PROJECTIVE INVARIANCE AS THE FOUNDATIONAL PRINCIPLE BENEATH DARK ENERGY AND INFLATION

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Brown University and the University of Iowa

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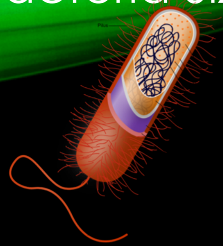
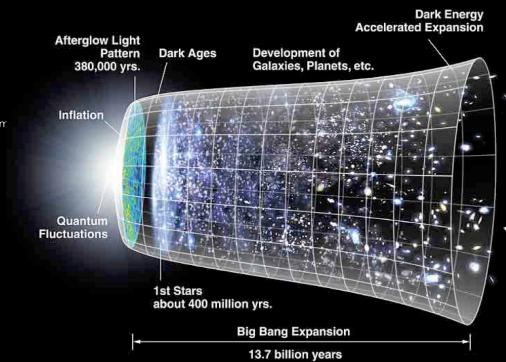
'Bates College, ^Colorado Mesa University, *The University of Iowa, undergraduate co-authors in yellow

Inflation at t~10^{-35} s

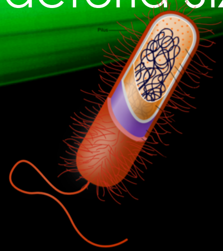
Bacteria Size





Galaxy Size

Bacteria Size



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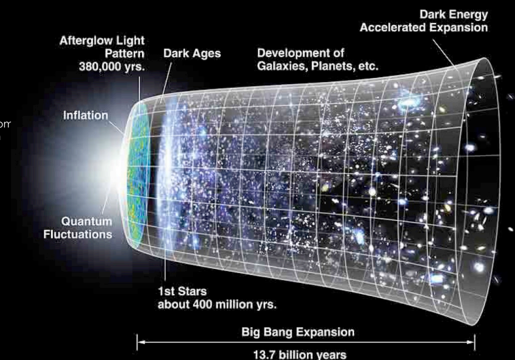
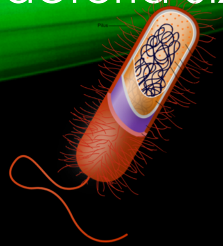
Inflation at $t \sim 10^{-35}$ s



Galaxy Size

- No known principle behind inflation: ad-hoc models

Bacteria Size



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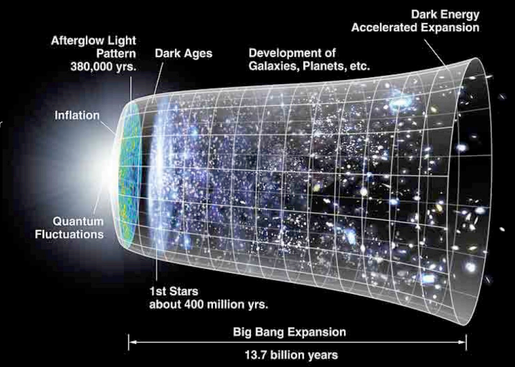
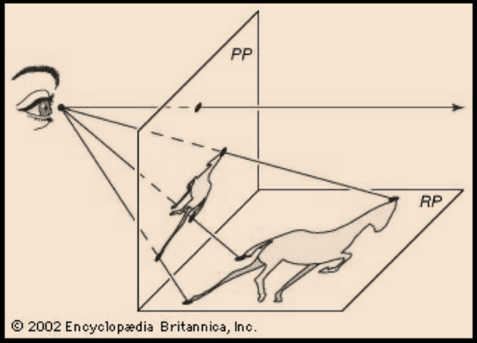


- No known principle behind inflation: ad-hoc models
- Thomas-Whitehead (TW) gravity: projective symmetry is this principle
    - Einstein-Hilbert (EH) gravity augmented with projective symmetry
    - Additional dynamical Rank two tensor and non-Levi-Civita connection



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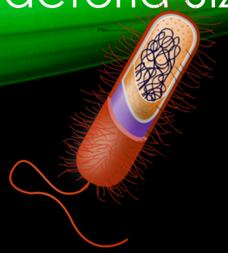

