Scattering Amplitudes and Unitarity for Gravitationally Mediated Dark Matter in Extra Dimensions

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With

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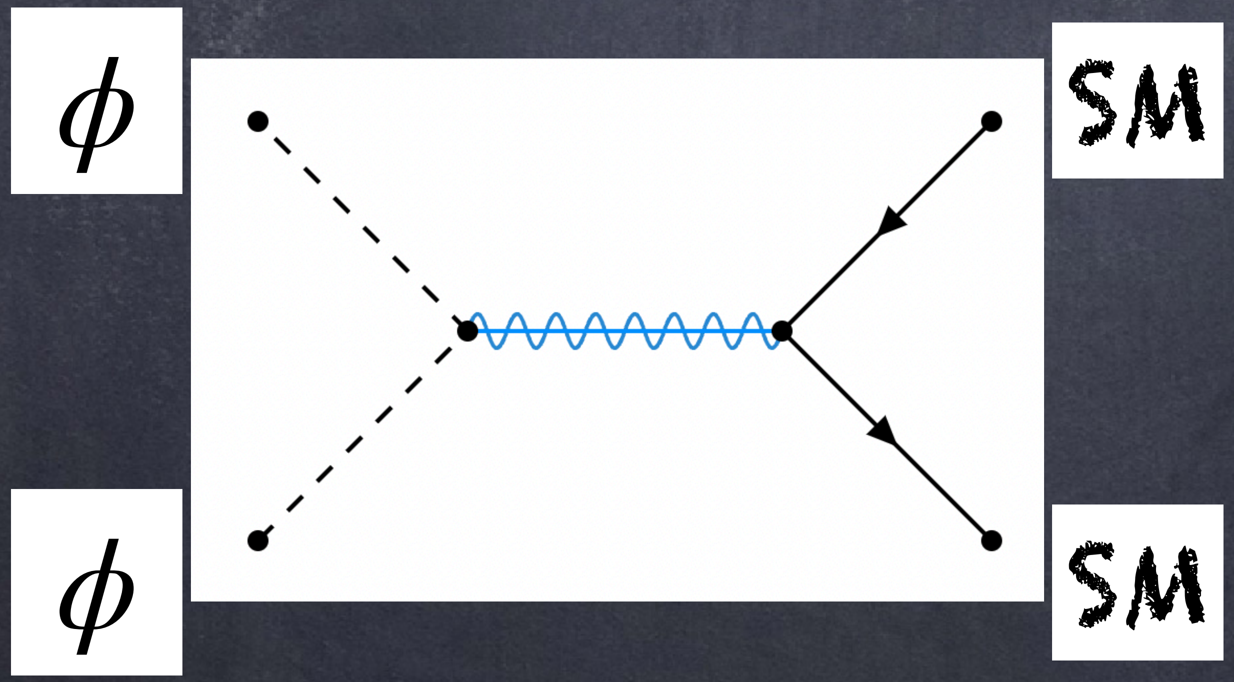
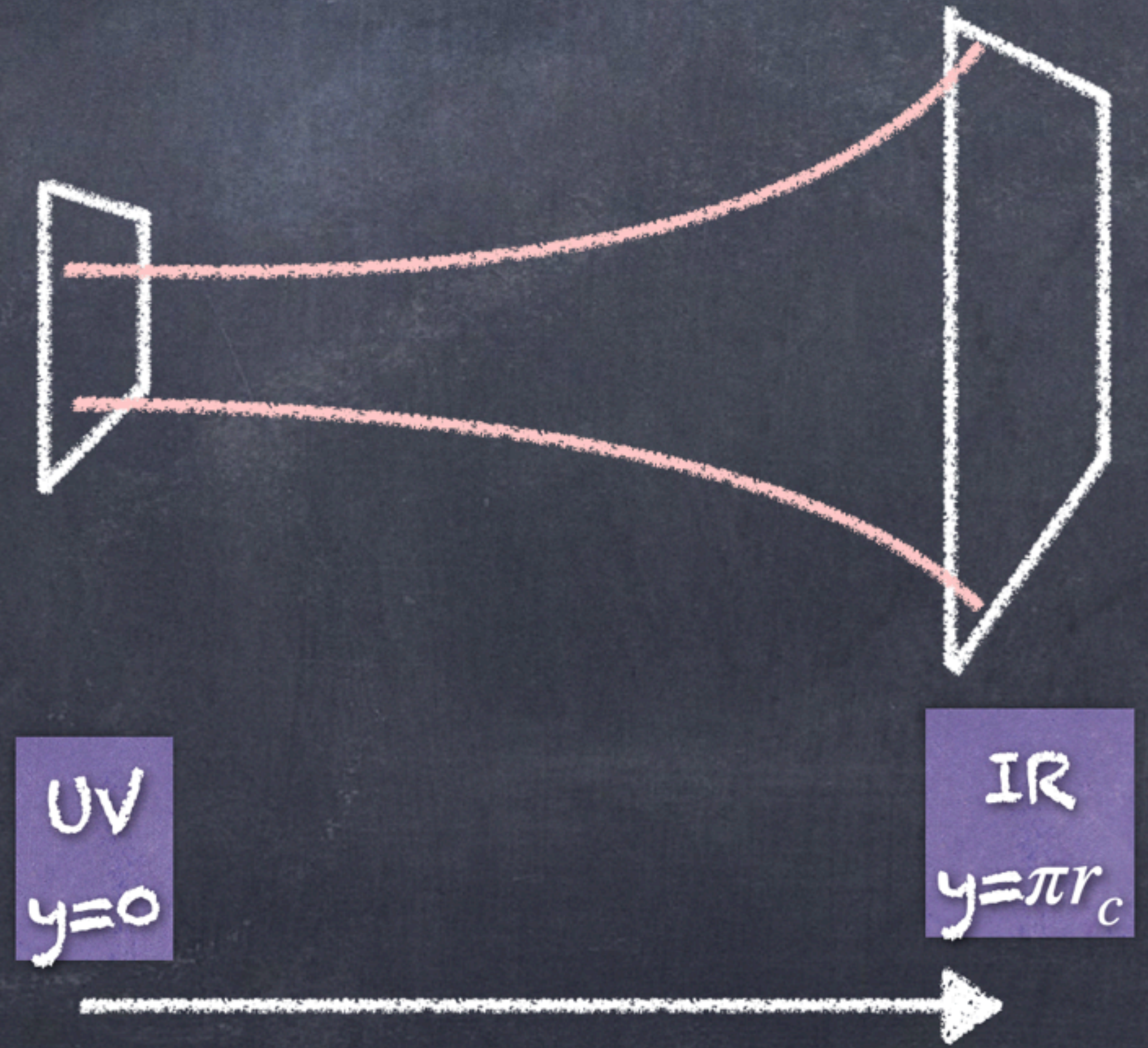
arXiv:1903.05650, arXiv:1910.06159, arXiv:2002.12458, arXiv:2104.08169, arXiv:2206:xxxxx

Motivation

- Can Dark Matter only have gravitational interactions
- Yes, but interactions are Planck scale suppressed.
- Can Dark Matter only have gravitational interactions and and still behave like a WIMP?
- Yes!

Simple Example:





$$\mathcal{L}_{\rm DM} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \; .$$

$$h^{\mu\nu}(T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu})$$

Particle Content

- Tower of massive spin-2 states $h_{\mu\nu}$
- Radion (r)
- SM +DM (localized on TeV Brane)
- SM +DM couple to $\frac{1}{\Lambda} h^{\mu\nu}(T^{(SM)}_{\mu\nu} + T^{(DM)}_{\mu\nu})$
- $\Lambda \simeq M_{pl}\, e^{-kr_c}$ (Usually a few TeV or more)

$$S = S_{\rm bulk} + S_{\rm UV} + S_{\rm IR}\ ,$$

$$S_{\rm bulk} = \frac{1}{2} M_5^3 \int d^4x \int_{-\pi}^{\pi} d\varphi \sqrt{G}(R - 2\Lambda_B),$$

$$S_{\rm UV} = \int d^4x \int_{-\pi}^{\pi} d\varphi \sqrt{-g_{\rm UV}}(-V_{\rm UV} + \mathcal{L}_{\rm UV})\delta(\varphi),$$

$$S_{\rm IR} = \int d^4x \int_{-\pi}^{\pi} d\varphi \sqrt{-g_{\rm IR}}(-V_{\rm IR} + \mathcal{L}_{\rm IR})\delta(\varphi - \pi)\ ,$$

$$[G_{MN}] = \begin{pmatrix} g_{\mu\nu}\ \exp\left[-2\left(A(y) + \frac{e^{2A(y)}}{2\sqrt{6}}\kappa\,\hat{r}(x,y)\right)\right] & 0 \\ 0 & -\left(1 + \frac{e^{2A(y)}}{\sqrt{6}}\kappa\,\hat{r}(x,y)\right)^2 \end{pmatrix}$$

