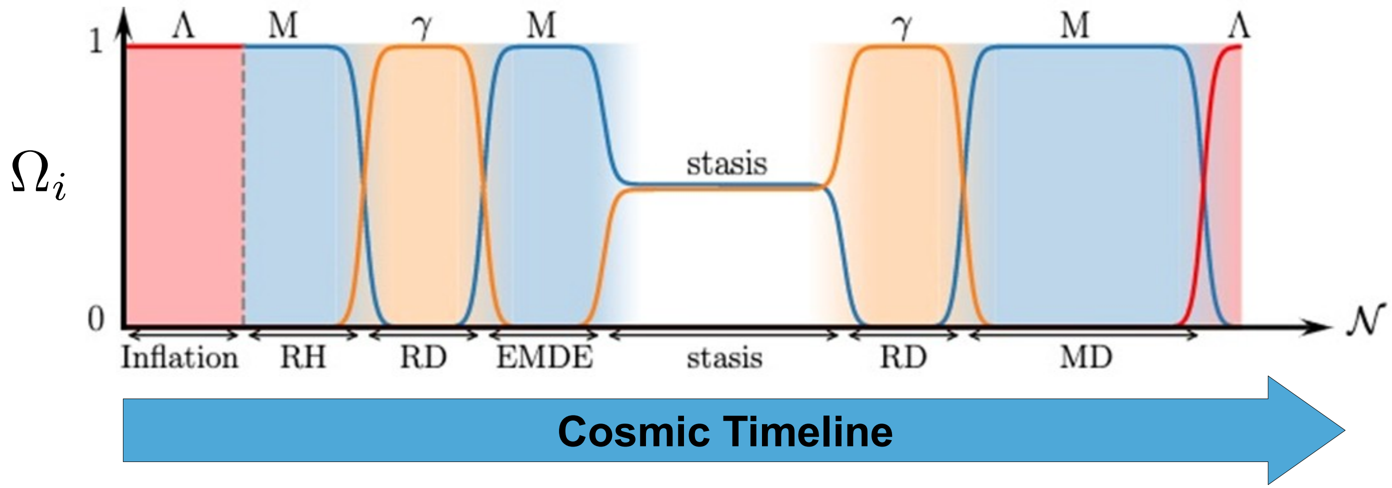
Stasis in an Expanding Universe



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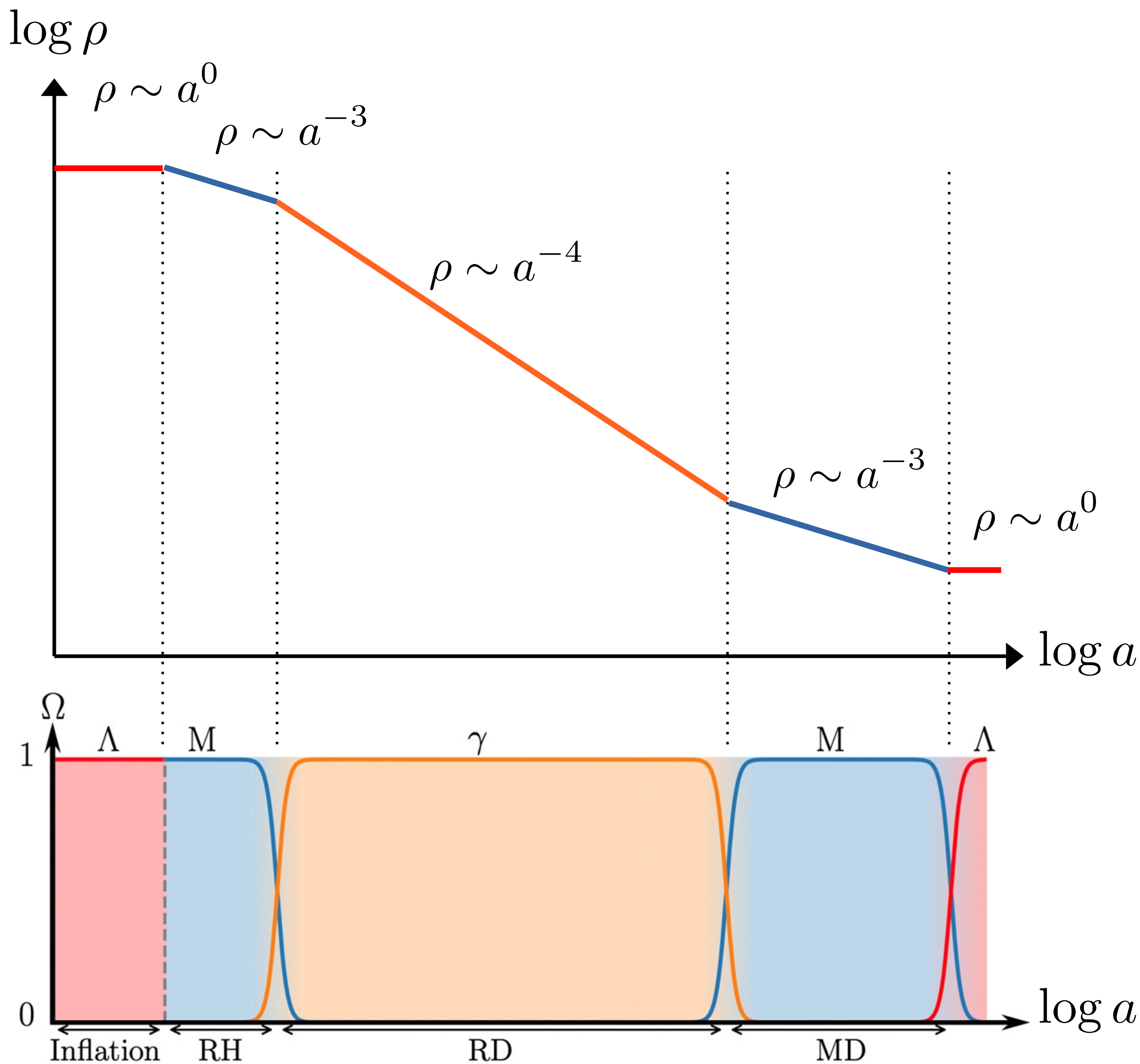
Based on work done in collaboration with:

- **Keith R. Dienes, Fei Huang, Lucien Heurtier, Doojin Kim, and Tim M. P. Tait [arXiv:2108.02204, arXiv:2206.xxxxx, arXiv:2206.xxxx(x+1)]**

PPC 2022, June 8th, 2022

A Succession of Single-Component Eras

- The energy densities associated with different ***cosmological components*** (matter, radiation, vacuum energy, etc.) with different equations of state scale behave differently under cosmic expansion.
- As a result, except during brief transition periods, the energy density of the universe is ***dominated by one such component***.
- This is certainly the case in the standard cosmology.
- Moreover, it's typically the case event in ***modified cosmologies*** (e.g., with epochs of early matter- or vacuum-energy-domination) as well.