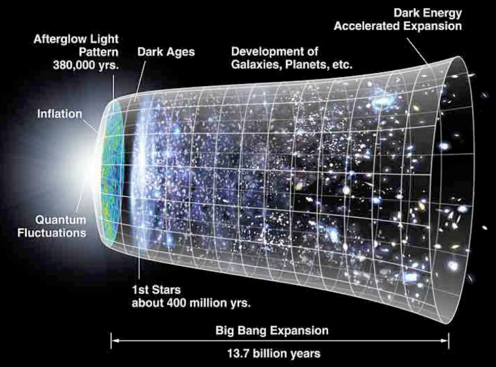

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- No known principle behind inflation: ad-hoc models
- Thomas-Whitehead (TW) gravity: projective symmetry is this principle
  - Einstein-Hilbert (EH) gravity augmented with projective symmetry
  - Additional dynamical Rank two tensor and non-Levi-Civita connection
- TW Rank two tensor => TW scalar mode => Inflaton field
  - Potential specified by TW gravity
  - Dark Energy considerations from constant modes

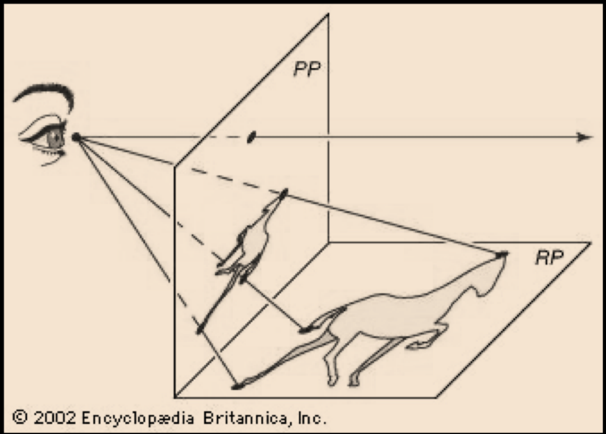
THOMAS-WHITEHEAD (TW) GRAVITY

$$S_{TW} = -\frac{1}{2\tilde{\kappa}_0}\int d\ell\, d^{\mathrm{d}}x\sqrt{|G|}(\mathcal{K} + 2\Lambda_0) - \tilde{J}_0 c\int d\ell\, d^{\mathrm{d}}x\sqrt{|G|}\left[\mathcal{K}^2 - 4\mathcal{K}_{\alpha\beta}\mathcal{K}^{\alpha\beta} + \mathcal{K}^{\alpha}{}_{\beta\mu\nu}\mathcal{K}_{\alpha}{}^{\beta\mu\nu}\right]$$

$$\mathcal{K}^{\mu}{}_{\nu\alpha\beta} \equiv \tilde{\Gamma}^{\mu}{}_{\nu[\beta,\alpha]} + \tilde{\Gamma}^{\rho}{}_{\nu[\beta}\tilde{\Gamma}^{\mu}{}_{\alpha]\rho}$$

$$G_{\alpha\beta} = \delta^{a}{}_{\alpha}\delta^{b}{}_{\beta}\, g_{ab} - \lambda_0^2 g_\alpha g_\beta$$

Projective Direction $\ell \equiv \lambda/\lambda_0$



Greek Indices $\alpha, \beta, \gamma, \ldots = 0, 1, \ldots \mathrm{d},$ excluding λ

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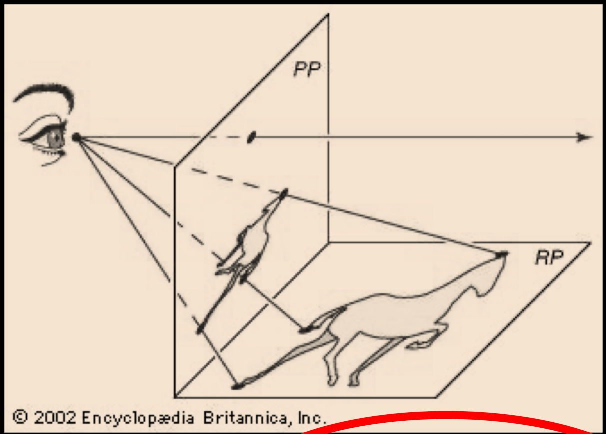