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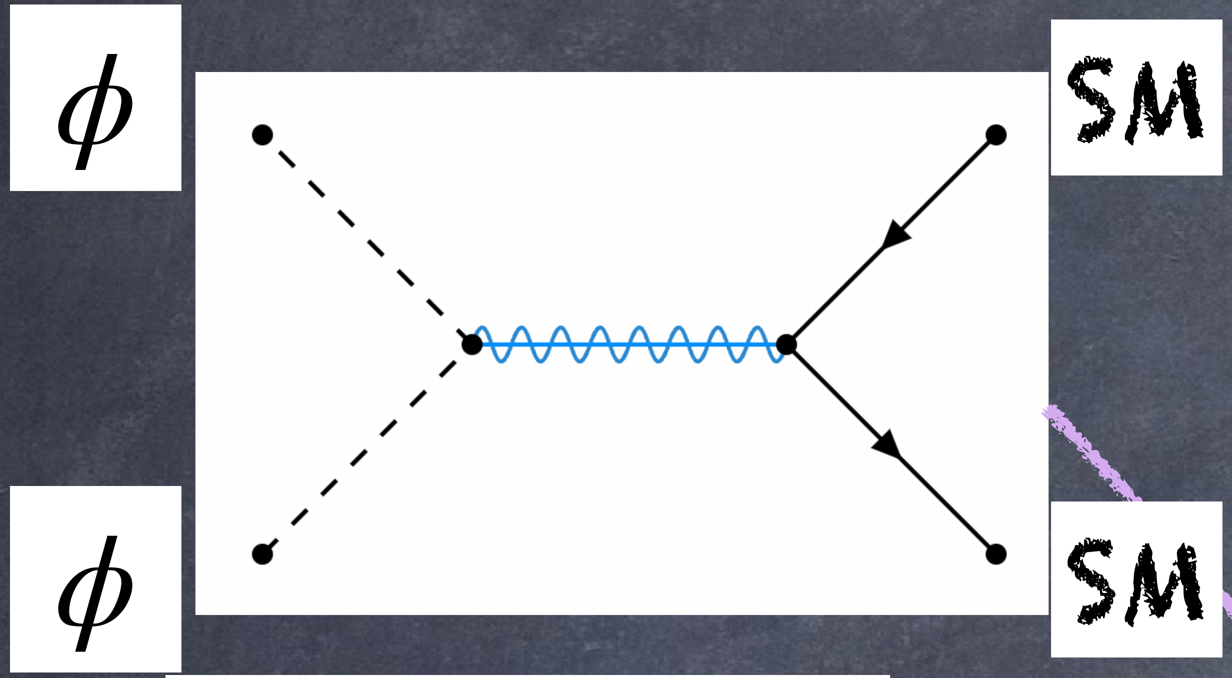
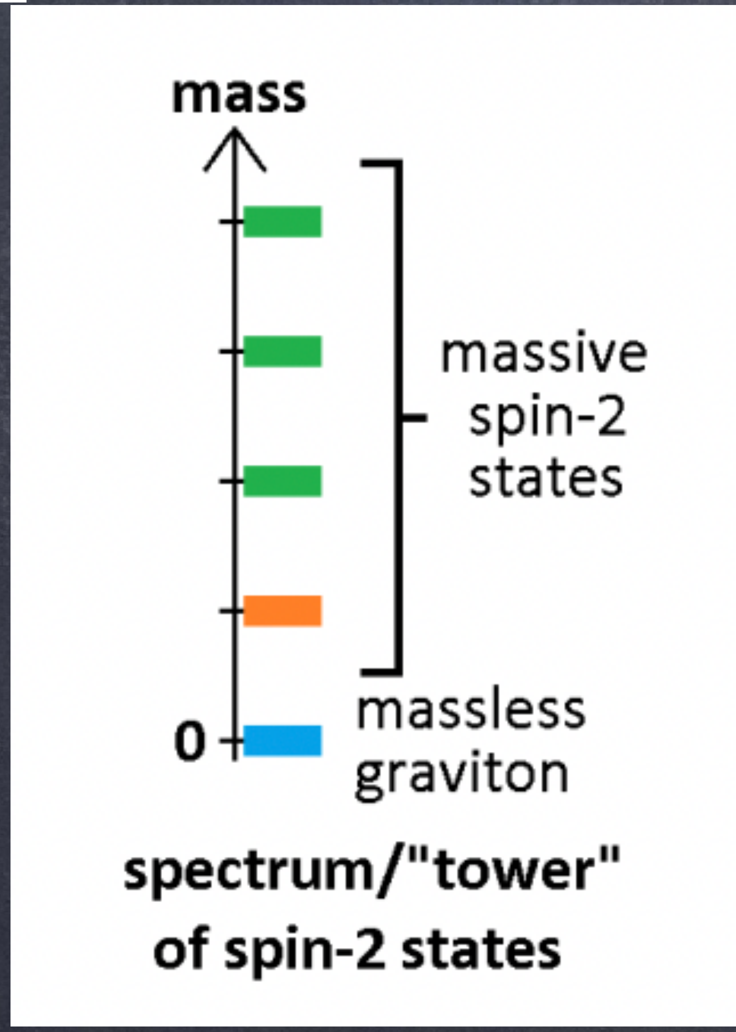
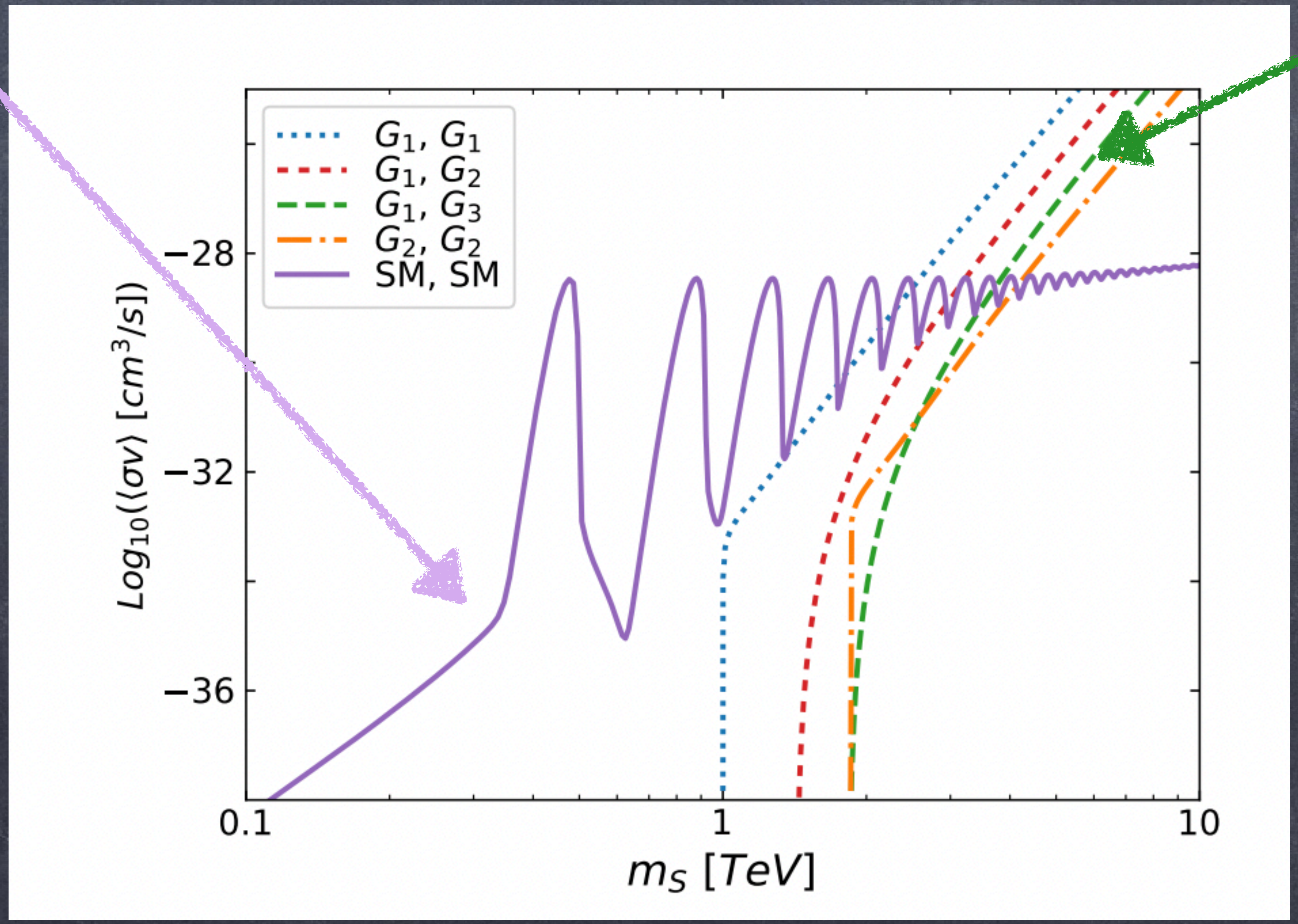
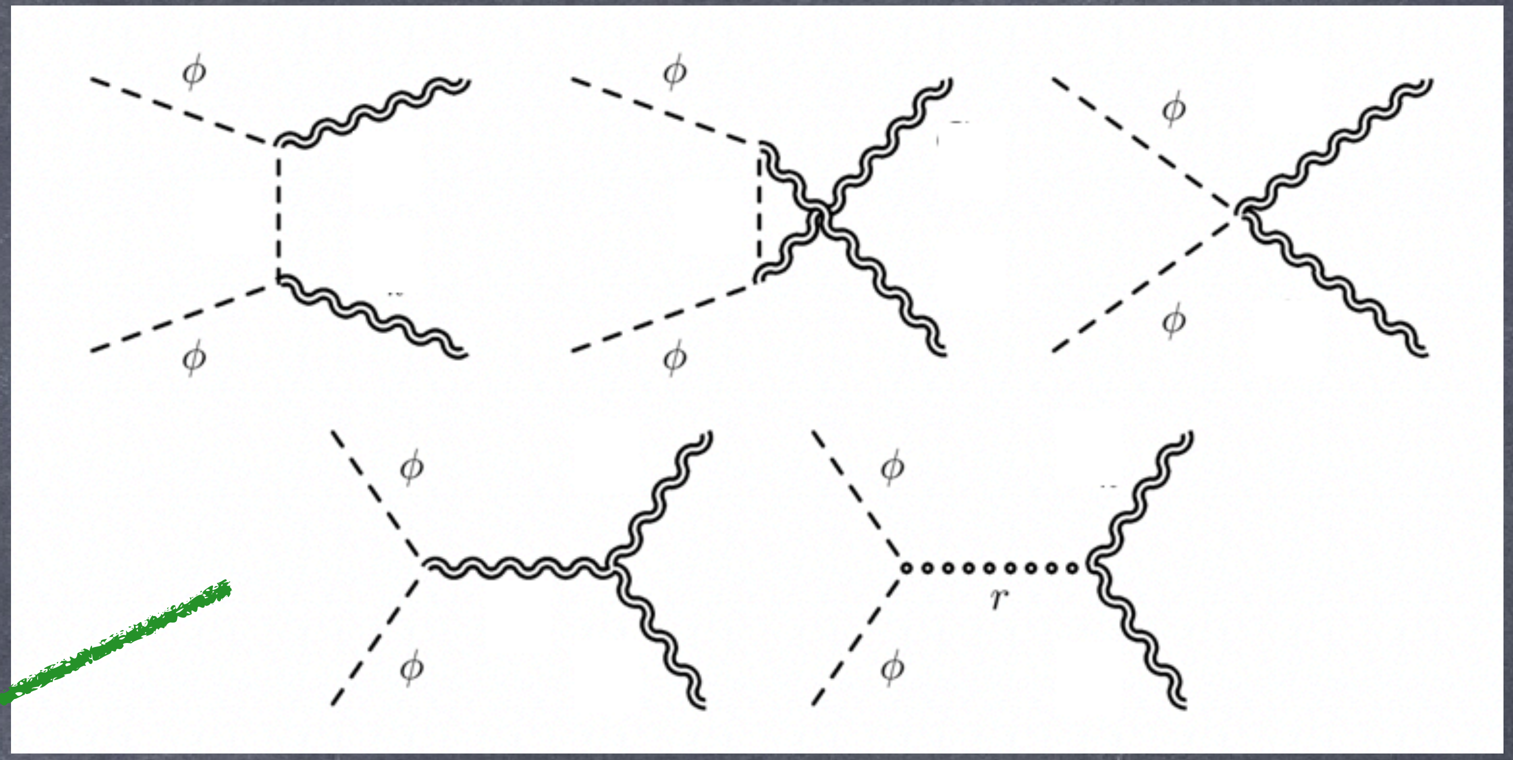
Let's Calculate the Relic







