ANALYTIC APPROXIMATIONS FOR THE VELOCITY SUPPRESSION OF DARK MATTER CAPTURE

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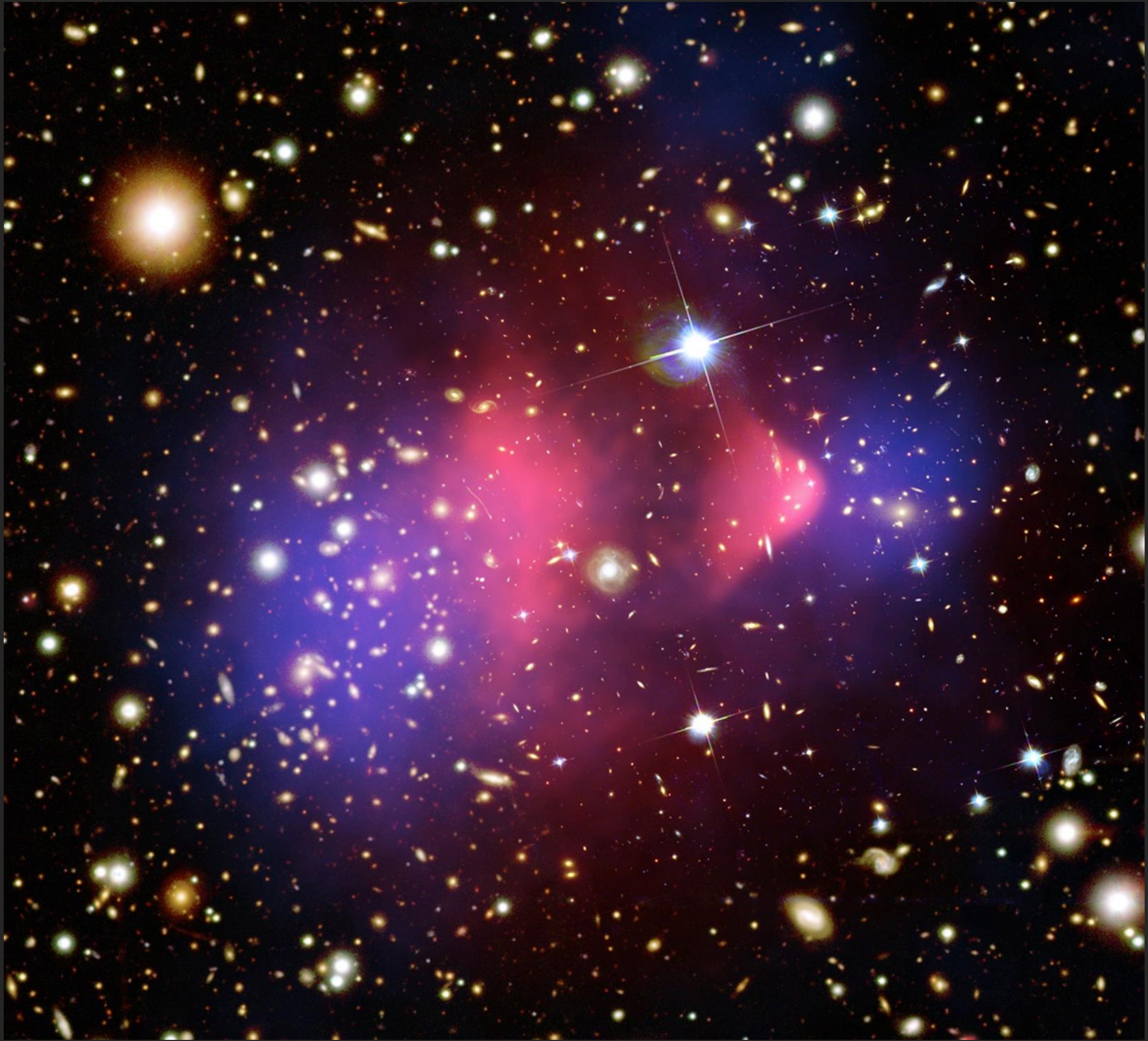


