Machine-Learning quantum entanglement with top quark pair production at the LHC

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Based on ongoing work in collaboration with
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Why top quark pair production?

- Growing interest in observing entanglement and violation of Bell's inequality at the LHC. 2106.01377 Barr 2102.11883 Fabbrichesi, Floreanini, Panizzo 2110.10112 Severi, Boschi, Maltoni, Sioli
- Top quark pair productions are good candidate because they decay before de-correlation and that the lepton from two-step decay is highly correlated with Top spin.

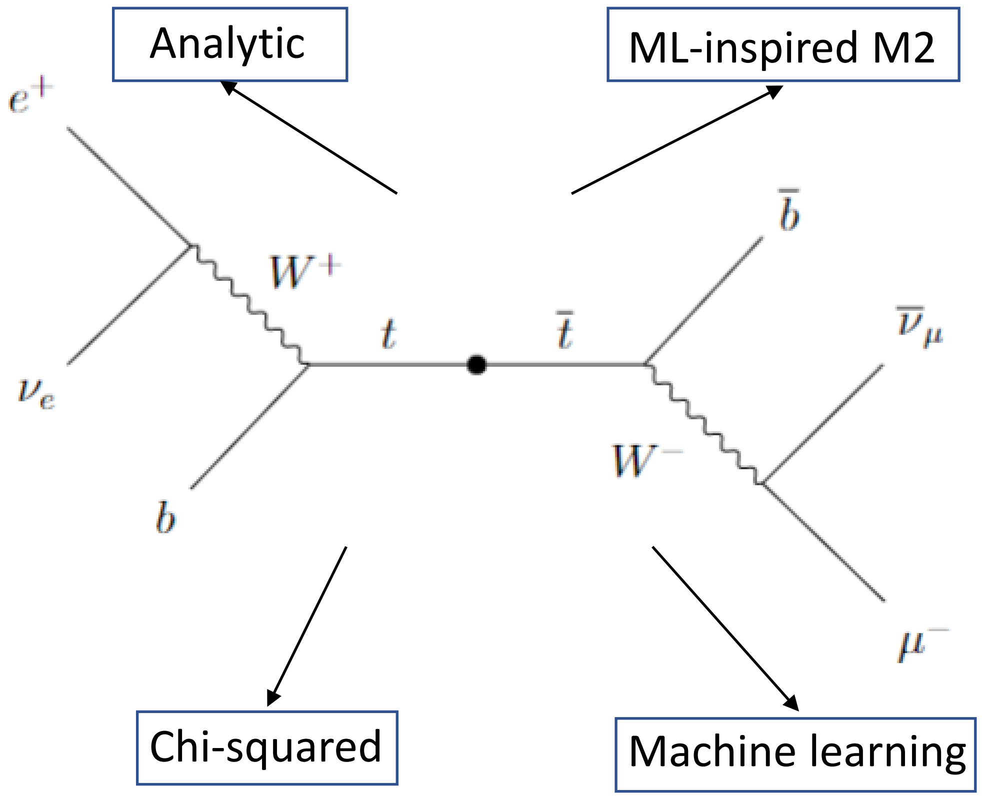
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\chi} = \frac{1+\alpha\cos\chi}{2} = \begin{cases} +1.0 & l^+ \text{ or } \bar{d} \\ -0.31 & \nu \text{ or } u \\ -0.41 & b \end{cases}$$

1001.3422 Mahlon, Parke

- χ is the angle between the decay product and top quark spin axis in the top quark rest frame.
- To obtain optimal basis for spin-correlation study, we need to reconstruct top quarks in dileptonic ttbar events.

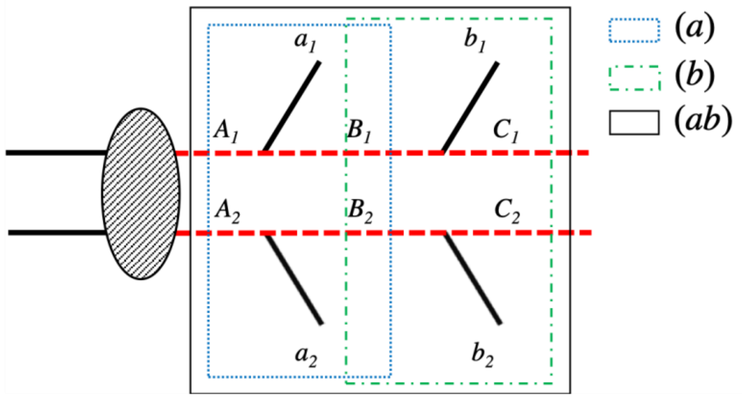
Top quark reconstruction

- To maximally exclude backgrounds, we study events with different flavored lepton final states.
- We need to find the neutrino momentum and the correct b quark and lepton pairing given all the visible momentum.
- We compared the performance of different reconstruction methods cross different statistics and chose one to do analysis.
- Event generation using MG5 for event generation. PYTHIA8 for parton-shower and hadronization. Delphes for detector effect



ML-inspired M2 reconstruction

- Generalization of MT2, which allows additional mass constraints.
- Obtain neutrino momentum from minimization.
- Reconstructed mass distribution of the non-constrained mass is not optimal.
- Accuracy of combinatorial assignment is better when using neural network, we used it to resolve combinatorics instead of M2.



$$M_{2CW}^{(b\ell)} \equiv \min_{\vec{q}_1, \vec{q}_2} \{\max[M_{P_1}(\vec{q}_1, \tilde{m}),\ M_{P_2}(\vec{q}_2, \tilde{m})]\}$$
$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{\not{P}}_T$$
$$M_{P_1} = M_{P_2}$$
$$M_{R_1}^2 = M_{R_2}^2 = m_W^2$$

$$M_{2Ct}^{(\ell)} \equiv \min_{\vec{q}_1, \vec{q}_2} \{\max[M_{P_1}(\vec{q}_1, \tilde{m}),\ M_{P_2}(\vec{q}_2, \tilde{m})]\}$$
$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{\not{P}}_T$$
$$M_{P_1} = M_{P_2}$$
$$M_{R_1}^2 = M_{R_2}^2 = m_t^2$$

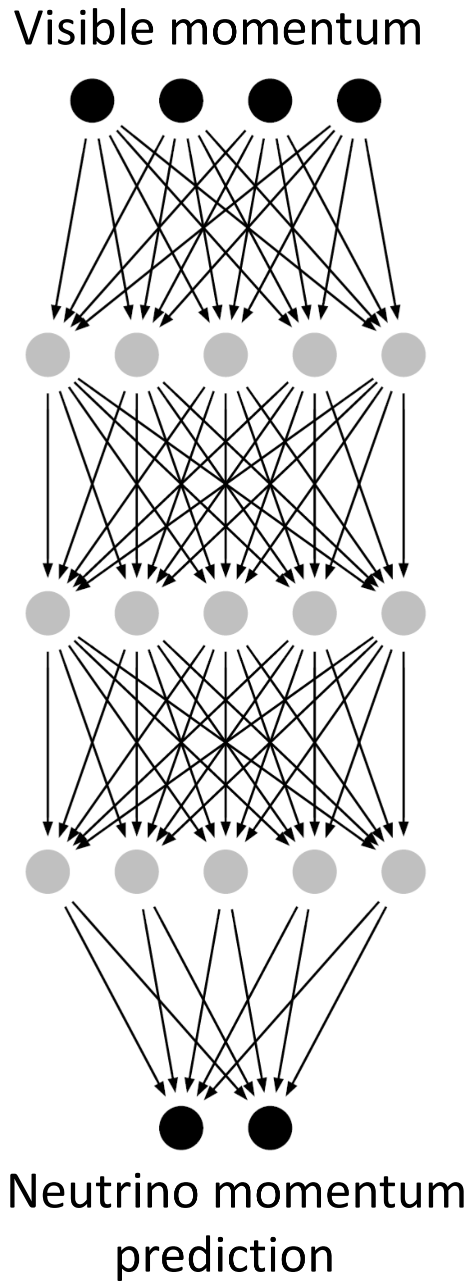
1703.06887, Kim, Matchev, Moortgat, Pape