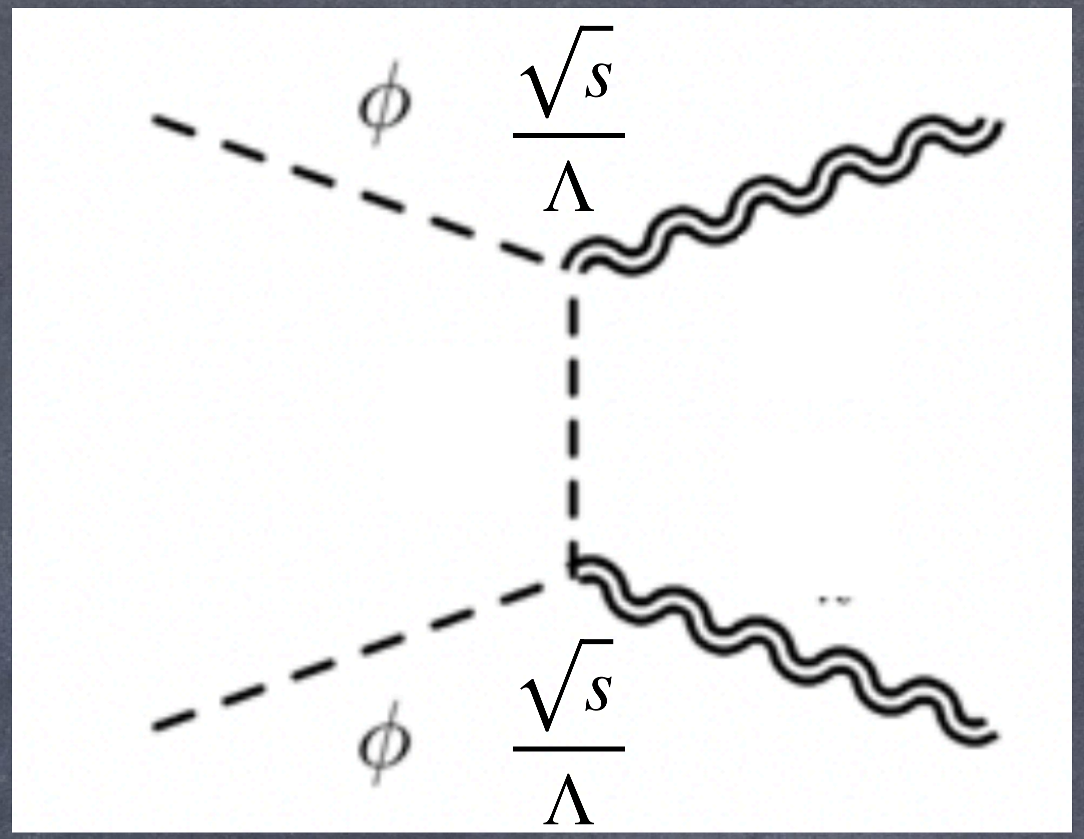


Bad High Energy Growth

$<\sigma v> \propto s^6$ or E^{12} ?

arXiv: 1511.03278, 1709.09688, 1907.04340, 2004.14403

Why the bad high energy growth?



