High-quality axions in solutions to the μ problem

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Based on work with Stephen P. Martin, arXiv:hep-ph/2106.14964

The Kim-Nilles mechanism

Consider MSSM (without -term) + two gauge-singlets X and Y:

$$W_{\mathrm{I}} \qquad \frac{}{M_P} X Y H_u H_d + \frac{}{6 M_P} X^3 Y;$$

respecting $U(1)_{\mathrm{PQ}}$.

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The total scalar potential (with the soft terms) has a local minimum for

$$\langle X \rangle \qquad \langle Y \rangle \qquad \sqrt{m_{\mathrm{soft}} M_P} \qquad M_{\mathrm{int}};$$

with m_{soft} TeV scale, and M_{int} in the range

$$10^9\ \mathrm{GeV} < M_{\mathrm{int}} < 10^{12}\ \mathrm{GeV}.$$

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The low-energy theory now contains:

- $= \frac{\mu}{M_P} \langle XY \rangle \quad m_{\mathrm{soft}}$
- An invisible DFSZ-type QCD axion

solving the problem and the strong CP problem!

The four base models[†]

Base model	Superpotential terms	PQ charges of (X; Y)
$\mathrm{B_I}$	$XYH_uH_d + X^3Y$	(1; 3)
$\mathrm{B_{II}}$	$X^2H_uH_d + X^3Y$	(1; 3)
$\mathrm{B_{III}}$	$Y^2H_uH_d + X^3Y$	$\frac{1}{3}$; 1
$\mathrm{B_{IV}}$	$X^2H_uH_d + X^2Y^2$	(1; 1)

[†] B_{I} proposed in H. Murayama, H. Suzuki, T. Yanagida Phys. Lett. B **291**, 418-425 (1992); B_{II} in K. Choi, E. J. Chun, J. E. Kim hep-ph/9608222; B_{III}; B_{IV} in S. P. Martin hep-ph/0005116;

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In terms of the PQ charges of the MSSM quark and lepton doublets Q_q, Q_ℓ

	H_u	H_d	$\overline{u}$	$\overline{d}$	$\overline{e}$
PQ charge	$2c^2$	$2s^2$	$2c^2$ Q_q	$2s^2$ Q_q	$2s^2$ Q_ℓ

where tan $= s$ $=c$ is the ratio of the Higgs VEVs.

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g_A suppressed in all four base models!

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Cosmological domain wall problem

$$U(1)_{\text{PQ}} \xrightarrow{\text{PQ breaking}} Z_{N_{\text{DW}}} \text{ discrete symmetry}$$

Domain wall number (N_{DW}): number of discrete set of inequivalent degenerate minima of the axion potential.

†See e.g. P. Sikivie Phys. Rev. Lett. **48**, 1156-1159 (1982)

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Formation of topological defects such as stable DWs, due to the different possible phases of the axion, which dominate the universe†

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Some solutions

- If PQ breaking happens before inflation
- $N_{\text{DW}} = 1$ (**our focus**)

$N_{\text{DW}} \neq 1$ in all four base models.

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Base model extensions

Consistent with gauge coupling unification, we consider the following extensions:

- $\mathbf{5}+\overline{\mathbf{5}}$ at TeV or M_{int}
- $\mathbf{10}+\overline{\mathbf{10}}$ at TeV
- $\mathbf{10}+\overline{\mathbf{10}}$ at M_{int}
- $(\mathbf{5}+\overline{\mathbf{5}})$ or $(\mathbf{10}+\overline{\mathbf{10}})$ at TeV, $(\mathbf{5}+\overline{\mathbf{5}})$ or $(\mathbf{10}+\overline{\mathbf{10}})$ at M_{int}

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(the last two items are bracketed with: $N_{\text{DW}}=1$ possible)

Here,

$$
\begin{aligned}
\overline{\mathbf{5}} &= \underbrace{(\overline{\mathbf{3}},\mathbf{1},1/3)}_{\overline{D}} + \underbrace{(\mathbf{1},\mathbf{2},-1/2)}_{L} \\
\mathbf{10} &= \underbrace{(\mathbf{3},\mathbf{2},1/6)}_{Q} + \underbrace{(\overline{\mathbf{3}},\mathbf{1},-2/3)}_{\overline{U}} + \underbrace{(\mathbf{1},\mathbf{1},1)}_{\overline{E}}
\end{aligned}
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We allow for different components of the $\mathbf{5}+\overline{\mathbf{5}}$ and/or $\mathbf{10}+\overline{\mathbf{10}}$ to have different mass source terms.

