# Traces of a Heavy Field in Gravitational Waves

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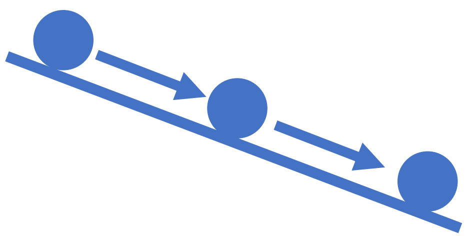
# Overview

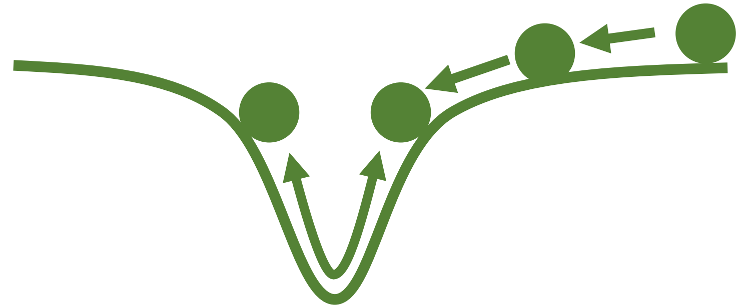
I discuss the situation where large GWs are produced during inflation.

**During inflation**

**Inflaton ($\phi$)**

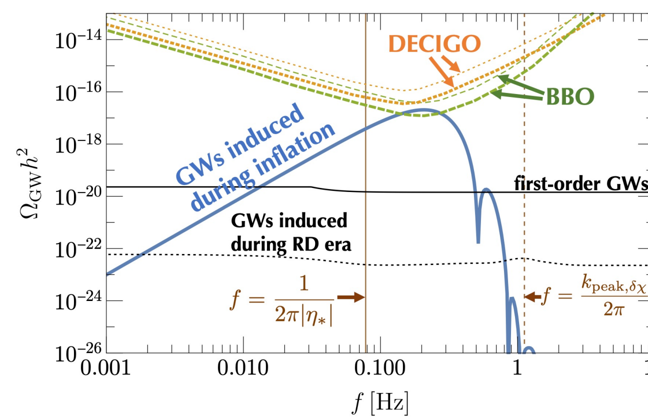


**Spectator field ($\chi$)**



We consider the case where a heavy spectator field starts to oscillate during inflation.

In this case, the spectator field fluctuations can be amplified because of the parametric resonance, which induces large GWs.



# Outline

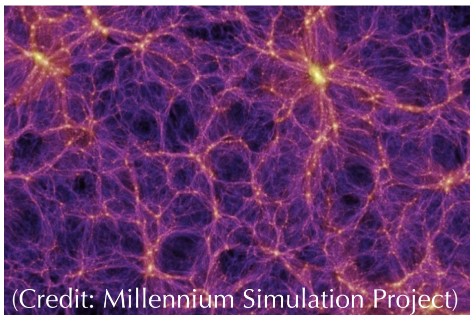
- Introduction
- GW production during inflation
- Summary

# Scalar perturbations in Cosmology

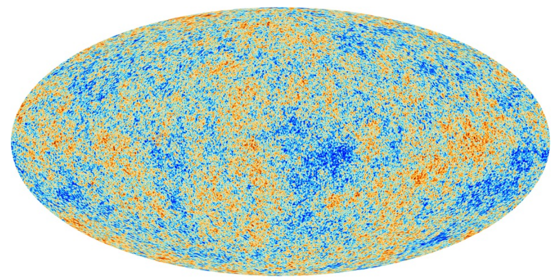
Scalar perturbations are one of the most important quantities in Cosmology.

Scalar perturbations are origins of many things.

Examples


(Credit: Millennium Simulation Project)

Large Scale Structure


(Credit: ESA and the Planck Collaboration)

CMB anisotropies

From the observations, we already know the amplitude of the scalar perturbations.