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$$\Gamma^{a}{}_{bc} = \hat{\Gamma}^{a}{}_{bc} + \delta^{a}{}_{b} v_c + \delta^{a}{}_{c} v_b$$

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Dynamical Fields Circled in Red

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$$g_\alpha = (g_a, 1/\lambda)$$

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$$\tilde{\Gamma}^{\lambda}{}_{bc} = \lambda(\mathcal{P}_{bc} + \partial_b\alpha_c - \Gamma^{e}{}_{bc}\alpha_e - \alpha_b\alpha_c)$$

Projective Schouten Tensor $\mathcal{P}_{bc}$

TW GRAVITY

Expand in terms of *P* and *R*

$$S_{TW} = -\frac{1}{2\tilde{\kappa}_0}\int d\ell\ell^{-1}\int d^{\mathrm{d}}x\sqrt{|g|}\left(R + (\mathrm{d}-1)\mathcal{P} + 2\Lambda_0\right) - \tilde{J}_0 c\int d\ell\ell^{-1} S_{GB}$$
$$+\tilde{J}_0 c\int d\ell\ell^{-1}\int d^{\mathrm{d}}x\sqrt{|g|}\left(\lambda_0^2 K_{abc}K^{abc} - \mathcal{P}_{ab}\tilde{\mathcal{P}}_*^{ab} - p(\mathrm{d})\mathcal{P}_{ab}\mathcal{P}^{[ab]}\right)$$

$$S_{GB} = \int d^{\mathrm{d}}x\sqrt{|g|}\left(R^2 - 4R_{ab}R^{ab} + R^a{}_{bcd}R_a{}^{bcd}\right) \quad , \quad p(\mathrm{d}) = 2(4\mathrm{d}^2 - 3\mathrm{d} - 2)$$

$$\tilde{\mathcal{P}}_{ab} \equiv (\mathrm{d}-1)\mathcal{P}_{ab} + 2R_{ab}$$
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Projective Cotton-York Tensor

$$K_{nab} = \nabla_{[a}\mathcal{P}_{b]n} - \Delta_n\mathcal{P}_{[ab]} + \Delta_{[a}\mathcal{P}_{b]n} + \Delta_m R^m{}_{nab}$$
$$\Delta_a = g_a - \alpha_a$$

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"Renormalize"

$$\frac{1}{\tilde{\kappa}_0}\int_{\ell_i}^{\ell_f} d\ell\ell^{-1} = \frac{\ln(\ell_f/\ell_i)}{\tilde{\kappa}_0} \quad \Rightarrow \kappa_0 \equiv \frac{\tilde{\kappa}_0}{\ln(\ell_f/\ell_i)}$$
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"On-shell (Levi-Civita)"

$$\Gamma^{m}{}_{ab} = \frac{1}{2}g^{mn}\left(g_{n(a,b)} - g_{ab,n}\right)$$

TW GRAVITY

"Renormalized and On-Shell (Levi-Civita)"

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- Natural Units $\kappa_0 = n_\kappa M_p^{-2}, \quad \lambda_0 = n_\lambda M_p^{-1}, \quad J_0 = n_J\hbar = n_J$
- Decompose in terms of trace and traceless $\mathcal{P}_{ab} = \frac{M_p}{n_\lambda}\phi g_{ab} + w_0 W_{ab}$