A Stable Mixed-Component Era?

Q Is it possible to achieve a ***stable, mixed-component cosmological era*** in which multiple Ω_i maintain non-neglible, effectively constant values over an extended period?

- In other words, can we arrange for the paritioning of the cosmic pie to ***remain effectively fixed*** over an extended period, with sizable slices corresponding to components with different equations of state?

- At first glance, arranging this may seem impossible – or at least attainable only with a ridiculous amount of fine-tuning.

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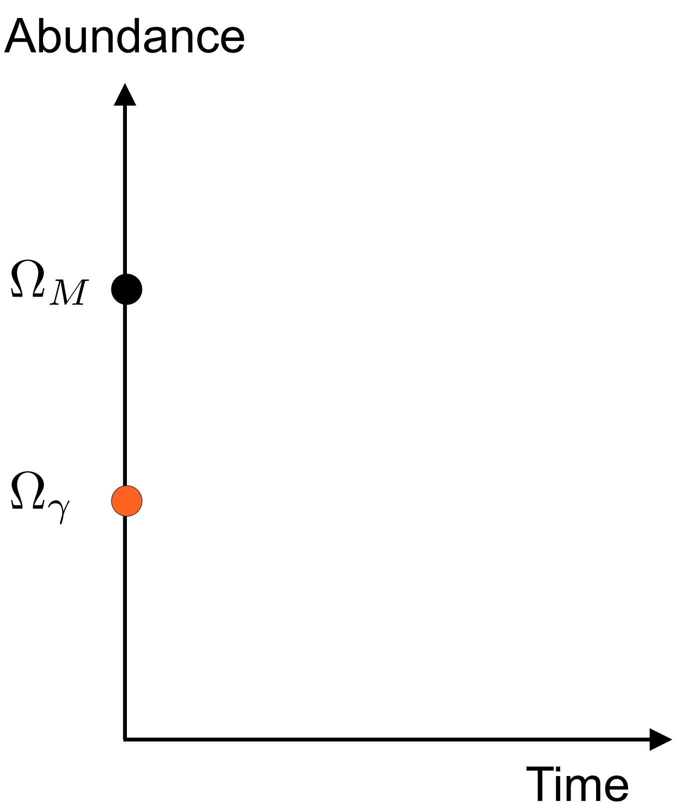
- At first glance, arranging this may seem impossible – or at least attainable only with a ridiculous amount of fine-tuning.

A However, it turns out that such eras, which we call periods of ***cosmic stasis***, can be realized in a straightforward manner.

- Stasis eras arise naturally in many extensions of the Standard model.
- Moreover, in such scenarios, stasis is actually a ***global attractor*** – the universe will evolve toward stasis regardless of the initial conditions.

The Challenge: Achieving Stasis

- To see how a stasis era can arise, let us consider a universe effectively consisting of matter and radiation alone, with all other Ω_i negligible.



Boltzmann Equations

$$\frac{d\rho_M}{dt} = -3H\rho_M$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma$$

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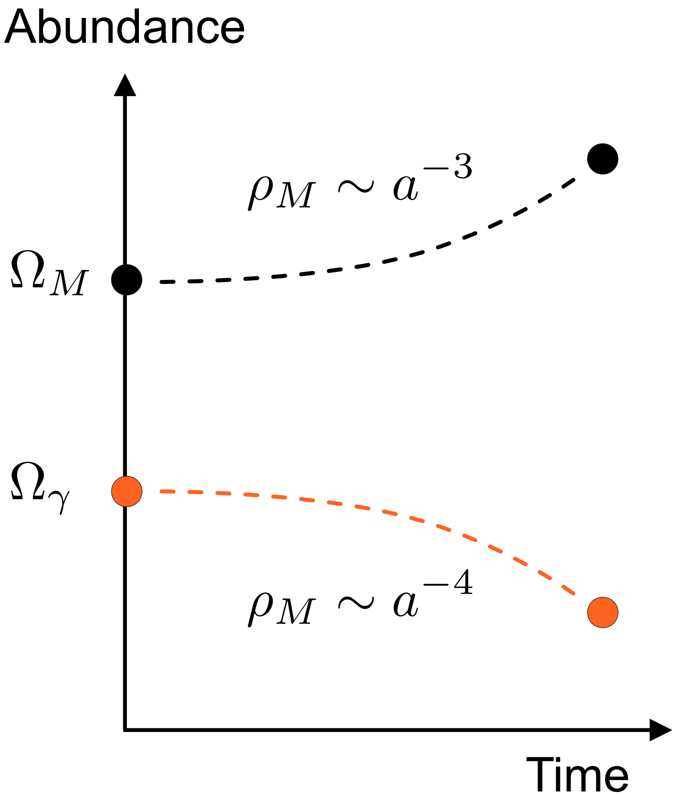

