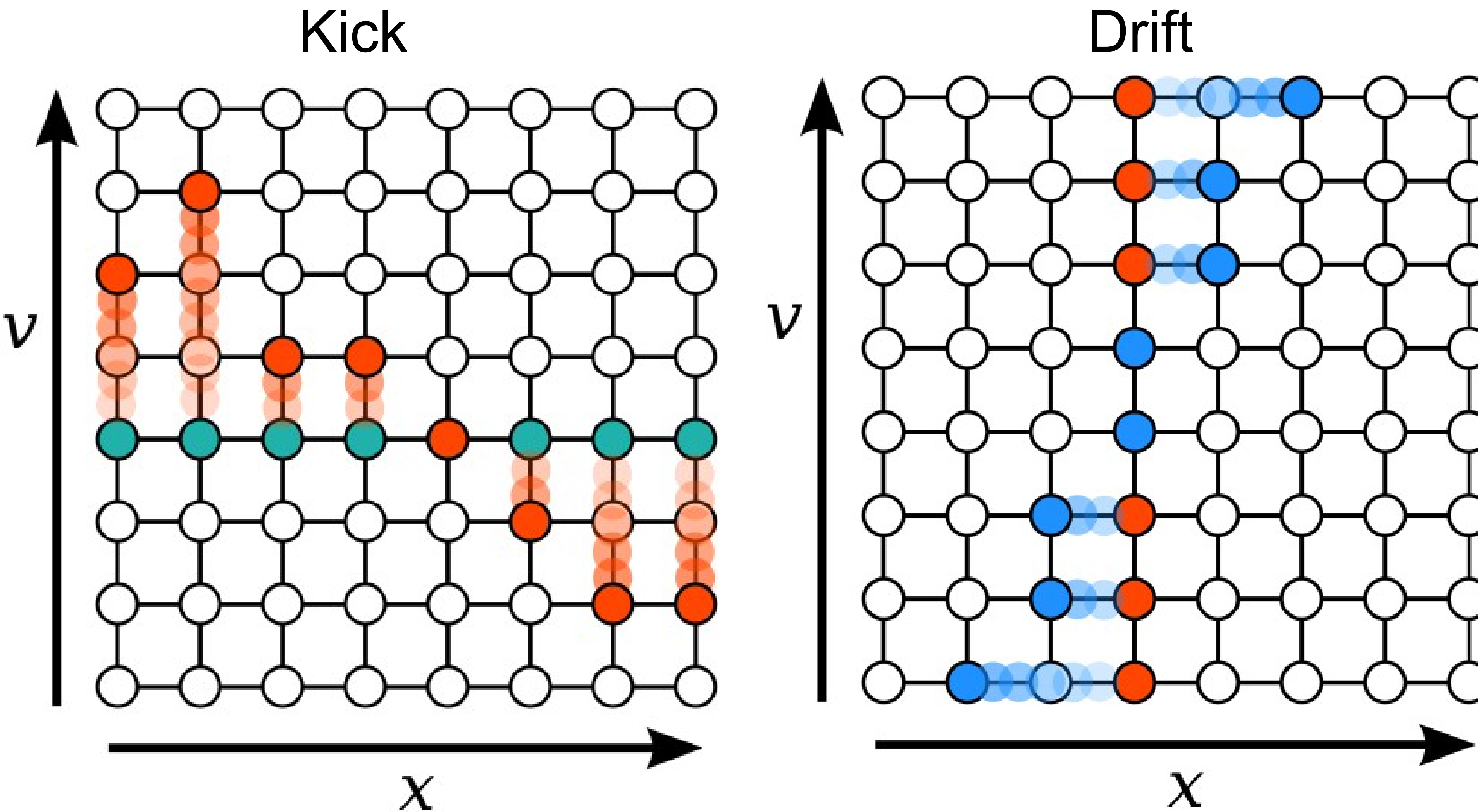
$$\nabla^2\Phi(x) = 4\pi G\rho(x)$$

$$a(x) = -\frac{\mathrm{d}\Phi(x)}{\mathrm{d}x}$$

$$v_{n+1} = v_n + \lfloor a_n\ \delta t \rceil$$

$$x_{n+1} = x_n + \lfloor v_n\ \delta t \rceil$$

Streaming Step



Collisional Term: the BGK Approximation

$$C[f] = -\frac{1}{\tau}(f - f_{eq})$$

$$f(x + v\delta t, v, t + \delta t) - f(x, v, t) =$$
$$-\frac{\delta t}{\tau}[f(x, v, t) - f_{eq}(x, v, t)]$$