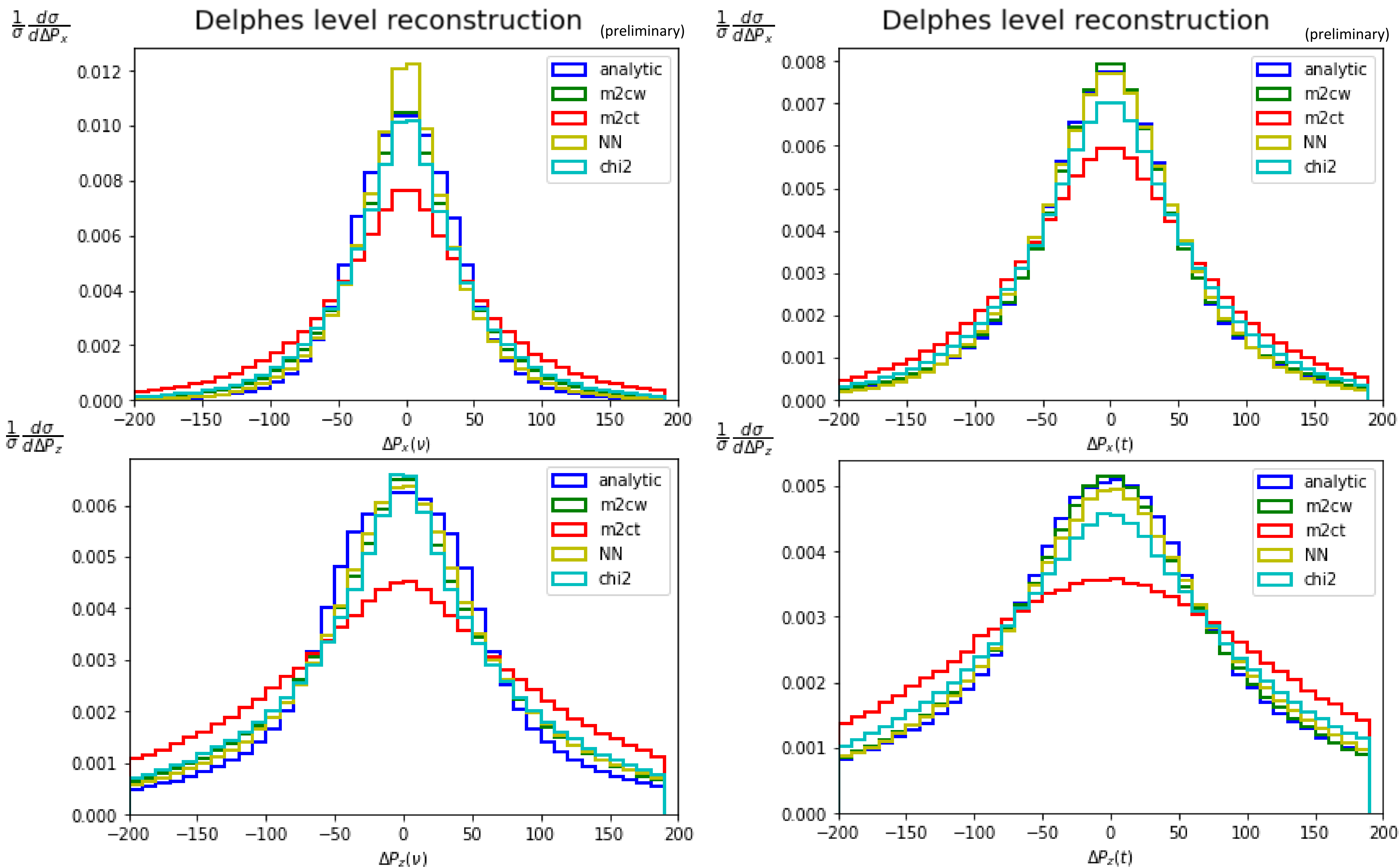
Machine learning reconstruction

- Fully connected DNN trained on parton level data.
- Uses mean squared loss of neutrino momentum.
- Adds additional mean squared loss of Top and W masses, as inspired from the chi-squared method.

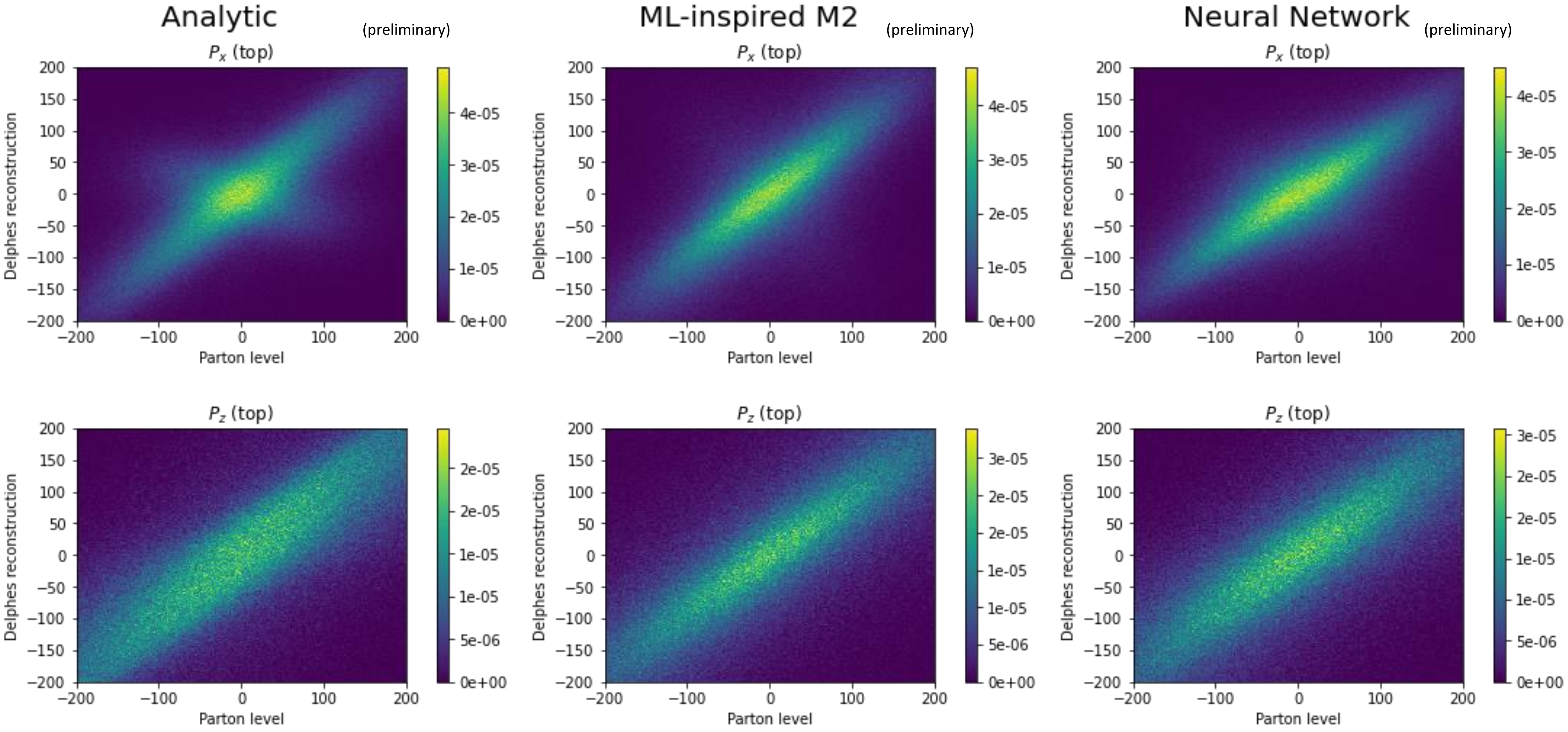
$$L = \sum_{i=1}^{6}(\hat{p}_i - p_i)^2 + \lambda_1\left[(\hat{m}_t - m_t)^2 + (\hat{m}_{\bar{t}} - m_t)^2\right] + \lambda_2\left[(\hat{m}_{W^+} - m_W)^2 + (\hat{m}_{W^-} - m_W)^2\right]$$

- A separate network resolves combinatorial assignment of b quark and leptons.





Delphes level reconstruction
(preliminary)
$\frac{1}{\sigma}\frac{d\sigma}{d\Delta P_x}$
analytic
m2cw
m2ct
NN
chi2
$\Delta P_x(\nu)$
$\Delta P_x(t)$
$\frac{1}{\sigma}\frac{d\sigma}{d\Delta P_z}$
$\Delta P_z(\nu)$
$\Delta P_z(t)$



Analytic
(preliminary)
ML-inspired M2
(preliminary)
Neural Network
(preliminary)
P_x (top)
P_z (top)
Delphes reconstruction
Parton level
200
150
100
50
0
−50
−100
−150
−200
4e-05
3e-05
2e-05
1e-05
5e-06
0e+00

Top quark production as a two-qubit system

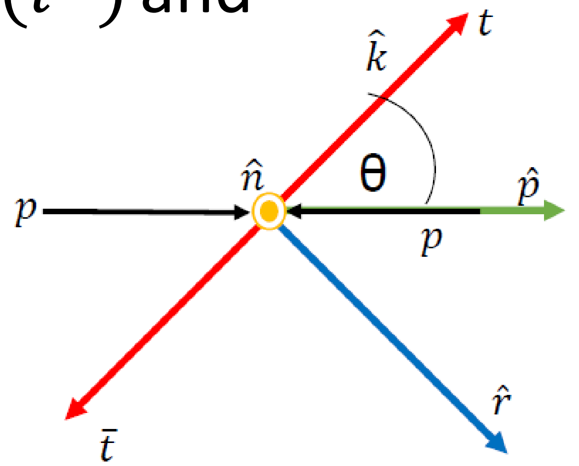
For a system of two spin-1/2 particles, the density matrix can be written as

$$\rho = \frac{I_4 + B_i^{+}\sigma^i \otimes I_2 + B_i^{-} I_2 \otimes \sigma^i + C_{ij}\sigma^i \otimes \sigma^j}{4}$$

$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$ Represents the spin correlation between the two particles.