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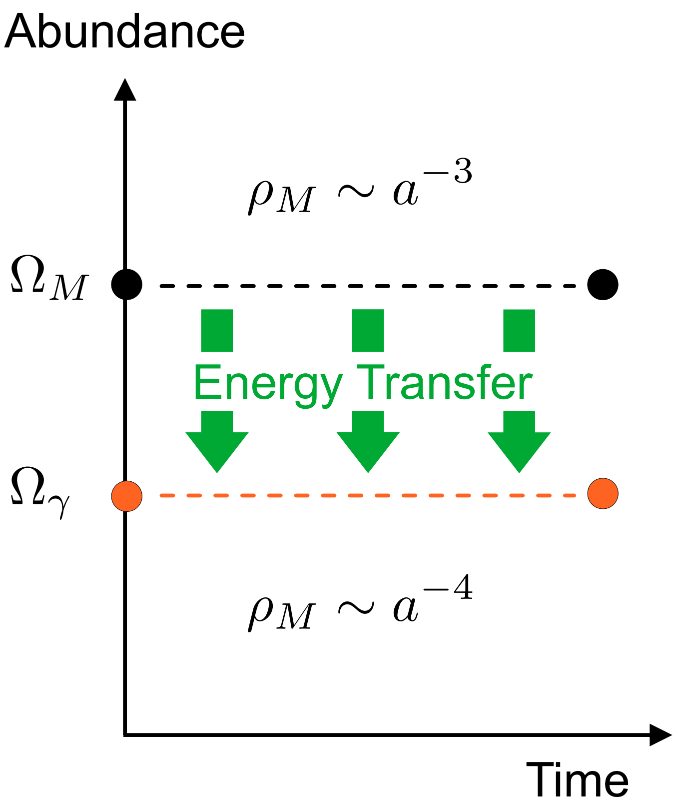
- In order to compensate for this effect, what's needed is a ***continuous transfer of energy density*** from matter to radiation.



Boltzmann Equations

$$\frac{d\rho_M}{dt} = -3H\rho_M - S(t)$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + S(t)$$

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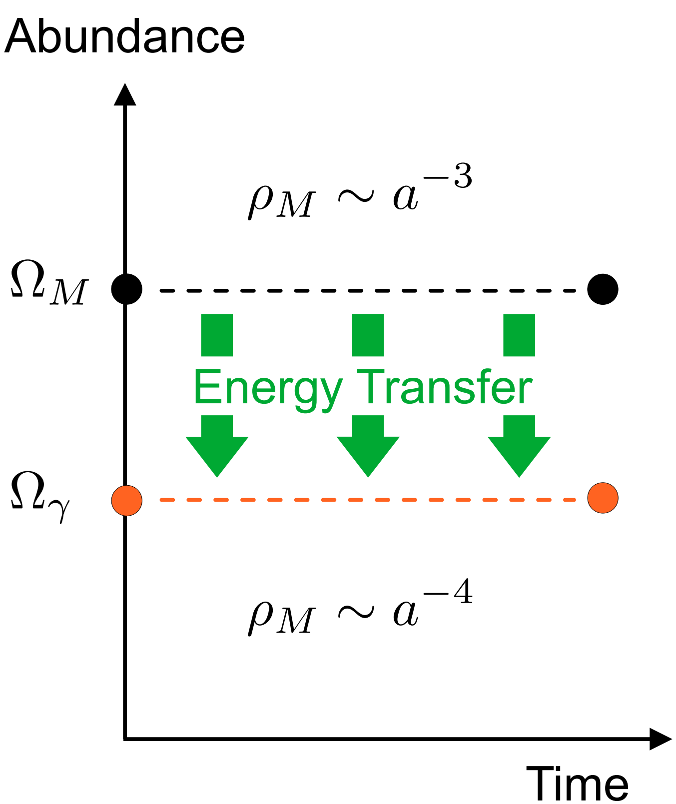

Boltzmann Equations

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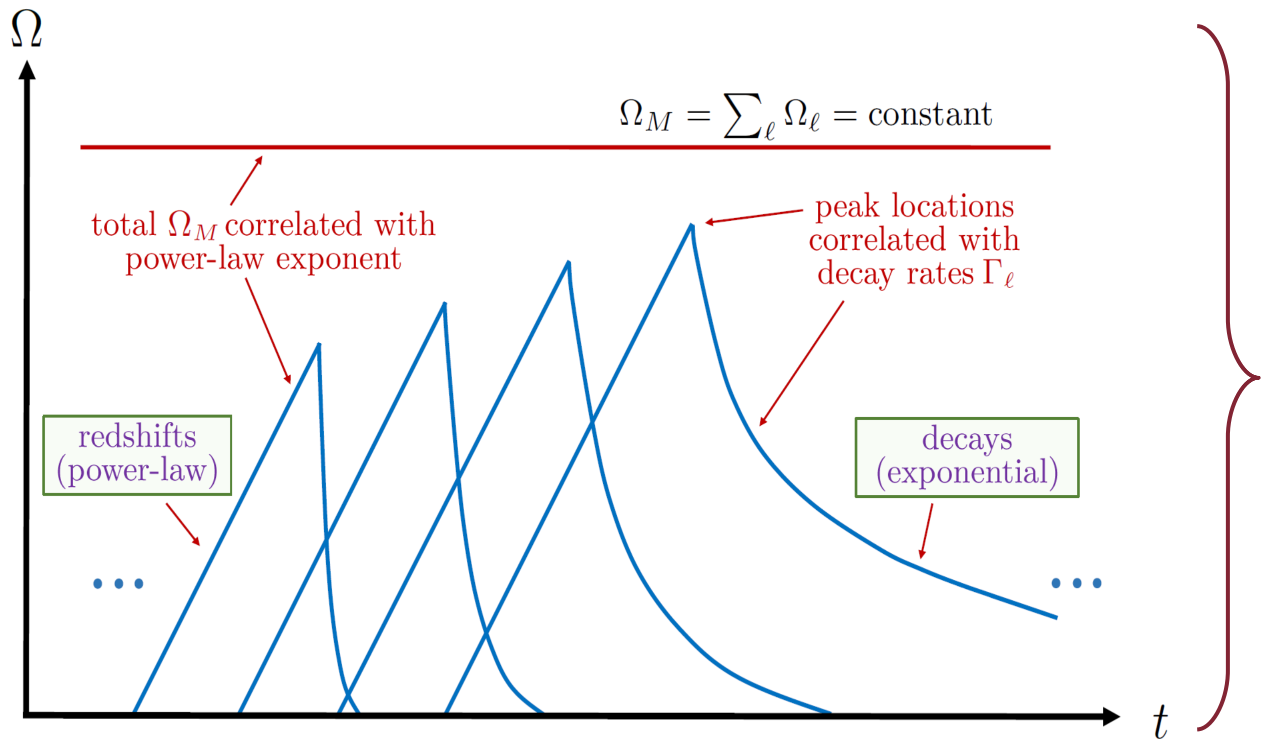
Particle decays provide a natural mechanism for obtaining these source/sink terms.