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$\Big\}$ $N_{\text{DW}} = 1$ possible (applies to the last two extensions)

Here,

$$\overline{\mathbf{5}} = \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, 1/3)}_{\overline{D}} + \underbrace{(\mathbf{1}, \mathbf{2}, -1/2)}_{L}$$

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Extensions with $N_{\text{DW}} = 1$ give rise to enhanced low-energy axion couplings!

Vectorlike mass terms

Assuming the same mechanism that gives a term also gives masses to vectorlike pairs of chiral superfields $\Phi + \overline{\Phi}$.

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TeV scale masses:

$$W_{\text{mass}} = \begin{cases} \frac{\Phi}{M_P} XY\Phi\overline{\Phi}; \\ \frac{\Phi}{2M_P} X^2\Phi\overline{\Phi}; \\ \frac{\Phi}{2M_P} Y^2\Phi\overline{\Phi}; \end{cases}$$

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Mass terms fix the PQ charge of the terms $\Phi\overline{\Phi}$ which in turn fix the low-energy axion couplings, independent of the Yukawa terms.

The axion quality problem

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Higher dimensional operators from quantum gravity can explicitly violate global $U(1)_{\text{PQ}}$ and reintroduce the strong CP problem

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In our case, consider

$$W = \frac{\quad}{M_P^{p\ \ 3}} X^j Y^{p\ \ j}$$

that contributes to the axion potential (with soft terms), giving rise to:

$$j\ {}_{\text{eff}} j \quad = \quad \frac{\quad}{(0{:}0754\ \text{GeV})^4} \frac{f_A^{p+2}}{M_P^{p\ \ 2}};$$

with a dimensionless quantity , and f_A identified with M_{int}.

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Solution

We find that $X^j Y^{p\ \ j}$ with $p < 7$ should be forbidden for $j\ \ _{\text{eff}} j < 10^{\ \ 10}$

Non-R and R discrete Z_n symmetries

	gauginos	W	chiral superfield Φ	fermion in Φ
Z_n^R charge (mod n)	r	$2r$	z_Φ	$z_\Phi \quad r$

For non-R symmetry $r = 0$, and for R-symmetry $0 < r < n$=2.

In both cases, $z_\Phi = 0; 1; : : : ; n \quad 1$.

† See e.g. L. E. Ibanez arXiv:hep-ph/9210211

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With a normalization where $Z_n^R \quad G_{\mathrm{SM}} \quad G_{\mathrm{SM}}$ anomalies are integers, we impose the following anomaly-free conditions:†

$$A_2 = A_3 = \quad_{\mathrm{GS}} \ (\mathrm{mod}\ n);$$

for the weaker condition

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for the weaker condition, with the additional stronger condition

$$A_1 = 5A_3 = 5 \ _{\mathrm{GS}} \ (\mathrm{mod}\ n);$$

which does not require the Green-Schwarz (GS) mechanism if $\ _{\mathrm{GS}} = 0$.

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Examples with non-R Z_n symmetries: Base models

Stronger constraints with $_{\mathrm{GS}} \neq 0$: (Here, $m = 0; 1; 2$)

Model	Z_n	X	H_u	p	$_{\mathrm{GS}}$
$\mathrm{B_{III}}$	36	1	$8 + 12m$	12	18
$\mathrm{B_{IV}}$	36	3	4	8	18

† proposed and studied in K. S. Babu, I. Gogoladze, K. Wang hep-ph/0212245.

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Weaker constraint with $_{\mathrm{GS}} \neq 0$: Lots of cases, e.g., a Z_{22} symmetry†

Model	Z_n	X	H_u	p	$_{\mathrm{GS}}$
$\mathrm{B_{IV}}$	22	2	2	11	12

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Examples with Z_n^R symmetries: Base models

Stronger constraints with $_{\mathrm{GS}} = 0$: Some examples,

Model	Z_n^R	r	X	H_u	p
$\mathrm{B_{III}}$	54	3	5	$1 + 18m$	10
$\mathrm{B_{IV}}$	12	1	8	$1 + 4m$	7

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Stronger constraints with $_{\text{GS}} \neq 0$: As a special case, we found a Z_{24}^R symmetry with $SU(5)$ invariance†

Model	Z_n^R	r	X	H_u	p	$_{\text{GS}}$
B_{II}	24	1	11	1	10	18
B_{III}	24	1	5	1	10	18

We do not impose $SU(5)$ invariance.