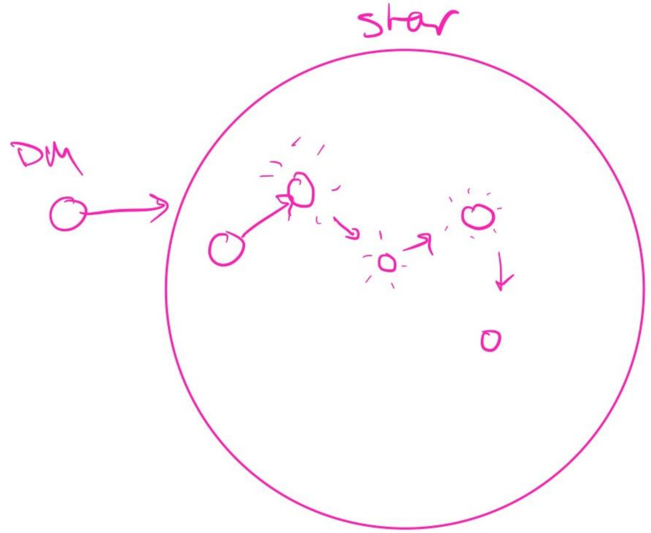
INTRODUCTION

- Dark Matter (DM)– one of the biggest mysteries in modern astrophysics & cosmology
- Indirect detection of DM with astrophysical objects is an important discovery method
- Impact of relative stellar velocity to DM halo on DM capture rates
- This presentation is based on our paper: C. Ilie and J. Paulin, *ApJ* (in press) arXiv:2107.00675
  - Accessible via the QR code

- Outline:
  - DM Capture
  - DM Distributions
  - Suppression Factor
  - Bounds on DM-nucleon cross section

DARK MATTER CAPTURE: CONCEPTUAL PICTURE

- DM scatters off nuclei & loses energy with each collision
- DM is captured once the DM velocity falls below the escape velocity of the capturing object



DARK MATTER CAPTURE

$$C_N = \underbrace{\pi R_\star^2}_{\text{Stellar radius}} \underbrace{p_N(\tau)}_{\text{Prob. For } N \text{ collisions}} \underbrace{\int_0^\infty f(u)\frac{du}{u}}_{\text{DM Distribution}} \underbrace{w^2 g_N(w)}_{\text{Prob. DM particle falls below } v_{esc}}$$

- The total capture rate: the sum of the capture rate after *N* scatters
- The focus for this project is the effect of altering the DM distribution

$$C_{tot} = \sum_{N=1}^{\infty} C_N$$

DARK MATTER VELOCITY DISTRIBUTION

- It is traditional to assume a Maxwell-Boltzmann (MB) distribution:

$$f_0(u)du = n_\chi \frac{4}{\sqrt{\pi}} x^2 \exp(-x^2)dx,$$

  - where n_χ is the number density of DM and *x* is the dimensionless particle velocity.

- When an object orbits with respect to ambient DM, we need to apply boosted MB distribution:

$$f_\eta(u) = f_0(u)\exp(-\eta^2)\frac{\sinh(2x\eta)}{2x\eta},$$

  - where η is a dimensionless parameter dependent on the velocity of the object with respect to the DM halo, and defined as:

$$\boxed{\eta \equiv \sqrt{\frac{3}{2}\frac{\tilde{v}}{\bar{v}}}.}$$

Stellar velocity

Average DM velocity

TOTAL CAPTURE RATE: WITH BOOSTED DISTRIBUTION

$$
\begin{aligned}
C_N = \frac{n_\chi \pi p_N(\tau) R^2}{2\sqrt{6}\bar{v}\eta} \Bigg[& \frac{1}{\sqrt{\pi}} \exp\left(\frac{-3(v_N^2 - v_{esc}^2)}{\bar{v}^2} - 2\eta^2\right) \bar{v} \Bigg\{ 4 \exp\left(\frac{3(v_N^2 - v_{esc}^2)}{\bar{v}^2} + \eta^2\right) \bar{v}\eta + \\
& \exp\left(\frac{-6v_{esc}^2 + 6v_N^2 - 4\sqrt{6}v_{esc}\bar{v}\eta\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}} + 4\bar{v}^2\eta^2}{4\bar{v}^2}\right) \left(\sqrt{6}v_{esc}\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}} - 2\bar{v}\eta\right) - \\
& \exp\left(\left(\frac{\sqrt{\frac{3}{2}}v_{esc}\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}}}{\bar{v}} + \eta\right)^2\right) \left(\sqrt{6}v_{esc}\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}} + 2\bar{v}\eta\right) \Bigg\} + \\
& \left(3v_{esc}^2 + \bar{v}^2\left(1 + 2\eta^2\right)\right) \operatorname{erf}\left(\frac{\sqrt{\frac{3}{2}}v_{esc}\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}}}{\bar{v}} - \eta\right) + \\