$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$

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- Counting energy from polarization and vertices , we see that the energy of the amplitude should grow like s^3 (E^6)
- $\Longrightarrow$Unitarity is violated at a scale much smaller than Λ

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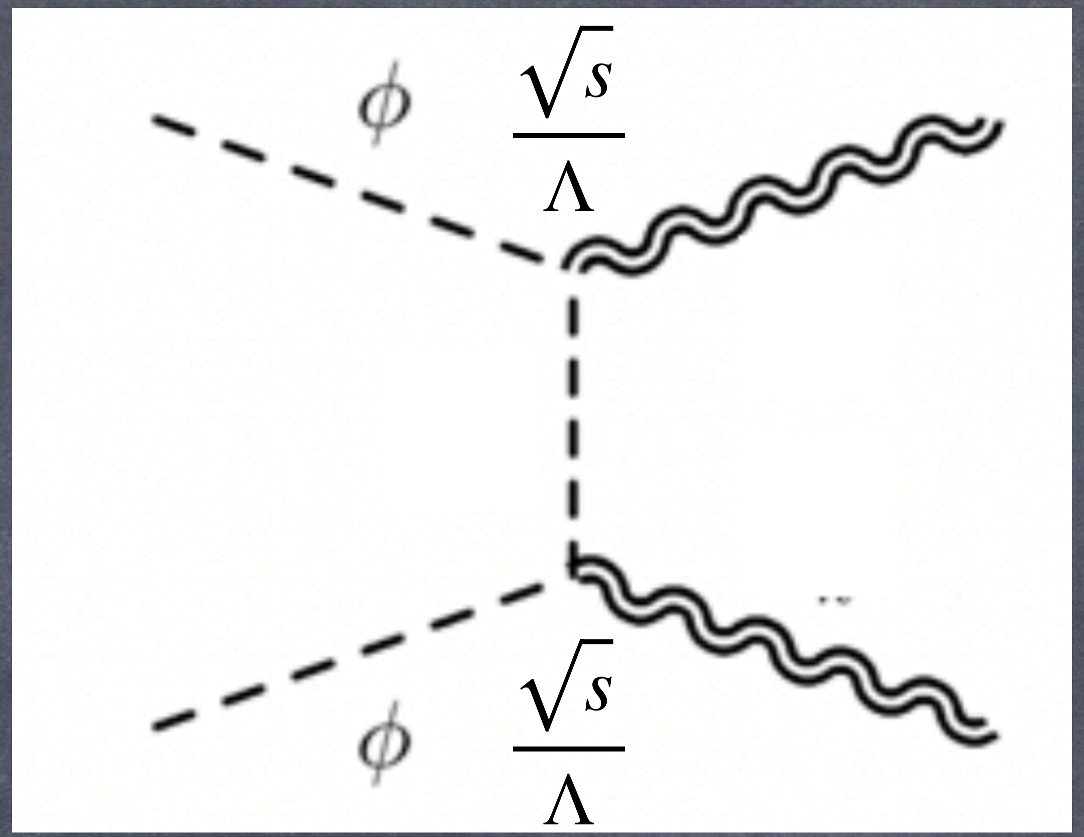

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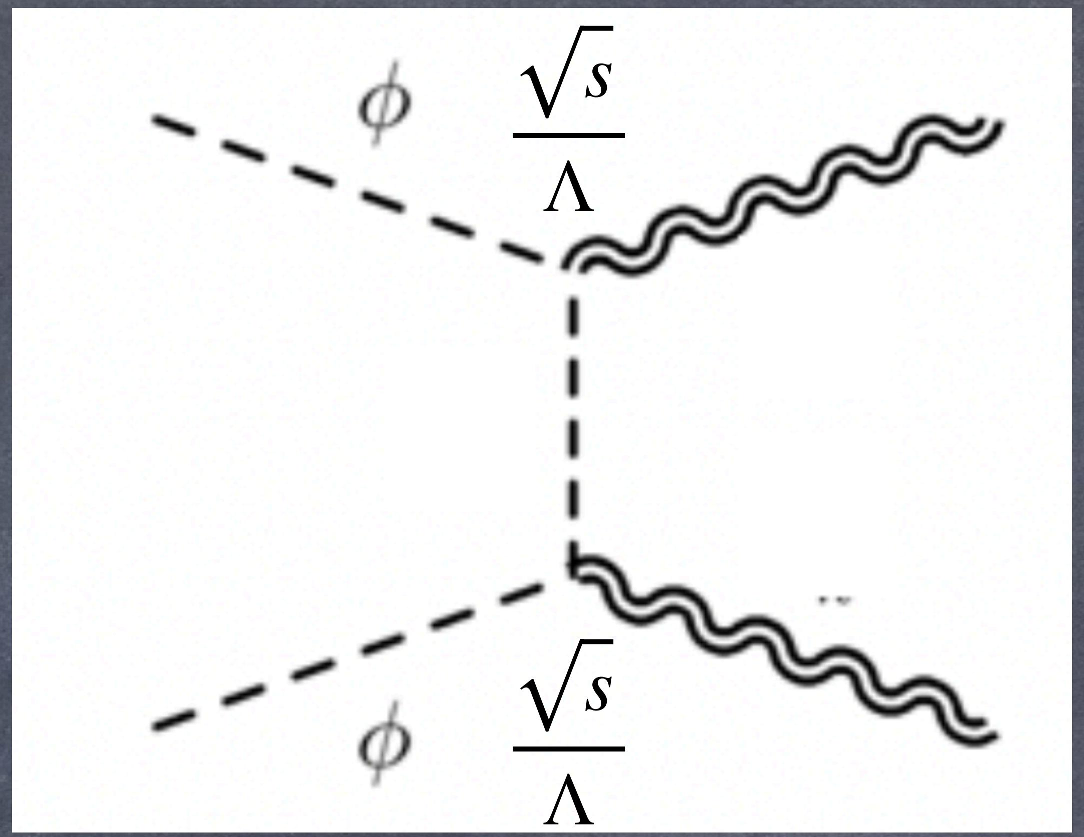

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Diffeomorphism invariance protects amplitude from bad high energy growth! How?