$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \quad \text{(Planck 2018)}$$

$$(\delta\rho/\rho \sim 10^{-5})$$

Scalar perturbations originate from vacuum fluctuations of fields during the inflation era.

# GWs induced by scalar perturbations (Tomita 1967)

Metric perturbations:

Scalar perturbations (related to curvature perturbations)

$$ds^2 = a^2\left[-(1+2\Phi)d\eta^2 + \left((1-2\Phi)\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j\right]$$

Tensor perturbations (GWs)

$h^i_i = 0$
$\partial_i h^i_j = 0$

Einstein equation:

$$G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}$$

$$G^i_j = a^{-2}\left[\frac{1}{4}(h^{i''}_j + 2\mathcal{H}h^{i'}_j - \Delta h^i_j) + 4\Phi\partial^i\partial_j\Phi + 2\partial^i\Phi\partial_j\Phi + A^i_j\right]$$

$$T^i_j = (\rho + P)\delta u^{,i}\delta u_{,j} + (P+\delta P)\delta^i_j$$

Terms irrelevant to tensor perturbations

E.o.m. for tensor perturbations:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}$$

$\hat{\mathcal{T}}_{ij}{}^{lm}$ : Projection operator onto the transverse and traceless tensor

$(w = P/\rho)$

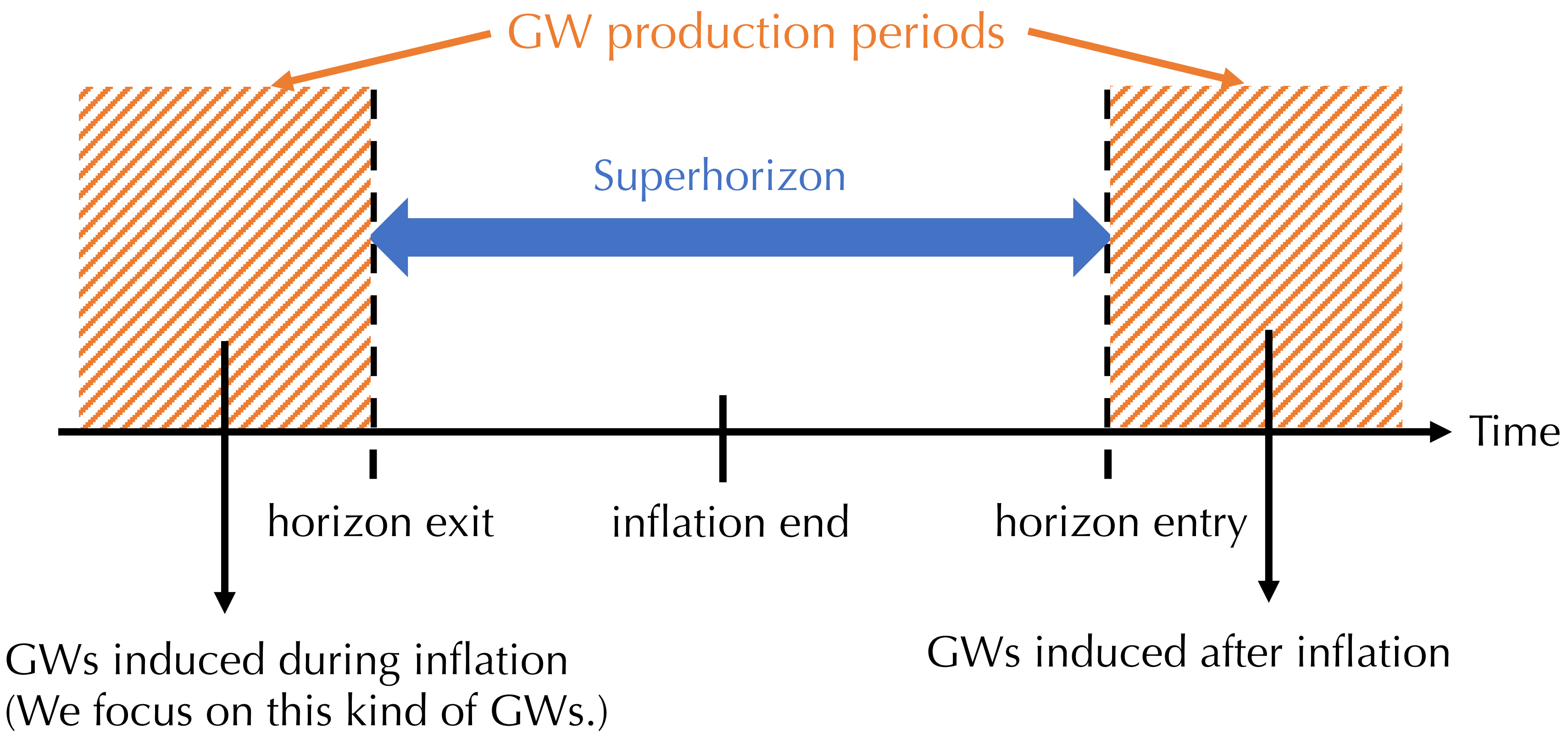
$$S_{ij} \simeq \begin{cases} -\frac{1}{M_{\rm Pl}^2}\partial_i\delta\phi\partial_j\delta\phi & \text{(during inflation)} \\ 4\Phi\partial_i\partial_j\Phi + 2\partial_i\Phi\partial_j\Phi - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Phi' + \mathcal{H}\Phi)\partial_j(\Phi' + \mathcal{H}\Phi) & \text{(after inflation)} \end{cases}$$