& 2\left(3v_{esc}^2 + \bar{v}^2\left(1 + 2\eta^2\right)\right) \operatorname{erf}(\eta) - \\
& \left(3v_{esc}^2 + \bar{v}^2\left(1 + 2\eta^2\right)\right) \operatorname{erf}\left(\frac{\sqrt{\frac{3}{2}}v_{esc}\sqrt{-1 + \frac{v_N^2}{v_{esc}^2}}}{\bar{v}} + \eta\right) \Bigg]
\end{aligned}
$$

In contrast:

$$
C_N = \pi\, R^2\, p_N(\tau) \frac{\sqrt{6}\, n_X}{3\sqrt{\pi}\bar{v}} \Big((2\,\bar{v}^2 + 3\, v_{esc}^2) - (2\,\bar{v}^2 + 3\, v_N^2) \exp\Big(- \frac{3(v_N^2 - v_{esc}^2)}{2\,\bar{v}^2} \Big) \Big),
$$

ANALYTIC SUPPRESSION FACTOR

$$\boxed{\xi_\eta \equiv \frac{C_{tot}(\eta)}{C_{tot}(\eta = 0)}}$$

- The following is a useful closed form solution for the suppression factor:

$$I_\eta(t) = \frac{e^{-\eta^2}}{16\eta}\left(4\eta + 2\sqrt{\pi}e^{\eta^2}\left(2\eta^2+1\right)\operatorname{erf}(\eta) + A(\eta;t) + B(\eta;t)\right)$$

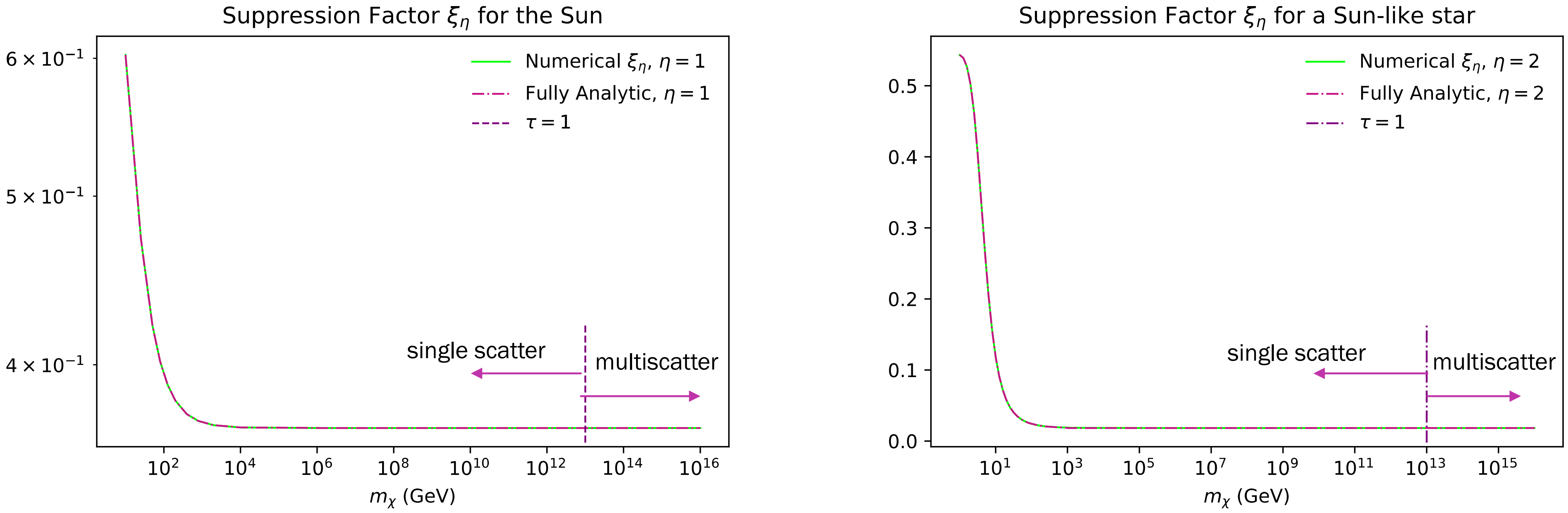
$$I_0(t) = \frac{1}{2}\left(\frac{3e^{-t^2}\left(e^{t^2}-1\right)v_{\rm esc}^2}{2\bar{v}^2} - e^{-t^2}\left(t^2+1\right) + 1\right)$$