The Challenge: Achieving Stasis

- However, the exponential decay of a single matter species, which occurs over a relatively short time period, is insufficient for achieving stasis.
- What we need is a ***tower of matter states*** ϕ_ℓ, where $\ell = 0, 1, 2, \ldots, N-1$, whose decay widths Γ_ℓ and initial abundances $\Omega_\ell^{(0)}$ scale across the tower in such a way that the effect of decays on Ω_M and Ω_γ compensates for the effect of cosmic expansion over a extended period.
- These states could be ***moduli***, ***composite states*** of a strongly-coupled theory, or the ***KK modes*** of a higher-dimensional field.



The Goal

Conditions for Stasis

- The Boltzmann equations for the individual ρ_ℓ, in conjunction with the relevant Friedmann equation, yield an equation of motion for Ω_M.

Boltzmann Equations
$$\begin{cases} \dfrac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell \rho_\ell \\ \dfrac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell \end{cases}$$

Friedmann Equation
$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_\gamma)$$

$$\Rightarrow \quad \frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H(\Omega_M - \Omega_M^2)$$

Stasis Condition (Instantaneous)

- To achieve stasis, we impose $\dfrac{d\Omega_M}{dt} = 0$ $\Rightarrow$

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

- In order to achieve an ***extended period*** of stasis, we need this instantaneous stasis condition to be satisfied over a significant range of t.

$\Rightarrow$ The left and right sides of this stasis-condition equation must have the same functional dependence on t.

Conditions for Stasis

- By construction, during a stasis era, $\frac{d\Omega_M}{dt} = 0$ ⟹ $\Omega_M = \overline{\Omega}_M = [\text{const.}]$

- The Friedmann acceleration equation therefore implies:

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4 - \overline{\Omega}_M) \quad \Longrightarrow \quad H(t) = \left(\frac{2}{4 - \overline{\Omega}_M}\right)\frac{1}{t}$$

- Substituting these results into our instantaneous stasis condition, we find that the conditions for realizing an ***extended period of stasis*** are:

$$\sum_\ell \Gamma_\ell \Omega_\ell = \frac{2\overline{\Omega}_M(1 - \overline{\Omega}_M)}{4 - \overline{\Omega}_M}\frac{1}{t}$$

$$\sum_\ell \Omega_\ell = \overline{\Omega}_M$$

- These conditions can also be combined to yield $\frac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} = \frac{2(1 - \overline{\Omega}_M)}{4 - \overline{\Omega}_M}\frac{1}{t}$

- During stasis, then, this ratio of sums must be inversely proportional to t.

A Model of Stasis

- Let's consider a tower of N such states states with...

Masses	Decay Widths	Initial Abundances
$m_\ell = m_0 + (\Delta m)\ell^\delta$	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

- Towers of states with mass spectra of this form arise naturally in many extensions of the Standard Model.
  - KK excitations of a 5D scalar: $\begin{cases} mR \ll 1 \longrightarrow \delta \sim 1 \\ mR \gg 1 \longrightarrow \delta \sim 2 \end{cases}$
  - Bound states of a strongly-coupled gauge theory: $\delta \sim \frac{1}{2}$
- Decay through ***contact operators*** of dimension d implies a scaling: $\mathcal{O}_\ell \sim \frac{c_\ell}{\Lambda^{d-4}} \phi_\ell \mathcal{F} \longrightarrow \gamma = 2d - 7$
- Scaling of initial abundances depends on how they're generated:
  - Misalignment production $\longrightarrow \alpha < 0$
  - Thermal freeze-out $\longrightarrow \alpha < 0 \text{ or } \alpha > 0$
  - Universal inflaton decay $\longrightarrow \alpha \sim 1$
  - ...

A Model of Stasis

- The abundance $\Omega_\ell(t)$ of each state at time t is a product of three factors.

$$\Omega_\ell(t) = \Omega_\ell^{(0)} \times h(t^{(0)}, t) \times e^{-\Gamma_\ell(t-t^{(0)})}$$

Initial abundance (established prior to stasis) — Redshift factor — Exponential-decay factor

- For sufficiently large N and small Δm, we can approximate the sum over $\Gamma_\ell \Omega_\ell$ with an integral:

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \Gamma_0 \Omega_0^{(0)} h(t^{(0)}, t) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma (t-t^{(0)})}$$

$$\approx \frac{\Gamma_0 \Omega_0^{(0)} h(t^{(0)}, t)}{\delta} \int_{m_0}^{m_{N-1}} \frac{dm}{m-m_0} \left(\frac{m-m_0}{\Delta m}\right)^{1/\delta} \left(\frac{m}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0 \left(\frac{m}{m_0}\right)^\gamma (t-t^{(0)})}$$

- For $t_{N-1} \ll t \ll t_0$, this is approximately

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) \approx \frac{\Gamma_0 \Omega_0^{(0)}}{\gamma\delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} h(t^{(0)}, t) \Gamma\left(\frac{\alpha+\gamma+1/\delta}{\gamma}\right) \left[\Gamma_0 (t-t^{(0)})\right]^{-(\alpha+\gamma+1/\delta)/\gamma}$$

Euler gamma function

A Model of Stasis

- Likewise, the sum over Ω_ℓ is well approximated by

$$\sum_\ell \Omega_\ell(t) \approx \frac{\Omega_0^{(0)}}{\gamma\delta}\left(\frac{m_0}{\Delta m}\right)^{1/\delta} h(t^{(0)}, t)\Gamma\left(\frac{\alpha+1/\delta}{\gamma}\right)\left[\Gamma_0(t-t^{(0)})\right]^{-(\alpha+1/\delta)/\gamma}$$