We can compute the C_{ij} by computing angles between $l^+(l^-)$ and the axis in $t(\bar{t})$ rest frames respectively.

$$\hat{k} = \text{top direction}, \qquad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \qquad \hat{n} = \hat{k} \times \hat{r}, \qquad \hat{p} = (0,0,1)$$



Entanglement

For a system of two spin-1/2 particles, the criterion for entanglement is $|C_{11} + C_{22}| - C_{33} > 1$, equivalent under permutation.

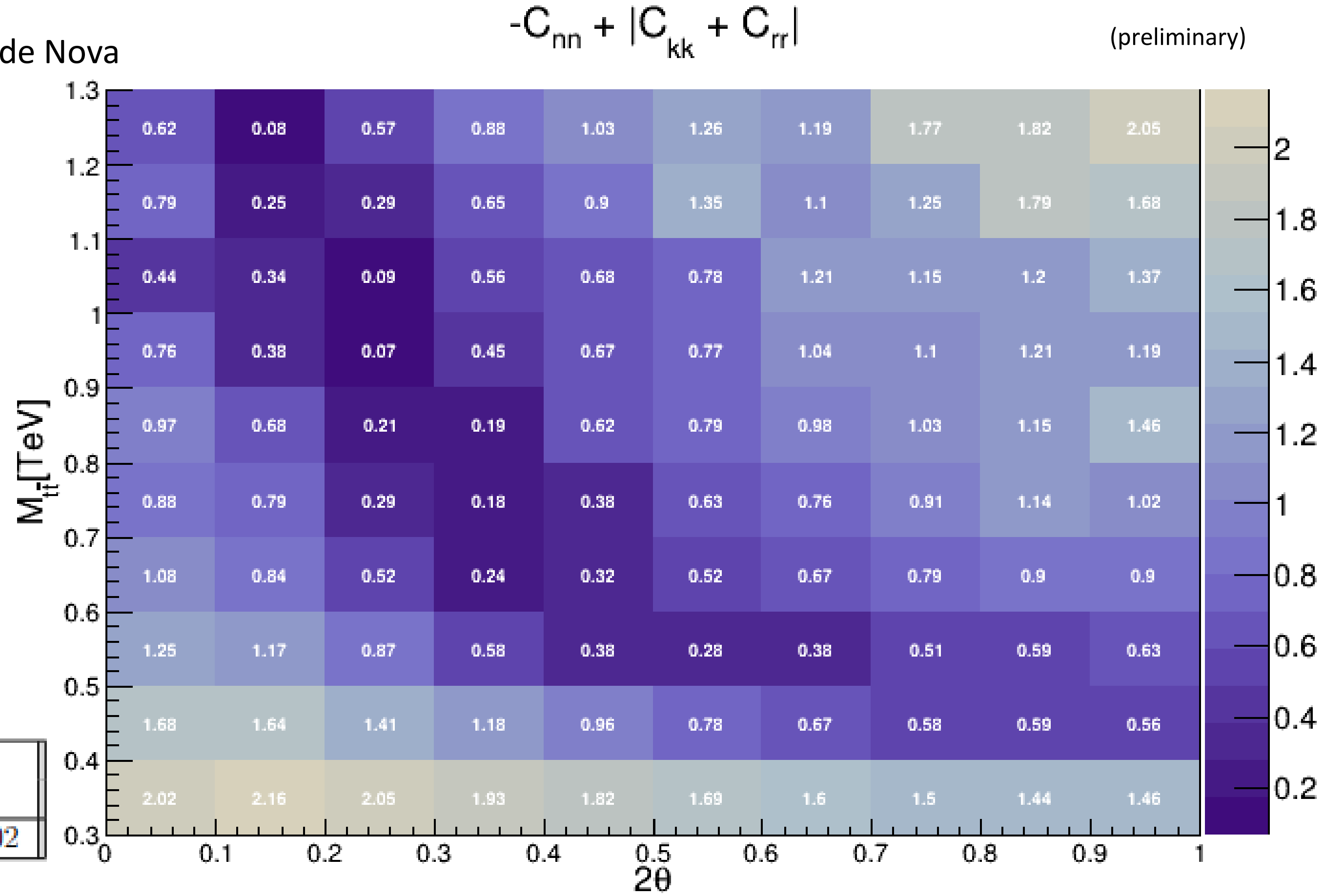
2203.05582, Y. Afik, J. R. M. de Nova
2110.10112. C. Severi et al.

For the basis we defined, we only need to check

$$|C_{kk} + C_{rr}| - C_{nn}$$

Region	$\lvert C_{kk} + C_{rr}\rvert - C_{nn}$ (Parton)
Threshold	1.60 ± 0.02

Region	$-C_{nn} + \lvert C_{kk} + C_{rr}\rvert$ (Delphes)				
	Analytic	ML-M2CW	ML-M2CT	χ^2	NN
Threshold	0.73 ± 0.02	1.33 ± 0.02	3.71 ± 0.02	1.00 ± 0.02	2.02 ± 0.02

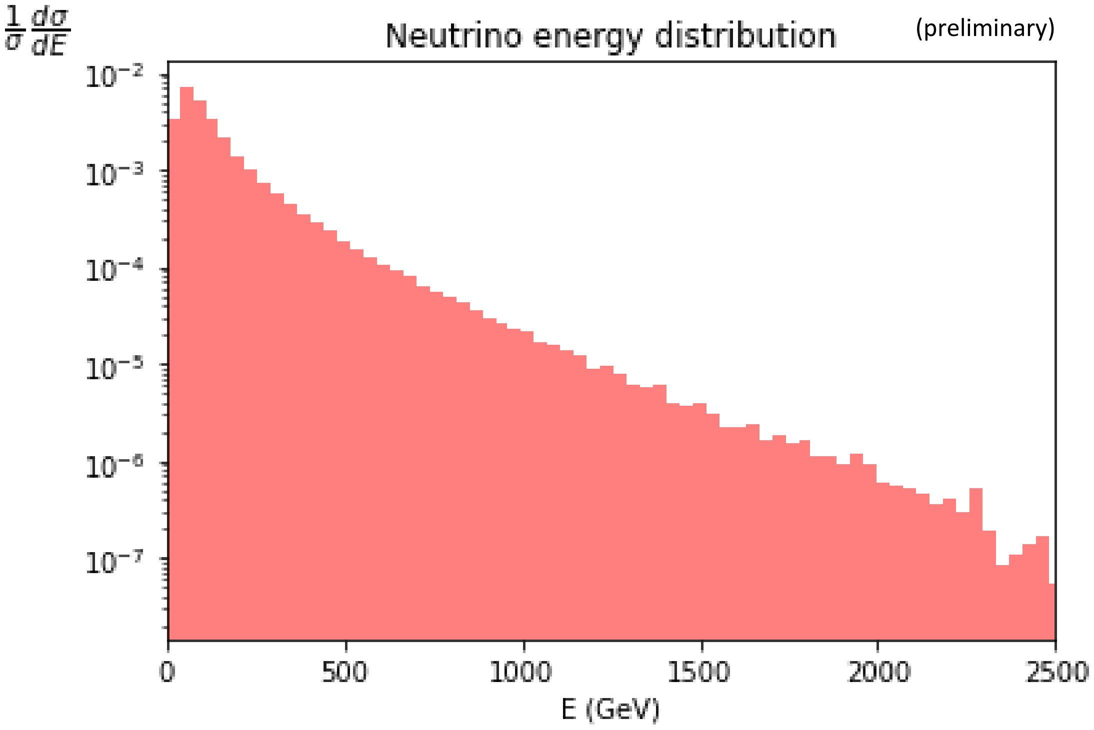


Summary

- Top quark pair-production at the LHC provides a window to study the foundations of QM in the high-energy regime.
- The dilepton final state is useful as the top (anti-top) and lepton spins are fully correlated.
- We should choose the reconstruction method that best fits our purpose.
- The ML-M2CW method performs better than others in the reconstruction of the entanglement signature.
- We should continue to explore semi-leptonic channel.