where:

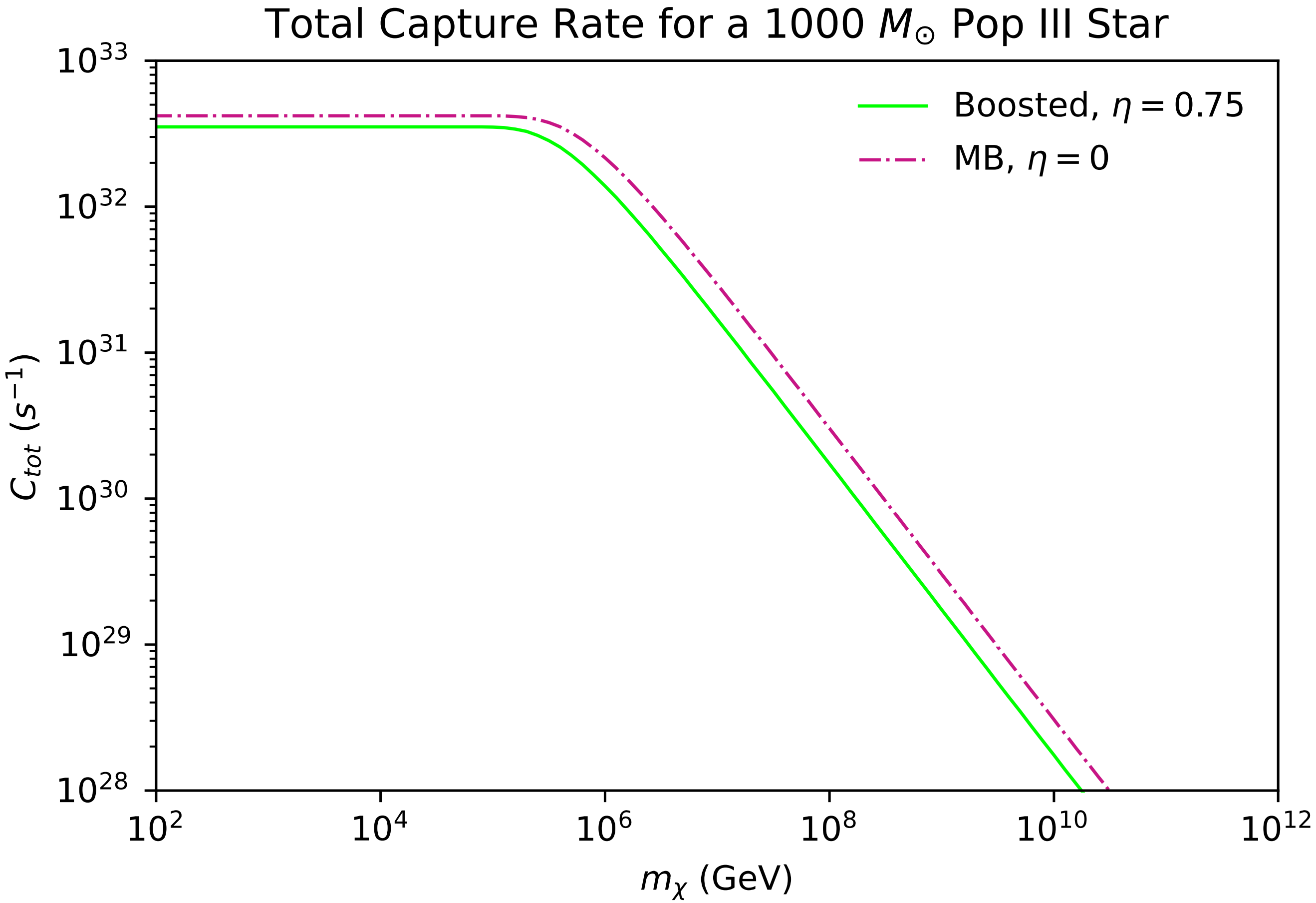
$$A(\eta;t) \equiv e^{-t(2\eta+t)}\left[\sqrt{\pi}\left(2\eta^2+1\right)e^{(\eta+t)^2}\left(\operatorname{erf}(t-\eta) - \operatorname{erf}(\eta+t)\right) - 2\eta - 2e^{4\eta t}(\eta+t) + 2t\right]$$

