High-quality axions in solutions to the μ problem

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Based on work with Stephen P. Martin, `arXiv:hep-ph/2106.14964`

The Kim-Nilles mechanism

Consider MSSM (without μ-term) + two gauge-singlets X and Y:

$$W_{\mathrm{I}} \supset \frac{\lambda_\mu}{M_P} X Y H_u H_d + \frac{\lambda}{6 M_P} X^3 Y,$$

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The total scalar potential (with the soft terms) has a local minimum for

$$\langle X \rangle \sim \langle Y \rangle \sim \sqrt{m_{\rm soft} M_P} \equiv M_{\rm int},$$

with $m_{\rm soft} \sim$ TeV scale, and $M_{\rm int}$ in the range

$$10^9 \text{ GeV} \lesssim M_{\rm int} \lesssim 10^{12} \text{ GeV}.$$

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The low-energy theory now contains:

- $\mu = \frac{\lambda_\mu}{M_P} \langle XY \rangle \sim m_{\text{soft}}$
- An invisible DFSZ-type QCD axion

solving the μ problem and the strong CP problem!

The four base models†

Base model	Superpotential terms	PQ charges of (X, Y)
$\mathrm{B_I}$	$XYH_uH_d + X^3Y$	$(-1,\ 3)$
$\mathrm{B_{II}}$	$X^2H_uH_d + X^3Y$	$(1,\ -3)$
$\mathrm{B_{III}}$	$Y^2H_uH_d + X^3Y$	$(-\frac{1}{3},\ 1)$
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In terms of the PQ charges of the MSSM quark and lepton doublets Q_q, Q_ℓ

	H_u	H_d	$\overline{u}$	$\overline{d}$	$\overline{e}$
PQ charge	$-2c_\beta^2$	$-2s_\beta^2$	$2c_\beta^2 - Q_q$	$2s_\beta^2 - Q_q$	$2s_\beta^2 - Q_\ell$

where $\tan\beta = s_\beta/c_\beta$ is the ratio of the Higgs VEVs.

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$g_{A\gamma}$ suppressed in all four base models!

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Cosmological domain wall problem

$$U(1)_{\text{PQ}} \xrightarrow{\text{PQ breaking}} Z_{N_{\text{DW}}} \text{ discrete symmetry}$$

Domain wall number (N_{DW}): number of discrete set of inequivalent degenerate minima of the axion potential.

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Formation of topological defects such as stable DWs, due to the different possible phases of the axion, which dominate the universe[†]

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Some solutions

- If PQ breaking happens before inflation
- $N_{\text{DW}} = 1$ (**our focus**)

$N_{\text{DW}} \neq 1$ in all four base models.

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Base model extensions

Consistent with gauge coupling unification, we consider the following extensions:

- $\mathbf{5}+\overline{\mathbf{5}}$ at TeV or $M_{\rm int}$
- $\mathbf{10}+\overline{\mathbf{10}}$ at TeV
- $\mathbf{10}+\overline{\mathbf{10}}$ at $M_{\rm int}$
- $(\mathbf{5}+\overline{\mathbf{5}})$ or $(\mathbf{10}+\overline{\mathbf{10}})$ at TeV, $(\mathbf{5}+\overline{\mathbf{5}})$ or $(\mathbf{10}+\overline{\mathbf{10}})$ at $M_{\rm int}$

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(last two items:) $N_{\text{DW}} = 1$ possible

Here,

$$\begin{aligned} \overline{\mathbf{5}} &= \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, 1/3)}_{\overline{D}} + \underbrace{(\mathbf{1}, \mathbf{2}, -1/2)}_{L} \\ \mathbf{10} &= \underbrace{(\mathbf{3}, \mathbf{2}, 1/6)}_{Q} + \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, -2/3)}_{\overline{U}} + \underbrace{(\mathbf{1}, \mathbf{1}, 1)}_{\overline{E}} \end{aligned}$$

We allow for different components of the $\mathbf{5} + \overline{\mathbf{5}}$ and/or $\mathbf{10} + \overline{\mathbf{10}}$ to have different mass source terms.

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Extensions with $N_{\text{DW}}=1$ give rise to enhanced low-energy axion couplings!

Vectorlike mass terms

Assuming the same mechanism that gives a μ term also gives masses to vectorlike pairs of chiral superfields $\Phi + \overline{\Phi}$.