Analytic reconstruction

- Based on the four on-shell conditions.
- Involves solving quartic polynomial with potential multiple real solutions.
- We compute weighted solution based on neutrino energy distribution.
- Does not consider widths of Top and W masses. Reconstructed mass distribution is not optimal.



Entanglement Calculation

- We can either calculate the spin correlation matrix by estimating the differential cross-section or using anti-symmetry.

$$\frac{1}{4}\alpha_a\alpha_b = \frac{N(\cos\theta_a^i \cos\theta_b^i > 0) - N(\cos\theta_a^i \cos\theta_b^i < 0)}{N(\cos\theta_a^i \cos\theta_b^i > 0) + N(\cos\theta_a^i \cos\theta_b^i < 0)}$$

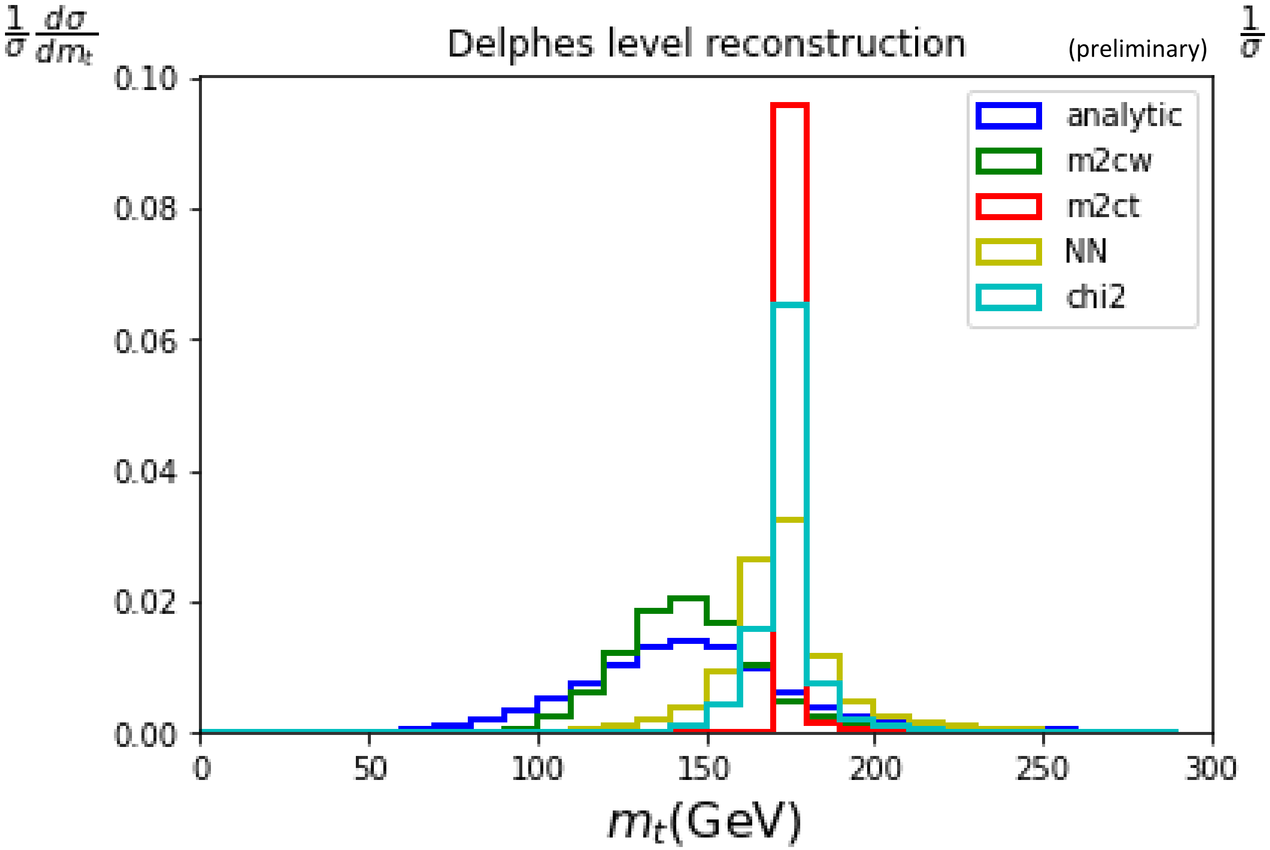
Chi-squared reconstruction

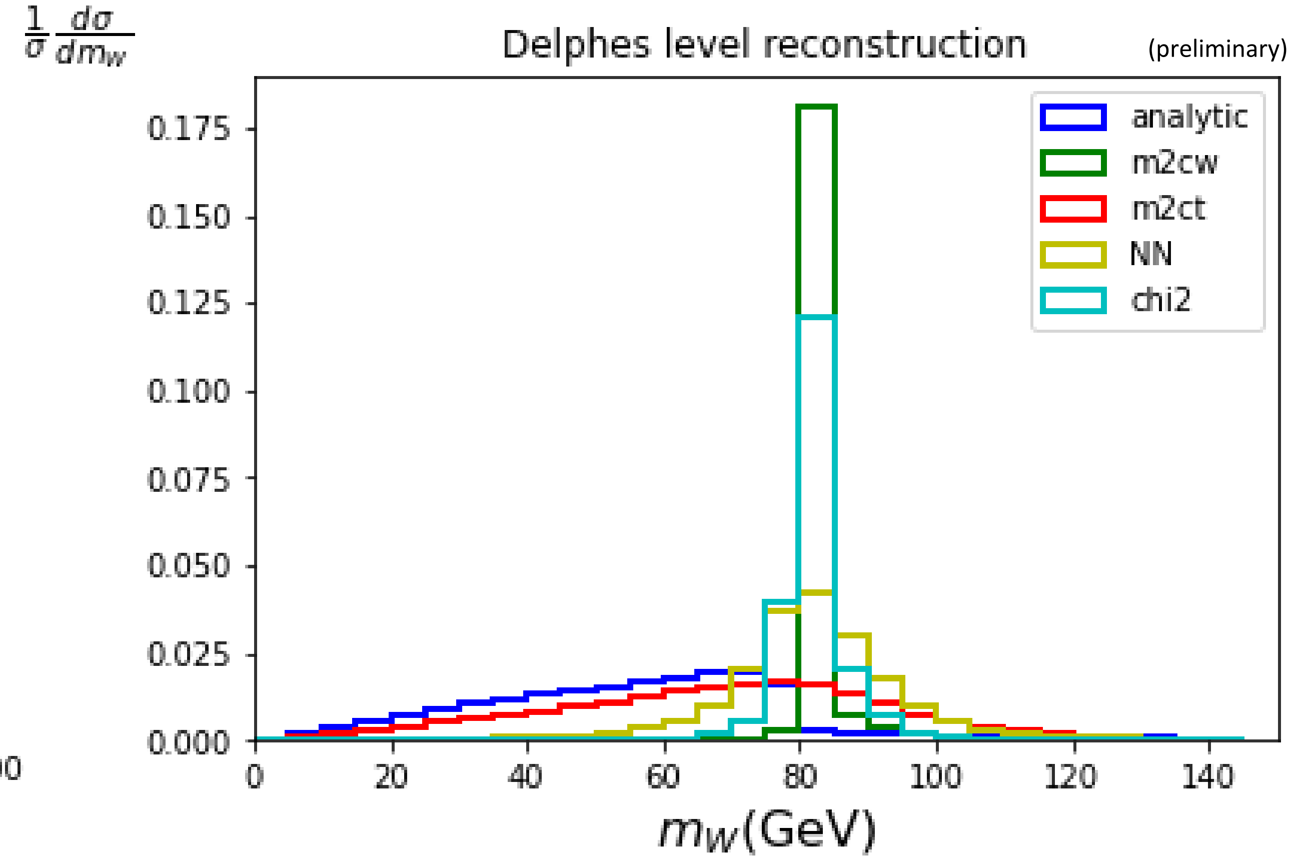
- Minimized over chi-squared statistic.
- Uses explicit mass information

$$\chi^2 \equiv \min_{\not{p}_T = p_{\nu T} + p_{\bar{\nu} T}} \left[\frac{\left(m_{bl^+\nu} - m_t\right)^2}{\sigma_t^2} + \frac{\left(m_{l^+\nu} - m_W\right)^2}{\sigma_W^2} + \frac{\left(m_{\bar{b}l^-\bar{\nu}} - m_t\right)^2}{\sigma_t^2} + \frac{\left(m_{l^-\bar{\nu}} - m_W\right)^2}{\sigma_W^2} + \frac{\left(|pT_t| - |pT_{\bar{t}}|\right)^2}{\sigma_{pT}^2} \right]$$

