Looking for Projective Black Holes

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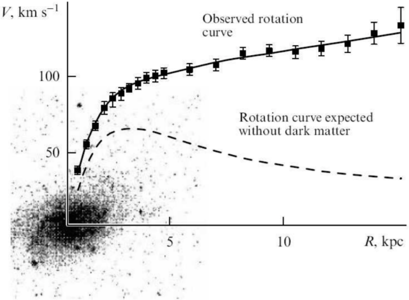
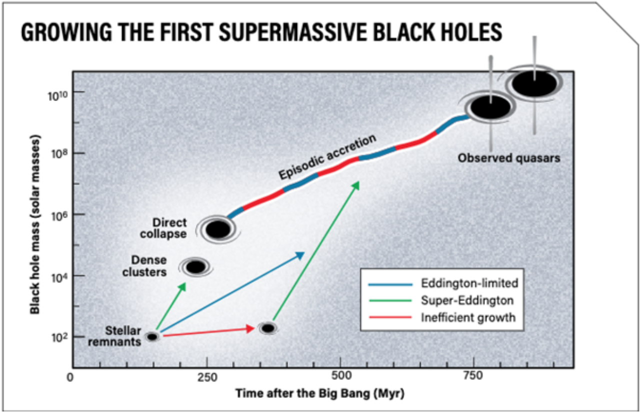
Galaxy Rotation Curves and Dark Matter Halos



From A.V. Zasov, A.S. Saburova, A.V. Khoperskov, S.A. Khoperskov, Dark Matter in Galaxies (2017)

Supermassive Black Hole Formation



Astronomy: Roen Kelly, after Smith & Bromm, 2019.
See Regan et al., MNRAS, Volume 486, Issue 3, July 2019, Pages 3892–3906

Tangherlini Source to Schwarzschild Solution

F. Tangherlini, Nonclassical Structure of the Energy-Momentum Tensor of a Point Mass Source for the Schwarzschild Field (1961)

$$T^a{}_b = \frac{M\delta(r)}{4\pi r^2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix} \tag{1}$$

$$S = S_{EH} + S_{Source} \tag{2}$$

$$T_{mn} = \frac{2}{\sqrt{|g|}}\frac{\delta\left(\sqrt{|g|}\mathcal{L}_{Source}\right)}{\delta g^{mn}} \tag{3}$$

Thomas-Whitehead Gravity

See PhysResD.103.044060 (2021)

$$\begin{aligned} S_{TW} =& S_{PEH} + S_{PGB} \\ S_{TW} =& -\frac{1}{2\kappa_0}\int d^d x\sqrt{|g|}\mathcal{K} + J_0 c\lambda_0^2\int d^d x\sqrt{|g|}K_{bcd}K^{bcd} \\ & - J_0 c\int d^d x\sqrt{|g|}\left(\mathcal{K}^a{}_{bcd}\mathcal{K}_a{}^{bcd} - 4\mathcal{K}_{ab}\mathcal{K}^{ab} + \mathcal{K}^2\right) \end{aligned} \tag{4}$$

With fundamental fields $(g_{ab},\ \mathcal{D}_{ab},\ \Pi^a{}_{bc})$.

Projective Invariant Tensor Form

$\hat{\nabla}_a$:

$$\hat{\nabla}_a g_{bc} = 0 \tag{5}$$

$$\hat{\Gamma}^a{}_{bc} = \frac{1}{2} g^{ai} \left(\partial_b g_{ic} + \partial_c g_{bi} - \partial_i g_{bc} \right) \tag{6}$$

∇_a :

$$\Gamma^a_{bc} = \hat{\Gamma}^a{}_{bc} + C^a{}_{bc} \tag{7}$$

Projective Invariant Tensor Form

$\bar{\nabla}_a$:

$$\bar{\Gamma}^a{}_{bc} \equiv \Gamma^a{}_{bc} - \delta^a{}_{(b} v_{c)} \tag{8}$$

$$= \Pi^a{}_{bc} - \delta^a{}_{(b} g_{c)} \tag{9}$$

$$v_c = g_c - \alpha_c, \quad g_c = -\frac{1}{d+1}\partial_c \log\sqrt{|g|}, \quad \alpha_c = -\frac{1}{d+1}\Gamma^m{}_{cm} \tag{10}$$

$$\mathcal{P}_{bc} = \mathcal{D}_{bc} - \partial_b \alpha_c + \Gamma^e{}_{bc}\alpha_e + \alpha_b \alpha_c \tag{11}$$

