Muon $g-2$: theory overview

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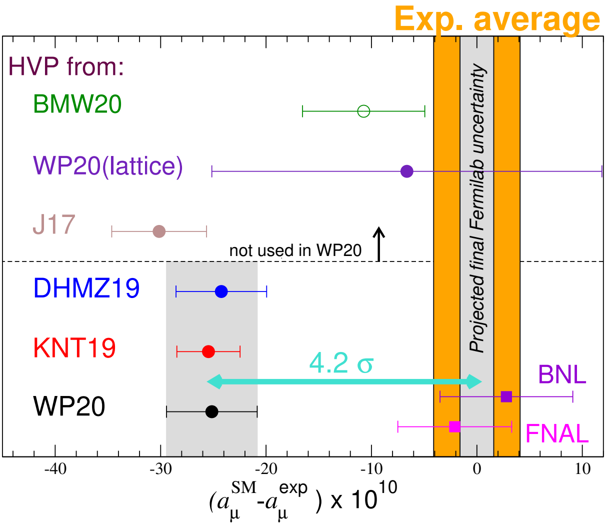
Particle Physics and Cosmology PPC 2022

St. Louis, Missouri, USA

The situation after the Fermilab announcement



This talk: **theory overview**

The Standard Model prediction for $a_\mu = (g-2)_\mu/2$

- **Dipole moments**: definition

$$\mathcal{H} = -\boldsymbol{\mu}_\ell \cdot \boldsymbol{B} \qquad \boldsymbol{\mu}_\ell = -g_\ell \frac{e}{2m_\ell}\boldsymbol{S} \qquad a_\ell = \frac{g_\ell - 2}{2}$$

- Overview of theory status (Standard Model)

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}} \qquad a_\mu^{\text{had}} = a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

- Comments on possible BSM explanations

QED: mass-independent terms

$$a_\mu^{\text{QED}} = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{j=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^j A_i^{(2j)}$$

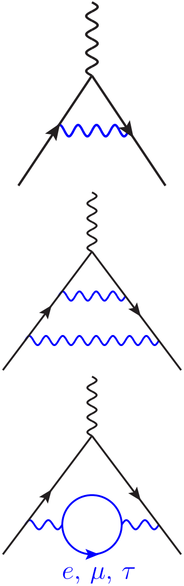
- **Mass-independent** term A_1 universal

$$A_1^{(2)} = 0.5$$
$$A_1^{(4)} = -0.328478965579193784582\ldots$$
$$A_1^{(6)} = 1.181241456587200\ldots$$
$$A_1^{(8)} = -1.912245764926445574\ldots \qquad \text{Laporta 2017}$$
$$A_1^{(10)} = 6.737(159) \qquad \text{Aoyama, Kinoshita, Nio 2019}$$

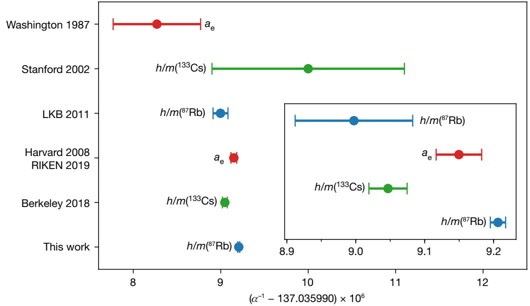
- 4.8σ discrepancy between

  $A_1^{(10)}$[no lepton loops] $= 7.668(159)$ Aoyama, Kinoshita, Nio 2019

  and $A_1^{(10)}$[no lepton loops] $= 6.793(90)$ Volkov 2019



QED: fine-structure constant



LKB 2020

- Tensions
  - Berkeley 2018 vs. LKB 2020: 5.4σ
  - LKB 2011 vs. LKB 2020: 2.4σ

- With new **Rb measurement** LKB 2020, Nature 588 (2020) 61

$$a_e^{\mathrm{SM}}[\mathrm{Rb}] = 1{,}159{,}652{,}180.25(1)_{\text{5-loop}}(1)_{\text{had}}(9)_{\alpha(\mathrm{Rb})} \times 10^{-12}$$

$$a_e^{\exp} - a_e^{\mathrm{SM}}[\mathrm{Rb}] = 0.48(30) \times 10^{-12}[1.6\sigma]$$

$$a_e^{\exp} - a_e^{\mathrm{SM}}[\mathrm{Cs}] = -0.88(36) \times 10^{-12}[-2.5\sigma]$$