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Examples with $Z_n^{(R)}$ symmetries: Base model extensions

Stronger constraints: Examples,

Base	Extension	Z_n^R	r	X	H_u	p	GS
B_{I}	$XYD\overline{D} + X^2L\overline{L}$	34	1	31	15	12	16
B_{II}	$Y^2D\overline{D} + Y^2L\overline{L}$	108	6	11	$22 + 36m$	20	0
B_{III}	$X^2Q\overline{Q} + X^2U\overline{U} + Y^2E\overline{E}$	42	0	1	$8 + 14m$	14	18
B_{IV}	$XD\overline{D} + YL\overline{L}$	20	0	1	8	12	5

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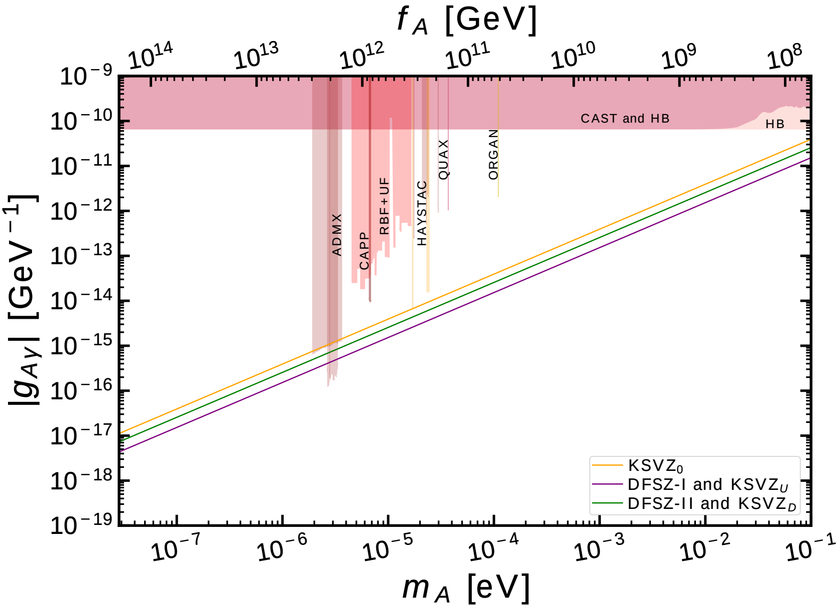
Can find an anomaly-free $Z_n^{(R)}$ symmetry protecting $U(1)_{\text{PQ}}$ for each model thus giving rise to a high-quality axion!

Axion-photon coupling (limits)



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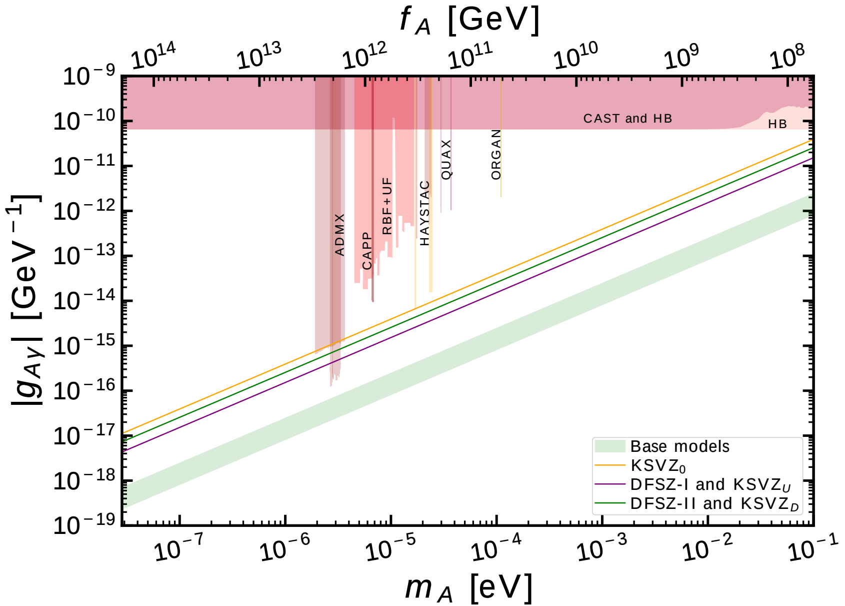