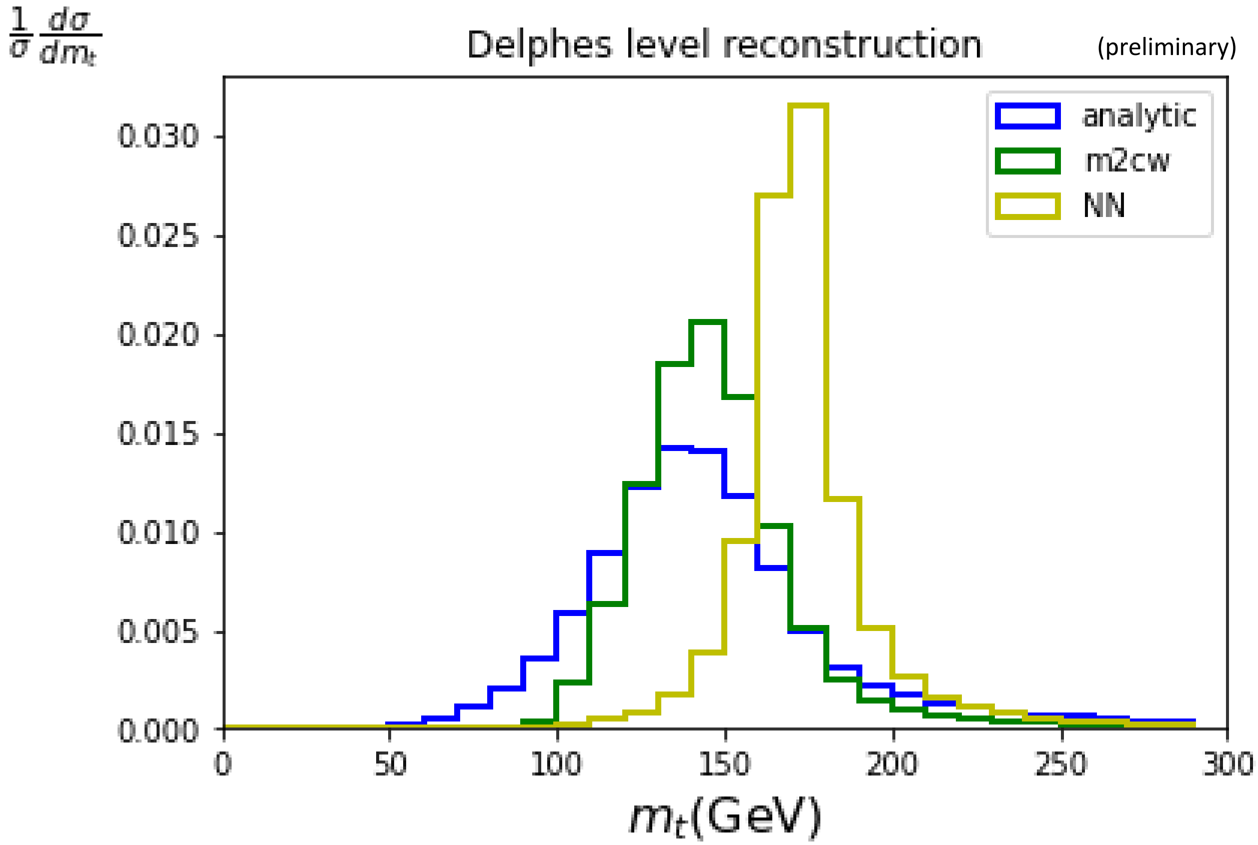
- Resolves combinatorics by choosing the combination with a smaller chi-squared value.
- More hyperparameters are used compared to other methods.
- Cannot generalize to other mass configurations.