QED: muon

- **5-loop QED** result Aoyama, Kinoshita, Nio 2018:

$$a_\mu^{\text{QED}} = 116\,584\,719.0(1) \times 10^{-11}$$

$\hookrightarrow$ insensitive to input for α (at this level)

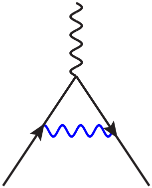
- QED coefficients enhanced by $\log m_\mu / m_e$
- Enhancement from naive RG expectation for 6-loop QED

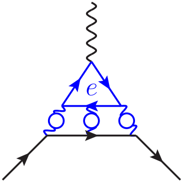
$$10 \times \frac{2}{3}\pi^2 \log \frac{m_\mu}{m_e} \times \left(\frac{2}{3} \log \frac{m_\mu}{m_e}\right)^3 \sim 1.6 \times 10^4$$

$\hookrightarrow$ would imply $a_\mu^{\text{6-loop}} \sim 0.2 \times 10^{-11}$

- Refined RG estimate Aoyama, Hayakawa, Kinoshita, Nio 2012

$$a_\mu^{\text{6-loop}} \sim 0.1 \times 10^{-11}$$





Electroweak contribution to $(g-2)_\mu$

- Electroweak contribution Gnendiger et al. 2013

$$a_\mu^{\text{EW}} = (194.8 - 41.2) \times 10^{-11} = 153.6(1.0) \times 10^{-11}$$

- Remaining uncertainty dominated by $q = u, d, s$ loops

  $\hookrightarrow$ nonperturbative effects Czarnecki, Marciano, Vainshtein 2003

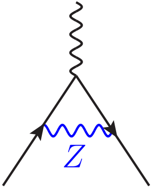
- Two-loop calculation revisited without asymptotic expansion

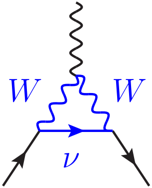
  Ishikawa, Nakazawa, Yasui 2019

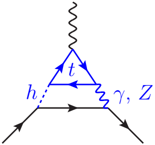
$$a_\mu^{\text{EW}} = 152.9(1.0) \times 10^{-11}$$

- 3-loop corrections?
  - 3-loop RG estimate accidentally cancels in scheme chosen by Gnendiger et al. 2013, with an error of 0.2×10^{-11}
  - α_s corrections to t-loop should scale as

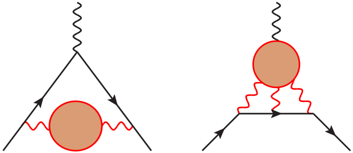
$$a_\mu^{t\text{-loop}}\big|_{\text{2-loop}} \times \frac{\alpha_s}{\pi} \sim 0.3 \times 10^{-11}$$







Hadronic effects



- **Hadronic vacuum polarization**: need hadronic two-point function

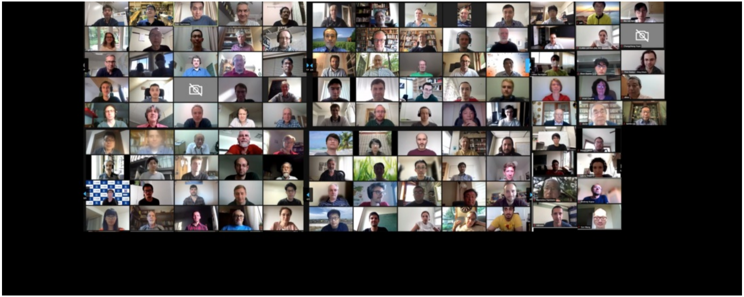
$$\Pi_{\mu\nu} = \langle 0 | T\{ j_\mu j_\nu \} | 0 \rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

$$\Pi_{\mu\nu\lambda\sigma} = \langle 0 | T\{ j_\mu j_\nu j_\lambda j_\sigma \} | 0 \rangle$$