**Point** Scalar perturbations can induce GWs at second order.

# When the GWs are induced

Scalar perturbations can induce GWs during and after the inflation.

Evolution of a scalar perturbation



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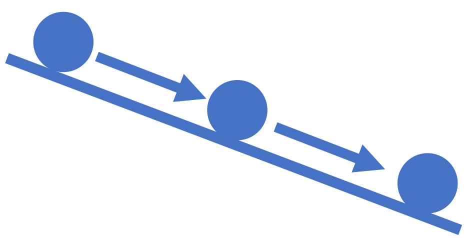
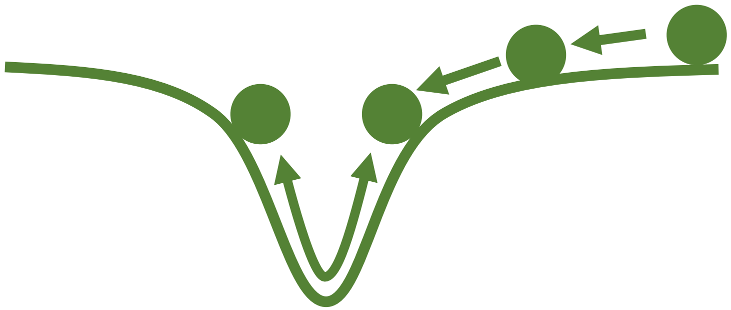
# Situation

**During inflation**




We consider the case where a spectator field starts to oscillate during inflation.

**Fiducial potential**

$$V(\phi, \chi) = V_1(\phi) + V_2(\chi)$$

$$V_1(\phi) = V_0(1 - \sqrt{2\epsilon_1}\phi/M_{\rm Pl}) + V_{\rm end}(\phi), \qquad (\Delta V/V_0 < 1)$$

$$V_2(\chi) = \Delta V \tanh^2\left(\frac{\chi}{\sqrt{6\alpha}M_{\rm Pl}}\right) + \tilde{V}_2(\chi).$$

α-attractor type potential

This is irrelevant to the potential around the minimum.