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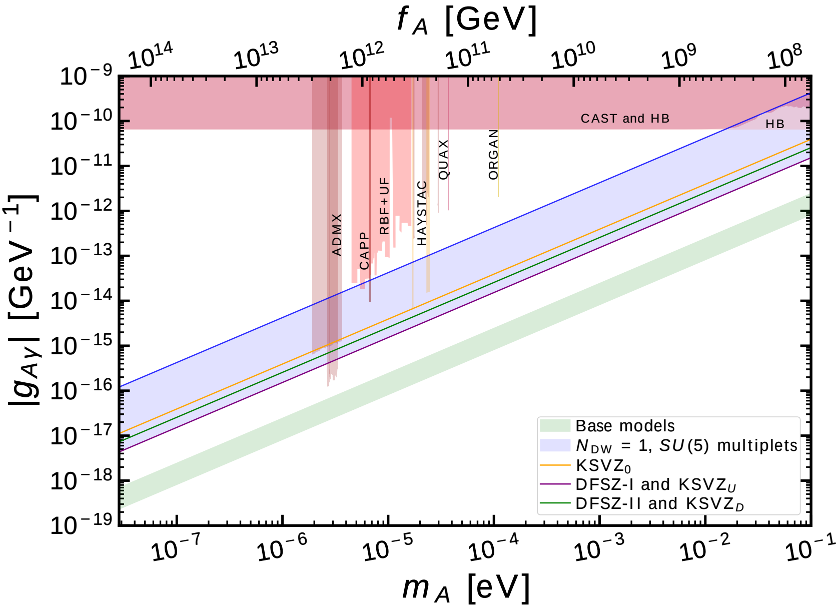

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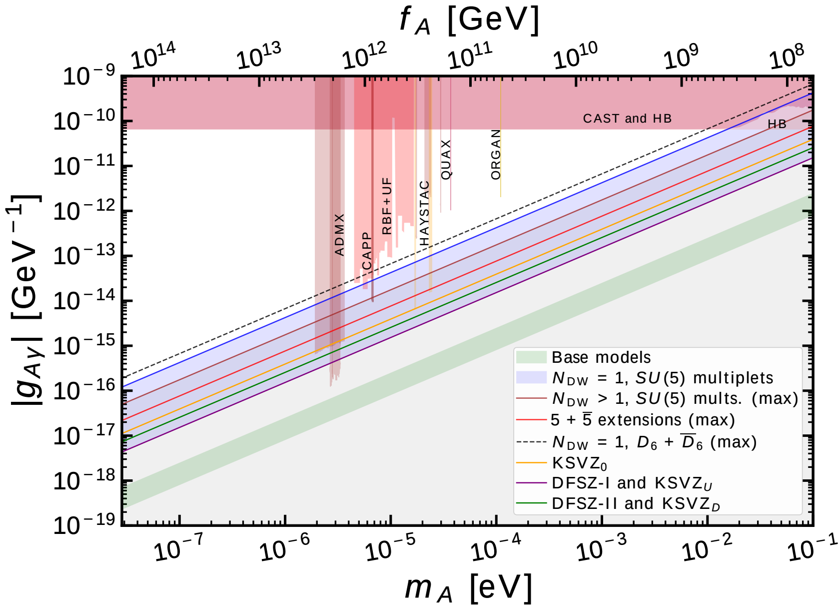
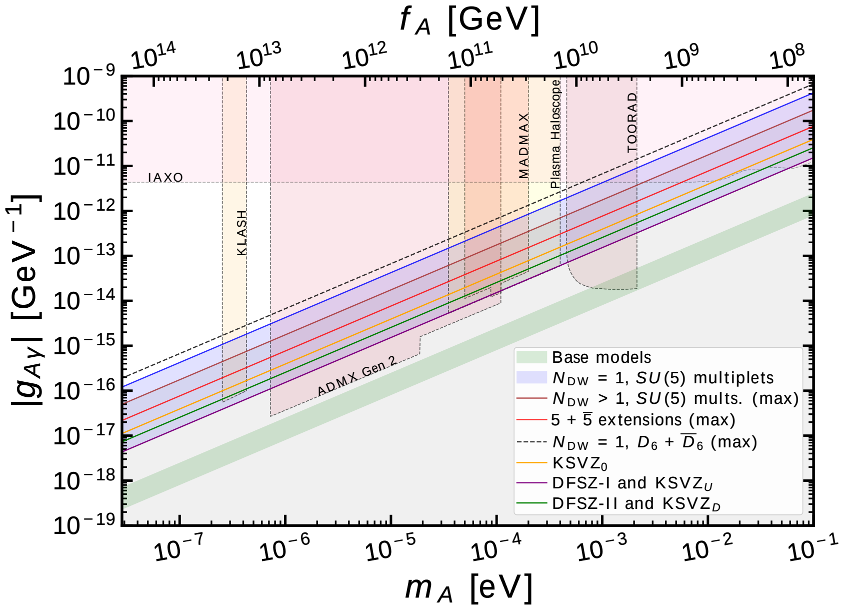
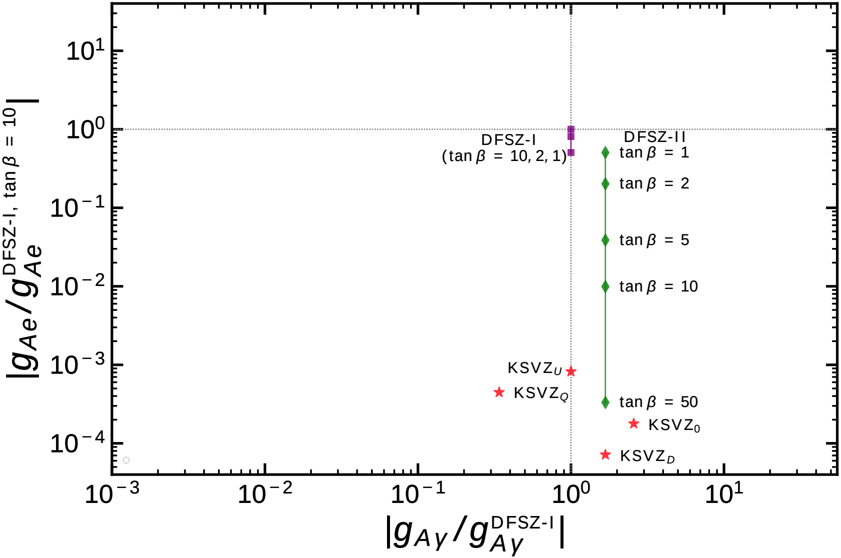