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TeV scale masses:

$$W_{\text{mass}} = \begin{cases} \frac{\lambda_\Phi}{M_P} XY\Phi\overline{\Phi}, \\ \frac{\lambda_\Phi}{2M_P} X^2\Phi\overline{\Phi}, \\ \frac{\lambda_\Phi}{2M_P} Y^2\Phi\overline{\Phi}, \end{cases}$$

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Mass terms fix the PQ charge of the terms $\Phi\overline{\Phi}$ which in turn fix the low-energy axion couplings, independent of the Yukawa terms.

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Higher dimensional operators from quantum gravity can explicitly violate global $U(1)_{\mathrm{PQ}}$ and reintroduce the strong CP problem

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In our case, consider

$$W = \frac{\kappa}{M_P^{p-3}} X^j Y^{p-j}$$

that contributes to the axion potential (with soft terms), giving rise to:

$$|\theta_{\mathrm{eff}}| \quad = \quad \frac{\delta}{(0.0754\ \mathrm{GeV})^4} \frac{f_A^{p+2}}{M_P^{p-2}},$$

with a dimensionless quantity δ, and f_A identified with M_{int}.

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Solution

We find that $X^j Y^{p-j}$ with $p < 7$ should be forbidden for $|\theta_{\text{eff}}| \lesssim 10^{-10}$

Non-R and R discrete Z_n symmetries

	gauginos	W	chiral superfield Φ	fermion in Φ
Z_n^R charge (mod n)	r	$2r$	z_Φ	$z_\Phi - r$

For non-R symmetry $r = 0$, and for R-symmetry $0 < r < n/2$.

In both cases, $z_\Phi = 0, 1, \ldots, n-1$.

† See e.g. L. E. Ibanez arXiv:hep-ph/9210211

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With a normalization where $Z_n^R \times G_{\text{SM}} \times G_{\text{SM}}$ anomalies are integers, we impose the following anomaly-free conditions:†

$$A_2 = A_3 = \rho_{\text{GS}} \ (\text{mod } n),$$

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for the weaker condition, with the additional stronger condition

$$A_1 = 5A_3 = 5\rho_{\rm GS} \text{ (mod } n),$$

which does not require the Green-Schwarz (GS) mechanism if $\rho_{\rm GS} = 0$.

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Examples with non-R Z_n symmetries: Base models

Stronger constraints with $\rho_{\text{GS}} \neq 0$: (Here, $m = 0, 1, 2$)

Model	Z_n	X	H_u	p	ρ_{GS}
B_{III}	36	1	$8 + 12m$	12	18
B_{IV}	36	3	4	8	18

† proposed and studied in K. S. Babu, I. Gogoladze, K. Wang hep-ph/0212245.

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Weaker constraint with $\rho_{GS} \neq 0$: Lots of cases, e.g., a Z_{22} symmetry†

Model	Z_n	X	H_u	p	ρ_{GS}
B_{IV}	22	2	2	11	12

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Examples with Z_n^R symmetries: Base models

Stronger constraints with $\rho_{GS} = 0$: Some examples,

Model	Z_n^R	r	X	H_u	p
B_{III}	54	3	5	$1 + 18m$	10
B_{IV}	12	1	8	$1 + 4m$	7

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Stronger constraints with $\rho_{\mathrm{GS}} \neq 0$: As a special case, we found a Z_{24}^R symmetry with $SU(5)$ invariance†

Model	Z_n^R	r	X	H_u	p	ρ_{GS}
$\mathrm{B_{II}}$	24	1	11	1	10	18
$\mathrm{B_{III}}$	24	1	5	1	10	18

We do not impose $SU(5)$ invariance.

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Examples with $Z_n^{(R)}$ symmetries: Base model extensions