TW GRAVITY

In Natural Units, Decomposed trace/traceless

$$S_{TW} = -\frac{M_p^2}{2n_\kappa}\int d^{\mathrm{d}}x\sqrt{|g|}\left[f(\phi)R + 2\Lambda_0\right] + w_0\int d^{\mathrm{d}}x\sqrt{|g|}\mathcal{L}_W$$
$$+ 4(\mathrm{d}-1)n_J\int d^{\mathrm{d}}x\sqrt{|g|}\left[\tfrac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi)\right] - J_0 c S_{GB}$$

$$\mathcal{L}_W = w_0\frac{2n_J n_\lambda^2}{M_p^2}\left[\nabla^m W^{nb}\nabla_{[m}W_{n]b} + \frac{(\mathrm{d}-1)(2\mathrm{d}-3)M_p^2}{n_\lambda^2}W^{ab}W_{ab}\right] + 4(2\mathrm{d}-3)n_J R_{ab}W^{ab}$$
$$-\frac{4n_J n_\lambda}{M_p}\nabla_a\phi\nabla_b W^{ab} - \frac{4n_J n_\lambda}{M_p}W\left[\Box\phi + \frac{(\mathrm{d}-1)M_p^3}{8n_\lambda n_J n_\kappa}f(\phi) + \frac{(\mathrm{d}-1)M_p}{2n_\lambda}R\right] + \hat{\lambda}W^2$$

$$f(\phi) = 1 + \frac{4(\mathrm{d}-2)(\mathrm{d}-3)n_\kappa n_J}{n_\lambda M_p}\phi \quad , \quad V(\phi) = \frac{\mathrm{d}M_p^4}{64n_J^2 n_\kappa^2(\mathrm{d}-2)(\mathrm{d}-3)}\left(f(\phi)^2 - 1\right) \quad .$$

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$$\mathcal{L}_W = w_0\frac{2n_J n_\lambda^2}{M_p^2}\left[\nabla^m W^{nb}\nabla_{[m}W_{n]b} + \frac{({\rm d}-1)(2{\rm d}-3)M_p^2}{n_\lambda^2}W^{ab}W_{ab}\right] + 4(2{\rm d}-3)n_J R_{ab}W^{ab}$$
$$- \frac{4n_J n_\lambda}{M_p}\nabla_a\phi\nabla_b W^{ab} - \frac{4n_J n_\lambda}{M_p}W\left[\Box\phi + \frac{({\rm d}-1)M_p^3}{8n_\lambda n_J n_\kappa}f(\phi) + \frac{({\rm d}-1)M_p}{2n_\lambda}R\right] + \hat{\lambda}W^2$$

$$f(\phi) = 1 + \frac{4({\rm d}-2)({\rm d}-3)n_\kappa n_J}{n_\lambda M_p}\phi \quad , \quad V(\phi) = \frac{{\rm d}M_p^4}{64n_J^2 n_\kappa^2({\rm d}-2)({\rm d}-3)}\left(f(\phi)^2 - 1\right) \quad .$$

Set $w_0 = 0$ → Non-minimal Coupled EH gravity

TW->NON-MINIMAL COUPLED EH GRAVITY

$$S=-\frac{M_p^2}{2n_\kappa}\int d^4x\sqrt{|g|}f(\phi)R+12n_J\int d^{\mathrm{d}}x\sqrt{|g|}\left[\tfrac{1}{2}\nabla_a\phi\nabla^a\phi-V(\phi)\right]$$

$$f(\phi)=1+\frac{8}{\hat{n}M_p}\phi\quad,\quad V(\phi)=\frac{M_p^4}{32n_J^2n_\kappa^2}(f(\phi)^2-1)\quad,\quad \hat{n}=\frac{n_\lambda}{n_Jn_\kappa}$$

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Conformal Transformation

$$g_{ab} = e^{-2\omega}\tilde{g}_{ab}$$

$$\omega = \ln\sqrt{\frac{f(\phi)}{n_\kappa}}$$

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$$g_{ab} = e^{-2\omega}\tilde{g}_{ab}$$

$$\omega = \ln\sqrt{\frac{f(\phi)}{n_\kappa}}$$

$$S = \int d^4x\sqrt{|\tilde{g}|}\left(-\frac{M_p^2}{2}\tilde{R} + A(\phi)\frac{1}{2}\tilde{\nabla}_a\phi\tilde{\nabla}^a\phi + B(\phi)\frac{1}{2}\tilde{\Box}\phi - \tilde{V}(\phi)\right)$$

$$A(\phi) = \frac{288}{\hat{n}^2 f(\phi)^2}\left(1 + \tfrac{\hat{n}n_\lambda}{24}f(\phi)\right) \quad , \quad B(\phi) = -\frac{24M_p}{\hat{n}f(\phi)}$$

$$\tilde{V}(\phi) \equiv \frac{3M_p^4}{8n_J}\left(1 - \frac{1}{f(\phi)^2}\right)$$