- In the following: status of the hadronic contributions

The Muon $g-2$ Theory Initiative



Last plenary meeting held virtually at KEK in June 2021, https://www-conf.kek.jp/muong-2theory/

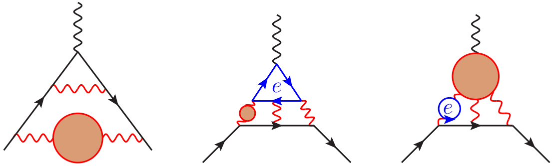
- Maximize the impact of the Fermilab and J-PARC experiments
  https://muon-gm2-theory.illinois.edu/

  $\hookrightarrow$ **quantify and reduce the theory uncertainties on the hadronic corrections**

- First white paper (WP20) Phys. Rept. 887 (2020) 1, 132 authors, 82 institutions, 21 countries

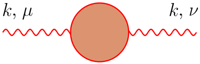
- Fifth plenary workshop @ Edinburgh: 5–9 Sep 2022 https://indico.ph.ed.ac.uk/event/112/

Higher-order hadronic effects



- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)\Pi\left(k^2\right)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi\left(k^2\right) - \Pi(0) = \frac{k^2}{\pi}\int\limits_{4M_\pi^2}^{\infty} \text{d}s\,\frac{\operatorname{Im}\Pi(s)}{s\left(s-k^2\right)}$$

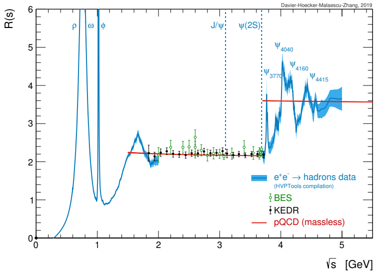
  - **Unitarity**

$$\operatorname{Im}\Pi(s) = \frac{s}{4\pi\alpha}\sigma_{\text{tot}}\left(e^+e^- \to \text{hadrons}\right) = \frac{\alpha}{3}R(s)$$

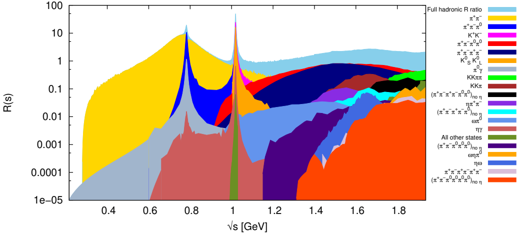
- Master formula

$$a_\mu^{\text{HVP LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds\,\frac{\hat{K}(s)}{s^2}R(s)$$

Hadronic vacuum polarization from e^+e^- data



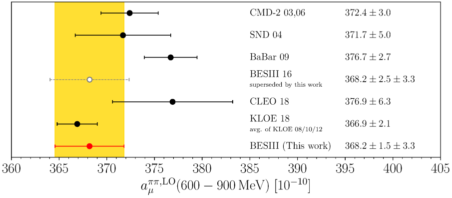
Davier, Hoecker, Malaescu, Zhang 2019



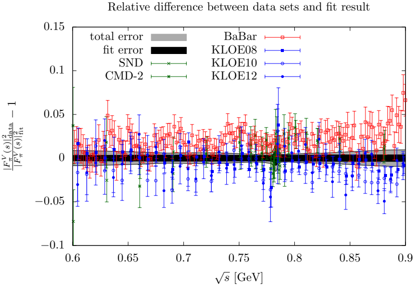
Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - Up to about 2 GeV: sum of exclusive channels
  - Above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

Hadronic vacuum polarization from e^+e^- data: 2π channel



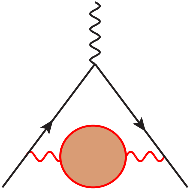
BESIII 2009.05011



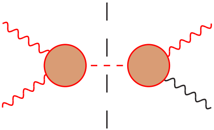
Colangelo, MH, Stoffer 2018

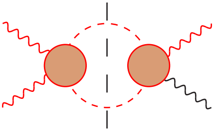
- Tension between KLOE and BaBar data:
  - Cross checks from **analyticity** and **unitarity** of the pion form factor
  - Affects the combination of data sets: different results depending on methodology
  - For white paper: adopt a **conservative merging procedure** that accounts for the 2π tension
- Our final recommendation: $a_\mu^{\text{HVP LO}}(e^+e^-) = 693.1(4.0)\times 10^{-10}$