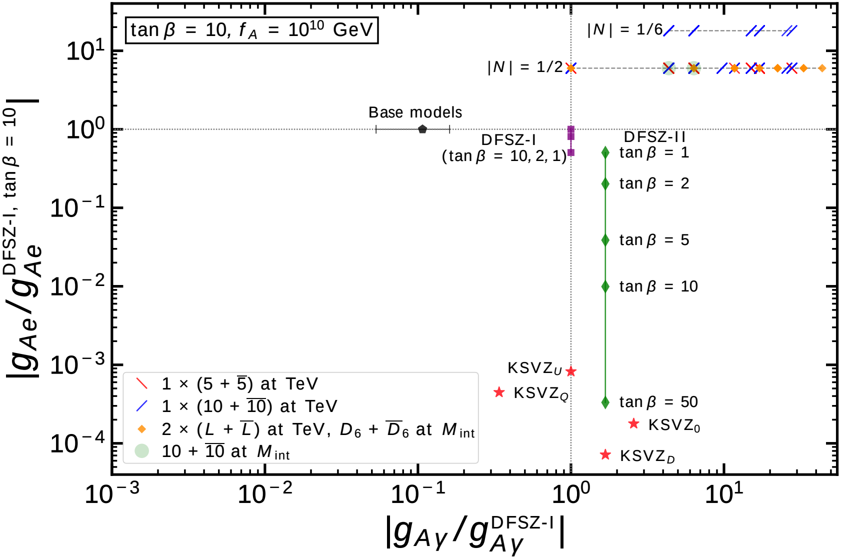
Axion-photon coupling (projections)

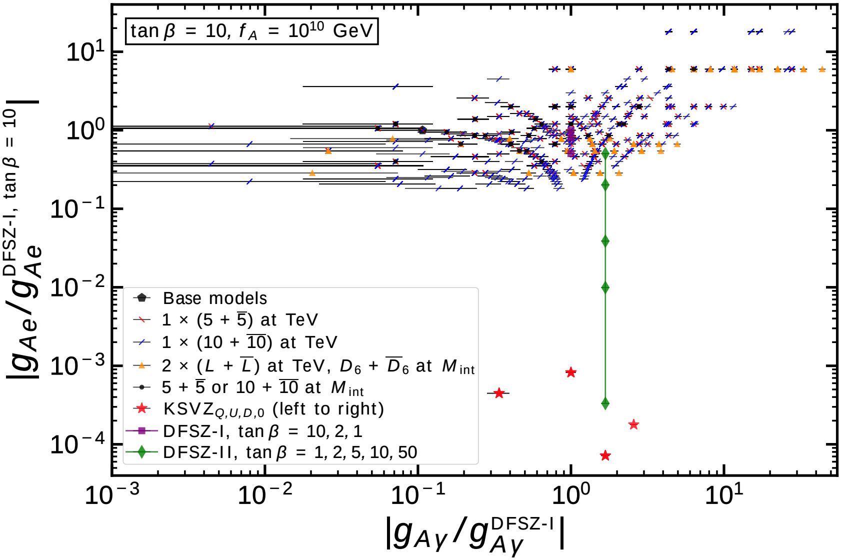


$|g_{Ae}/g_{Ae}^{\text{DFSZ-I},\tan\beta=10}|$ vs. $|g_{A}/g_{A}^{\text{DFSZ-I}}|$