- The ratio of the two sums is

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} \approx \left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t-t^{(0)}} \quad \xrightarrow{t \gg t^{(0)}} \quad \frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} \approx \left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t}$$

- Thus, our condition for extended stasis is satisfied! Indeed, we have

$$\left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}$$

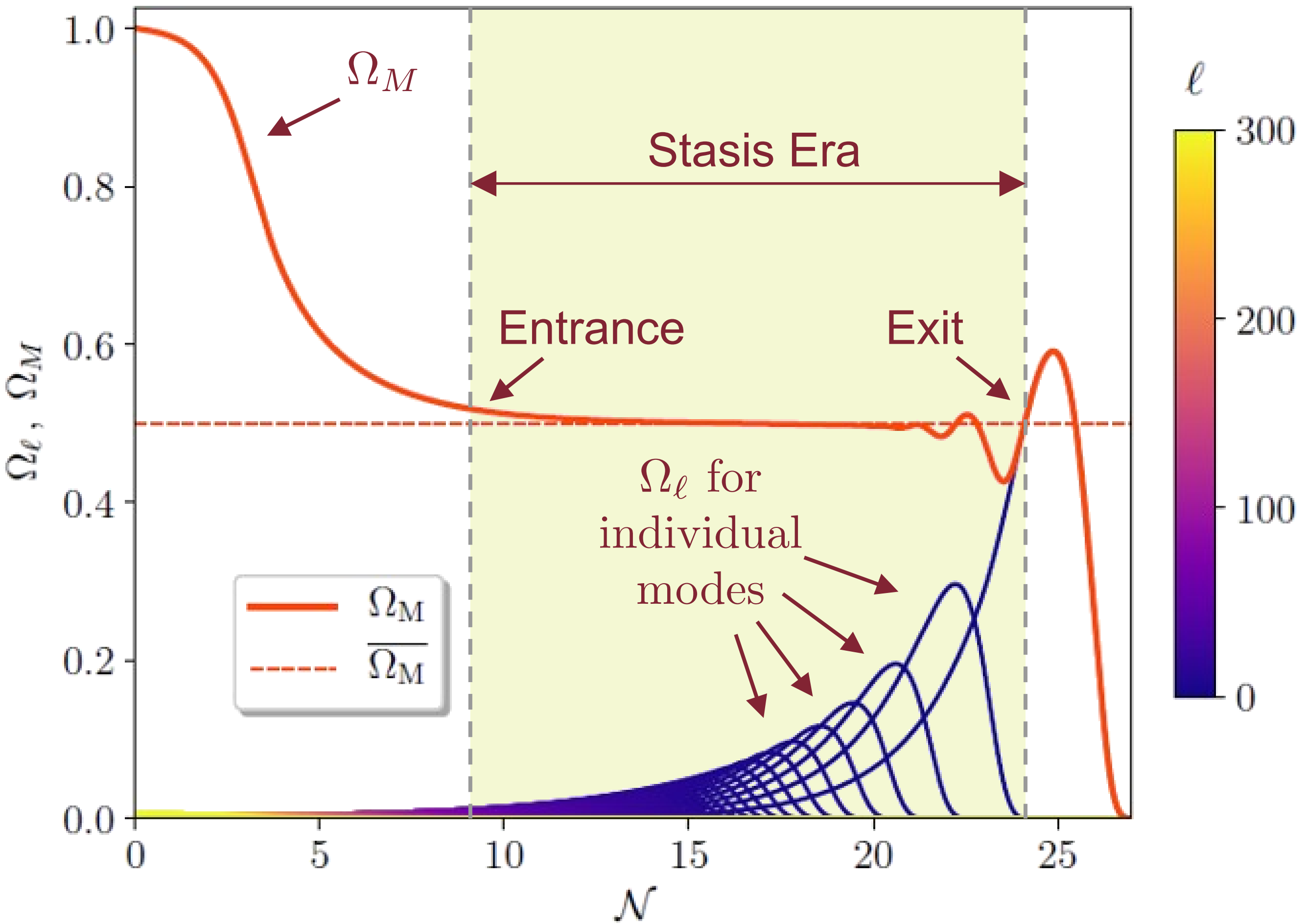
Both sides inversely proportional to t, as desired!

- Solving for $\overline{\Omega}_M$, we find that the ***matter and radiation abundances*** in such a stasis era are

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)} \qquad \overline{\Omega}_\gamma = 1 - \overline{\Omega}_M$$

Numerical Results

- These analytic results can be cross-checked by solving the Boltzmann equations numerically.
- The results of this analysis confirm our findings and provide additional information about how the stasis epoch ***begins*** and ***ends***.