Equilibrium Distribution

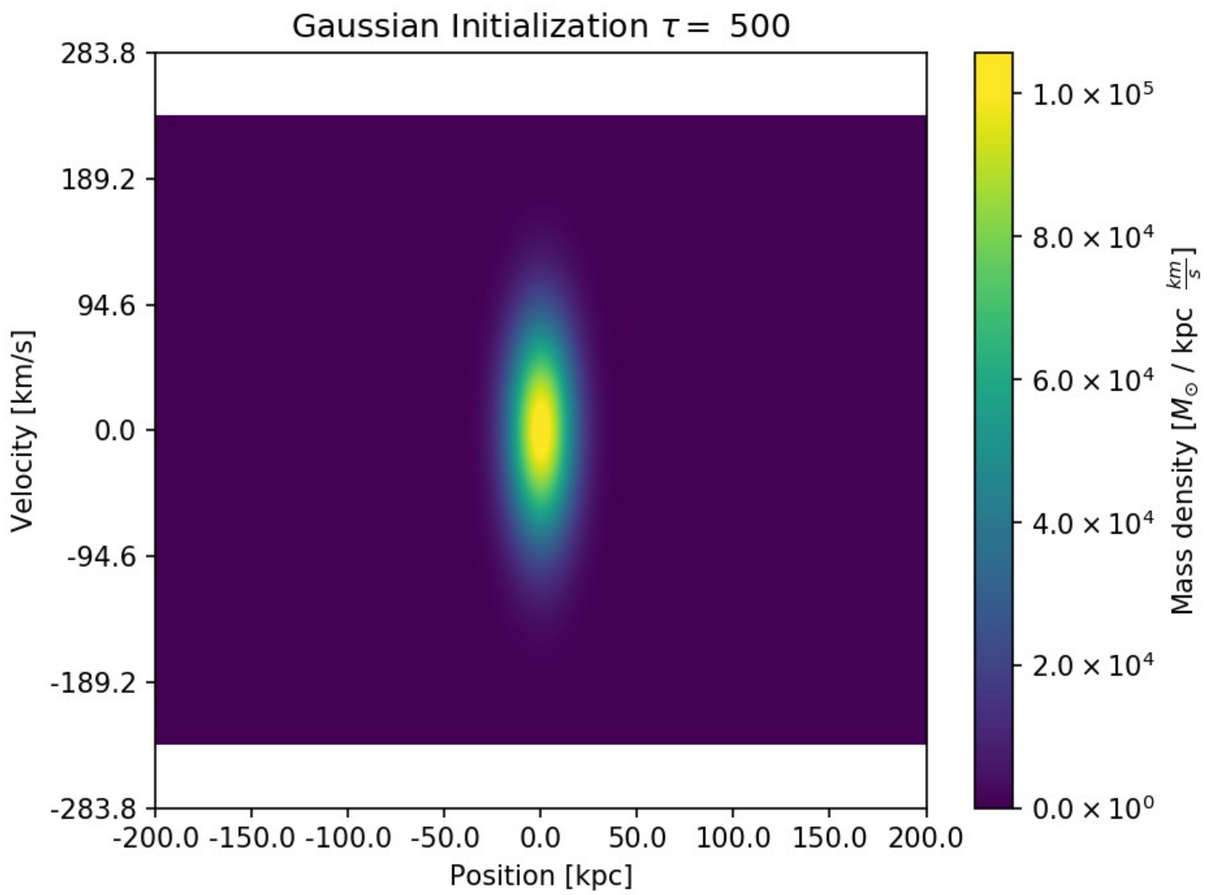
$$\rho(x,v,t) = \sum_{V_{min}}^{V_{max}} f(x,v,t)\Delta v$$

$$\rho(x,t)u(x,t) = \sum_{Vmin}^{Vmax} v f(x,v,t)\Delta v$$

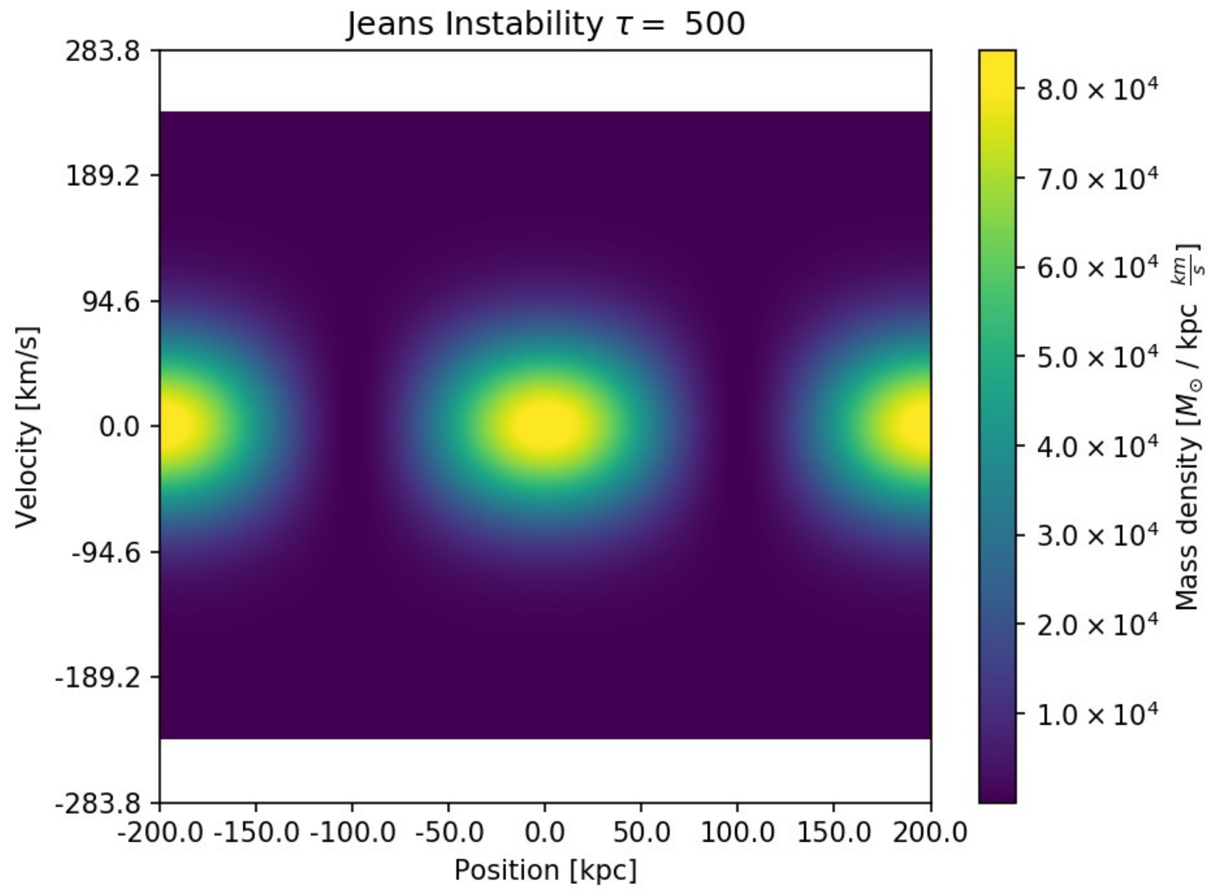
$$\rho(x,t)e(x,t) = \frac{1}{2}\sum_{Vmin}^{Vmax} [v - u(x,t)]^2 f(x,v,t)\Delta v$$

$$f_{eq}(x,v,t) = \frac{\rho(x,t)}{m\sqrt{2\pi e(x,t)}} \exp\left[-\frac{[v-u(x,t)]^2}{2e(x,t)}\right]$$