Projective Invariant Tensor Form

$$\bar{\mathcal{P}}_{ab} = \mathcal{P}_{ab} - \nabla_a v_b - v_a v_b \tag{12}$$

$$\bar{R}^a{}_{bcd} = \bar{\Gamma}^a{}_{b[d,c]} + \bar{\Gamma}^m{}_{b[d}\bar{\Gamma}^a{}_{c]m} \tag{13}$$

$$= R^a{}_{bcd} + \delta^a{}_{[c}\nabla_{d]} v_b + \delta^a{}_{[c} v_{d]} v_b - \delta^a{}_b \nabla_{[c} v_{d]} \tag{14}$$

$$\bar{R}_{ab} = \bar{R}^c{}_{acd} = \frac{1}{2} R_{(ab)} + (d-1)\,\nabla_{(a} v_{b)} + (d-1)\, v_a v_b \tag{15}$$

$$\bar{R} = g^{mn}\bar{R}_{mn} = R + 2\,(d-1)\, g^{ma}\nabla_a v_m + (d-1)\, v_m v^m \tag{16}$$

Projective Invariant Tensor Form

$$\mathcal{K}^{a}{}_{bcd} = \bar{R}^{a}{}_{bcd} + \delta^{a}{}_{[c}\bar{\mathcal{P}}_{d]b}, \quad K_{bcd} \equiv g_{\alpha}\mathcal{K}^{\alpha}{}_{bcd} = \bar{\nabla}_{[c}\bar{\mathcal{P}}_{d]b} \quad (17)$$

$$\mathcal{K}_{mn} = \bar{R}_{mn} + (d-1)\,\bar{\mathcal{P}}_{mn}, \quad \mathcal{K} \equiv g^{mn}\mathcal{K}_{mn} = \bar{R} + (d-1)\,\bar{\mathcal{P}} \quad (18)$$

$\frac{\delta S}{\delta C^a{}_{mn}} = 0$ Field Equations

$$E_a{}^{mn} - \frac{1}{d+1}\delta_a{}^{(m} E_b{}^{n)b} = 0 \tag{19}$$

$$\begin{aligned} E_a{}^{mn} = E_a{}^{nm} =& \bar{\nabla}_a \left[g^{mn} \left(\mathcal{K} + \frac{1}{4J_0\kappa_0} \right) - 4\mathcal{K}^{mn} \right] - \bar{\nabla}_i \mathcal{K}_a{}^{(mn)i} \\ & - \lambda_0^2 \bar{\mathcal{P}}_{ia} K^{(mn)i} \end{aligned} \tag{20}$$

$\frac{\delta S}{\delta \mathcal{P}_{ab}} = 0$ Field Equations

$$\lambda_0^2 \bar{\nabla}_c K^{(ab)c} = \frac{d-1}{4J_0\kappa_0} g^{ab} + (d-1)\, g^{ab}\mathcal{K} - 2\,(2d-3)\,\mathcal{K}^{ab} \qquad (21)$$

$\frac{\delta S}{\delta g_{ab}} = 0$ Field Equations

$$\bar{R}_{pq} - \frac{1}{2}\bar{R}g_{pq} = \kappa_0 \bar{\Theta}_{pq} + \Lambda_0 g_{pq}$$

$$\mathcal{L}_S = -\frac{1}{2\kappa_0}(d-1)\mathcal{P} + cJ_0\lambda_0^2 K_{bcd}K^{bcd} - cJ_0\left(\mathcal{K}^a{}_{bcd}\mathcal{K}_a{}^{bcd} - 4\mathcal{K}_{ab}\mathcal{K}^{ab} + \mathcal{K}^2\right) \tag{22}$$

$$\Theta_{mn} = -\frac{d-1}{\kappa_0}\left(\bar{\mathcal{P}}_{mn} - \frac{1}{2}\bar{\mathcal{P}}g_{mn}\right) - g_{mn}\left[\frac{2J_0 c\lambda_0^2}{d+1}\bar{\nabla}_a\mathcal{K}^a{}_{bcd}K^{bcd} - \mathcal{L}_S\right] + 2J_0 c\lambda_0^2\left(K_{mcd}K_n{}^{cd} + 2K_{bcm}K^{bc}{}_n\right) + 2J_0 c\left(8\mathcal{K}_{mb}\mathcal{K}^b{}_n - 2\mathcal{K}\mathcal{K}_{mn}\right) + 2J_0 c\left(\mathcal{K}_{mbcd}\mathcal{K}_n{}^{bcd} - \mathcal{K}^a{}_{mcd}\mathcal{K}_{an}{}^{cd} - 2\mathcal{K}^{abc}{}_m\mathcal{K}_{abcn}\right) \tag{23}$$

Stationary Spherical Symmetric Geometry

$$\mathcal{L}_X g_{ab} = 0$$
$$X^m \partial_m g_{ab} + (\partial_a X^m)\, g_{mb} + (\partial_b X^m)\, g_{am} = 0 \tag{24}$$

$$\mathcal{L}_X \mathcal{P}_{ab} = 0$$
$$X^m \partial_m \mathcal{P}_{ab} + (\partial_a X^m)\, \mathcal{P}_{mb} + (\partial_b X^m)\, \mathcal{P}_{am} = 0 \tag{25}$$

$$\mathcal{L}_X C^a{}_{bc} = 0$$
$$X^m \partial_m C^a{}_{bc} + (\partial_b X^m)\, C^a{}_{bc} + (\partial_c X^m)\, C^a{}_{bc} - (\partial_m X^a)\, C^m{}_{bc} = 0 \tag{26}$$