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

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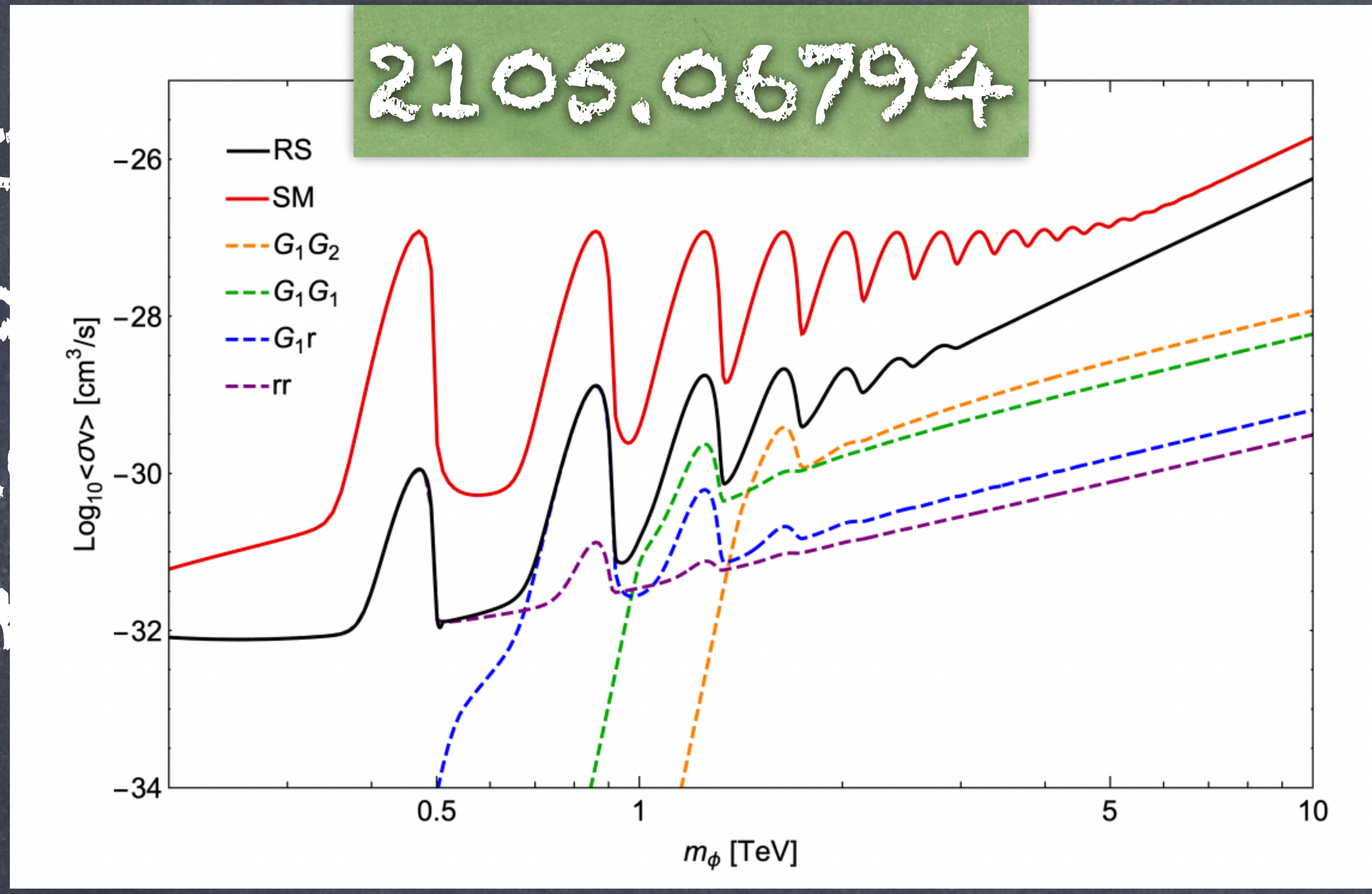
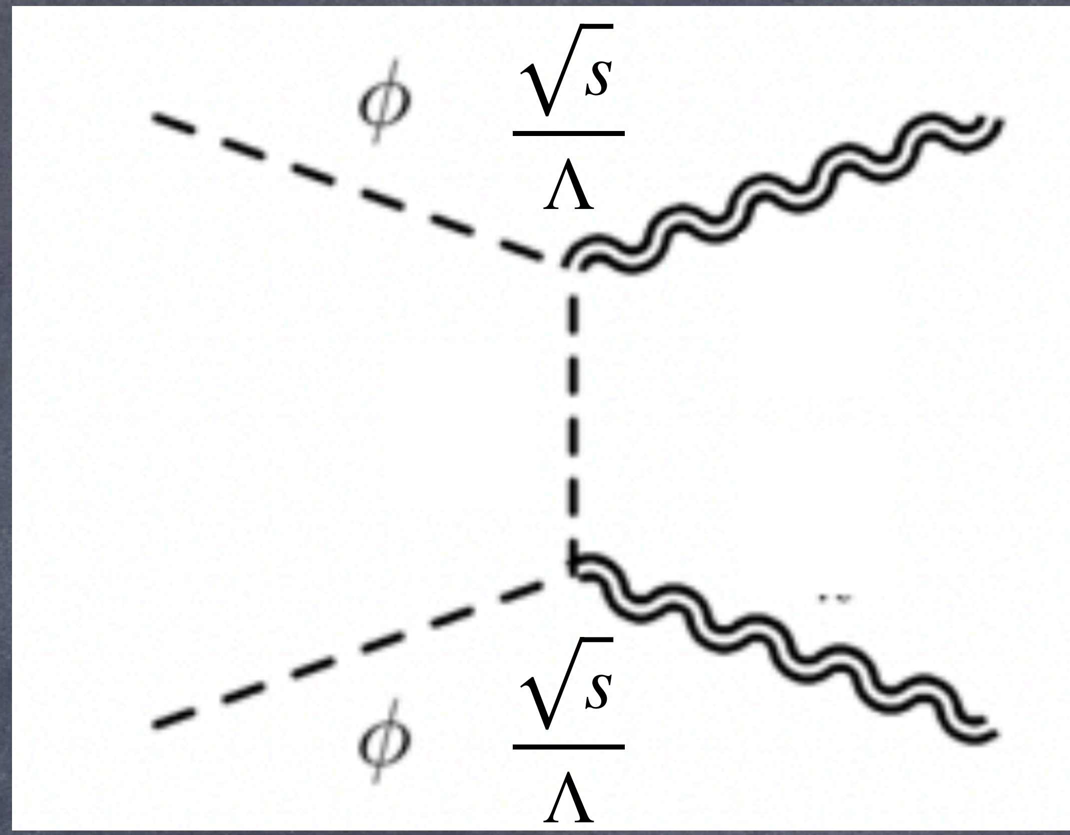


- Co… po…ces , we se… the an…w like s^3



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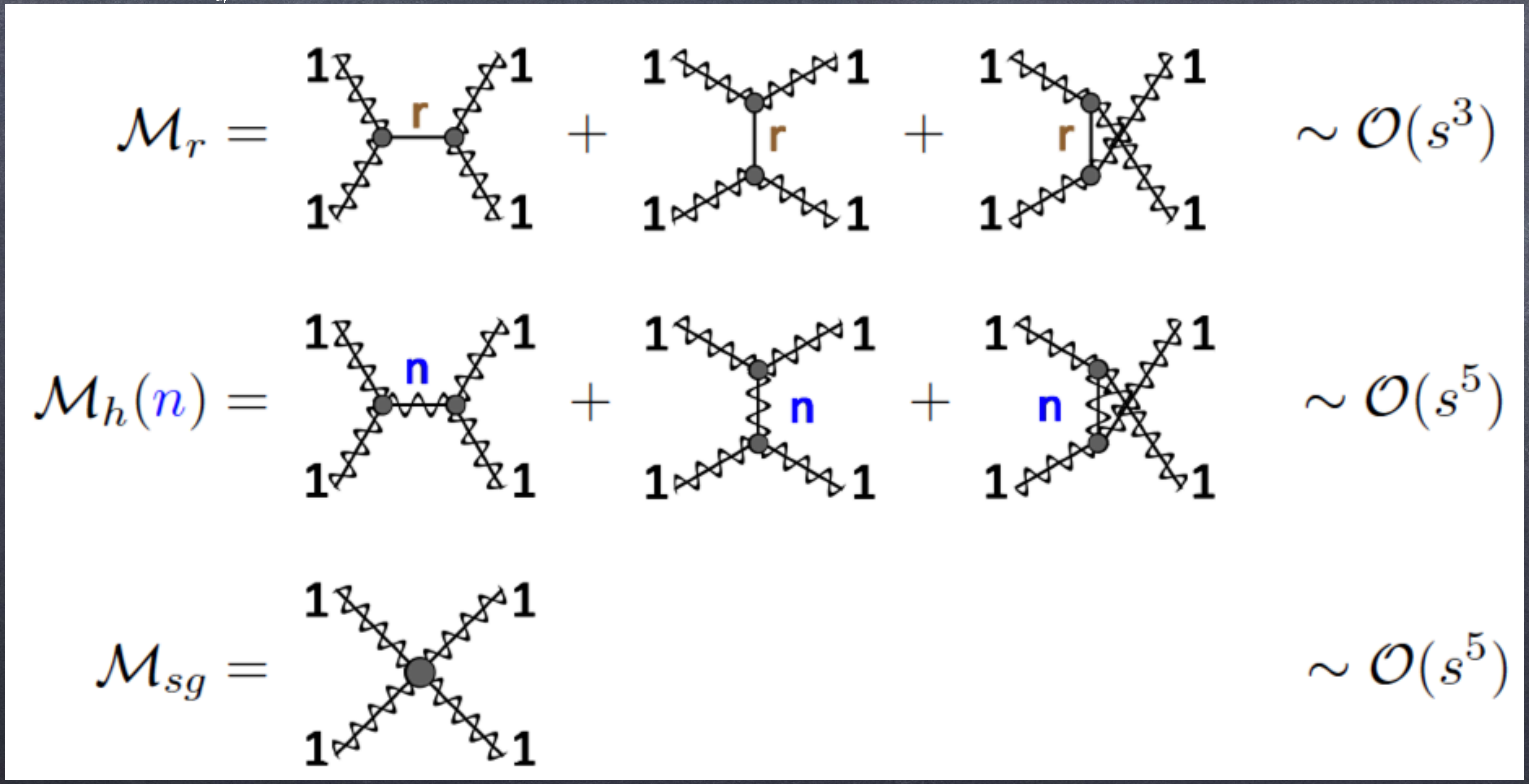
Unitarity in Extra Dimensional Models

Unitarity in Extra Dimensions

- Let's look at $2 \to 2$ scattering of gravitons - worst high energy behavior



Preliminaries

- Decompose the 5D graviton field into a tower of 4D fields

$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}_{\mu\nu}^{(n)}(x)\, \psi_n(\varphi) \,.$$

- Determine the fifth dimensional wave function for each mode from the Sturm-Liouville problem