$|g_{Ae}/g_{Ae}^{\text{DFSZ-I},\tan\beta=10}|$ vs. $|g_{A\gamma}/g_{A\gamma}^{\text{DFSZ-I}}|$




tan β = 10, f_A = 10^10 GeV
Base models
1 × (5 + 5̄) at TeV
1 × (10 + 10̄) at TeV
2 × (L + L̄) at TeV, D_6 + D̄_6 at M_int
5 + 5̄ or 10 + 10̄ at M_int
KSVZ_{Q,U,D,0} (left to right)
DFSZ-I, tan β = 10, 2, 1
DFSZ-II, tan β = 1, 2, 5, 10, 50
|g_Ae/g_Ae^{DFSZ-I, tan β = 10}|
|g_Aγ/g_Aγ^{DFSZ-I}|

Conclusion

Supersymmetry by itself addresses the electroweak hierarchy puzzle.

We considered extensions with extra vectorlike content that:

- have high-quality QCD axions within the reach of future axion searches
- simultaneously solve the problem
- evade cosmological domain wall problem
- maintain gauge coupling unification

BACKUP SLIDES

Non-trivial QCD vacuum structure requires the term:

$$L_{\text{QCD}} \quad \frac{g_s^2}{32\ ^2} G^a \quad \tilde{G}^a \ ;$$

where the QCD vacuum angle   is expected to be $O(1)$.

> "Everything not forbidden is compulsory."

However, experimentally:

$$j\ j < 10\ ^{10} :$$

Why so small?   strong CP problem

Peccei-Quinn (PQ) solution: promote   to a dynamical field

Lightning review: Peccei-Quinn (PQ) solution

Consider a global $U(1)_{\mathrm{PQ}}$ axial symmetry:

$$\partial_\mu j^\mu_{\mathrm{PQ}} = \underbrace{\frac{g_s^2 N}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}}_{\text{QCD anomaly}} + \underbrace{\frac{e^2 E}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\text{EM anomaly}},$$

with left-handed fermions with PQ charge Q_f, $SU(3)_c$ index $T(R_f)$, and EM charge q_f contributing to:

$$\begin{aligned} N &= \mathrm{Tr}\left[Q_f T(R_f)\right], \\ E &= \mathrm{Tr}\left[Q_f q_f^2\right]. \end{aligned}$$

$U(1)_{\mathrm{PQ}}$ can be spontaneously broken by scalars with PQ charge Q_s

$$\varphi_s \supset \frac{v_s}{\sqrt{2}} e^{i a_s / v_s}.$$

With $V^2 = \sum_s Q_s^2 v_s^2$, the axion field is given by:

$$A = \frac{1}{V} \sum_s Q_s v_s a_s.$$

Ensuring the axion is massless at tree-level by imposing:

$$\sum_s Y_s Q_s v_s^2 = 0;$$

where Y_s: weak hypercharge of φ_s.
QCD vaccum term now becomes:

$$L_{\rm QCD} \quad + \frac{A}{f_A} \quad \frac{g_s^2}{32\ ^2} G^a \quad \tilde{G}^a \ ;$$

with the axion decay constant

$$f_A \quad \frac{V}{2N}.$$

Under $U(1)_{\rm PQ}$ transformations:

$$A \to A + (\text{constant})\, f_A;$$

Thus solving the strong CP problem.