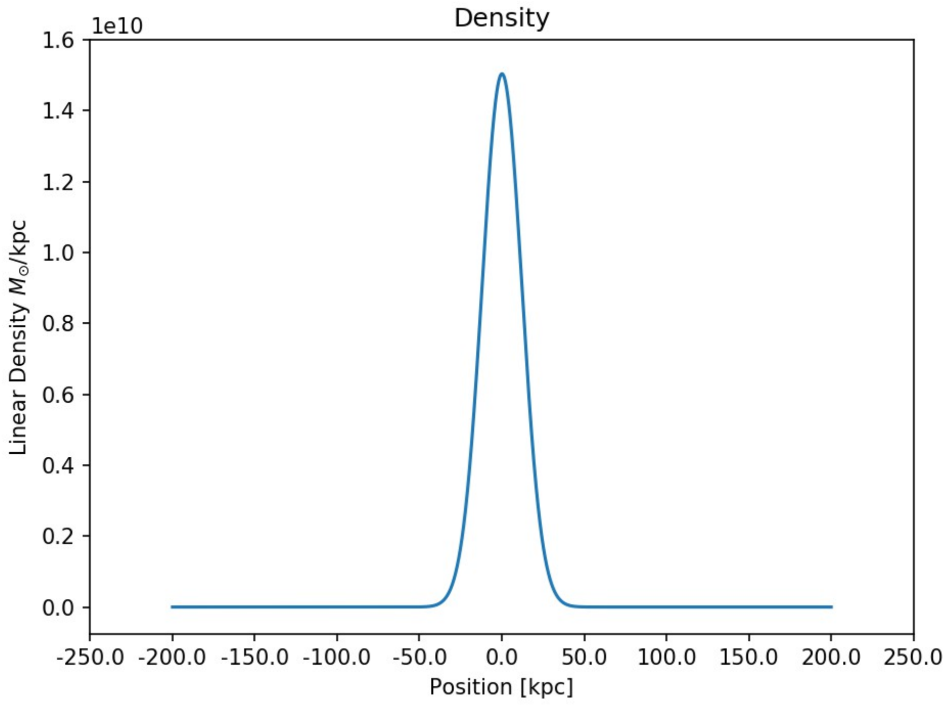
Initial Conditions

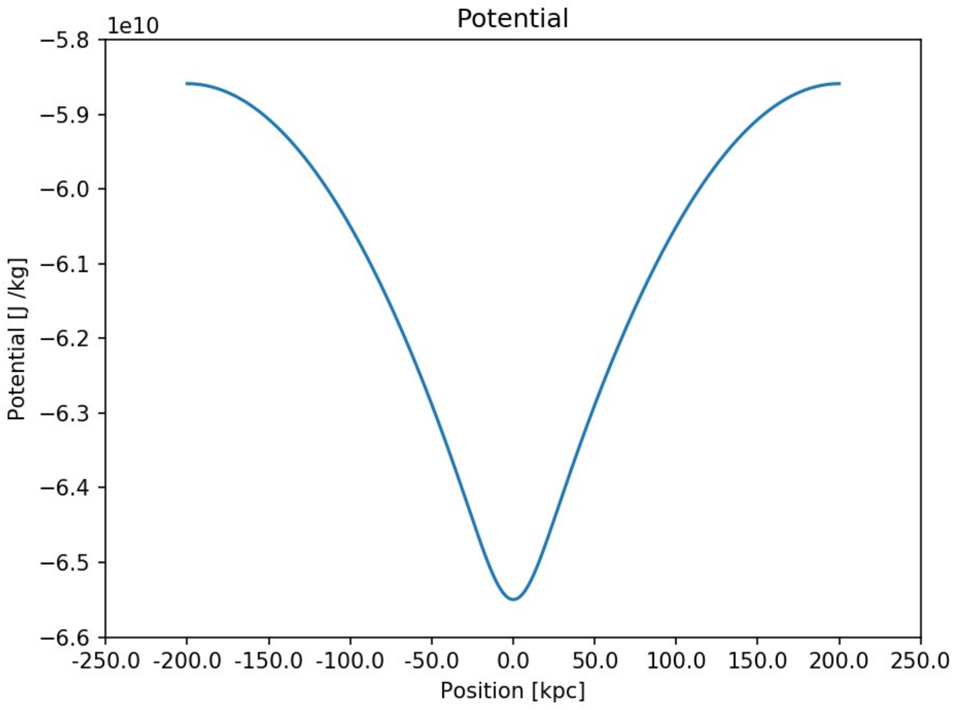


Initial Conditions