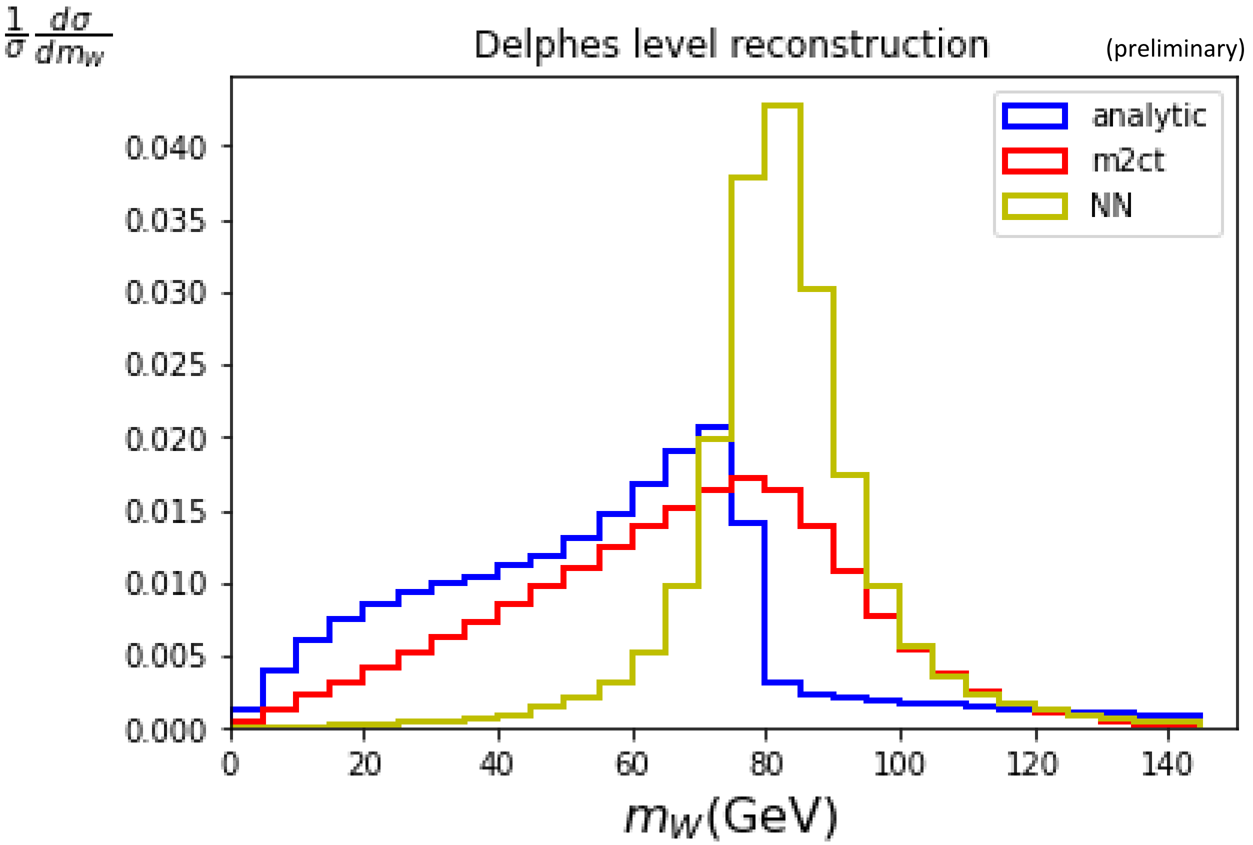
Comparisons

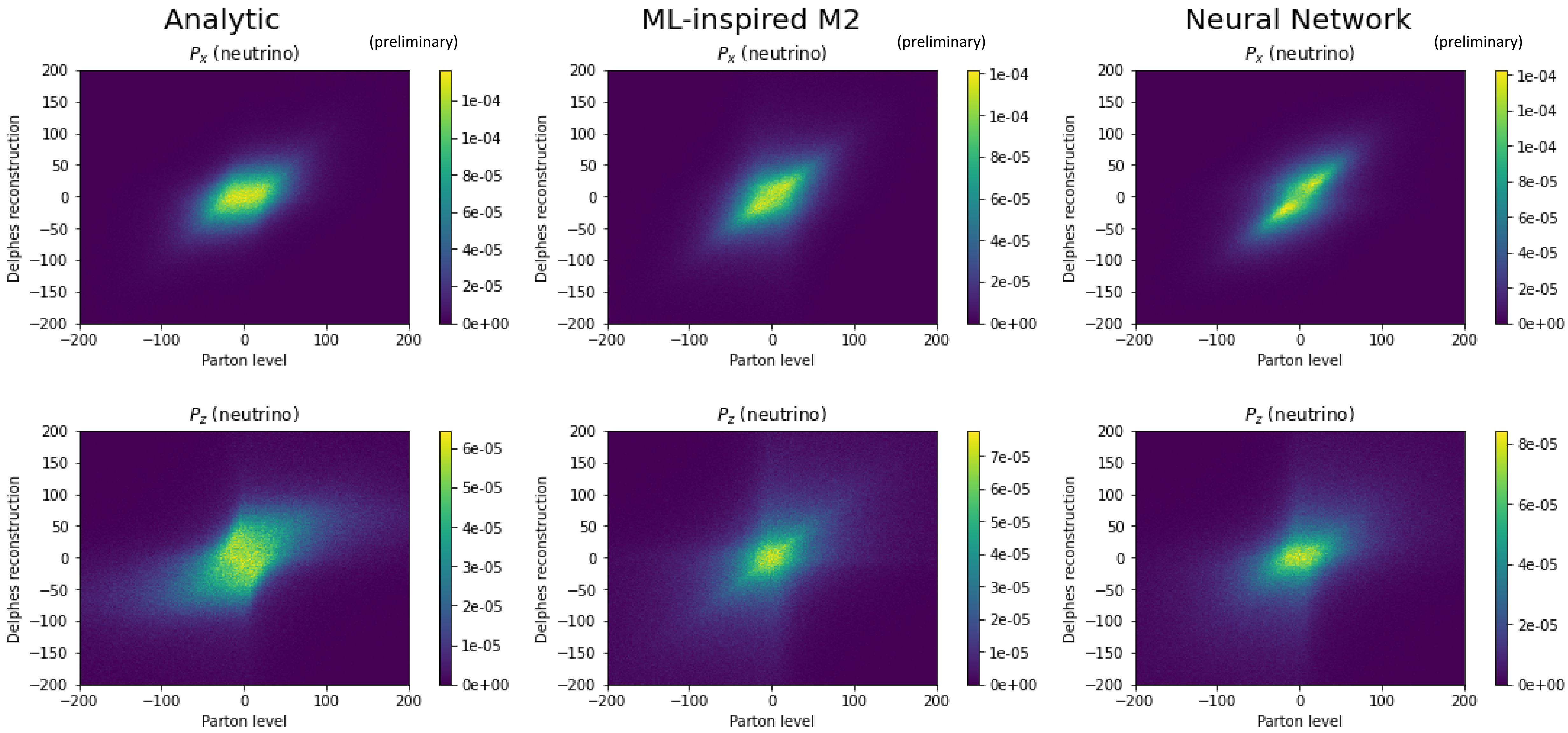


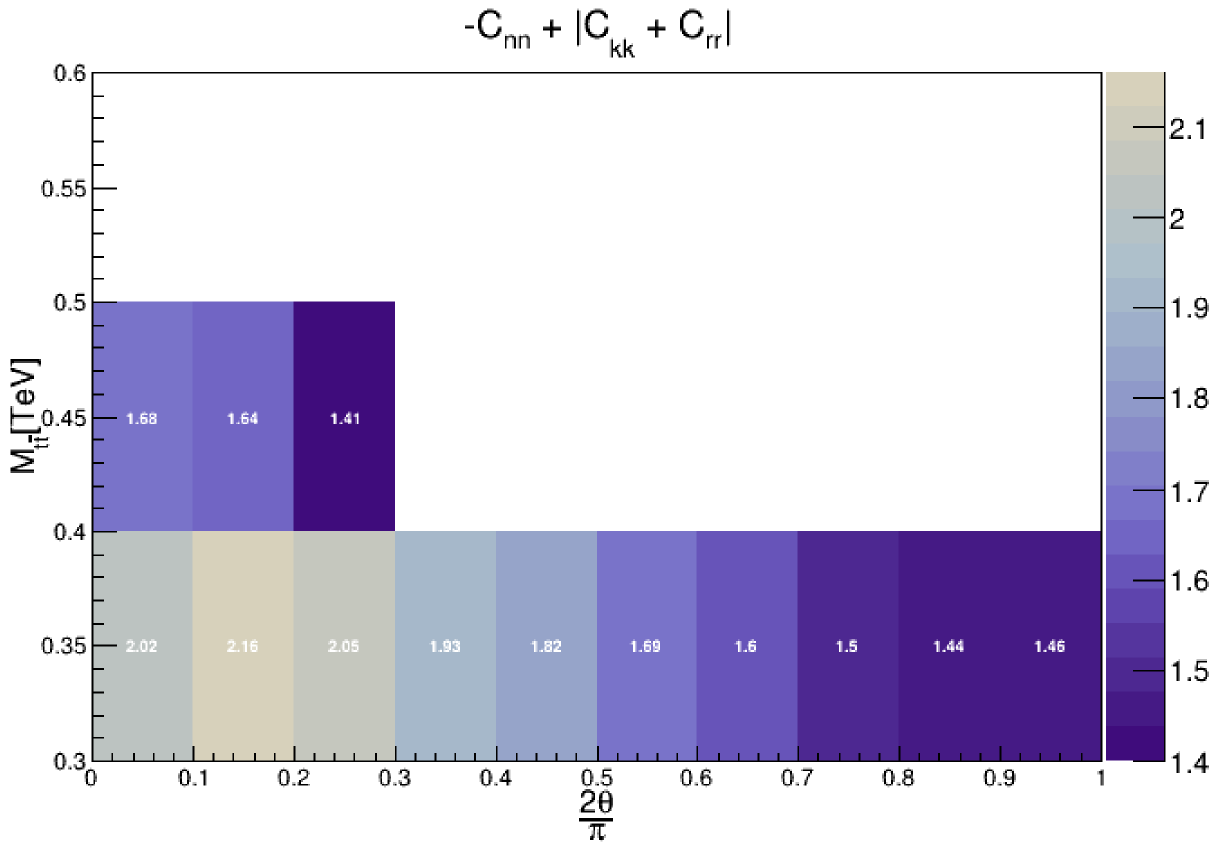


Comparisons






Analytic
(preliminary)
P_x (neutrino)
P_z (neutrino)
ML-inspired M2
(preliminary)
P_x (neutrino)
P_z (neutrino)
Neural Network
(preliminary)
P_x (neutrino)
P_z (neutrino)
Delphes reconstruction
Parton level
200
150
100
50
0
−50
−100
−150
−200
1e-04
8e-05
6e-05
4e-05
2e-05
0e+00
7e-05
5e-05
3e-05
1e-05

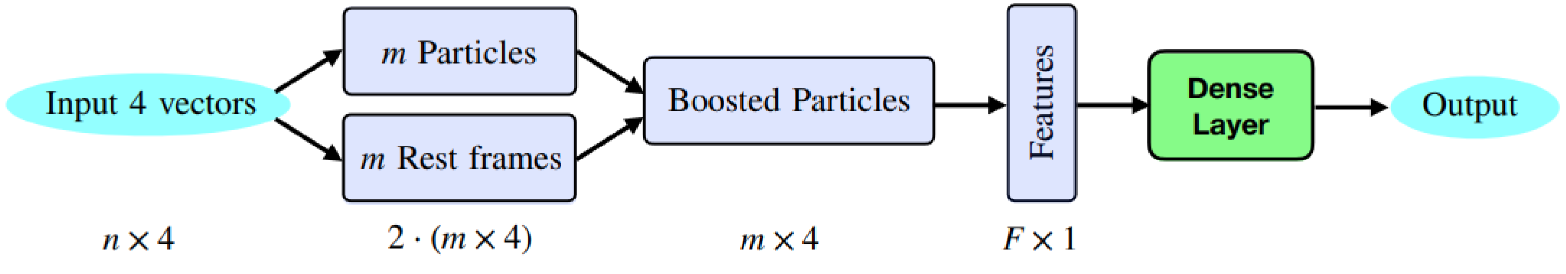


$-C_{nn} + |C_{kk} + C_{rr}|$
$M_{t\bar{t}}$[TeV]
$\frac{2\theta}{\pi}$
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.68
1.64
1.41
2.02
2.16
2.05
1.93
1.82
1.69
1.6
1.5
1.44
1.46
2.1
2
1.9
1.8
1.7
1.6
1.5
1.4

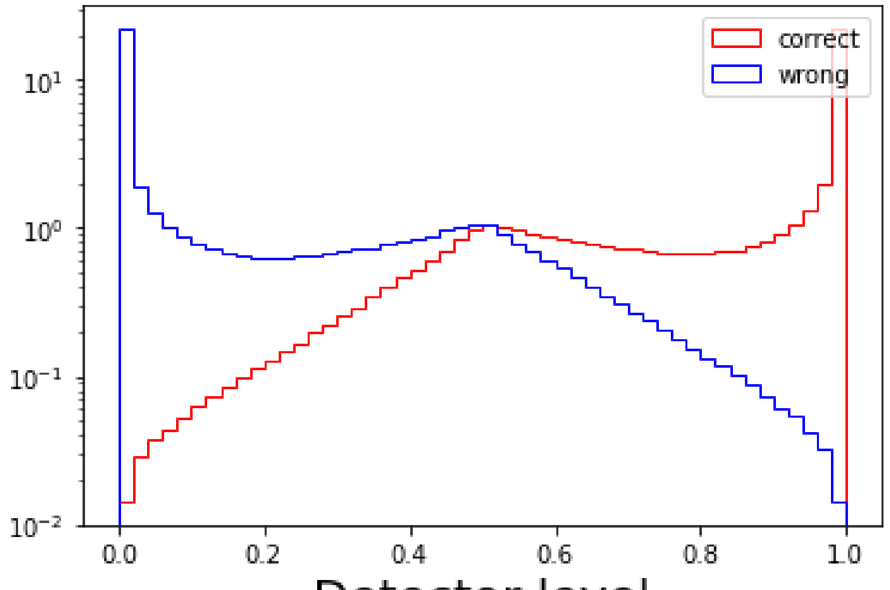
Combinatorial Classification: Lorentz Boosted Network