Lightning review: Low-energy axion couplings

$$L^{A}_{\text{int}} \quad \frac{1}{4} g_{A} \;\; AF \quad \tilde{F} \qquad \underset{f=e;n;p}{\text{X}} \; i g_{Af} A \overline{\Psi}_f \;\; {}_5 \Psi_f$$

where,

$$g_{A} \quad = \quad \frac{e}{2 \;\; f_A} \left(c \qquad 1{:}92(4) \right);$$

$$g_{Ae} \quad = \quad \frac{m_e}{f_A} \;\; c_e + \frac{3}{4} \frac{{}^2_e}{{}^2} \;\; c \;\; \log \frac{f_A}{m_e} \qquad 1{:}92(4) \log \frac{\text{GeV}}{m_e} \qquad ;$$

$$g_{An} \quad = \quad \frac{m_n}{f_A} \left(\quad 0{:}02(3) + 0{:}833(30) c_d \qquad 0{:}406(21) c_u \right):$$

with

$$c \;\; = \frac{E}{N}; \quad c_e = \frac{Q_{\grave{}} + Q_{\overline{e}}}{2N}; \quad c_u = \frac{Q_q + Q_{\overline{u}}}{2N}; \quad c_d = \frac{Q_q + Q_{\overline{d}}}{2N}:$$

Axion can accidentally decouple from photons if $E{=}N \qquad 1{:}92$.

Non-supersymmetric benchmark QCD axion models[†]

Benchmark	PQ charged fermions	N	c	c_u	c_d	c_e
KSVZ_0	$(\mathbf{3}; \mathbf{1}; 0) + (\overline{\mathbf{3}}; \mathbf{1}; 0)$	$\frac{1}{2}$	0	0	0	0
KSVZ_D	$D + \overline{D}$	$\frac{1}{2}$	$\frac{2}{3}$	0	0	0
KSVZ_U	$U + \overline{U}$	$\frac{1}{2}$	$\frac{8}{3}$	0	0	0
KSVZ_Q	$Q + \overline{Q}$	1	$\frac{5}{3}$	0	0	0
DFSZ-I	SM fermions	3	$\frac{8}{3}$	$\frac{c_\beta^2}{3}$	$\frac{s_\beta^2}{3}$	$\frac{s_\beta^2}{3}$
DFSZ-II	SM fermions	3	$\frac{2}{3}$	$\frac{c_\beta^2}{3}$	$\frac{s_\beta^2}{3}$	$\frac{c_\beta^2}{3}$

where tan $= s$ $=c$ is the ratio of Higgs VEVs in the DFSZ models.

[†] J. E. Kim Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, V. I. Zakharov Nucl. Phys. B **166**, 493-506 (1980); M. Dine, W. Fischler, M. Srednicki Phys. Lett. B **104**, 199-202 (1981); A. R. Zhitnitsky Sov. J. Nucl. Phys. **31**, 260 (1980)

Quixotic extension

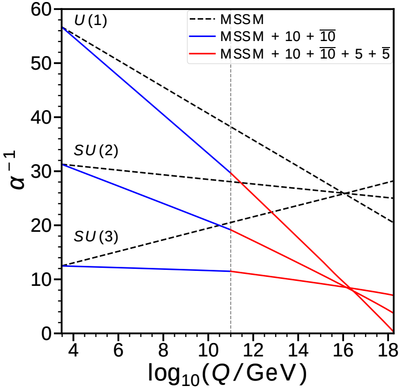
Consistent with gauge coupling unification, we can also consider

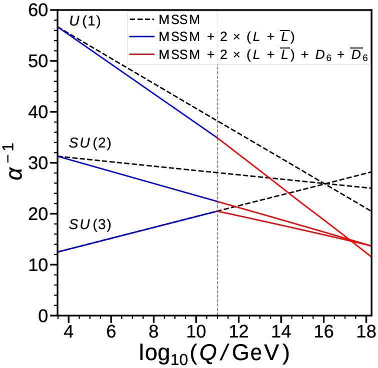
***D_6LL* models**: 2 $(L+\overline{L})$ at TeV, $D_6+\overline{D}_6$ at M_{int} 10^{11} GeV ($N_{\text{DW}}=1$ possible)

where

$$D_6+\overline{D}_6 \quad = \quad (\mathbf{6}; \mathbf{1}; 1{=}3) + (\overline{\mathbf{6}}; \mathbf{1}; \;\; 1{=}3) \quad \text{is an exotic quix pair}$$

Gauge coupling unification





$N_{\mathrm{DW}} = 1$ in base model extensions

$$N_{\mathrm{DW}} \equiv \text{minimum integer}\left(2N \sum_s \frac{n_s Q_s v_s^2}{V^2}\right),$$

where $n_s \in \mathbb{Z}$.[†] Using the above formula:

$$N_{\mathrm{DW}} = \begin{cases} \text{minimum integer}\,|2Nn_x| \text{ in } \mathrm{B_I},\ \mathrm{B_{II}},\ \mathrm{B_{IV}}, \text{ and extensions}, \\ \text{minimum integer}\,|6Nn_x| \text{ in } \mathrm{B_{III}} \text{ and extensions}. \end{cases}$$