Stronger constraints: Examples,

Base	Extension	Z_n^R	r	X	H_u	p	ρ_{GS}
B_I	$XYD\overline{D} + X^2L\overline{L}$	34	1	31	15	12	16
B_{II}	$Y^2D\overline{D} + Y^2L\overline{L}$	108	6	11	$22 + 36m$	20	0
B_{III}	$X^2Q\overline{Q} + X^2U\overline{U} + Y^2E\overline{E}$	42	0	1	$8 + 14m$	14	18
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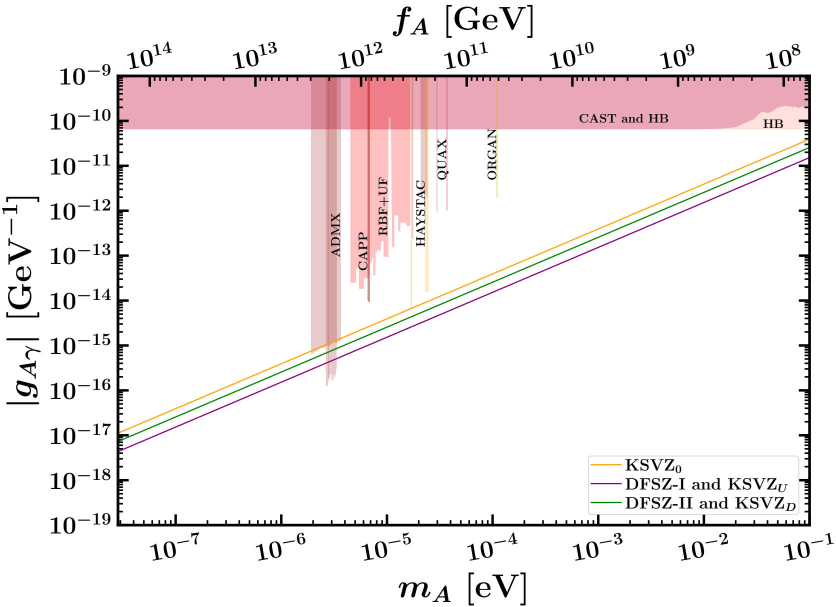
Can find an anomaly-free $Z_n^{(R)}$ symmetry protecting $U(1)_{\mathrm{PQ}}$ for each model thus giving rise to a high-quality axion!

Axion-photon coupling (limits)



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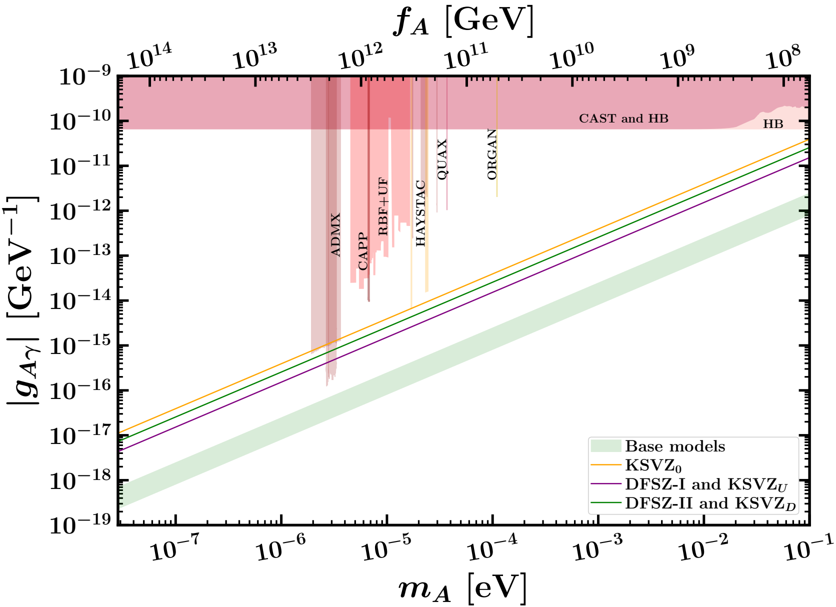

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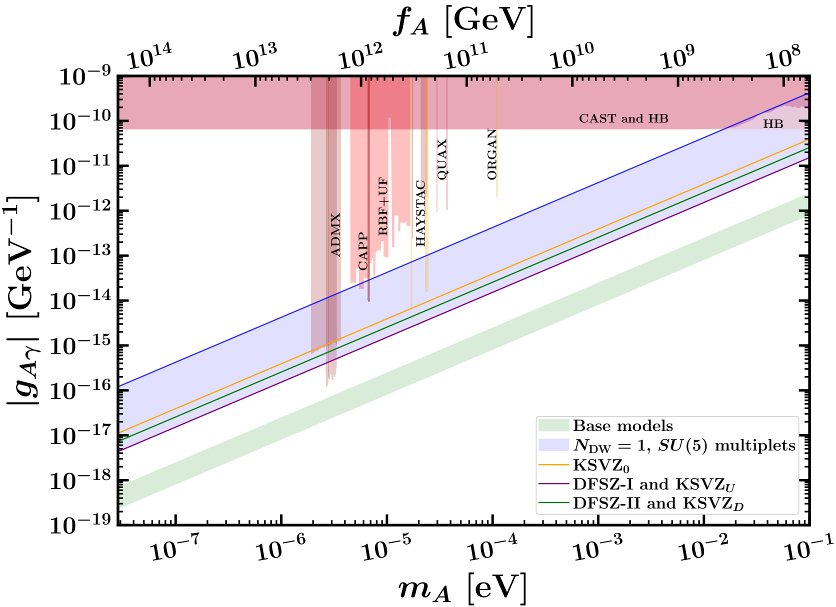