$$B(\eta;t) \equiv \frac{3\sqrt{\pi}e^{\eta^2}v_{\rm esc}^2\left(2\operatorname{erf}(\eta) + \operatorname{erf}(t-\eta) - \operatorname{erf}(\eta+t)\right)}{\bar{v}^2}.$$

SUPPRESSION FACTOR FOR THE SUN

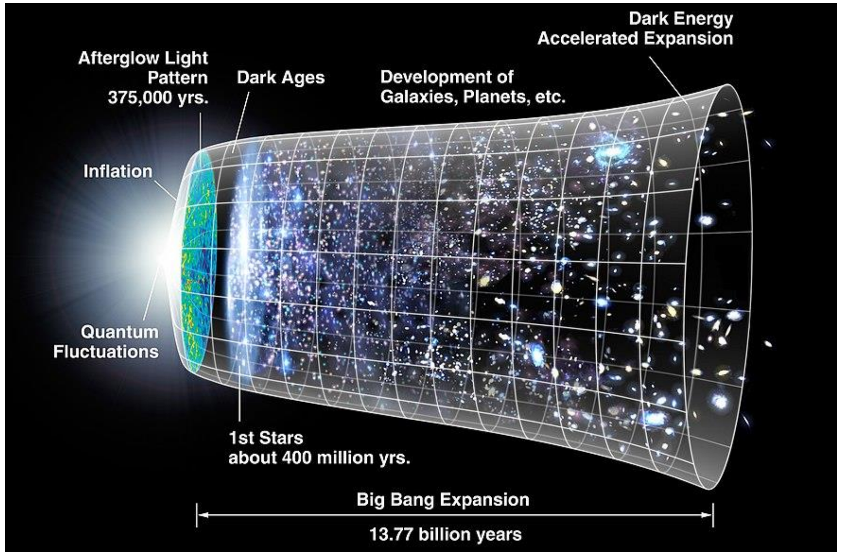


SUPPRESSION OF THE TOTAL CAPTURE RATE

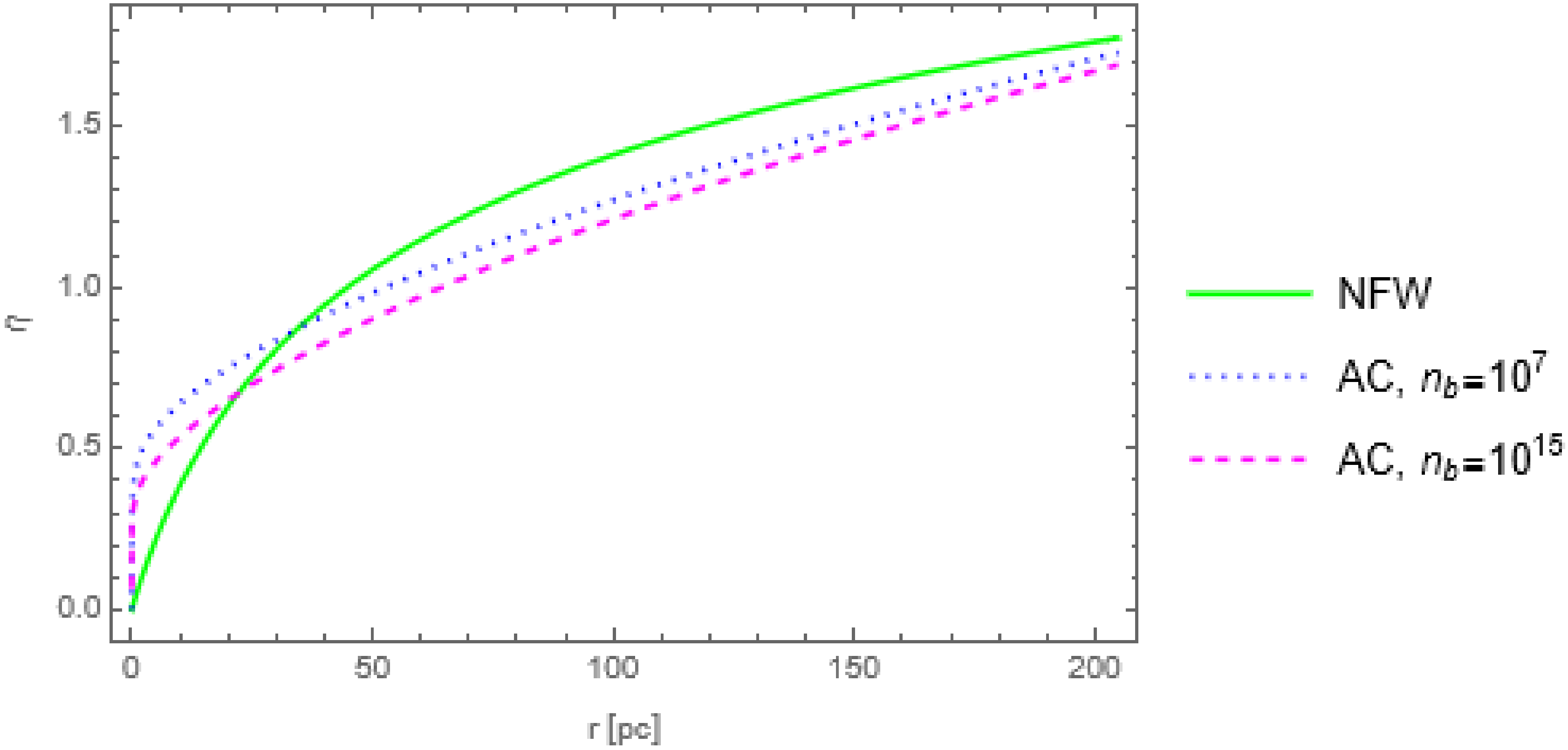


WHAT ABOUT THE EARLIEST STARS?