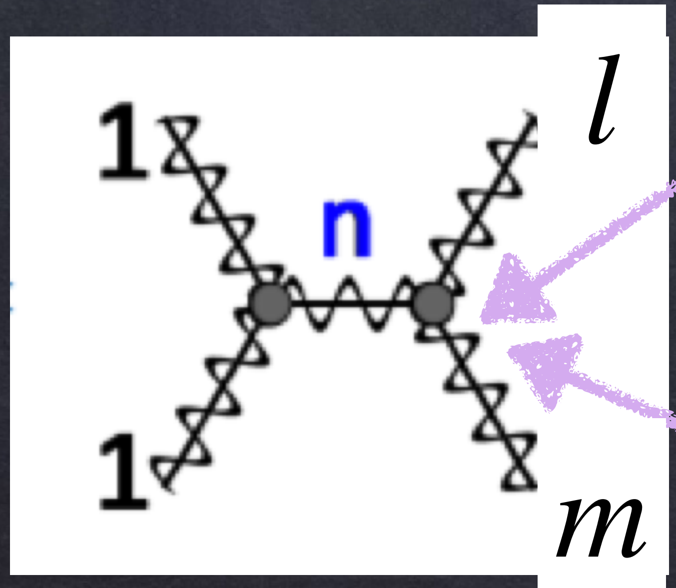
$$\partial_\varphi[e^{-4A}\, \partial_\varphi \psi_n] = -\mu_n^2 e^{-2A} \psi_n$$

- Wave functions are orthogonal and complete

$$\frac{1}{\pi} \int_{-\pi}^{+\pi} d\varphi \quad e^{-2A}\, \psi_m\, \psi_n = \delta_{m,n}$$

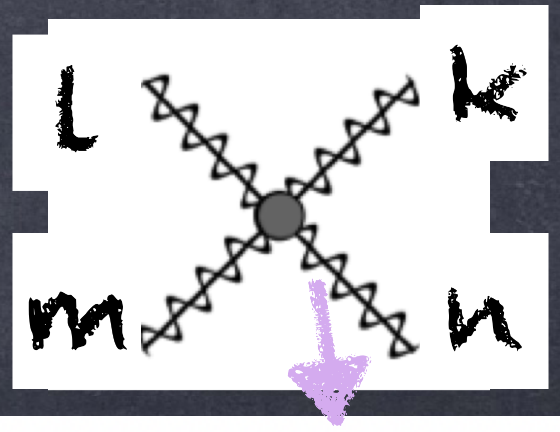
$$\delta(\varphi_2 - \varphi_1) = \frac{1}{\pi}\, e^{-2A} \sum_{j=0}^{+\infty} \psi_j(\varphi_1)\, \psi_j(\varphi_2)$$

- Substitute wave functions into overlap integrals that set the couplings of the vertices



$$a_{l'm'n'} = \frac{1}{\pi} \int d\varphi \;\, e^{-2A} (\partial_\varphi \psi_k)(\partial_\varphi \psi_l)(\partial_\varphi \psi_m)$$

$$a_{lmn} \equiv \frac{1}{\pi} \int_{-\pi}^{+\pi} d\varphi \quad e^{-2A}\, \psi_l\, \psi_m\, \psi_n$$



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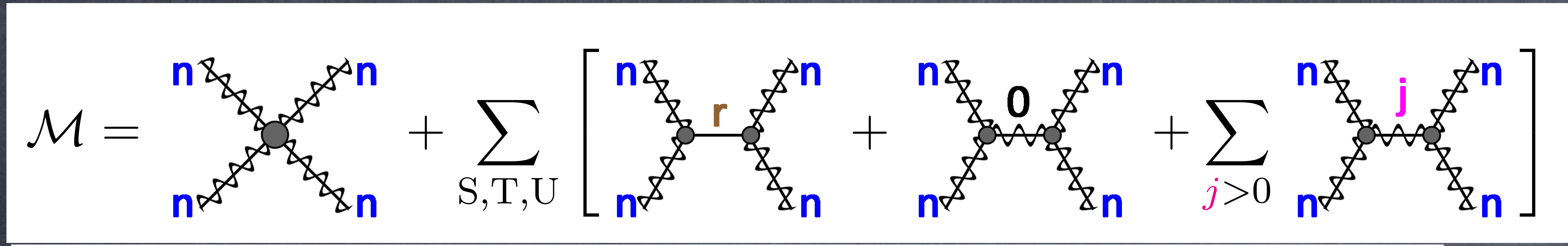
SUM RULES



$$\mathcal{M}(s,\theta) = \sum_{\sigma\in\frac{1}{2}\mathbb{Z}} \overline{\mathcal{M}}^{(\sigma)}(\theta)\cdot s^{\sigma}$$

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8}\cdot\left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8}\cdot\left(4m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

$$\overline{\mathcal{M}}^{(3)} = \frac{5\,\kappa^2\,\sin^2\theta}{1152\,\pi r_c\,m_n^4}\left\{\sum_j \frac{m_j^4}{m_n^4}a_{nnj}^2 - \frac{16}{15}a_{nnnn} - \frac{4}{5}\left[\frac{9\,b_{nnr}^2}{(m_n r_c)^4} - a_{nn0}^2\right]\right\}.$$

$$\overline{\mathcal{M}}^{(2)} = \frac{\kappa^2\,[7+\cos(2\theta)]}{864\,\pi r_c\,m_n^2}\left\{\sum_j\left[\frac{m_j^2}{m_n^2} - \frac{5}{2}\right]\frac{m_j^4}{m_n^4}a_{nnj}^2 + \frac{8}{3}a_{nnnn} - 2\left[\frac{9\,b_{nnr}^2}{(m_n r_c)^4} - a_{nn0}^2\right]\right\}$$

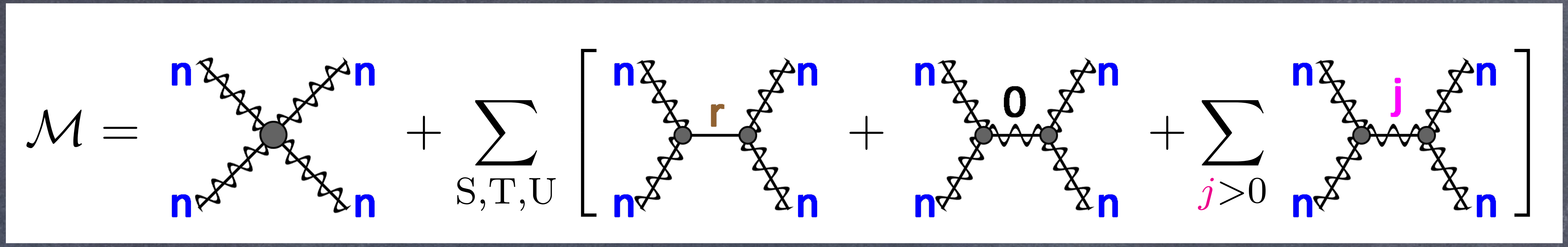
- At each order of s, relations between overlap integrals ensure that the bad high energy growth vanishes
- Example: $\sum_j a_{nnj}^2 = a_{nnnn}$
- Possible to prove analytically using completeness and orthogonality
- Finally, as expected, we find that the amplitude grows as $\mathcal{O}(s)$ $\Longrightarrow$ Unitarity violated at $\sim\Lambda \simeq M_{pl}\,e^{-kr_c}$

What about a Stabilized Extra dimension?

- In its original formulation RS consists of a massless radion $\Longrightarrow$ Casimir force that renders the extra-dimension unstable.
- Alternately, a massless radion leads to a Brans-Dicke (Scalar-Tensor) theory of gravity and contradicts experiments when there is matter on the TeV Brane.
- Goldberger-Wise mechanism can provide mass to the radion and stabilize the extra-dimension.