EINSTEIN FRAME

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_p^2}{2} \tilde{R} + A(\phi) \frac{1}{2} \tilde{\nabla}_a \phi \tilde{\nabla}^a \phi + B(\phi) \frac{1}{2} \tilde{\Box} \phi - \tilde{V}(\phi) \right)$$

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Integrate by Parts

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_p^2}{2}\tilde{R} + C(\phi)\frac{1}{2}\tilde{\nabla}_a\phi\tilde{\nabla}^a\phi - \tilde{V}(\phi) \right)$$

$$C(\phi) \equiv \frac{12}{\hat{n}^2 f(\phi)^2}[8 + \hat{n}n_\lambda f(\phi)]$$

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Absorb C: $\frac{dh}{d\phi} = \sqrt{C(\phi)}$

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{\nabla}_a h\tilde{\nabla}^a h - \tilde{V}(\phi(h)) \right)$$

TW CANONICAL SCALAR FIELD INFLATON

$$S = \int d^4x \sqrt{|\tilde{g}|}\left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{\nabla}_a h \tilde{\nabla}^a h - \tilde{V}(\phi(h))\right)$$

Specific Potential Predicted by Projective Symmetry

$$\tilde{V}(\phi) \equiv \frac{3M_p^4}{8n_J}\left(1 - \frac{1}{f(\phi)^2}\right)$$

$$f(\phi) = 1 + \frac{8}{\hat{n}M_p}\phi$$

$$\frac{dh}{d\phi} = \sqrt{C(\phi)}$$

$$C(\phi) \equiv \frac{12}{\hat{n}^2 f(\phi)^2}[8 + \hat{n}n_\lambda f(\phi)]$$

$$\hat{n} = \frac{n_\lambda}{n_J n_\kappa}$$

Three free parameters remaining from Thomas-Whitehead Gravity: n_λ n_J n_κ

TW INFLATION AND DARK ENERGY

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{\nabla}_a h \tilde{\nabla}^a h - \tilde{V}(\phi(h)) \right)$$

Specific Potential Predicted by Projective Symmetry

$$\tilde{V}(\phi) \equiv \frac{3M_p^4}{8n_J}\left(1 - \frac{1}{f(\phi)^2}\right)$$

$$f(\phi) = 1 + \frac{8}{\hat{n}M_p}\phi$$

Matches Inflation Data

$$n_J \sim 10^{10}$$

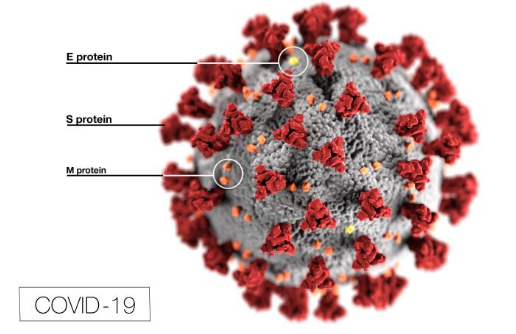


Image Credit: phil.cdc.gov

TW INFLATION AND DARK ENERGY

$$S = \int d^4x \sqrt{|\tilde{g}|} \left(-\frac{M_p^2}{2} \tilde{R} + \frac{1}{2} \tilde{\nabla}_a h \tilde{\nabla}^a h - \tilde{V}(\phi(h)) \right)$$