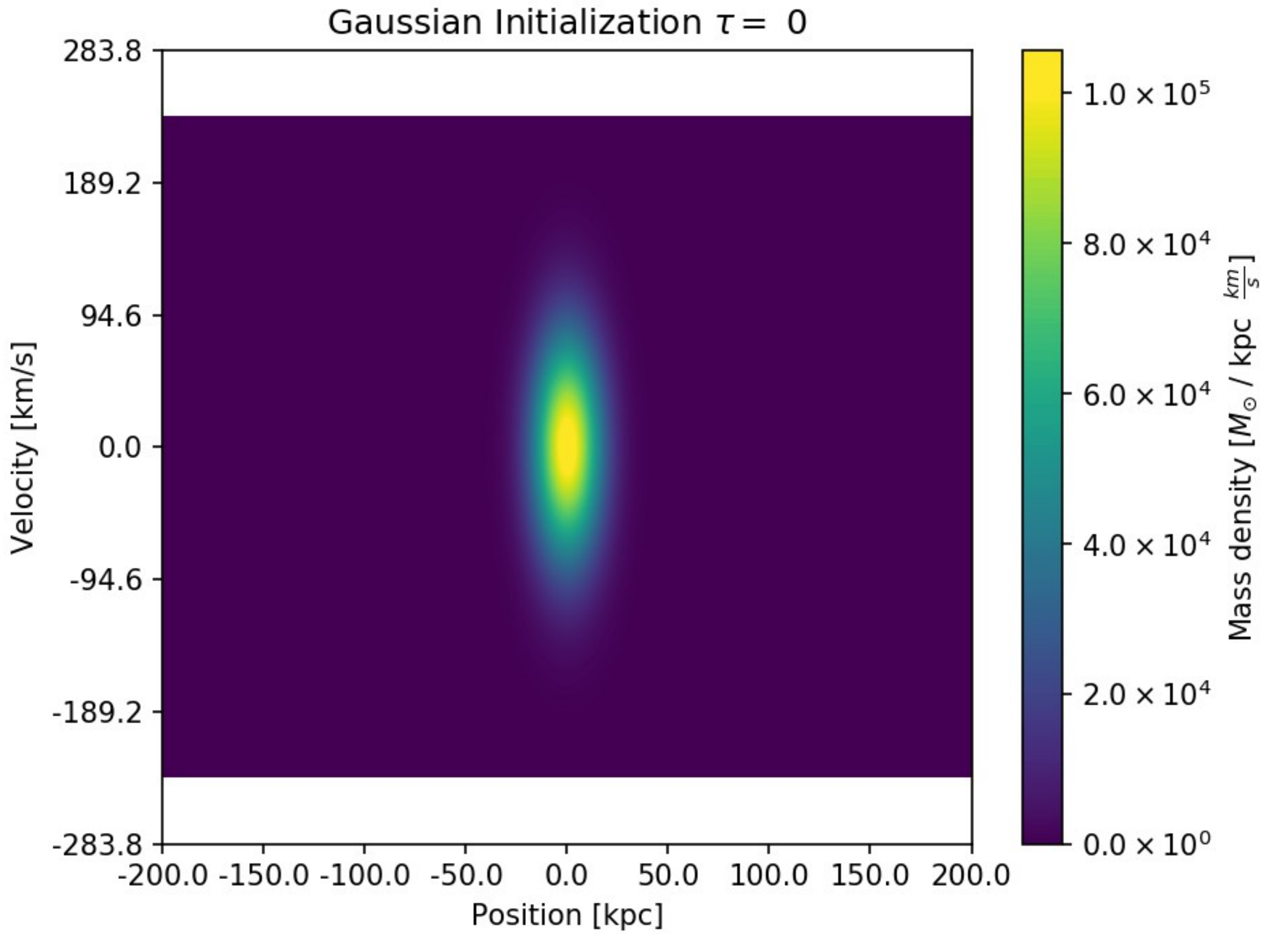


Density and Potential

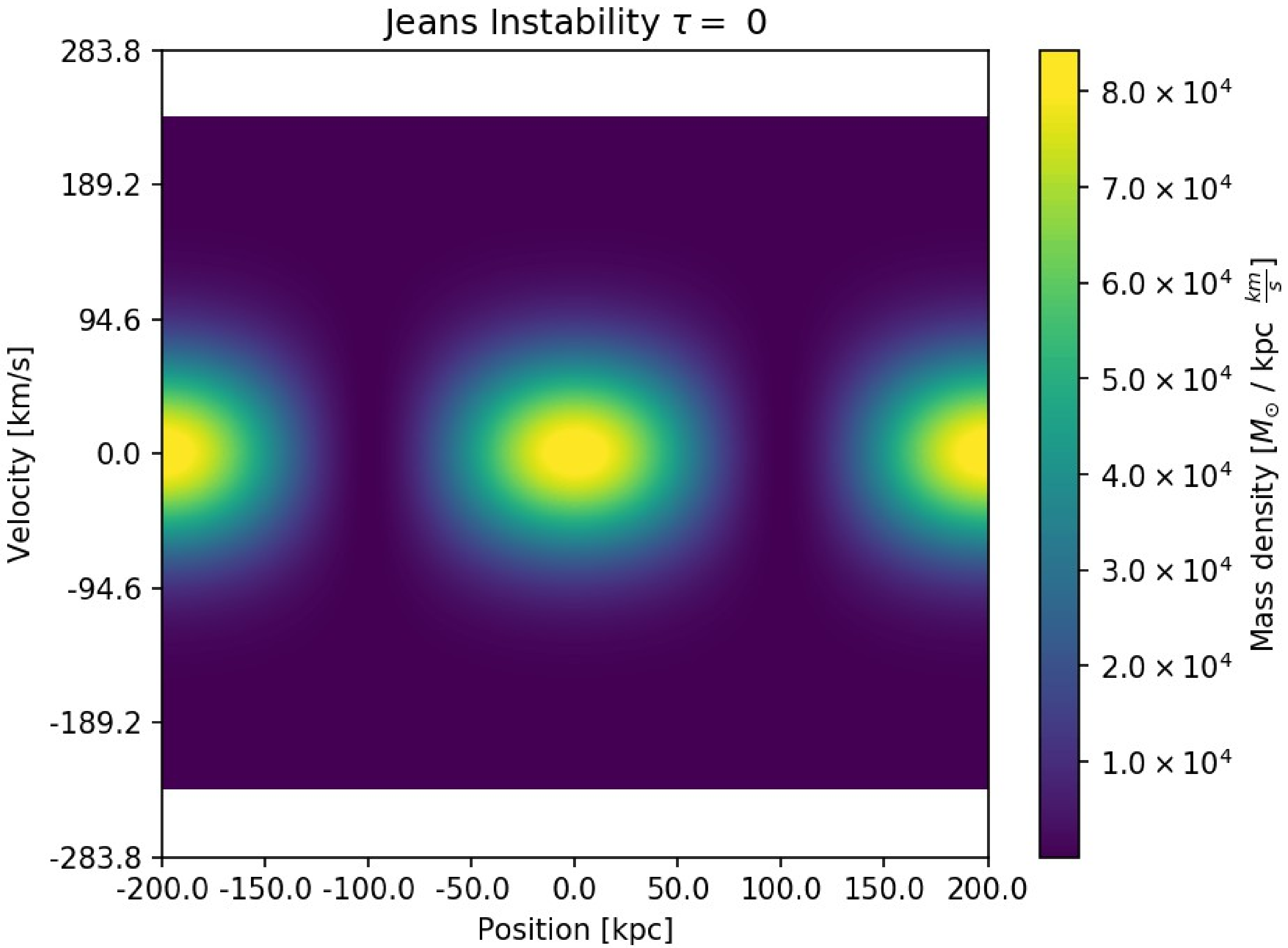




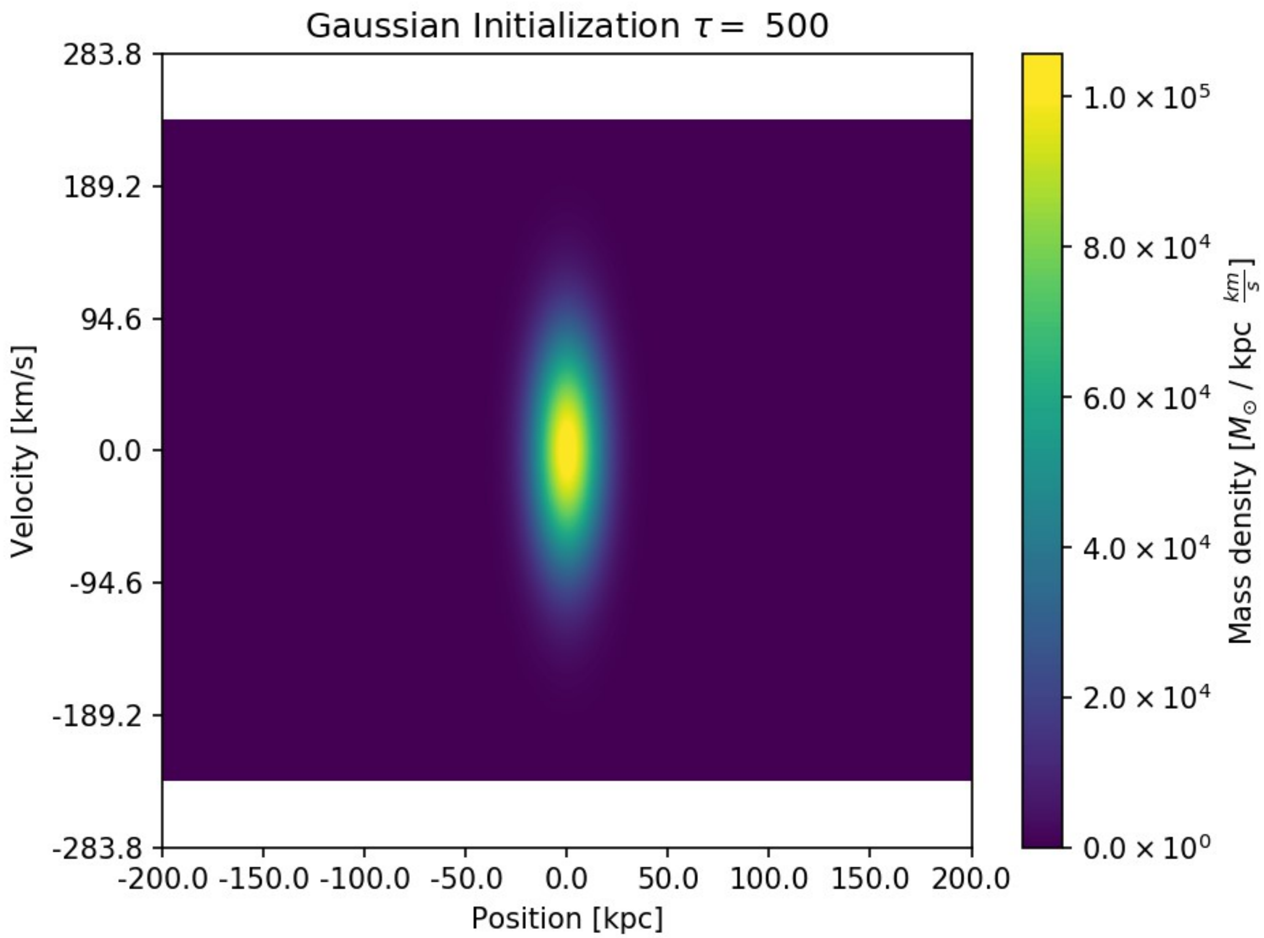