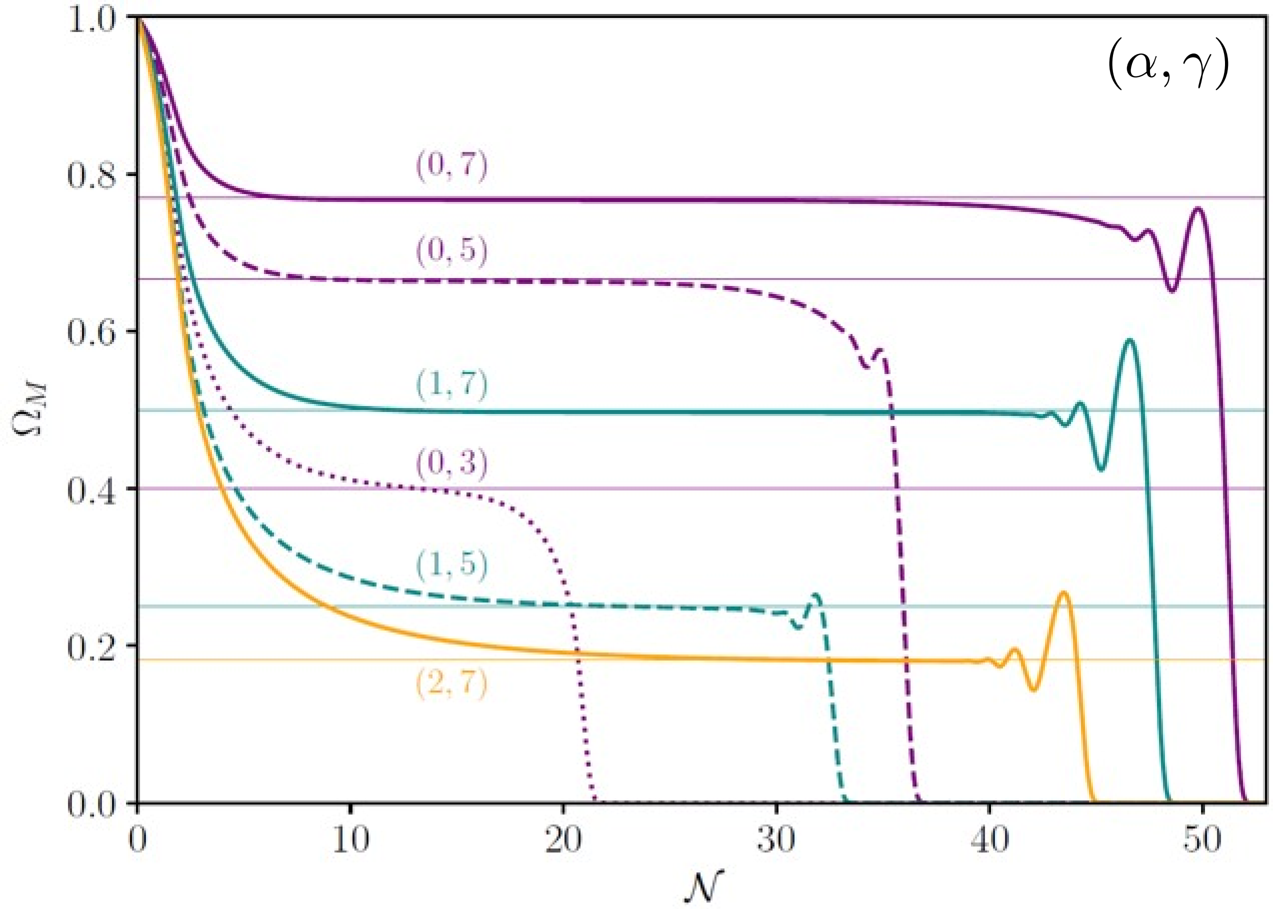


Parameter Choices

$$\alpha = 1$$
$$\gamma = 7$$
$$\delta = 1$$
$$N = 300$$
$$\frac{m_0}{\Delta m} = 1$$
$$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.01$$

Numerical Results

- We obtain similar results for different combinations of α and γ, which yield stasis eras with different values for $\overline{\Omega}_M$.



Parameter Choices

$$\delta = 1$$
$$N = 10^5$$
$$\frac{m_0}{\Delta m} = 1$$
$$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.1$$

- Indeed, our extended stasis condition implies that stasis can arise whenever the scaling parameters satisfy the following criterion:

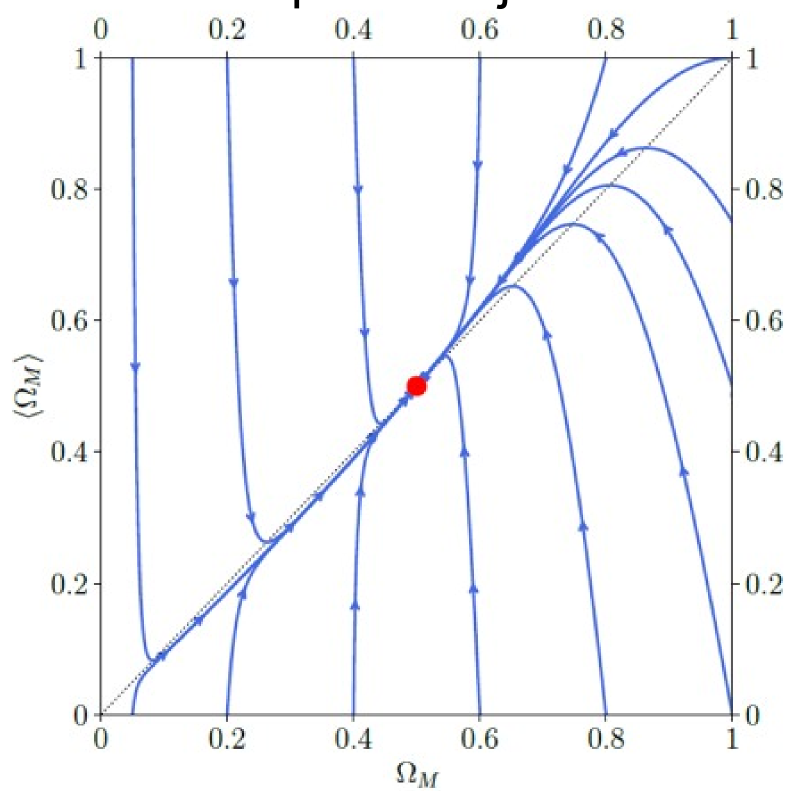
$$-\frac{1}{\delta} < \alpha < \frac{\gamma}{2} - \frac{1}{\delta}$$