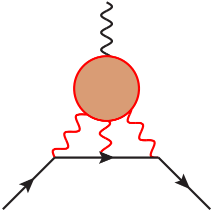
Hadronic light-by-light scattering



- In the past: hadronic models, inspired by various QCD limits, but error estimates difficult
- Dispersive approach: use again **analyticity**, **unitarity**, **crossing**, and **gauge invariance** for data-driven approach Colangelo, MH, Procura, Stoffer 2014, ...
- For simplest intermediate states: relation to $\pi^0 \to \gamma^*\gamma^*$ **transition form factor** and $\gamma^*\gamma^* \to \pi\pi$ **partial waves**







HLbL scattering: white paper

- Reference points:

$$a_\mu^{\text{HLbL}}\Big|_{\text{“Glasgow consensus” 2009}} = 105(26) \times 10^{-11}$$
$$a_\mu^{\text{HLbL}}\Big|_{\text{Jegerlehner, Nyffeler 2009}} = 116(39) \times 10^{-11}$$

- Strategy in the white paper
  - Take well-controlled results for the low-energy contributions
  - Combine errors in quadrature
  - Take best guesses for medium-range and short-distance matching
  - Add these errors linearly, since errors hard to disentangle at the moment
- **Recommended value**

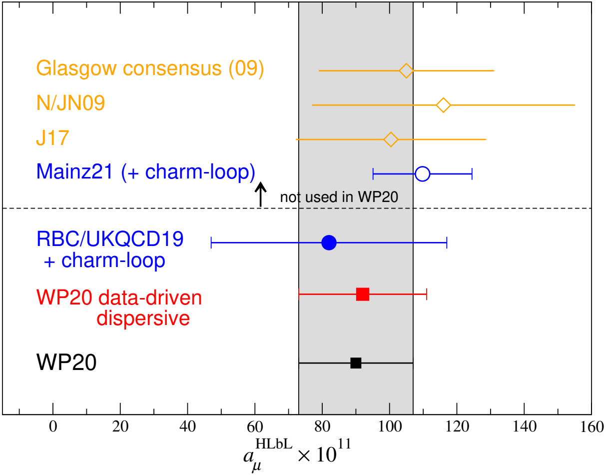
$$a_\mu^{\text{HLbL}}\,(\text{phenomenology}) = 92(19) \times 10^{-11}$$

- **Lattice QCD**: first complete calculation RBC/UKQCD 2019

$$a_\mu^{\text{HLbL}}\,(\text{lattice}, \mathit{uds}) = 79(35) \times 10^{-11}$$

$\hookrightarrow$ can combine with phenomenological value

Status of HLbL scattering



The anomalous magnetic moment of the muon in the Standard Model

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

Hadronic vacuum polarization: lattice QCD

- **White paper average**: $a_\mu^{\text{HVP LO}}$ (lattice) $= 711.6(18.4) \times 10^{-10}$

  $\hookrightarrow$ large uncertainty, consistent with both e^+e^- data and "no new physics"
- Does not include $a_\mu^{\text{HVP LO}}$ (BMWc) $= 707.5(5.5) \times 10^{-10}$ 2002.12347, first lattice result at $< 1\%$ precision

  $\hookrightarrow$ 2.1σ above e^+e^-, 1.6σ below "no new physics"
- How to resolve this?
  - Scrutiny by other lattice collaborations ongoing
  - Need to know at which energies the changes to the e^+e^- cross section occur

    $\hookrightarrow$ 2002.12347 points to low energies below 2 GeV
  - Would require changes to 2π cross section much bigger than the KLOE/BaBar tension
  - **New 2π data**: SND (published), CMD3 (forthcoming), BaBar (reanalysis on larger data set), Belle II, BESIII
  - **MUonE project**: extract space-like HVP from μe scattering