# Parametric resonance

**Simple example**

Mathieu's equation:

$$\frac{dy^2}{dx^2} + [A - 2q\cos(2x)]y = 0$$

If the frequencies match (A=1,4,...), the quantity grows exponentially:

$$y \propto \mathrm{e}^{\mu(A,q)x}$$

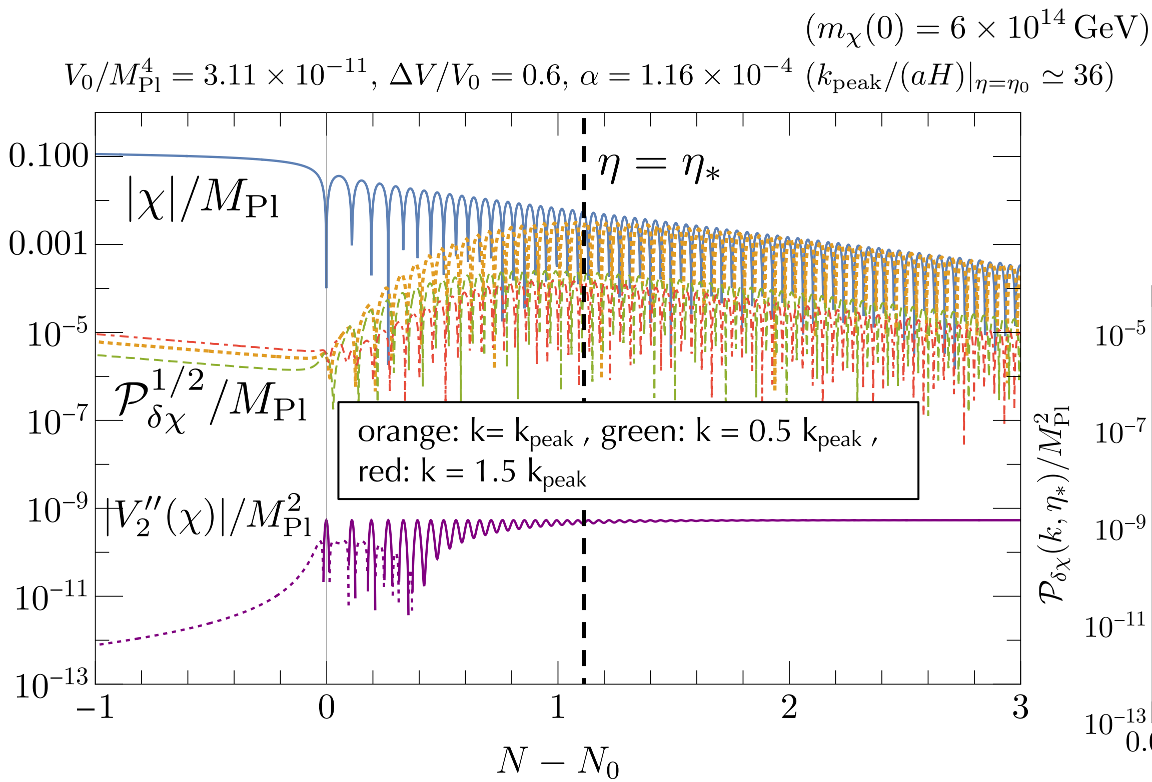
**Our setup**

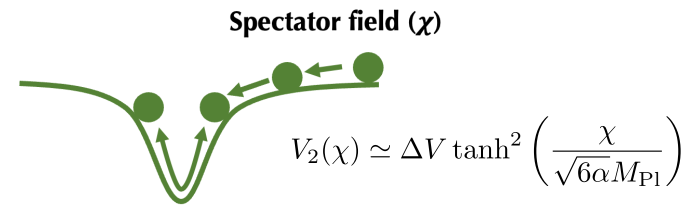
$$\delta\chi_k'' + 2\mathcal{H}\delta\chi_k' + k^2\delta\chi_k + a^2 m_\chi^2(\chi)\delta\chi_k \simeq 0 \qquad (m_\chi^2(\chi) \equiv V_2''(\chi))$$