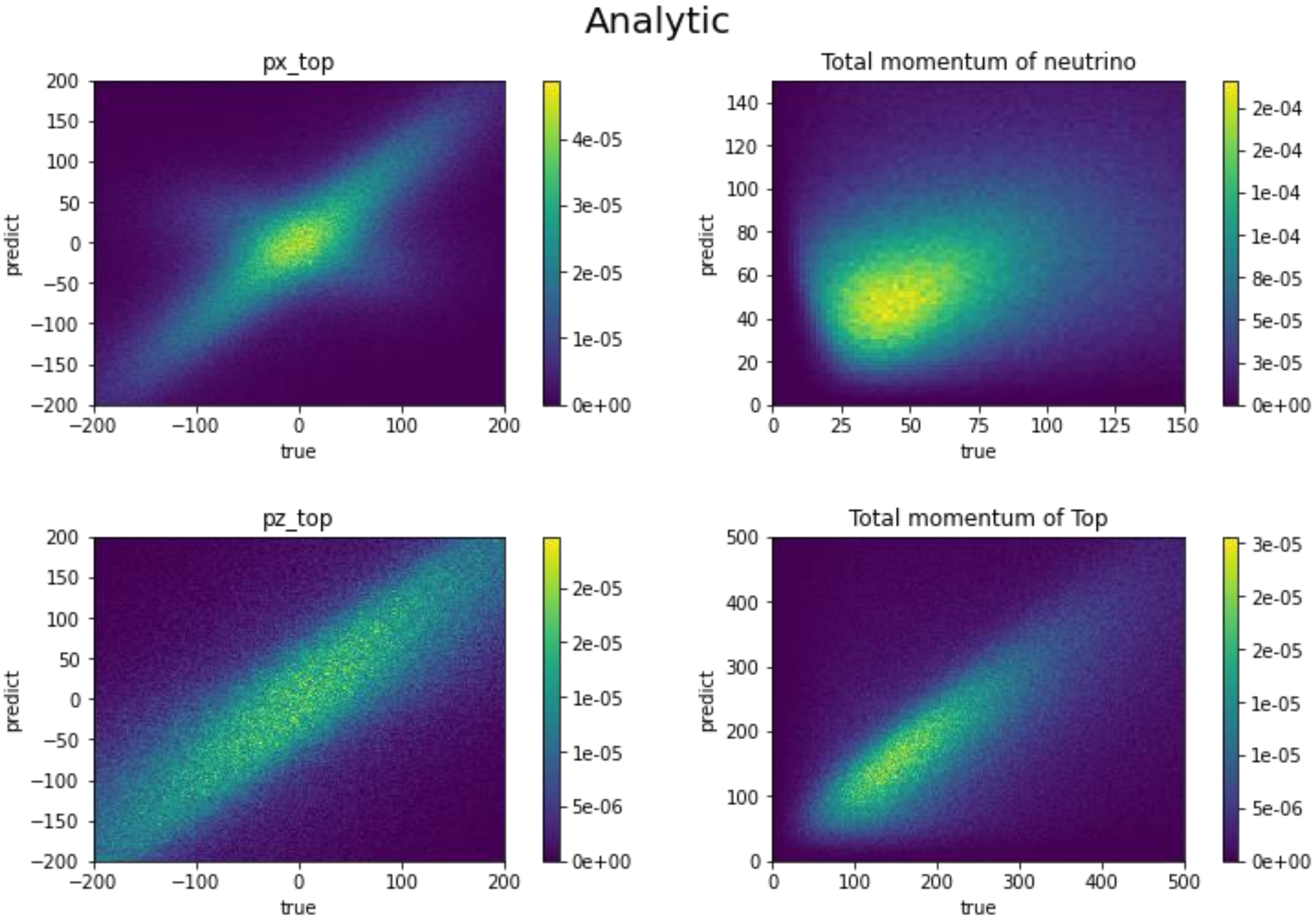
- Motivated by particle kinematics in the rest frames of various particle combinations.
- Form linear combinations of input momentums and boosts into rest frames.
- Output features of the boosted particles.
- We used the network structure studied in our previous work.

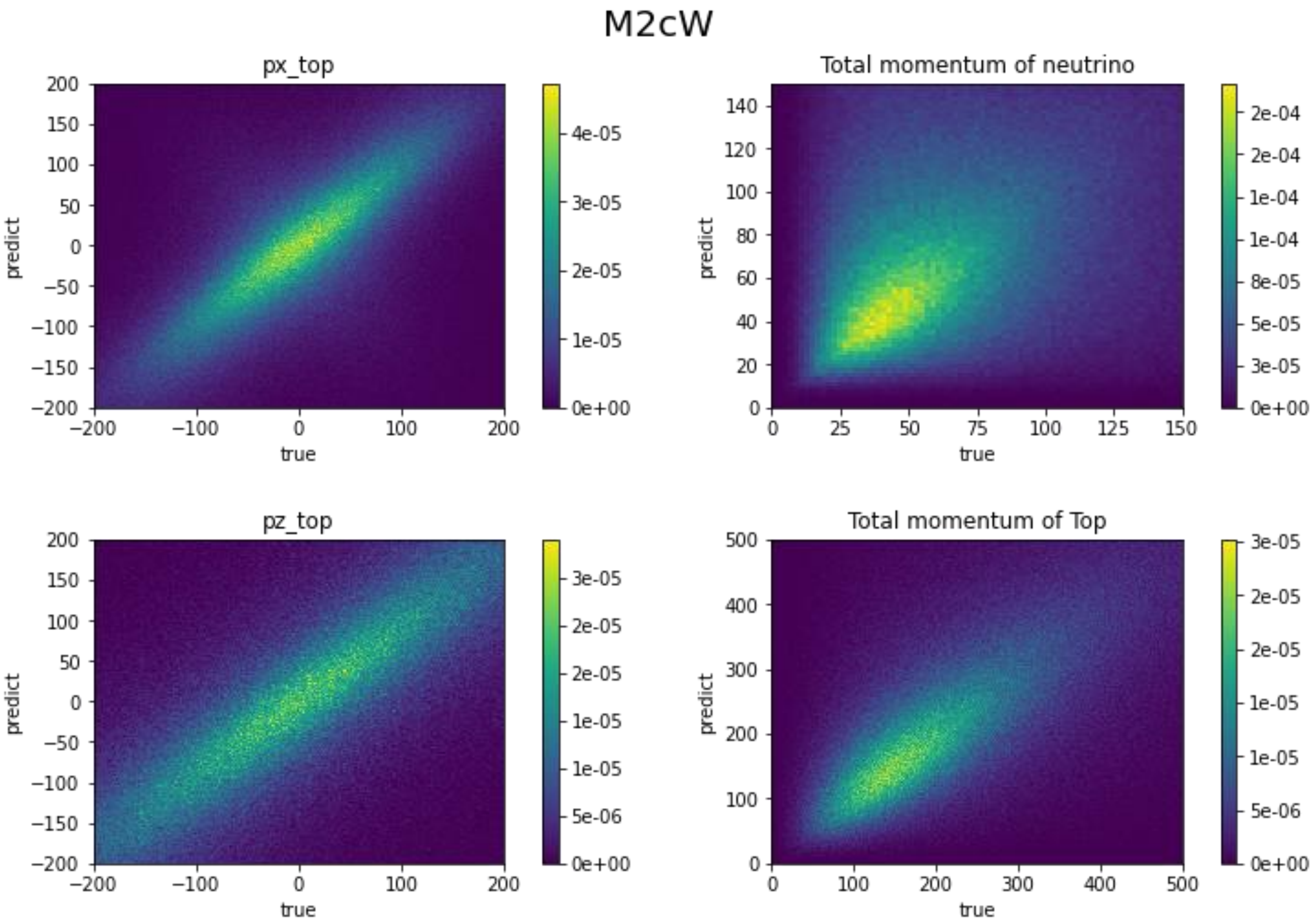
2202.05849 Alhazmi, Dong, Huang, Kim, Kong, Shih