Collisionless examples



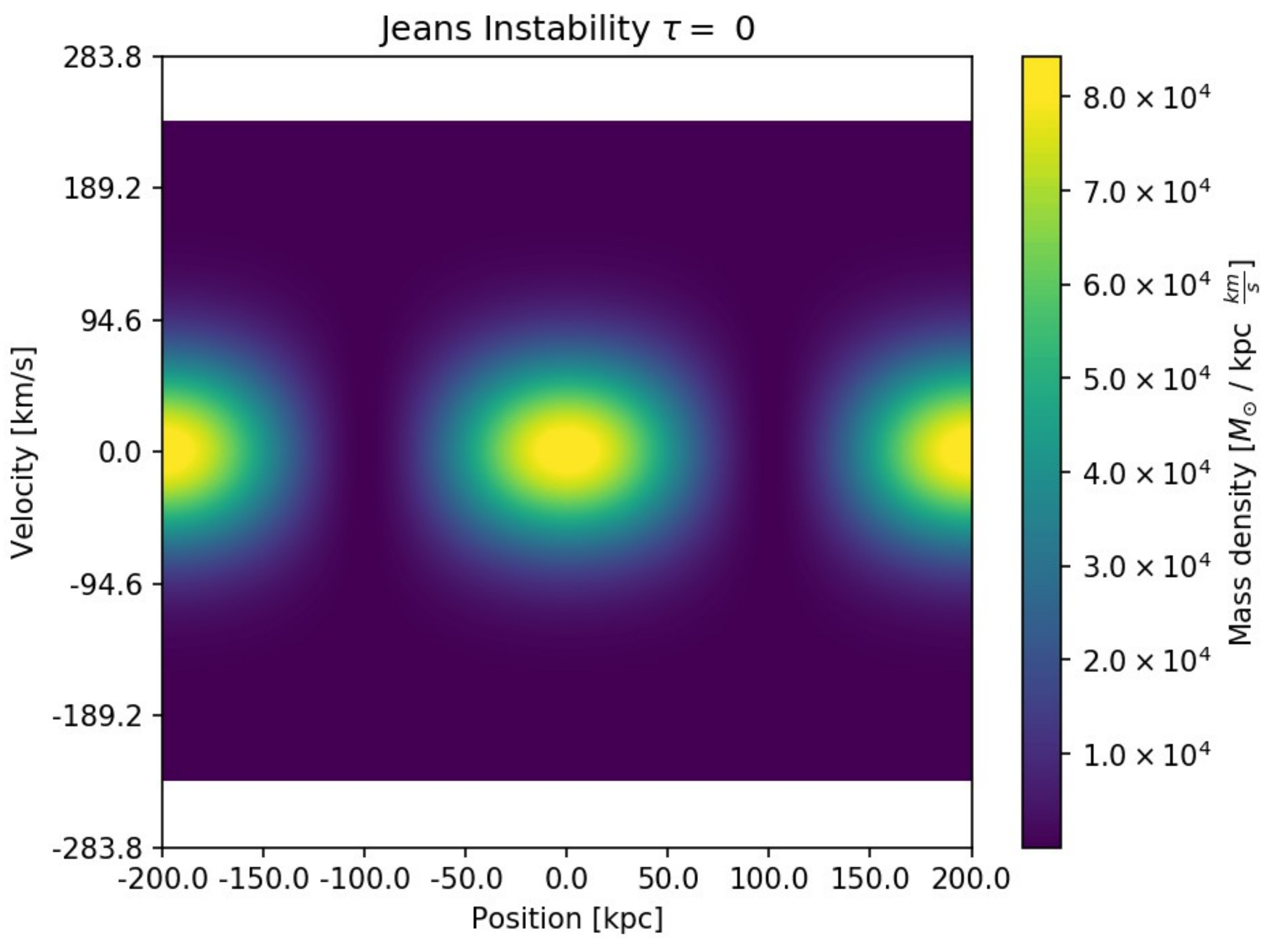
Collisionless examples



Collisional Examples