Stasis as a Global Attractor

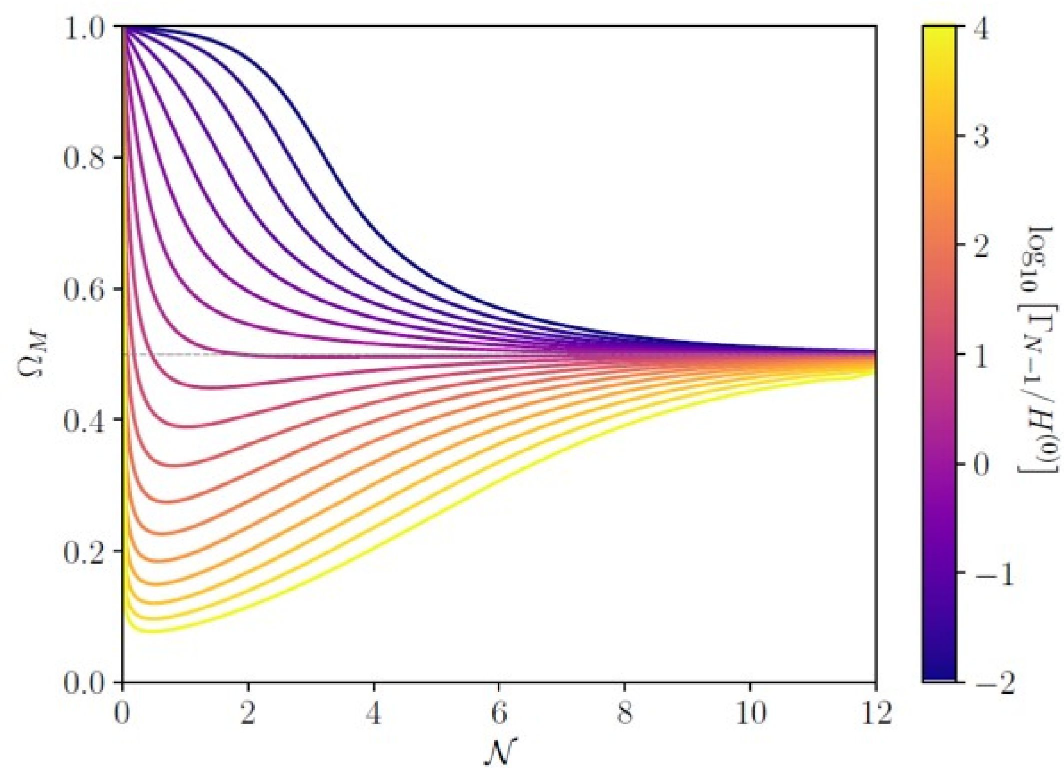
Q Does achieving cosmological stasis require a fine-tuning of the initial conditions for Ω_M and Ω_γ, or for the ratio $\Gamma_{\text{N-1}}/H^{(0)}$?

A No it doesn't. In fact, stasis is a ***global attractor*** in the sense that regardless of what $\Omega_M(t)$ and its time-average $\langle\Omega_M\rangle(t)$ from $t^{(0)}$ to t are at a given $t \geq t^{(0)}$, Ω_M and Ω_γ will ***evolve toward their stasis values***. Stasis doesn't require any special $\Gamma_{\text{N-1}}/H^{(0)}$ value either.

State-Space Trajectories



Ω_M vs. t For Different $\Gamma_{\text{N-1}}/H^{(0)}$

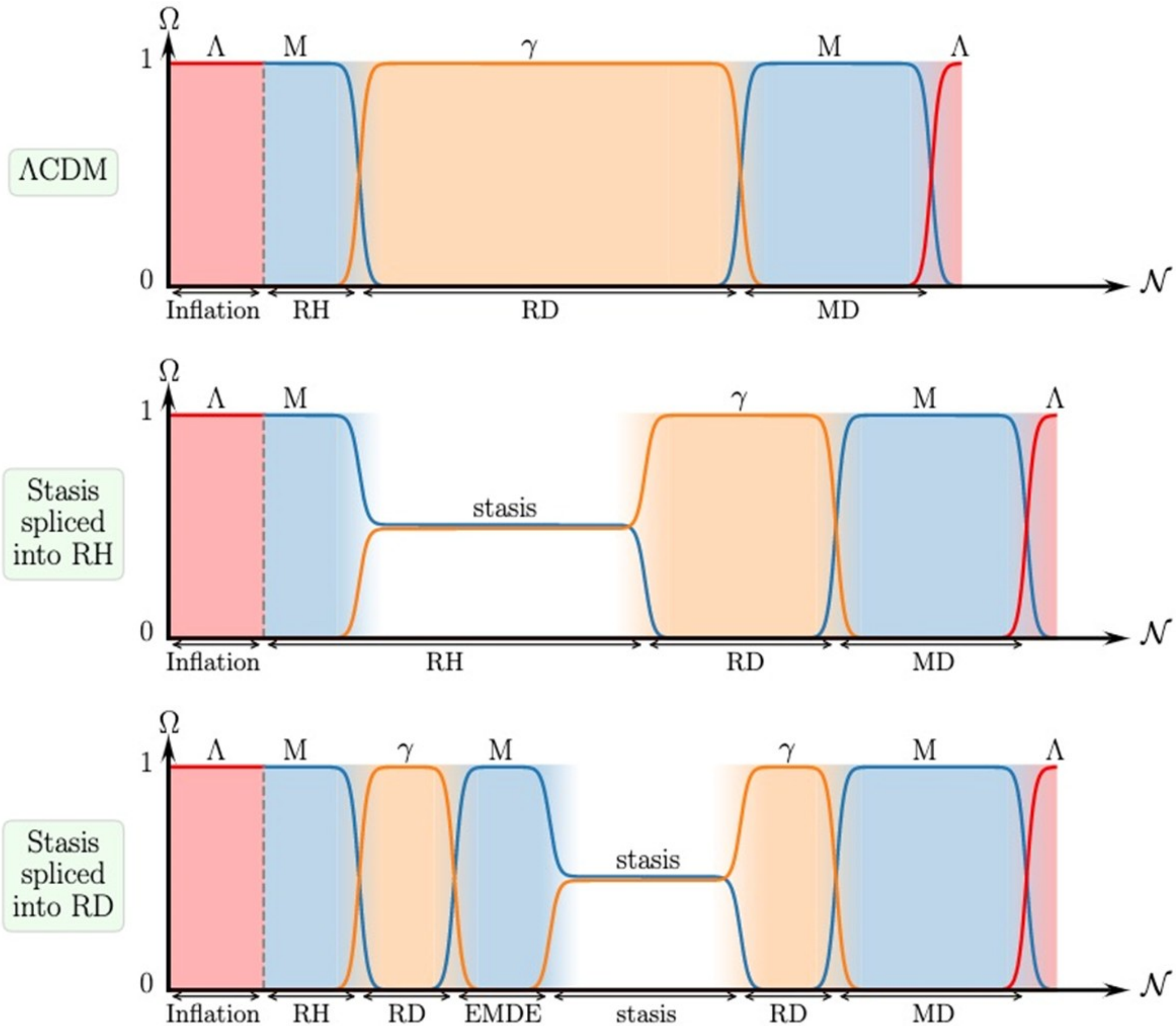


Splicing Stasis Into the Cosmological Timeline

- There are two primary ways in which a stasis epoch can be incorporated into the standard cosmological timeline.