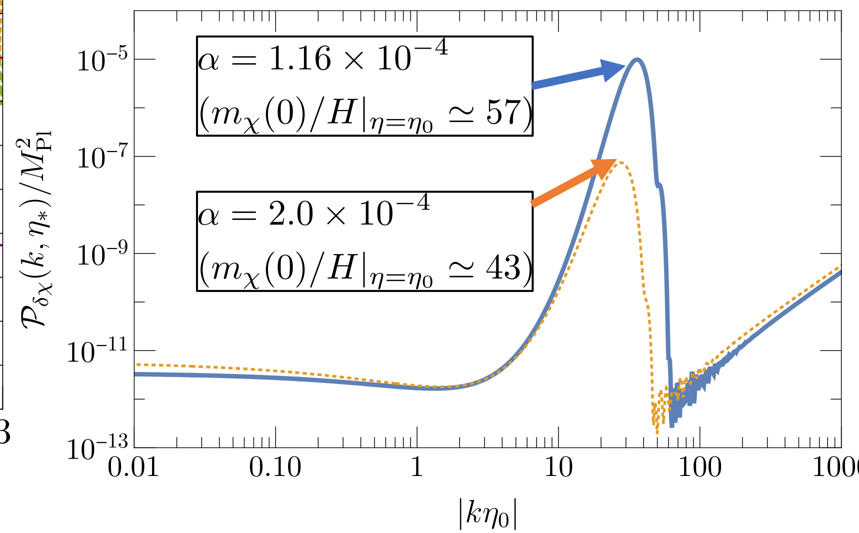
If the effective mass ($m_\chi$) oscillates, the perturbations on the resonance scale can grow exponentially.

Finally, the universe expansion stops the exponential growth.