Stationary Spherically Symmetric Ansatz

$$\{g_{tt}(r),\ g_{tr}(r),\ g_{rr}(r),\ g_{\theta\theta}(r)\}$$

$$\{\mathcal{P}_{tt}(r),\ \mathcal{P}_{tr}(r),\ \mathcal{P}_{rr}(r),\ \mathcal{P}_{\theta\theta}(r)\}$$

$$\{C^t{}_{tt}(r),\ C^t{}_{tr}(r),\ C^t{}_{rr}(r),\ C^t{}_{\theta\theta}(r),\ C^r{}_{tt}(r),\ C^r{}_{tr}(r),\ C^r{}_{rr}(r),\ C^r{}_{\theta\theta}(r)\}$$

Static Spherically Symmetric Solution Ansatz to TW Gravity

Measure $ds^2 = g_{ab}dx^a dx^b$ is invariant under $t \to -t$.

$$\{g_{tt}(r),\ g_{rr}(r),\ g_{\theta\theta}(r)\}$$

Static Spherically Symmetric Ansatz

For the static condition on $\mathcal{P}_{ab}$ and $C^a{}_{bc}$ we consider allowing only the components that allow for the geodesics on the spacetime to be invariant under $t \to -t$.

$$\frac{d^2x^a}{du^2} + \Pi^a{}_{bc}\frac{dx^b}{du}\frac{dx^c}{du} = -\frac{2}{\lambda}\left(\frac{d\lambda}{du}\right)\frac{dx^a}{du}$$

$$\frac{d^2\lambda}{du^2} + \mathcal{D}_{bc}\frac{dx^a}{du}\frac{dx^b}{du} = 0 \tag{27}$$

$$\{\mathcal{P}_{tt}(r),\ \mathcal{P}_{rr}(r),\ \mathcal{P}_{\theta\theta}(r)\}$$

$$\{C^r{}_{tt}(r),\ C^r{}_{rr}(r),\ C^r{}_{\theta\theta}(r)\}$$

Rotating Stationary Spherically Symmetric Solution Ansatz to TW Gravity

Just as with the spherically symmetric spacetime, we can enforce the rotating condition by ensuring the geodesics on the spacetime satisfy the rotating condition, $(t, \phi) \to (-t, -\phi)$. Likewise, the $\mathcal{P}_{ab}$ field has nonvanishing components:

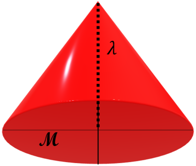
$$\{g_{tt}(r,\theta),\ g_{t\phi}(r,\theta),\ g_{rr}(r,\theta),\ g_{\theta\theta}(r,\theta),\ g_{\phi\phi}(r,\theta)\}$$

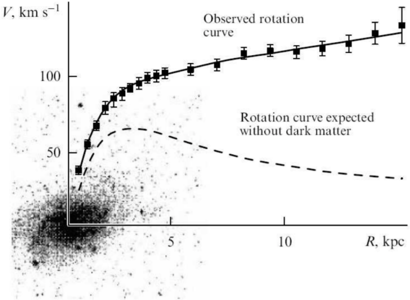
$$\{\mathcal{P}_{tt}(r,\theta),\ \mathcal{P}_{t\phi}(r,\theta),\ \mathcal{P}_{rr}(r,\theta),\ \mathcal{P}_{r\theta}(r,\theta),\ \mathcal{P}_{\theta\theta}(r,\theta),\ \mathcal{P}_{\phi\phi}(r,\theta)\}$$

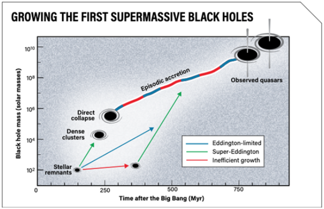
$$\{C^t_{tr}(r,\theta),\ C^t_{t\theta}(r,\theta),\ C^t_{r\phi}(r,\theta),\ C^t_{\theta\phi}(r,\theta)\}$$
$$\{C^r_{tt}(r,\theta),\ C^r_{t\phi}(r,\theta),\ C^r_{rr}(r,\theta),\ C^r_{r\theta}(r,\theta),\ C^r_{\theta\theta}(r,\theta),\ C^r_{\phi\phi}(r,\theta)\}$$

Summary

$$T^a{}_b = \frac{M\delta(r)}{4\pi r^2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix} \tag{28}$$







Thank You!

Any Questions?