Goldberger, Wise 2000
De-Wolfe, Freedman, Karch, Gubser 2001
Peloso, Koffman 2001
Tanaka, Garriga 2001
Smolyakov, Volobuev 2002

Massive Radion and Scattering Amplitudes



- Naively adding a mass term by hand to the radio propagator messes up the sum rules and the amplitude grows as $\mathcal{O}(s^2)$
- This too must cancel due to diffeomorphism invariance

Goldberger-Wise Mechanism

$$\mathcal{L}_{5D} \equiv \mathcal{L}_{EH} + \mathcal{L}_{\Phi\Phi} + \mathcal{L}_{pot} \ ,$$

$$\mathcal{L}_{\Phi\Phi} \equiv \sqrt{G}\left[\frac{1}{2}\tilde{G}^{MN}\,(\partial_M\hat{\Phi})\,(\partial_N\hat{\Phi})\right] \ .$$

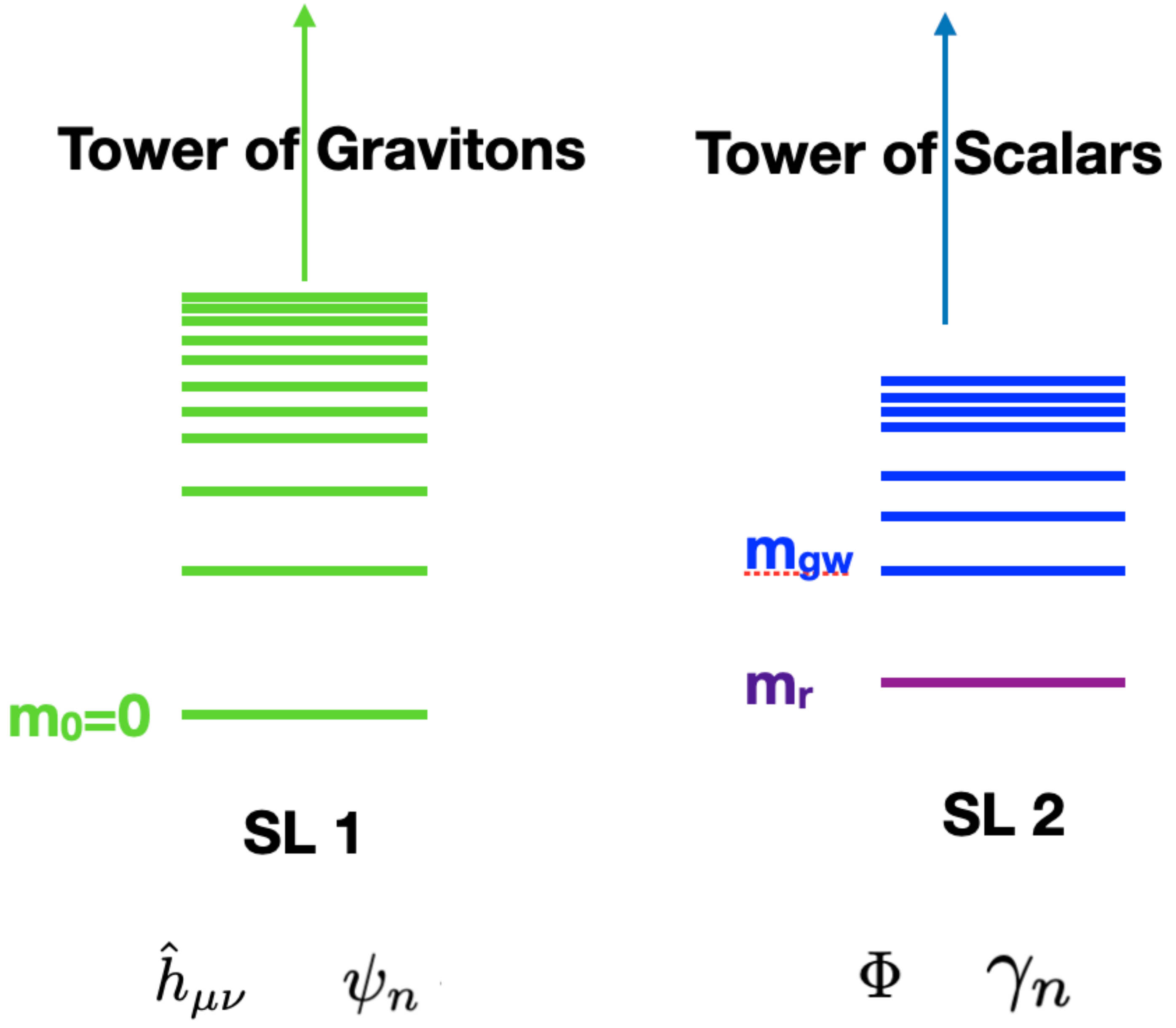
$$\mathcal{L}_{pot} \equiv -\frac{4}{\kappa^2}\left[\sqrt{G}\,V[\hat{\Phi}] + \sqrt{\overline{G}}\,V_1[\hat{\Phi}]\delta(\varphi) + \sqrt{\overline{G}}\,V_2[\hat{\Phi}]\delta(\varphi-\pi)\right] \ .$$

$$\hat{\Phi}(x,y) \equiv \frac{1}{\kappa}\hat{\phi}(x,y) \equiv \frac{1}{\kappa}\left[\phi_0(y) + \hat{f}(x,y)\right] \ ,$$

- Introduce a Bulk Scalar field that acquires a vev $\phi_0(y)$ that varies with the dimension 5 coordinate.
- This kinetically mixes with the radion field and is the source of its mass.
- We end up with two towers of scalar fields one corresponding to the modulus and the other to the bulk scalar. We can use the residual gauge freedom to collapse this into a single tower

Two Towers

- The radion is identified as the lightest mode of the tower of scalars.
- The order s^2 growth of the amplitude due to the mass of the radion is cancelled by the remainder of the scalar tower



$$\sum_{i=0}^{+\infty} a^2_{n'n'(i)} = \frac{5}{9} c_{n'n'n'n'} + \frac{1}{9}\mu_n^4 a^2_{nn0} + \frac{1}{27}\mu_n^4 a_{nnnn}$$

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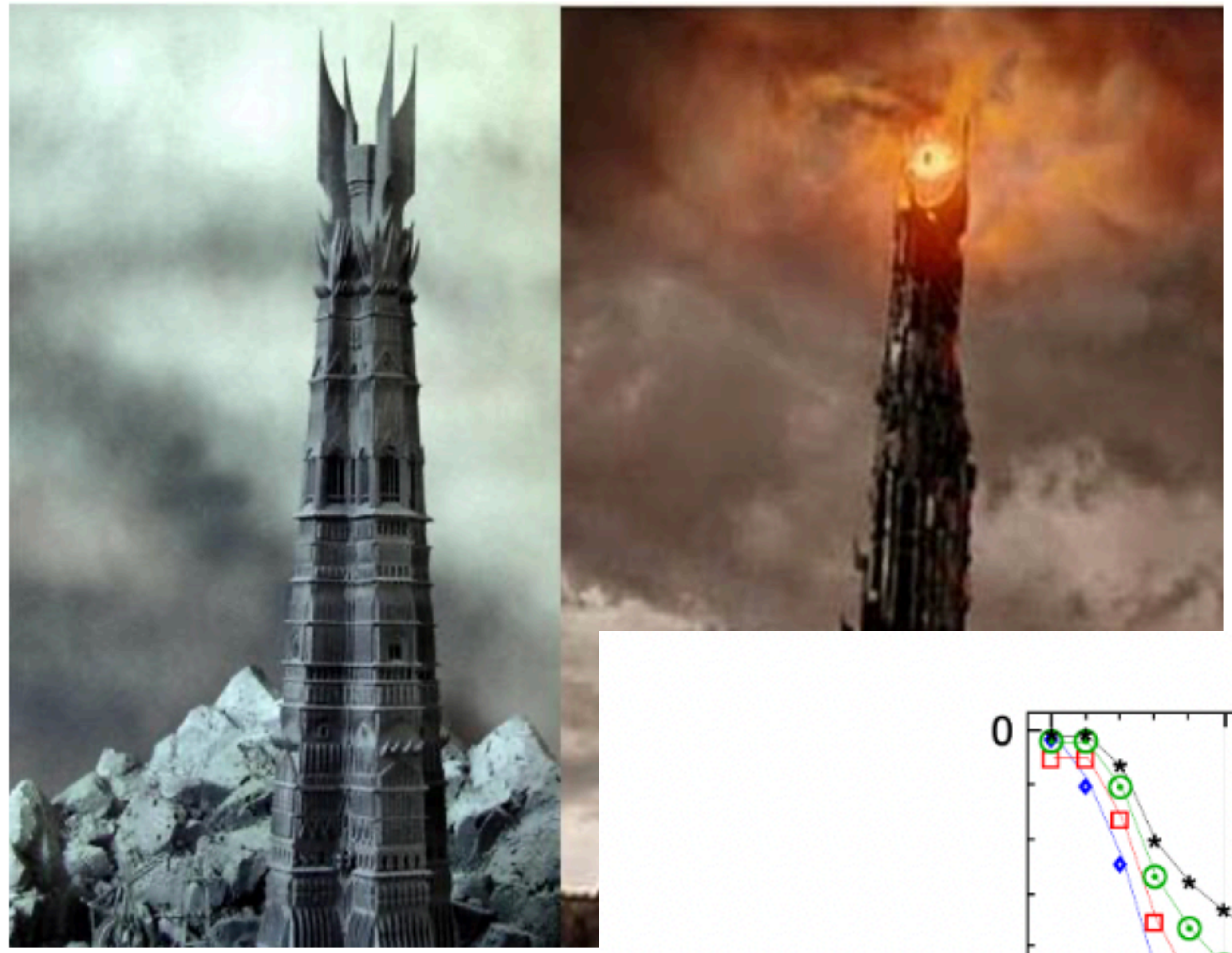