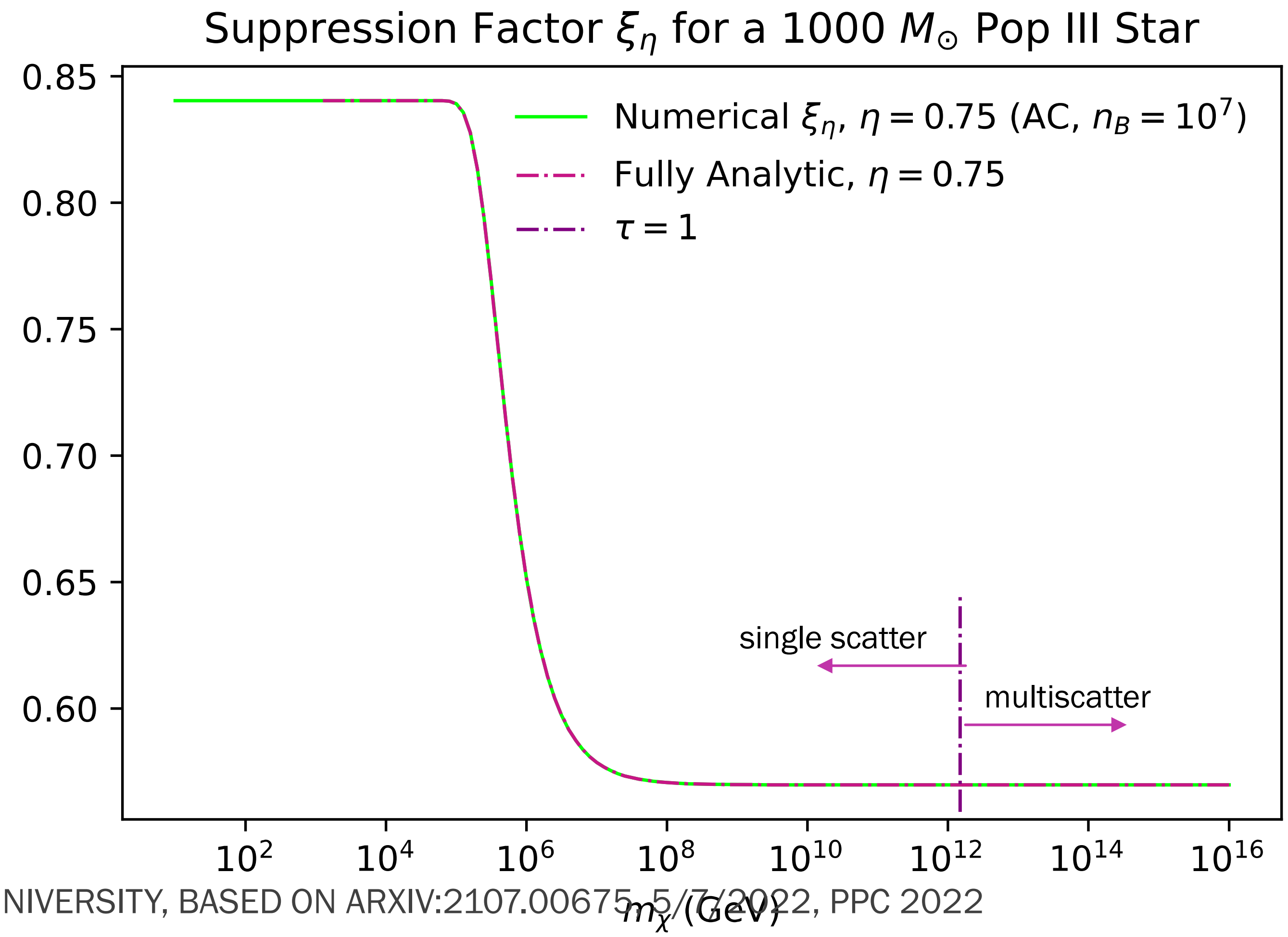
- Pop III stars:
  - Theoretical class of stars
  - Only contain H and He
  - Redshifts z=10-50 (universe was approx. 200 million yrs old)
  - 1 or a few (due to fragmentation) per DM halo (halo mass ~ 10^5-10^6 $M_\odot$)
  - Masses up to ~1000 $M_\odot$
  - Very interesting candidates to constrain DM properties