BSM: general remarks

- **BSM effect sizable**

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11} > a_\mu^{\text{EW}}$$

- Requires some form of enhancement:
  - **Chiral enhancement**: chirality flip $\propto m_\mu^2$ in SM

    $\hookrightarrow$ enhancement by $\tan\beta \sim 50$ in SUSY, $m_t/m_\mu \sim 1600$ in leptoquark models
  - **Light BSM**: axion-like particles, Z', $L_\mu - L_\tau$, light scalars
- Connections to other recent hints for the **violation of lepton flavor universality**?
  - B anomalies: $b \to s\ell\ell$ ($R(K^{(*)}$, P_5', $\ldots$), $b \to c\tau\nu$ ($R(D^{(*)})$)
  - First-row CKM unitarity, CMS dilepton data
  - Anomalous magnetic moment of the electron (?)

BSM: many possible models

http://pheno.csic.es/g-2Days21/program/

13:40 CEST · 14:25 CEST · 15:00 CEST · 15:10 CEST · 15:45 CEST · 15:55 CEST · 16:30 CEST · 16:40 CEST

Monday (31/05)

- **Data-driven evaluations of a_μ^{HVP}: Introduction, basics and main features** — T. Teubner, 15:00 - 15:45, *SM/HVP session*, slides
- **Aspects of the data-driven evaluation of HVP** — M. Hoferichter, 15:45 - 16:30, *SM/HVP session*, slides
- **Muon g-2 and $\Delta\alpha$ connection** — M. Passera, 16:30 - 17:15, *SM/HVP session*, slides

Tuesday (01/06)

- **g-2 and lepton flavour universality violation** — A. Crivellin, 13:40 - 14:25, *B physics session*, slides
- **Muon g-2 and B anomalies from Dark Matter** — L. Calibbi, 14:25 - 15:10, *B physics session*, slides
- **A model of muon anomalies** — A. Greljo, 15:10- 15:55, *B physics session*, slides
- **Leptoquark for (g-2) and B-meson anomalies** — S.C. Park, 15:55 - 16:40, *B physics session*, slides
- **TU Dresden colloquium**

Wednesday (02/06)

- **g-2 in the general MSSM** — D. Stöckinger, 15:00 - 15:45, *SUSY session*, slides
- **The Tiny (g-2) Muon Wobble from Small-μ Supersymmetry** — N. Shah, 15:45 - 16:30, *SUSY session*, slides
- **Anomalous muon magnetic moment, supersymmetry,naturalness, LHC search limits and the landscape**

Thursday (03/06)

- **Minimal models for g-2 and dark matter confront asymptotic safety** — K. Kowalska, 15:00 - 15:45, *non-SUSY session*, slides
- **Leptophilic bosons and muon g-2 in 2HDM** — E.J. Chun, 15:45 - 16:30, *non-SUSY session*, slides
- **Naturalness, the deus-ex-machina of the muon g-2 anomaly and lepton non-universality**

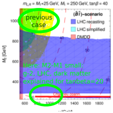
Friday (04/06)

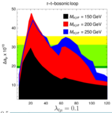
- **Constraints on "invisible" Feebly-Interacting Particles and g-2 anomalies** — L. Darmé, 15:00 - 15:45, *low-energy session*, slides
- **Challenges for an axion explanation for muon g-2** — J. Fan, 15:45 - 16:30, *low-energy session*, slides
- **Muon and electron g-2, proton and cesium weak charges implications on dark Z_d models**

There are many more examples. . .

SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



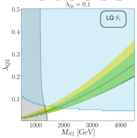


Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned

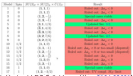
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

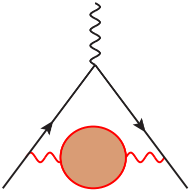
- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)

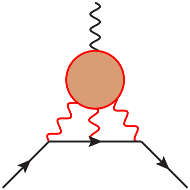


[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

Conclusions

- QED and EW contribution well under control
- **Hadronic vacuum polarization**
  - Presently largest systematic uncertainty in $\pi\pi$ **channel**
  - Comparison with lattice QCD just beginning
  - New data: SND, CMD-3, BaBar, Belle II, BESIII
- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible (π^0 and leading $\pi\pi$ effects done)
  - Evaluation of subleading terms and comparison to lattice-QCD calculations in progress
- Current theory matches expected experimental precision after first E989 release, **but need to go further**!
- Plethora of BSM explanations, possible relation to other lepton-flavor-universality violating "anomalies"