# Evolution



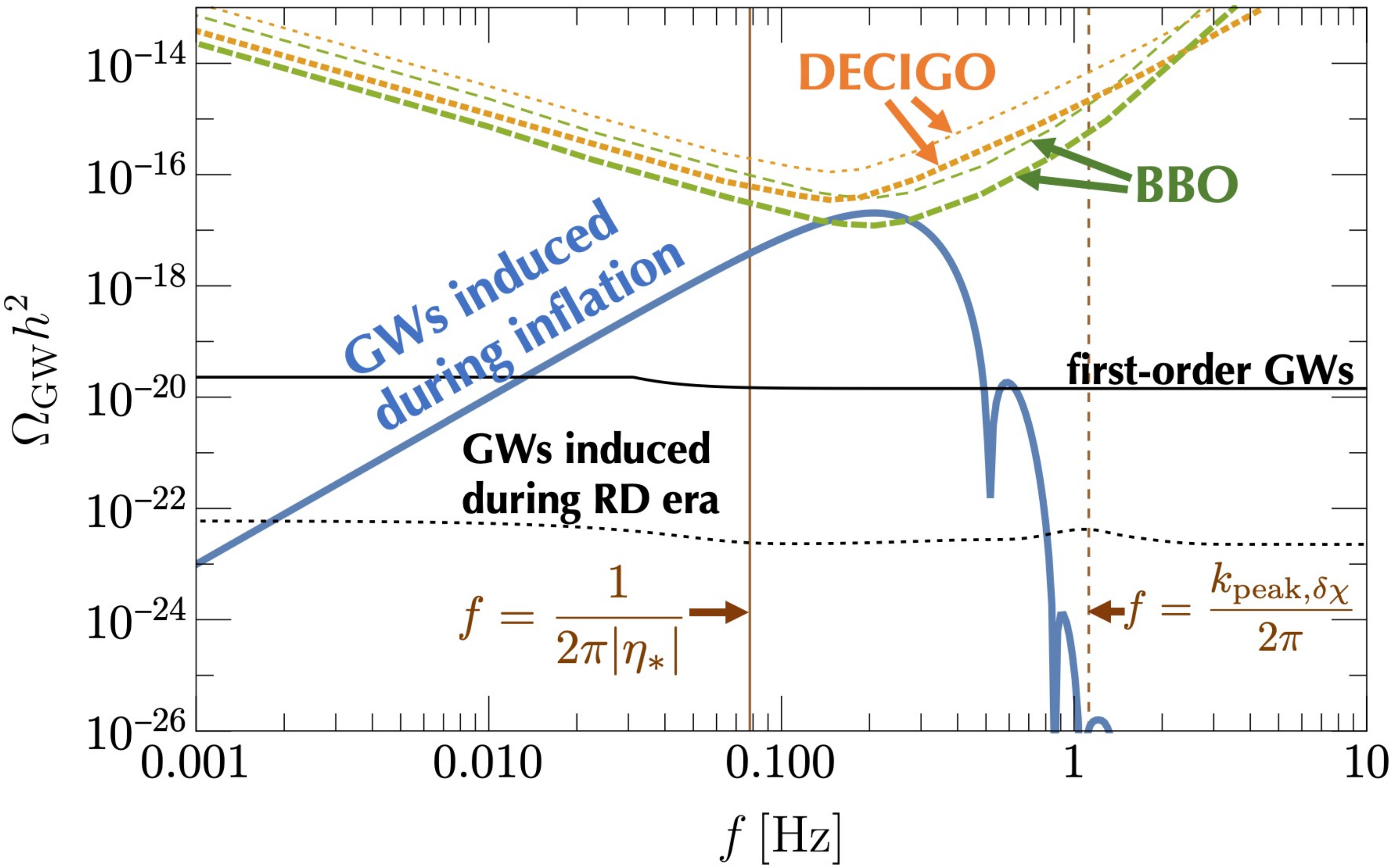




A larger mass causes a stronger resonance, but a too large mass causes the backreaction problem.

Similar resonances are often discussed in the context of preheating. (Khlebnikov and Tkachev (1997), Easther and Lim (2006), Garcia-Bellido and Figueroa (2007) )

# GW and curvature perturbation spectrum

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} \simeq \frac{4}{M_{\rm Pl}^2}\hat{\mathcal{T}}_{ij}{}^{lm}\partial_l\delta\chi\partial_m\delta\chi$$



# Necessary conditions for large GWs

**1. Oscillation time scale must be short enough compared to the Hubble time.**
Otherwise, the resonance stops due to universe expansion before the perturbations get enhanced significantly. So, the spectator field mass should be heavy.

**2. Oscillation time scale must not be too short.**
The energy density of the field fluctuations is upper-bounded as

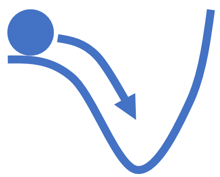
$$\rho_f \sim \mathcal{O}((\partial_i \delta\chi)^2) \sim \mathcal{O}(k_{\rm peak}^2 \delta\chi^2) < \Delta V$$

potential energy of the spectator

The shorter oscillation time scale leads to the smaller peak scale (larger $k_{\rm peak}$), which leads to stronger upper bound on $\delta\chi$. $(m_\chi(0)/H|_{\eta=\eta_0} \simeq 57$ in the fiducial setup)

**3. $\Delta V/V_0$ must be large enough.**
Once we fix $m_\chi/H$ , $\Delta V/V_0$ ($\sim O(m_\chi^2\chi^2/(H^2M_{Pl}^2))$) mainly determines the initial amplitude of the oscillation, which is the upper bound of the field fluctuations.

**4. The potential form must cause the oscillation of the effective mass.**
This kind of resonance can occur even in the hilltop type potential.
This condition can be avoided if the spectator has interactions with other fields as $\lambda\chi^2\varphi^2$.