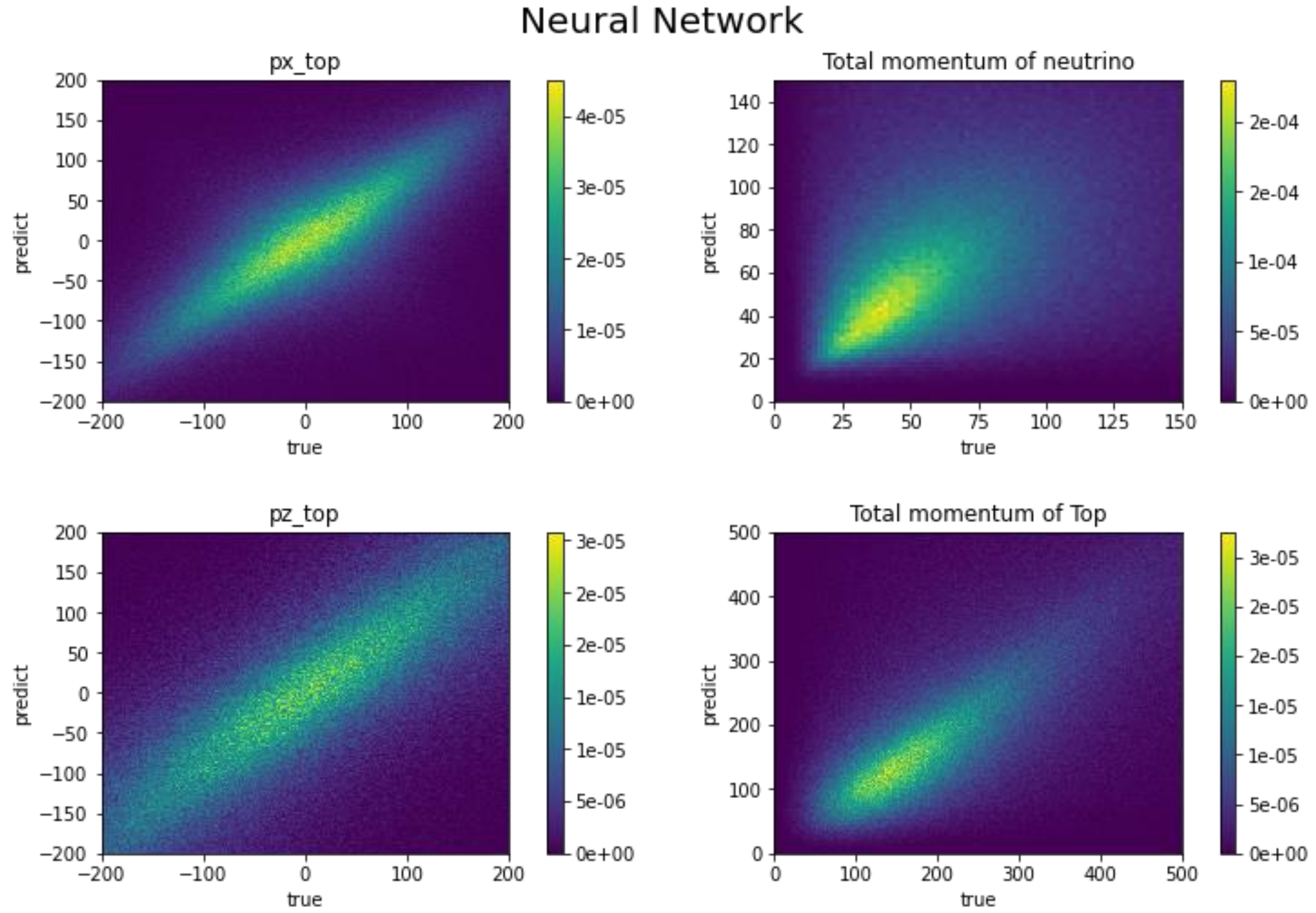
1812.09722 Erdmann, Geiser, Rath, Rieger

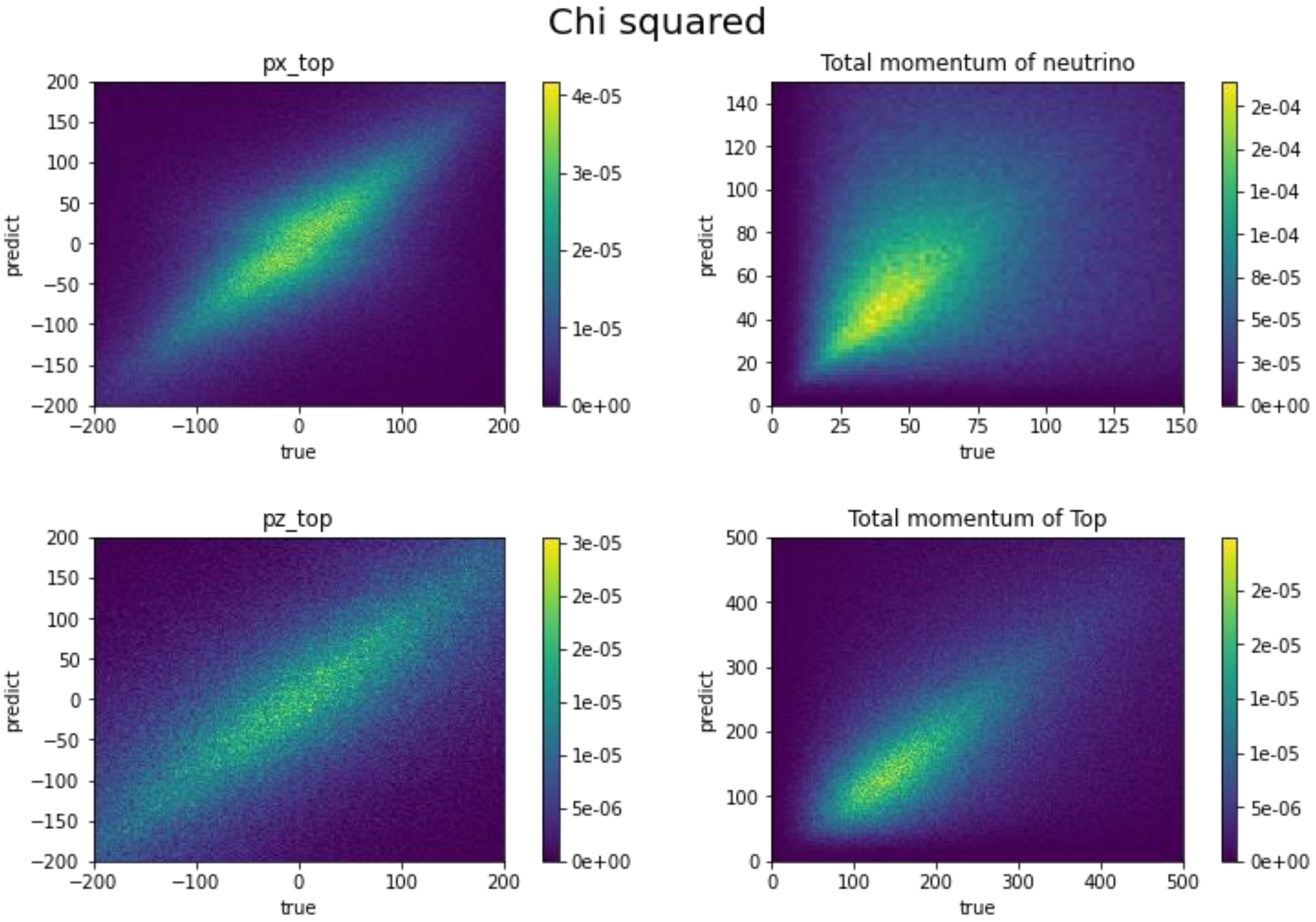



Analytic
px_top
Total momentum of neutrino
pz_top
Total momentum of Top
predict
true
4e-05
3e-05
2e-05
1e-05
0e+00
2e-04
1e-04
8e-05
5e-05
3e-05
5e-06
3e-05



M2cW
px_top
predict
true
200
150
100
50
0
−50
−100
−150
−200
−200
−100
0
100
200
4e-05
3e-05
2e-05
1e-05
0e+00
Total momentum of neutrino
predict
true
140
120
100
80
60
40
20
0
0
25
50
75
100
125
150
2e-04
2e-04
1e-04
1e-04
8e-05
5e-05
3e-05
0e+00
pz_top
predict
true
200
150
100
50
0
−50
−100
−150
−200
−200
−100
0
100
200
3e-05
2e-05
2e-05
1e-05
1e-05
5e-06
0e+00
Total momentum of Top
predict
true
500
400
300
200
100
0
0
100
200
300
400
500
3e-05
2e-05
2e-05
1e-05
1e-05
5e-06
0e+00


Neural Network
px_top
Total momentum of neutrino
pz_top
Total momentum of Top
predict
true


Chi squared
px_top
Total momentum of neutrino
pz_top
Total momentum of Top
predict
true
4e-05
3e-05
2e-05
1e-05
0e+00
2e-04
1e-04
8e-05
5e-05
3e-05
5e-06