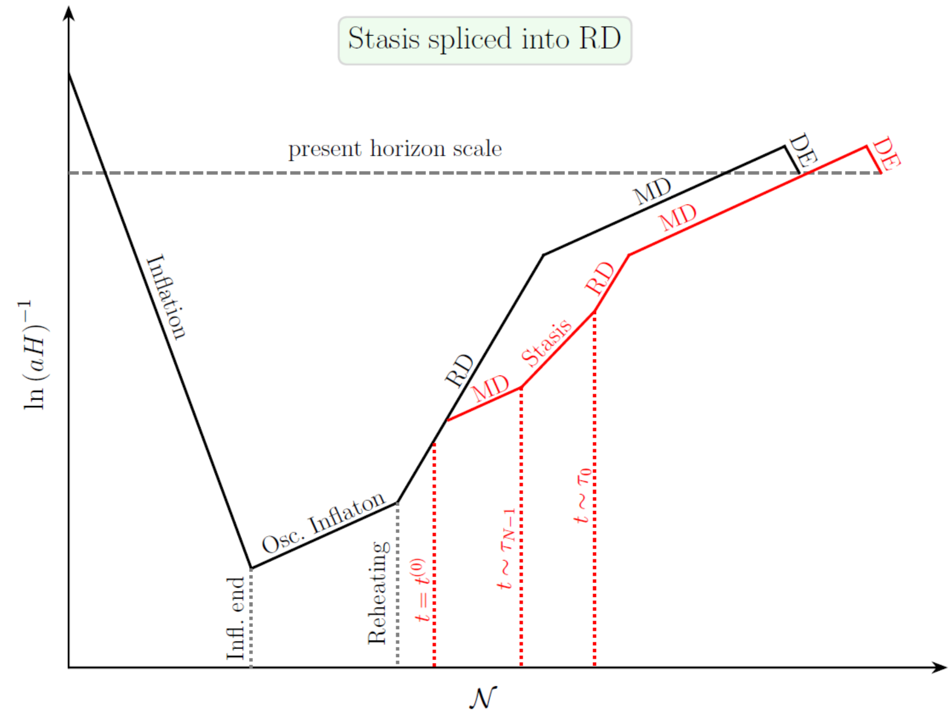
- The stasis epoch ***follows inflation***. The inflaton produces the ϕ_ℓ directly, and their decays reheat the universe.

- Stasis occurs at some point ***after reheating***, following an EMDE where the ϕ_ℓ dominate the energy density of the universe.



Implications of Stasis

- The comoving Hubble radius grows more slowly in cosmologies with a stasis era, so perturbation modes re-enter the horizon at a later time. This has implications for ***inflationary observables***.



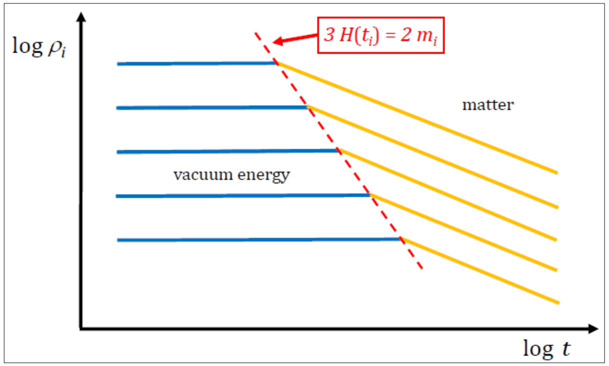
- ***Density perturbations*** grow more quickly during stasis than in an RD era. As a result, compact objects such as PBH or compact minihalos can potentially form during stasis, as they do in an EMDE.

- ***The dark-matter (DM) relic abundance*** would be affected if DM is produced prior to or during stasis, due to the modified expansion history and to the injection of entropy by ϕ_ℓ. The DM could potentially also be produced by the decays of the ϕ_ℓ directly.

Other Realizations of Stasis

Stasis with Other Components

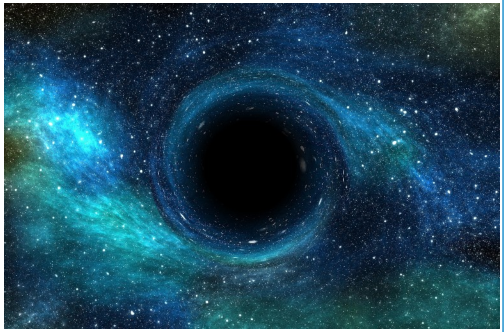
[Dienes, Heurtier, Huang, Kim, Tait, BT: 2206.xxxxx]



- We've seen how a stasis era involving matter and radiation can be realized, but one can also be realized involving ***matter and vacuum energy***.
- If a set of initially massless scalars acquire a spectrum of masses and abundances from misalignment production, their staggered transitions from overdamped to underdamped oscillation effectively convert vacuum energy to matter.

Stasis from Primordial-Black-Hole Evaporation

[Dienes, Heurtier, Huang, Kim, Tait, BT: 2206.xxxx(x+1)]



- A population of ***primordial black holes*** (PBH), whose evaporation via Hawking radiation transfers energy density from matter to radiation, can likewise give rise to a period of stasis.
- The spectrum of PBH formed from scale-invariant density fluctuations yields exactly the initial PBH number density per unit mass needed to achieve stasis.

Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* ***stasis eras*** – are indeed a viable cosmological possibility – and one that can arise naturally in many extensions of the Standard Model and even from PBH.
- Stasis is a ***global attractor***, and achieving it does not require any fine-tuning of initial conditions.
- A period of stasis has a variety of potential implications and can have an impact on inflationary observables, the evolution of density perturbations, and the abundance of dark matter.
- Stasis epochs involving either matter and radiation or matter and vacuum energy can be realized in a straightforward manner.