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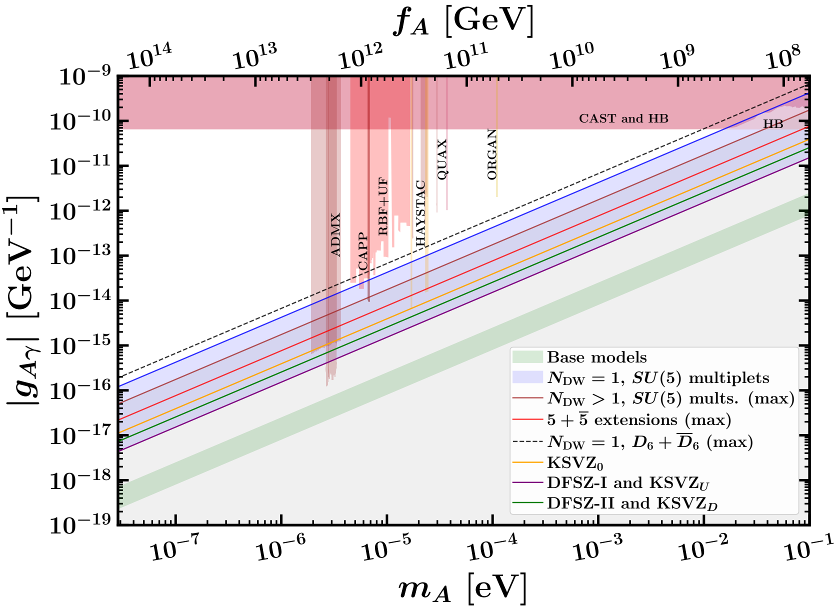
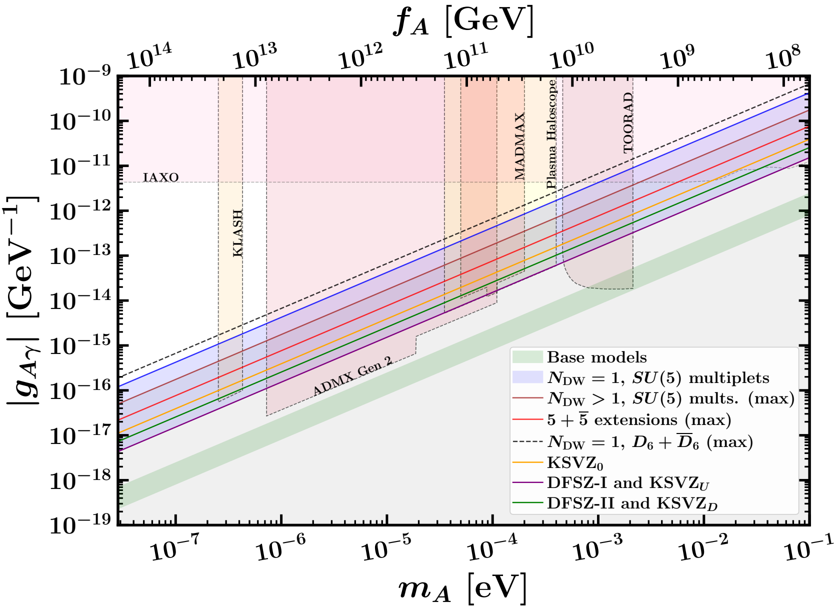
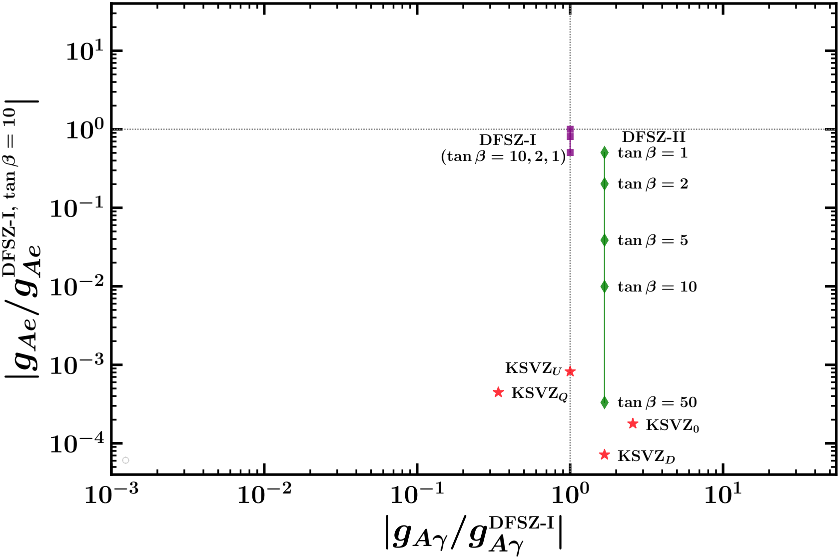