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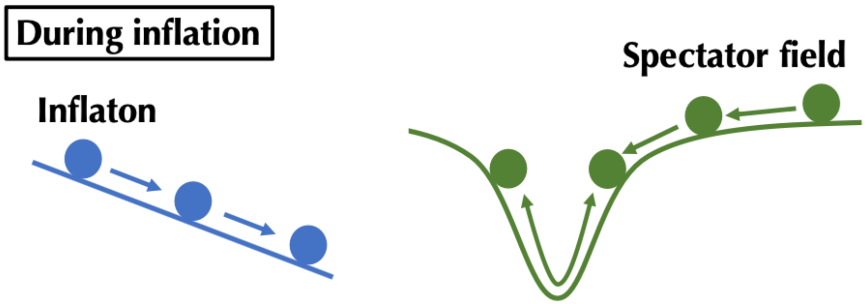
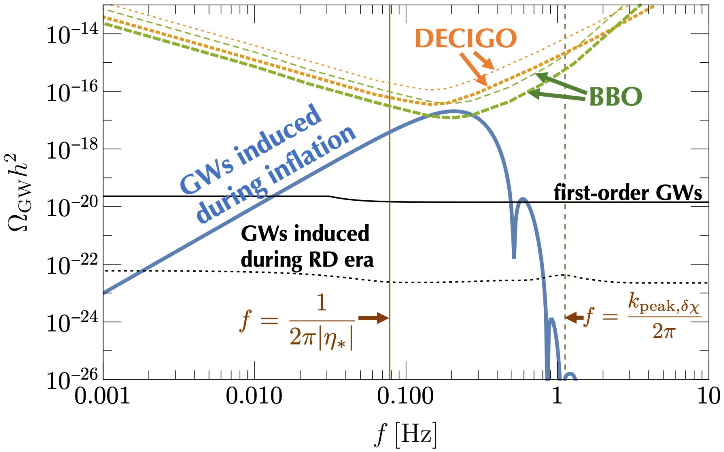



We have considered the case where a heavy spectator field starts to oscillate during inflation.

In this case, the spectator field fluctuations can be amplified because of the parametric resonance, which induces large GWs.

This kind of GWs could be used as detectable traces of a heavy field that is not coupled to any other fields.