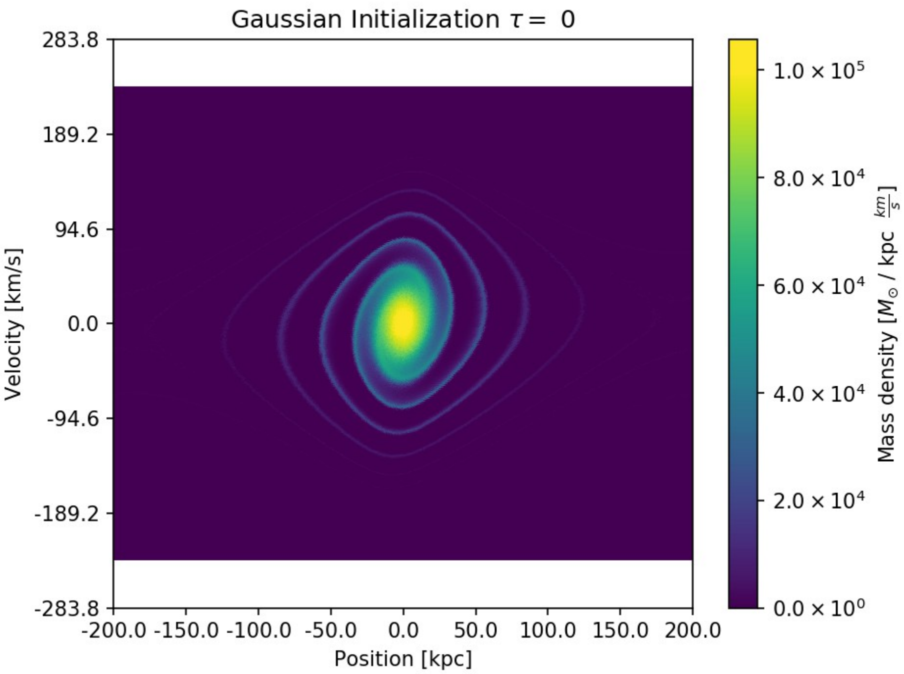


Collisional Examples

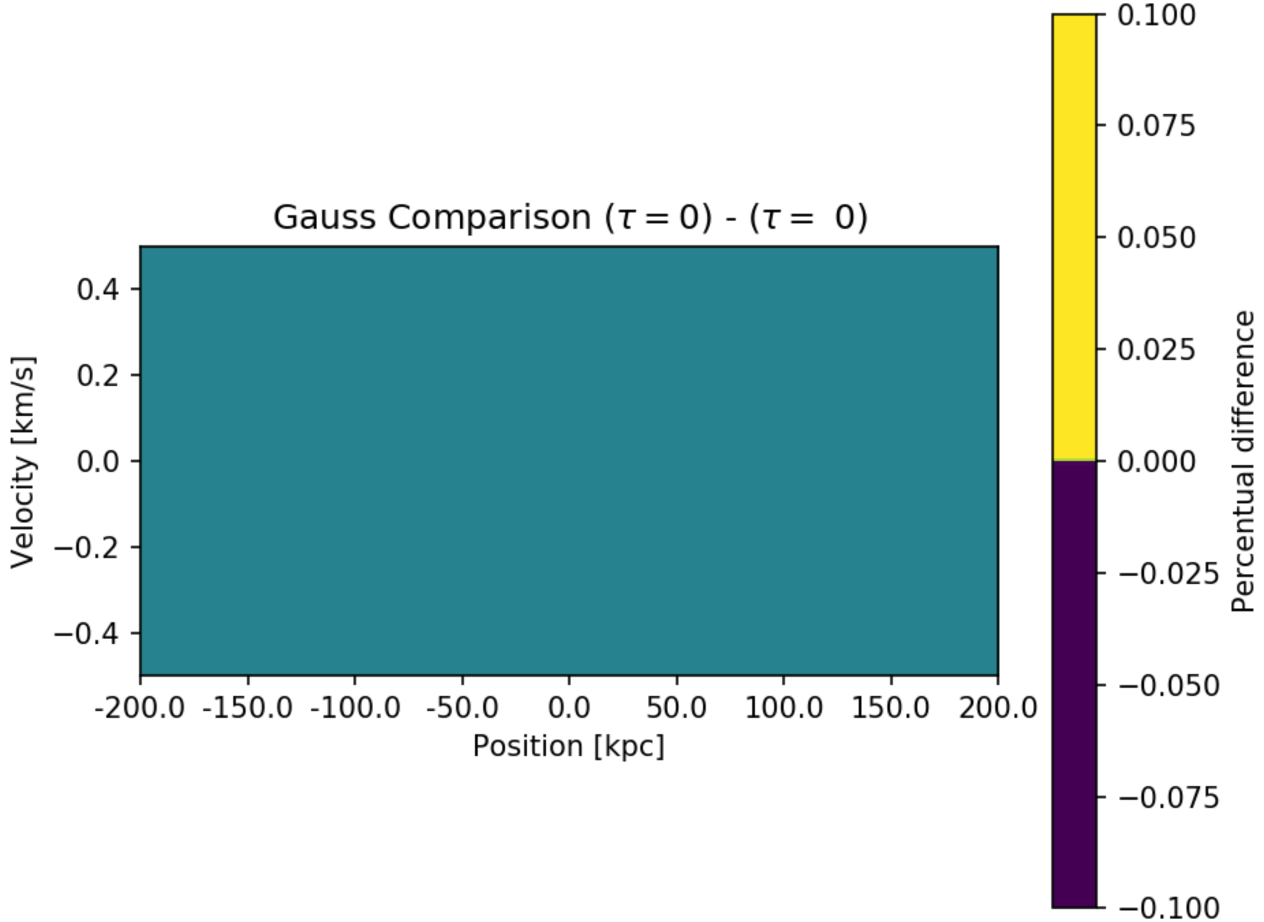


Gaussian distribution