Clearly, $N_{\mathrm{DW}} \neq 1$ in all four base models.
In the base model extensions,

$$\text{For } N_{\mathrm{DW}} = 1: \quad N = \begin{cases} \frac{1}{2} \text{ in model extensions of } \mathrm{B_I},\ \mathrm{B_{II}}, \text{ and } \mathrm{B_{IV}}, \\ \frac{1}{6} \text{ in model extensions of } \mathrm{B_{III}}. \end{cases}$$

[†] See A. Ernst, A. Ringwald, C. Tamarit 1801.04906

Lower bound on the axion decay constant

The red giant bound on the axion-electron coupling

$$g_{Ae} > 1.3 \times 10^{-13},$$

sets the most stringent astrophysical constraint throughout our supersymmetric DFSZ axion model space:

$$f_A > \frac{\sin^2\beta}{|N|}\left(3.9 \times 10^{9}\ \mathrm{GeV}\right).$$

For large tan β, the lower bound on the axion decay constant for

$$|N| = 1/6: \quad f_A \gtrsim 2.3 \times 10^{10}\ \mathrm{GeV},$$
$$|N| = 1/2: \quad f_A \gtrsim 7.8 \times 10^{9}\ \mathrm{GeV},$$
$$|N| = 3: \quad f_A \gtrsim 1.3 \times 10^{9}\ \mathrm{GeV}.$$

B and L violating operators

Renormalizable operators:

$$W_{\text{L-violating}} \quad = \quad H_u \ell + q \ell \overline{d} + \ell \ell \overline{e}; \qquad W_{\text{B-violating}} = \overline{u}\,\overline{d}\,\overline{d}:$$

The most common way of avoiding rapid proton decay due to these operators is to impose R-parity.

There are also non-renormalizable operators that mediate proton decay:

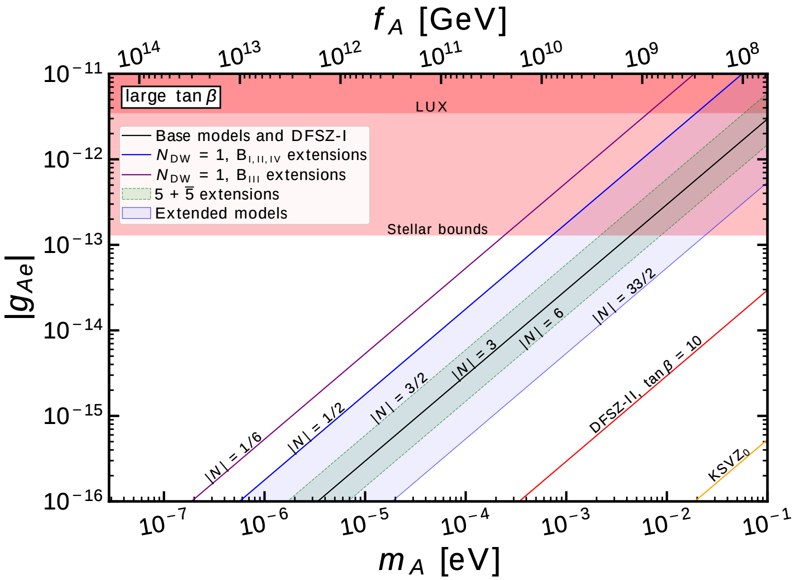
$$W \quad = \quad \frac{1}{M_P} qqq\ell + \frac{1}{M_P} \overline{u}\,\overline{u}\,\overline{d}\,\overline{e}:$$

The discrete charges $z_O \quad 2r$ charges:

O	B_I	B_{II}, B_{IV}	B_{III}
$H_u \ell$	r	r	r
$\ell \ell \overline{e}$, $q \ell \overline{d}$	$2x + r$	$2x \quad r$	$6x + 3r$
$\overline{u}\,\overline{d}\,\overline{d}$	$h \quad 4x + 4r$	$h + 4x$	$h \quad 12x + 8r$
$qqq\ell$	$h \quad r$	$h \quad r$	$h \quad r$
$\overline{u}\,\overline{u}\,\overline{d}\,\overline{e}$	$h \quad 4x + 5r$	$h + 4x + r$	$h \quad 12x + 9r$

Here, $x; h$ are the Z_n^R charges of $X; H_u$ superfields.

Axion-electron coupling



Axion-neutron coupling