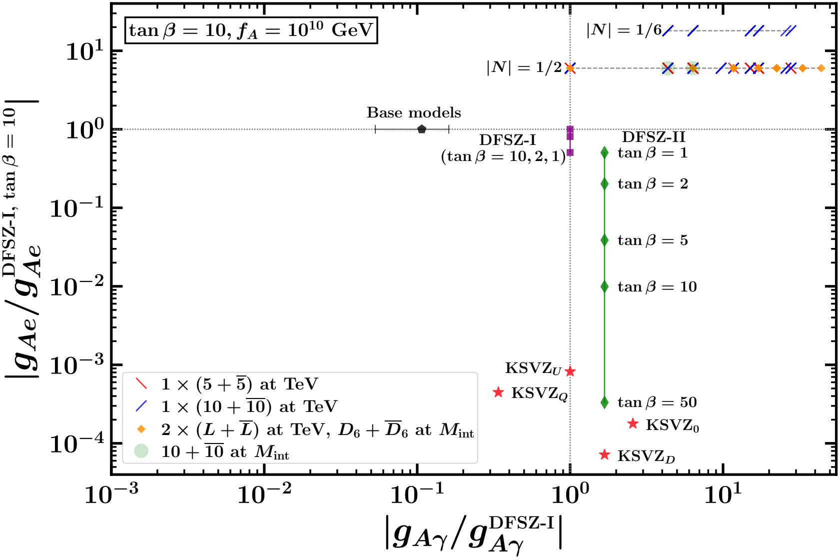
Axion-photon coupling (projections)

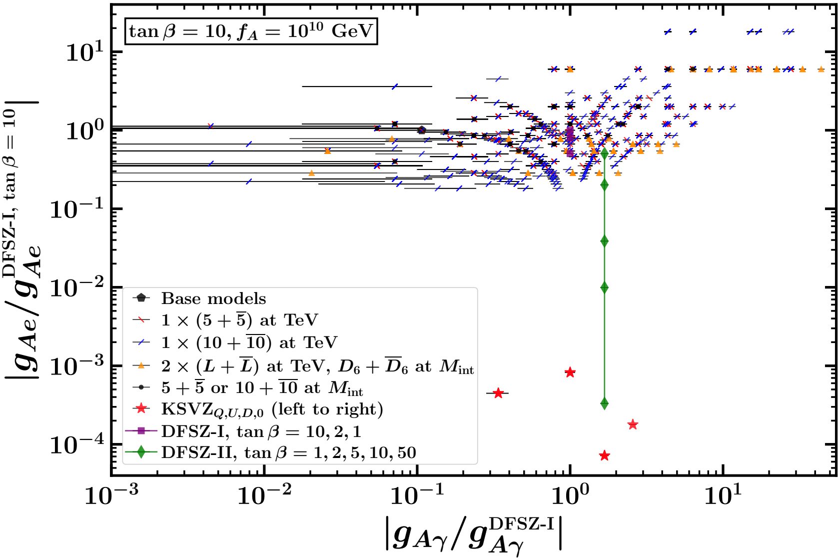


$|g_{Ae}/g_{Ae}^{\text{DFSZ-I},\tan\beta=10}|$ vs. $|g_{A\gamma}/g_{A\gamma}^{\text{DFSZ-I}}|$