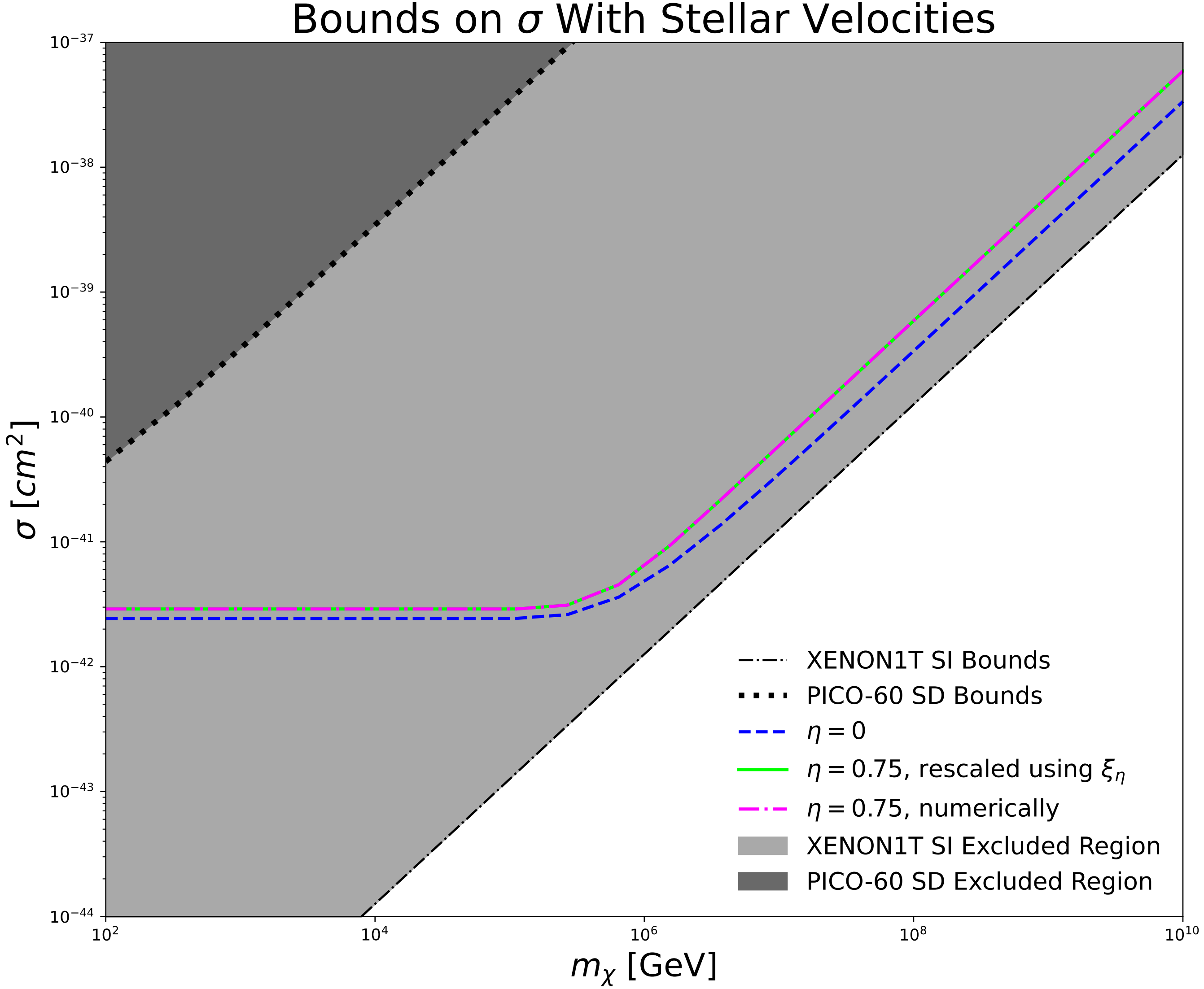
DARK MATTER HALO PROFILES



SUPPRESSION FACTOR FOR POP III STARS



BOUNDS ON THE DM-NUCLEON CROSS SECTION



CONCLUSIONS

- Stellar velocity has a negligible impact on DM constraints from Pop III stars
- Stellar velocity has a more significant impact for more local objects where η is higher
- We have developed useful analytic formulae which are applicable in a variety of astrophysical contexts
- We plan to study exoplanets and brown dwarfs next, in which we may account for the objects' velocities using the given formulae. We plan to also explore the effects of DM evaporation in such objects.

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QUESTIONS?

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