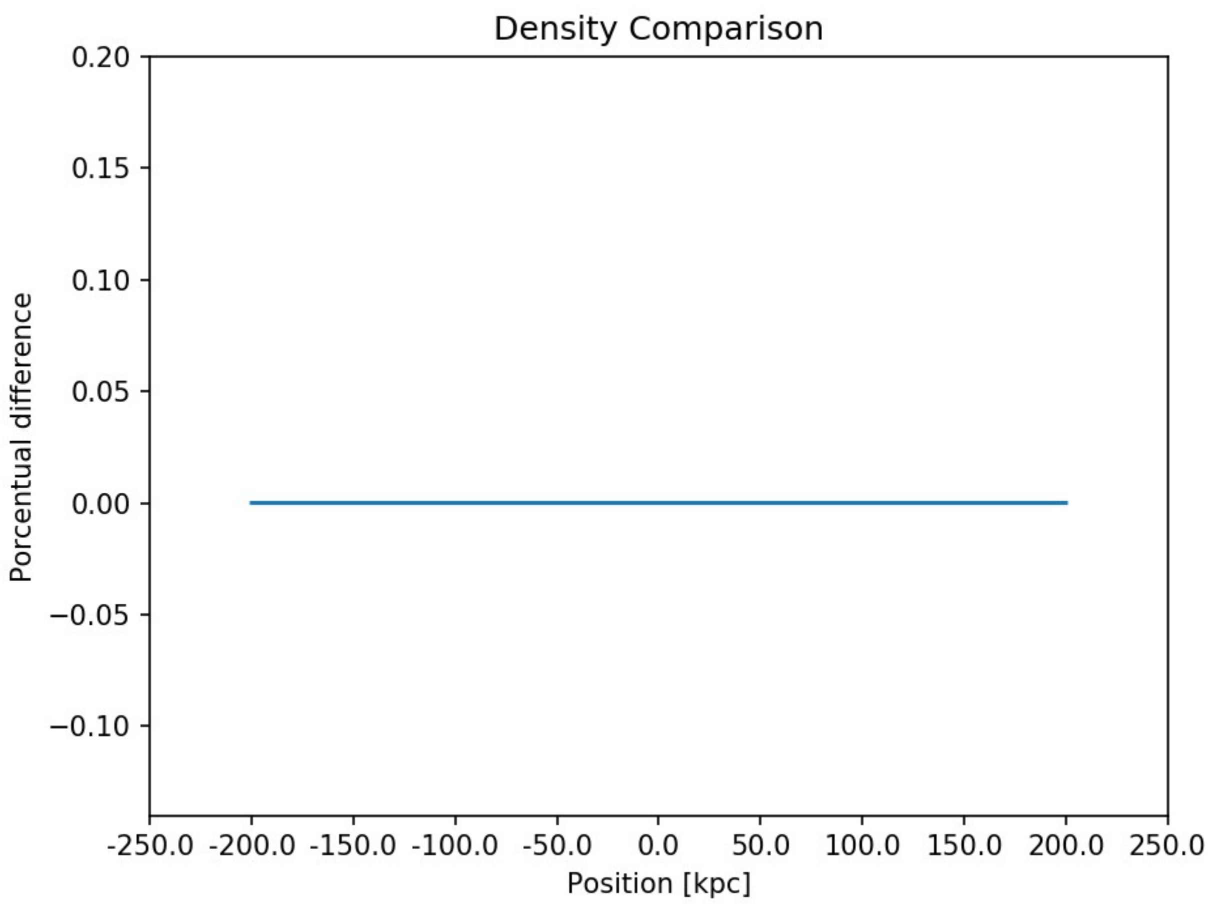
$$f_(x, v, 0) = A \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{v^2}{2\sigma_v^2}\right]$$