$|g_{Ae}/g_{Ae}^{\text{DFSZ-I},\tan\beta=10}|$ vs. $|g_{A\gamma}/g_{A\gamma}^{\text{DFSZ-I}}|$





$\tan\beta = 10, f_A = 10^{10}$ GeV
Base models
$1 \times (5 + \overline{5})$ at TeV
$1 \times (10 + \overline{10})$ at TeV
$2 \times (L + \overline{L})$ at TeV, $D_6 + \overline{D}_6$ at $M_{\rm int}$
$5 + \overline{5}$ or $10 + \overline{10}$ at $M_{\rm int}$
KSVZ$_{Q,U,D,0}$ (left to right)
DFSZ-I, $\tan\beta = 10, 2, 1$
DFSZ-II, $\tan\beta = 1, 2, 5, 10, 50$
$|g_{A\gamma}/g_{A\gamma}^{\rm DFSZ\text{-}I}|$
$|g_{Ae}/g_{Ae}^{\rm DFSZ\text{-}I,\ \tan\beta=10}|$
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{-4}

Conclusion

Supersymmetry by itself addresses the electroweak hierarchy puzzle.

We considered extensions with extra vectorlike content that:

- have high-quality QCD axions within the reach of future axion searches
- simultaneously solve the μ problem
- evade cosmological domain wall problem
- maintain gauge coupling unification

BACKUP SLIDES

Lightning review: The strong CP problem

Non-trivial QCD vacuum structure requires the term:

$$\mathcal{L}_{\text{QCD}} \supset \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu},$$

where the QCD vacuum angle θ is expected to be $\mathcal{O}(1)$.

"Everything not forbidden is compulsory."

However, experimentally:

$$|\theta| \lesssim 10^{-10}.$$

Why so small? $-$ strong CP problem

Peccei-Quinn (PQ) solution: promote θ to a dynamical field

Lightning review: Peccei-Quinn (PQ) solution

Consider a global $U(1)_{\text{PQ}}$ axial symmetry:

$$\partial_\mu j^\mu_{\text{PQ}} = \underbrace{\frac{g_s^2 N}{16\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}}_{\text{QCD anomaly}} + \underbrace{\frac{e^2 E}{16\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}}_{\text{EM anomaly}},$$

with left-handed fermions with PQ charge Q_f, $SU(3)_c$ index $T(R_f)$, and EM charge q_f contributing to:

$$\begin{aligned} N &= \text{Tr}\left[Q_f T(R_f)\right], \\ E &= \text{Tr}\left[Q_f q_f^2\right]. \end{aligned}$$

$U(1)_{\text{PQ}}$ can be spontaneously broken by scalars with PQ charge Q_s

$$\varphi_s \supset \frac{v_s}{\sqrt{2}} e^{i a_s / v_s}.$$

With $V^2 = \sum_s Q_s^2 v_s^2$, the axion field is given by:

$$A = \frac{1}{V} \sum_s Q_s v_s a_s.$$

Ensuring the axion is massless at tree-level by imposing:

$$\sum_s Y_s Q_s v_s^2 = 0,$$

where Y_s: weak hypercharge of φ_s.
QCD vaccum term now becomes:

$$\mathcal{L}_{\text{QCD}} \supset \left(\theta + \frac{A}{f_A}\right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu},$$

with the axion decay constant

$$f_A \equiv \frac{V}{2N}.$$

Under $U(1)_{\text{PQ}}$ transformations:

$$A \to A + (\text{constant})\ f_A,$$

Thus solving the strong CP problem.

Lightning review: Low-energy axion couplings

$$\mathcal{L}^{A}_{\text{int}} \supset \frac{1}{4} g_{A\gamma} A F^{\mu\nu} \tilde{F}_{\mu\nu} - \sum_{f=e,n,p} i g_{Af} A \overline{\Psi}_f \gamma_5 \Psi_f$$

where,

$$\begin{aligned} g_{A\gamma} &= \frac{\alpha_e}{2\pi f_A} \left(c_\gamma - 1.92(4) \right), \\ g_{Ae} &= \frac{m_e}{f_A} \left[c_e + \frac{3\alpha_e^2}{4\pi^2} \left(c_\gamma \log \frac{f_A}{m_e} - 1.92(4) \log \frac{\text{GeV}}{m_e} \right) \right], \\ g_{An} &= \frac{m_n}{f_A} \left(-0.02(3) + 0.833(30) c_d - 0.406(21) c_u \right). \end{aligned}$$

with

$$c_\gamma = \frac{E}{N}, \quad c_e = \frac{Q_\ell + Q_{\overline{e}}}{2N}, \quad c_u = \frac{Q_q + Q_{\overline{u}}}{2N}, \quad c_d = \frac{Q_q + Q_{\overline{d}}}{2N}.$$

Axion can accidentally decouple from photons if $E/N \approx 1.92$.