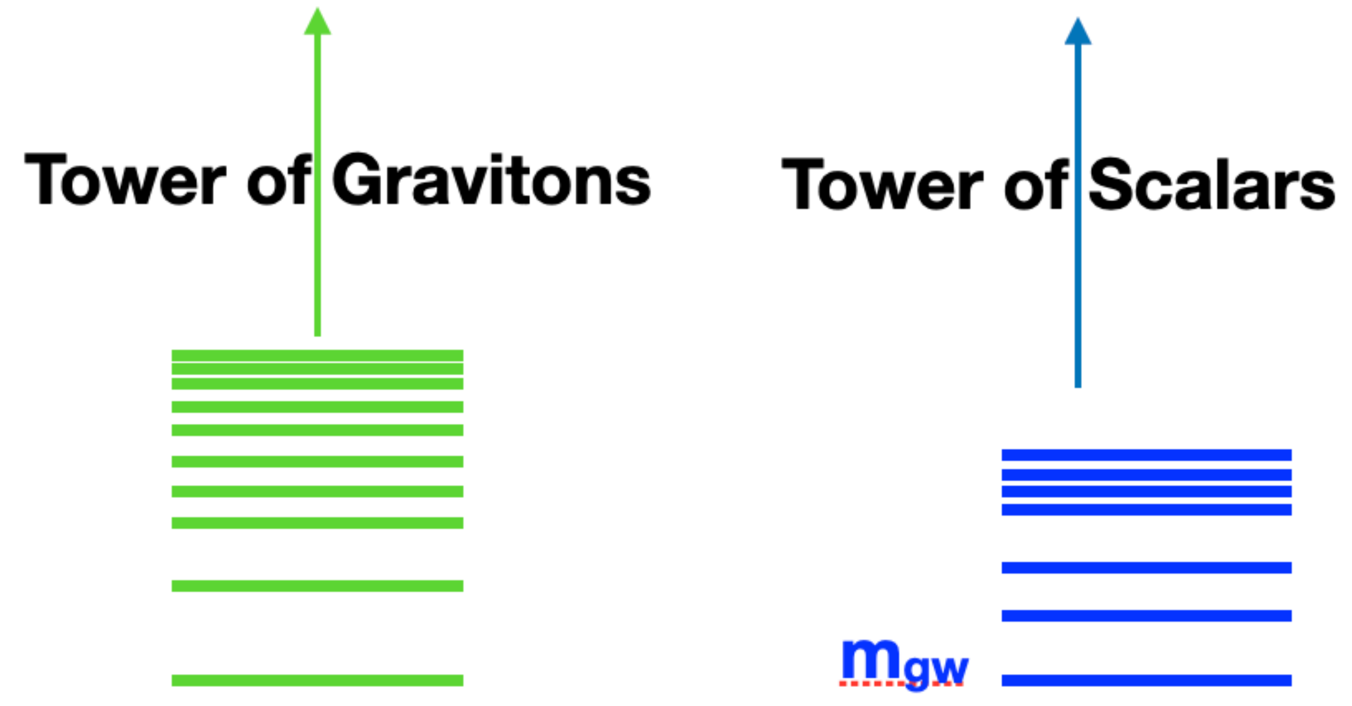

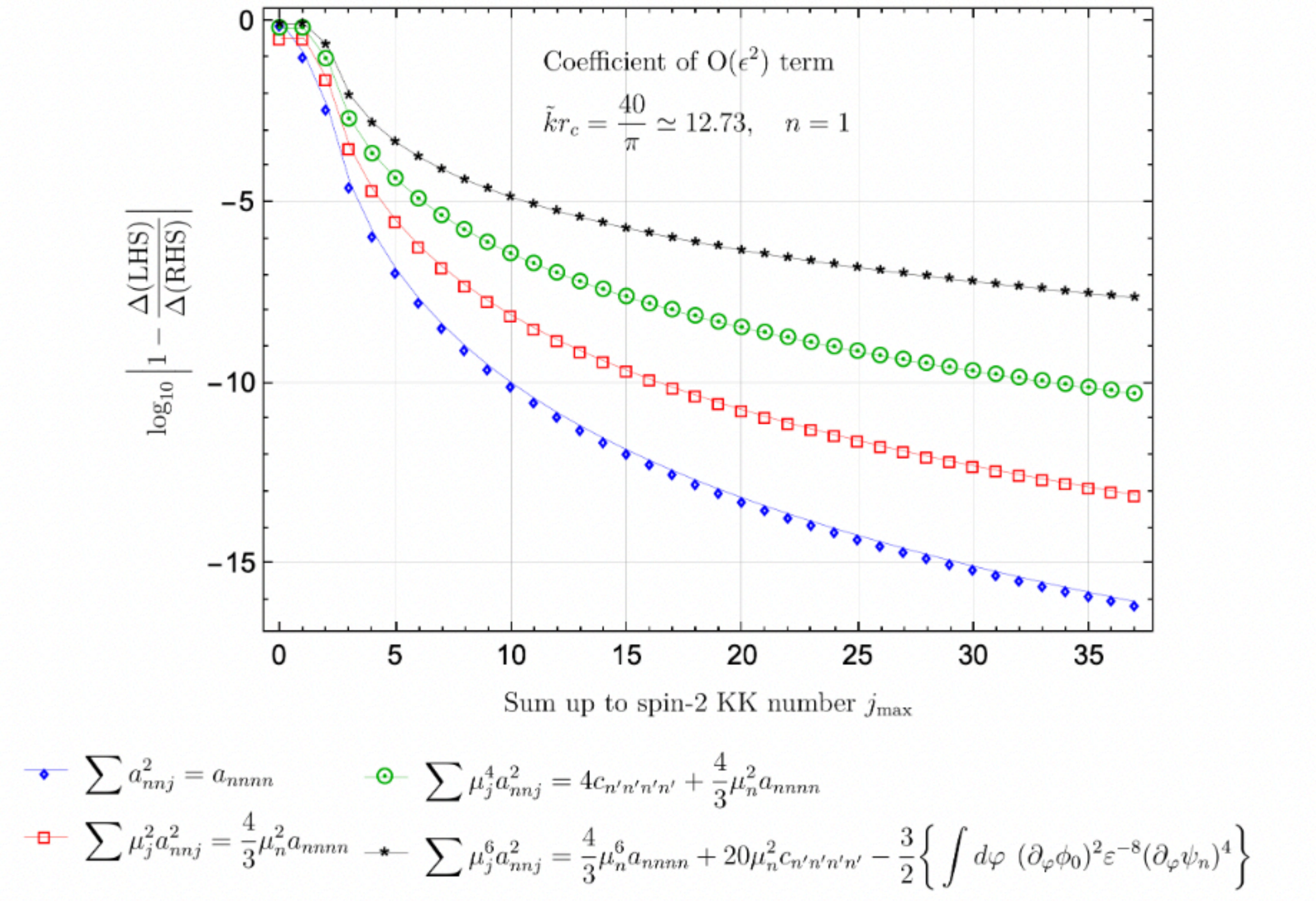

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Summary

- Apparent bad high energy growth of massive graviton scattering amplitudes in compactifield Extra dimensional theories are tamed by diffeomorphism invariance.
- Uncovered sum rules that ensure this cancellation. This also provides a cross-check for our wavefunctions, couplings and masses- important for implementing in phenomenological applications.
    - Unstabilized model: Cancellations between single tower of states and couplings.
    - Stabilized model: Cancellations between two towers
    - Validity of 4D EFT - does not depend on modulus stabilization
        - 5D Planck Scale for flat stabilized models
        - $\Lambda \simeq M_{pl}\, e^{-kr_c}$ for warped RS model
- In order to demonstrate this
    - Determined the Lagrangian of the stabilized model up to quartic order - this is non-trivial
    - First calculation to involve full dynamics of a stabilized model.