Specific Potential Predicted by Projective Symmetry

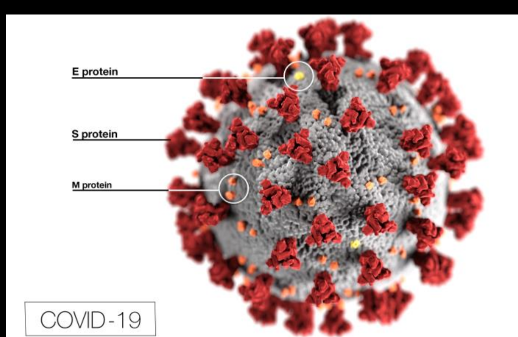
$$\tilde{V}(\phi) \equiv \frac{3M_p^4}{8n_J} \left(1 - \frac{1}{f(\phi)^2} \right)$$

$$f(\phi) = 1 + \frac{8}{\hat{n} M_p} \phi$$

Matches Inflation Data **OR** **Matches Dark Energy Data**

$$n_J \sim 10^{10}$$

$$n_J \sim 10^{120}$$



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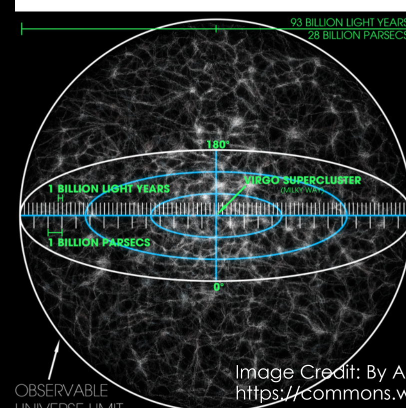

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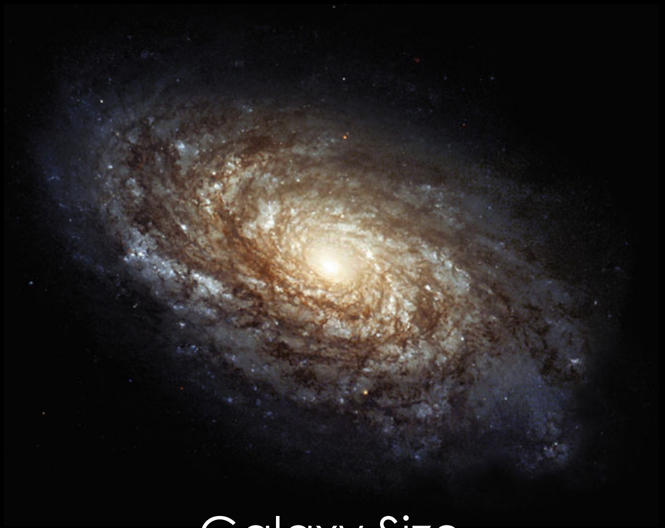
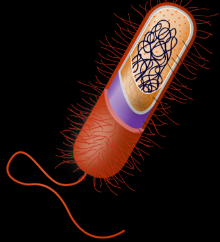
INFLATION

- Objects the size of atoms inflated to the size of small galaxies
- Occurs somewhere between 10^{-36} s and 10^{-32} s



Galaxy Size

Bacteria Size



?

TW Gravity: "Renormalized"

$$\frac{1}{\tilde{\kappa}_0}\int_{\ell_i}^{\ell_f} d\ell\ell^{-1} = \frac{\ln(\ell_f/\ell_i)}{\tilde{\kappa}_0} \quad \Rightarrow \kappa_0 \equiv \frac{\tilde{\kappa}_0}{\ln(\ell_f/\ell_i)}$$

$$\tilde{J}_0\int_{\ell_i}^{\ell f} d\ell\ell^{-1} = \tilde{J}_0\ln(\ell_f/\ell_i) \quad \Rightarrow J_0 \equiv \tilde{J}_0\ln(\ell_f/\ell_i)$$

SUMMARY AND FUTURE OUTLOOK

- Proposed principle behind inflation: projective symmetry

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  - inflaton field that match current data, constraining TW parameters
  - "Renormalized" TW parameters possibly speak to Dark Energy considerations and cosmological constant problem

Future Work

- Does TW inflation influence the projective direction?
- Contributions from traceless sector?
- Contributions from non-Levi-Civita connection dynamics?