Non-supersymmetric benchmark QCD axion models†

Benchmark	PQ charged fermions	N	c_γ	c_u	c_d	c_e
KSVZ$_0$	$(\mathbf{3},\mathbf{1},0)+(\overline{\mathbf{3}},\mathbf{1},0)$	$\frac{1}{2}$	0	0	0	0
KSVZ$_D$	$D+\overline{D}$	$\frac{1}{2}$	$\frac{2}{3}$	0	0	0
KSVZ$_U$	$U+\overline{U}$	$\frac{1}{2}$	$\frac{8}{3}$	0	0	0
KSVZ$_Q$	$Q+\overline{Q}$	1	$\frac{5}{3}$	0	0	0
DFSZ-I	SM fermions	3	$\frac{8}{3}$	$\frac{c_\beta^2}{3}$	$\frac{s_\beta^2}{3}$	$\frac{s_\beta^2}{3}$
DFSZ-II	SM fermions	3	$\frac{2}{3}$	$\frac{c_\beta^2}{3}$	$\frac{s_\beta^2}{3}$	$-\frac{c_\beta^2}{3}$

where tan $\beta = s_\beta/c_\beta$ is the ratio of Higgs VEVs in the DFSZ models.

† J. E. Kim Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, V. I. Zakharov Nucl. Phys. B **166**, 493-506 (1980); M. Dine, W. Fischler, M. Srednicki Phys. Lett. B **104**, 199-202 (1981); A. R. Zhitnitsky Sov. J. Nucl. Phys. **31**, 260 (1980)

Quixotic extension

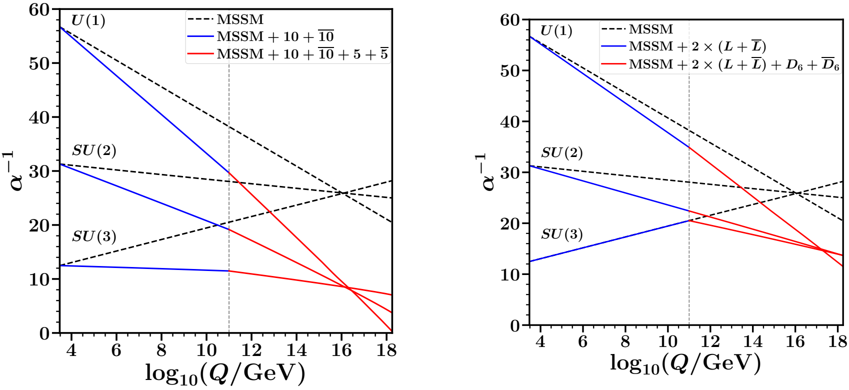
Consistent with gauge coupling unification, we can also consider

***$D_6 LL$* models**: $2 \times (L + \overline{L})$ at TeV, $D_6 + \overline{D}_6$ at $M_{\text{int}} \sim 10^{11}$ GeV ($N_{\text{DW}} = 1$ possible)

where

$$D_6 + \overline{D}_6 \quad = \quad (\mathbf{6}, \mathbf{1}, 1/3) + (\overline{\mathbf{6}}, \mathbf{1}, -1/3) \quad \text{is an exotic quix pair}$$

Gauge coupling unification



$N_{\mathrm{DW}} = 1$ in base model extensions

$$N_{\mathrm{DW}} \equiv \text{minimum integer} \left(2N \sum_s \frac{n_s Q_s v_s^2}{V^2} \right),$$

where $n_s \in \mathbb{Z}$.[†] Using the above formula:

$$N_{\mathrm{DW}} = \begin{cases} \text{minimum integer } |2Nn_x| \text{ in } \mathrm{B_I},\ \mathrm{B_{II}},\ \mathrm{B_{IV}}, \text{ and extensions,} \\ \text{minimum integer } |6Nn_x| \text{ in } \mathrm{B_{III}} \text{ and extensions.} \end{cases}$$