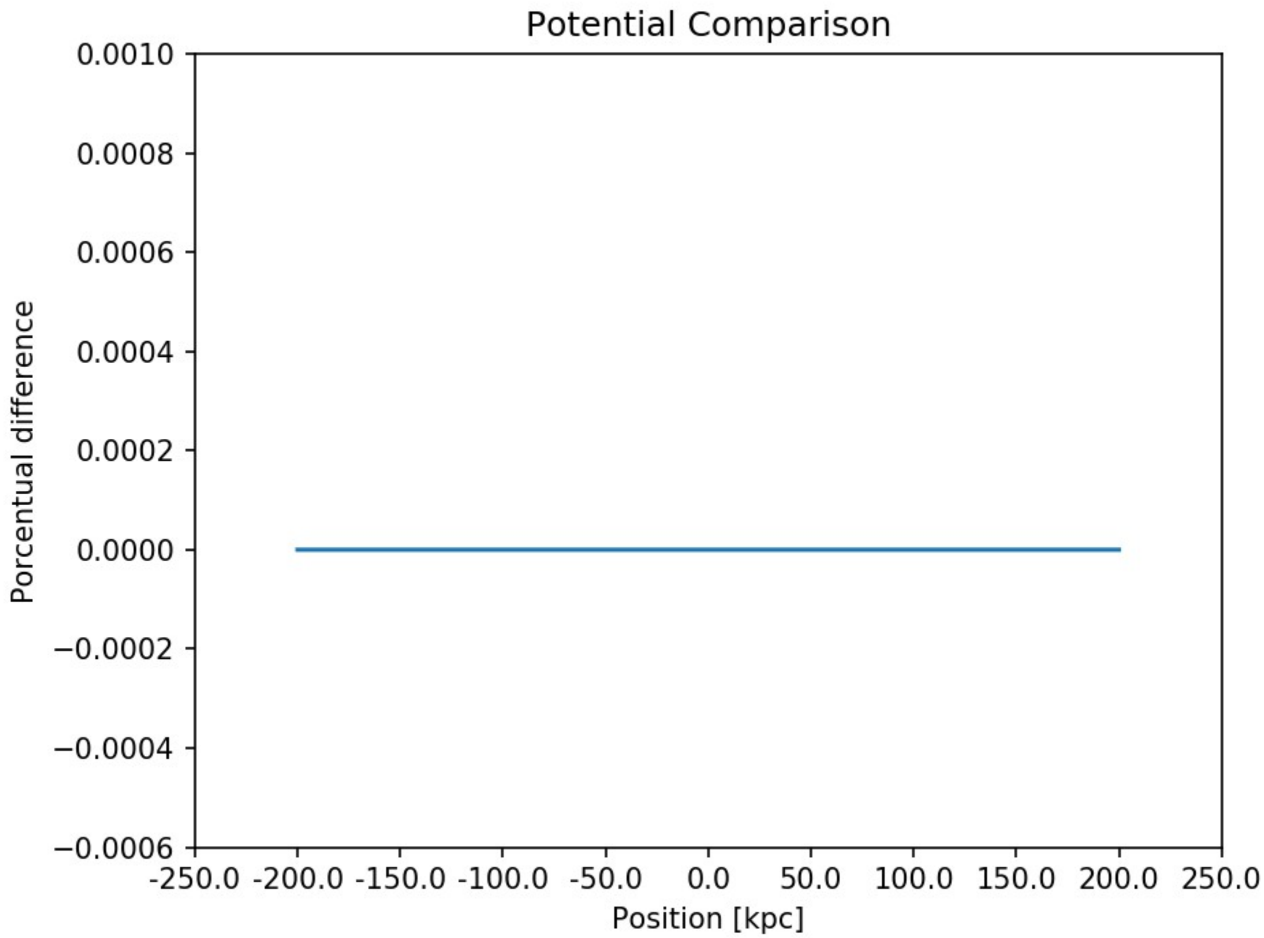
Results on phase-space



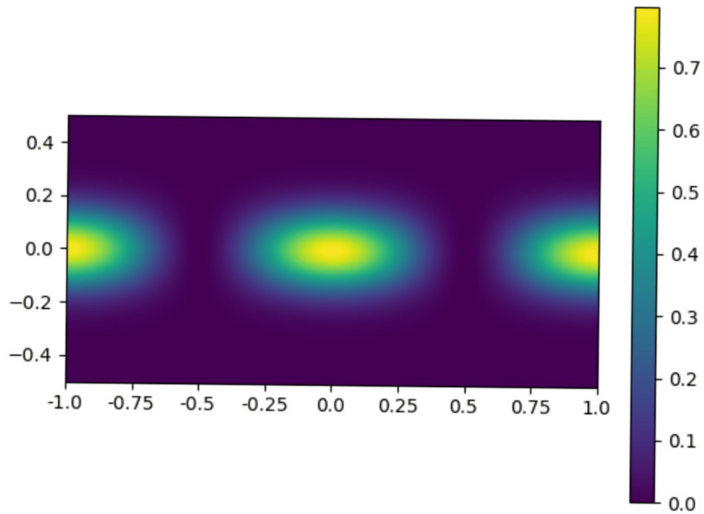
Results on Density

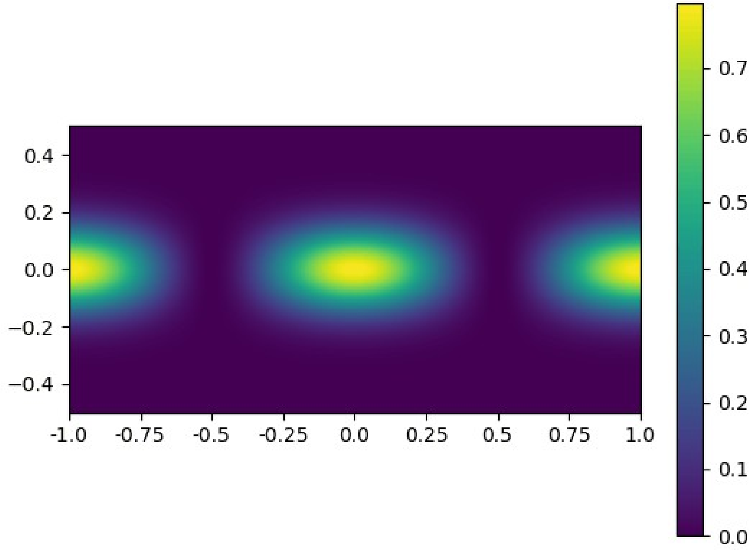


Results on Potential



Jeans instability





$$f(x, v, 0) = \frac{\bar{\rho}}{\sqrt{2\pi\sigma^2}} \exp(-\frac{v^2}{2\sigma^2})(1 + A\cos(kx))$$

Relating Tau with the cross-section

$$dim\ (\tau) = time \quad dim\ (\lambda) = length$$

$$\left[\frac{< \sigma v >}{\lambda^3}\right]^{-1} \propto \tau \quad \lambda = \alpha[n\ d]^{-1}$$

Conclusions

It is possible to simulate the phase space of a dark matter fluid, whether it is collisional or not.

The toy models implemented suggests a relaxation time (Tau) with the same magnitude order as the age of the universe. Implying a very small collisional nature.