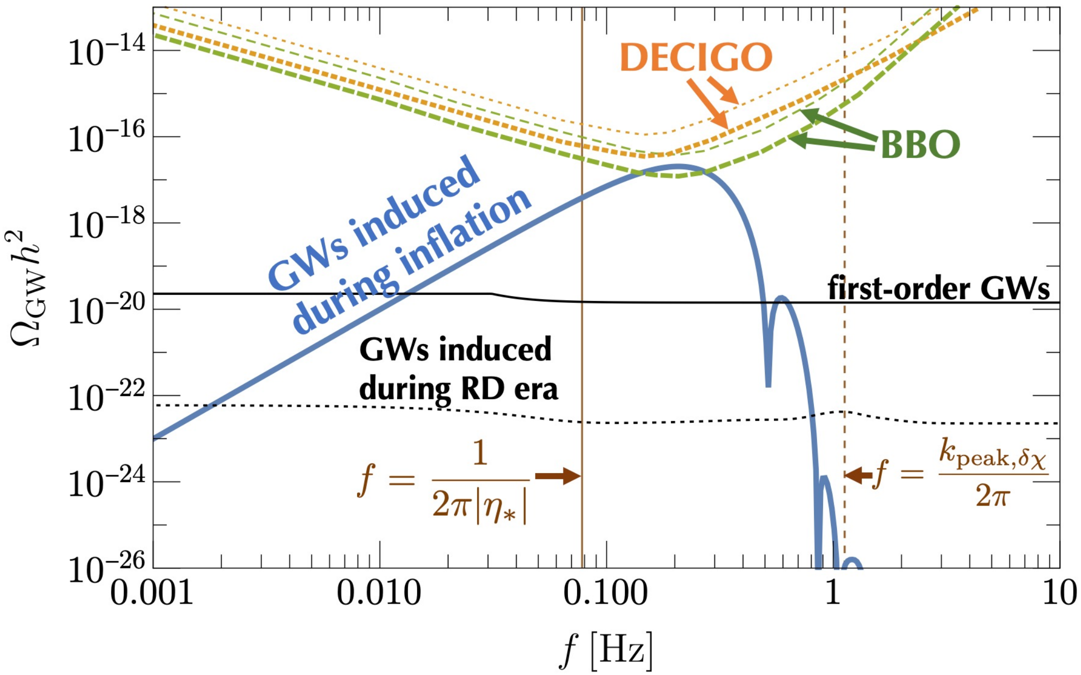
# Backup

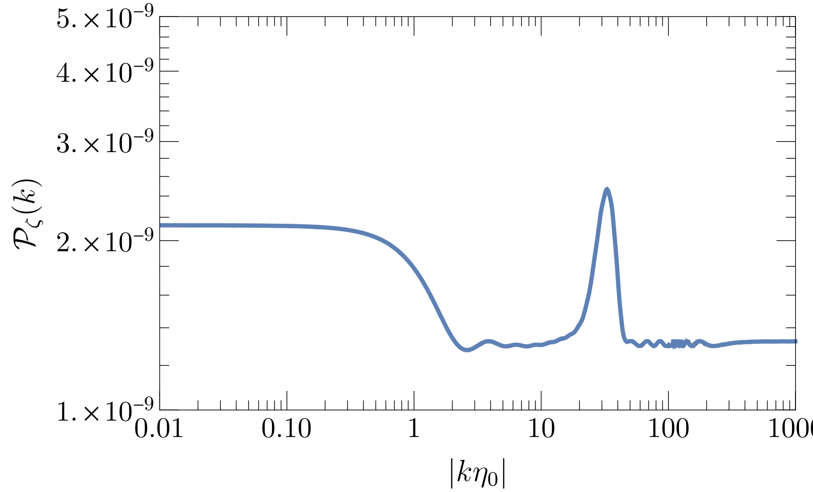
# GW and curvature perturbation spectrum

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} \simeq \frac{4}{M_{\rm Pl}^2}\hat{\mathcal{T}}_{ij}{}^{lm}\partial_l \delta\chi \partial_m \delta\chi$$

$$\delta\phi_J'' + 2\mathcal{H}\delta\phi_J' - \nabla^2\delta\phi_J + a^2\frac{\partial^2 V}{\partial\phi_J\partial\phi_I}\delta\phi^I = -2a^2\frac{\partial V}{\partial\phi_J}\Phi + 4\Phi'\phi_J',$$

$$(\phi_1 = \phi, \phi_2 = \chi)$$




Although the feature in the curvature power spectrum is small, the GWs can be large enough to be investigated by the future observations.

The GW peak scale is different from the field fluctuation peak scale because

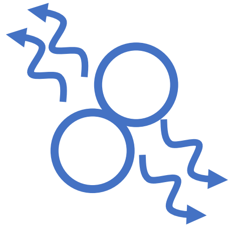
1. the GWs get redshifted after their production.
2. the scale dependence of GW production itself: $k^2 h_k \sim \mathcal{O}(\mathcal{S}_{k_{\rm peak}})$

# GWs produced during inflation

Sources of gravitational waves **during inflation**:

1. Quantum fluctuations of tensor field (Grishchuk (1974), Starobinsky (1979), Rubakov, Sazhin, and Veryaskin (1982), Febbri and Pollock (1983), Abbott and Wise (1984))

2. First-order phase transition (Jiang et al (2015), Wang, Cai, and Piao (2018), An et al. (2020)

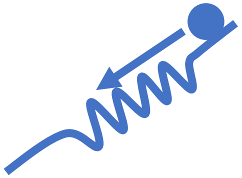


3. Gauge fields coupled to a rolling scalar field $(\phi F\tilde{F})$ (Sorbo (2011), Cook, Sorbo(2011), Barnaby, Pajer, and Peloso (2011))

4. **<u>Scalar fields</u>**

(1) Field with a small sound speed (Biagetti, Fasiello, and Riotto (2013), Fujita, J. Yokoyama, and S. Yokoyama (2014))

(2) Oscillatory features in the potential or sound speed (Cai et al. (2018), Zhou et al. (2020))



(3) A rapid turn in field trajectory (Fumagalli et al. (2021))

**(4) Oscillation of a spectator field (This work)**