Clearly, $N_{\mathrm{DW}} \neq 1$ in all four base models.
In the base model extensions,

$$\text{For } N_{\mathrm{DW}} = 1: \quad N = \begin{cases} \pm\frac{1}{2} \text{ in model extensions of } \mathrm{B_I},\ \mathrm{B_{II}}, \text{ and } \mathrm{B_{IV}}, \\ \pm\frac{1}{6} \text{ in model extensions of } \mathrm{B_{III}}. \end{cases}$$

[†] See A. Ernst, A. Ringwald, C. Tamarit 1801.04906

Lower bound on the axion decay constant

The red giant bound on the axion-electron coupling

$$g_{Ae} \quad > \quad 1.3 \times 10^{-13},$$

sets the most stringent astrophysical constraint throughout our supersymmetric DFSZ axion model space:

$$f_A \quad > \quad \frac{\sin^2\beta}{|N|}\,(3.9 \times 10^9 \text{ GeV}).$$

For large $\tan\beta$, the lower bound on the axion decay constant for

$$\begin{aligned} |N| = 1/6: &\quad f_A \gtrsim 2.3 \times 10^{10} \text{ GeV}, \\ |N| = 1/2: &\quad f_A \gtrsim 7.8 \times 10^{9} \text{ GeV}, \\ |N| = 3: &\quad f_A \gtrsim 1.3 \times 10^{9} \text{ GeV}. \end{aligned}$$

B and L violating operators

Renormalizable operators:

$$W_{\text{L-violating}} = H_u \ell + q\ell\overline{d} + \ell\ell\overline{e}, \qquad W_{\text{B-violating}} = \overline{u}\,\overline{d}\,\overline{d}.$$

The most common way of avoiding rapid proton decay due to these operators is to impose R-parity.

There are also non-renormalizable operators that mediate proton decay:

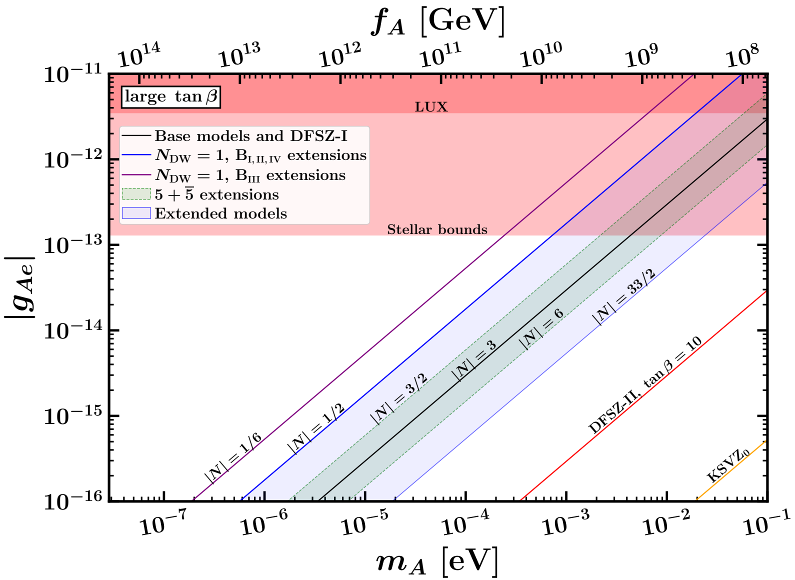
$$W = \frac{1}{M_P} qqq\ell + \frac{1}{M_P}\overline{u}\,\overline{u}\,\overline{d}\,\overline{e}.$$

The discrete charges $z_{\mathcal{O}} - 2r$ charges:

$\mathcal{O}$	B_{I}	B_{II}, B_{IV}	B_{III}
$H_u\ell$	$-r$	$-r$	$-r$
$\ell\ell\overline{e}$, $q\ell\overline{d}$	$-2x+r$	$2x-r$	$-6x+3r$
$\overline{u}\,\overline{d}\,\overline{d}$	$h-4x+4r$	$h+4x$	$h-12x+8r$
$qqq\ell$	$-h-r$	$-h-r$	$-h-r$
$\overline{u}\,\overline{u}\,\overline{d}\,\overline{e}$	$h-4x+5r$	$h+4x+r$	$h-12x+9r$

Here, x, h are the Z_n^R charges of X, H_u superfields.

Axion-electron coupling



Axion-neutron coupling