QED: fine-structure constant

- Input from **atom interferometry**

$$\alpha^2 = \frac{4\pi R_\infty}{c} \times \frac{m_{\text{atom}}}{m_e} \times \frac{\hbar}{m_{\text{atom}}}$$

- With **Rb measurement** LKB 2011

$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}$$

$$a_e^{\text{SM}} = 1{,}159{,}652{,}182.03(1)_{\text{5-loop}}(1)_{\text{had}}(72)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = -1.30(77) \times 10^{-12} [1.7\sigma]$$

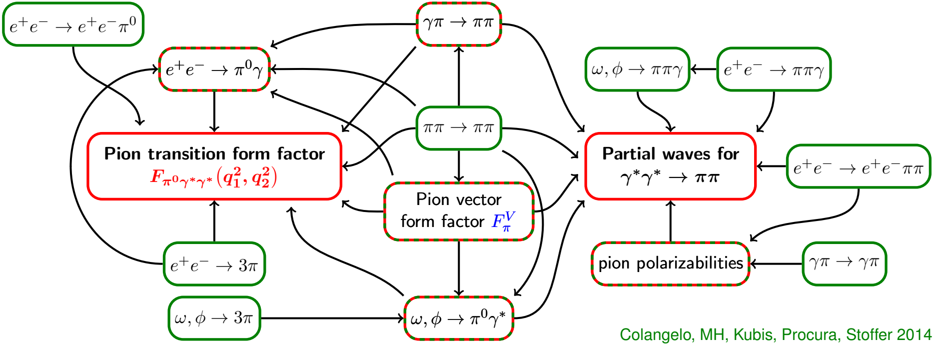
$\hookrightarrow$ α limiting factor, but more than an order of magnitude to go in theory

- With **Cs measurement** Berkeley 2018, Science 360 (2018) 191

$$a_e^{\text{SM}} = 1{,}159{,}652{,}181.61(1)_{\text{5-loop}}(1)_{\text{had}}(23)_{\alpha(\text{Cs})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = -0.88(36) \times 10^{-12} [2.5\sigma]$$

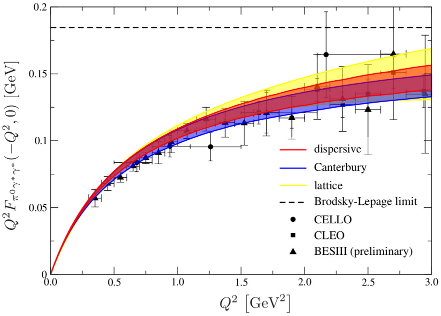
$\hookrightarrow$ for the first time a_e^{exp} limiting factor



Colangelo, MH, Kubis, Procura, Stoffer 2014

- Reconstruction of $\gamma^*\gamma^* \to \pi\pi, \pi^0$: combine experiment and theory constraints
- Need input on $\gamma^*\gamma^*$ matrix elements for as many states as possible

Hadronic light-by-light scattering: pion pole



- Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puerto 2017 and lattice QCD Gérardin et al. 2019

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{dispersive}} = 63.0^{+2.7}_{-2.1} \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{Canterbury}} = 63.6(2.7) \times 10^{-11}$$

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice+PrimEx}} = 62.3(2.3) \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice}} = 59.7(3.6) \times 10^{-11}$$

$\hookrightarrow$ agree within uncertainties well below Fermilab goal

- Singly-virtual results agree well with BESIII measurement

Hadronic vacuum polarization from e^+e^- data

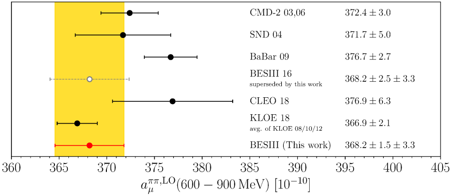
HVP from e^+e^- data

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} \text{d}s \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons}(+\gamma))(s)$$

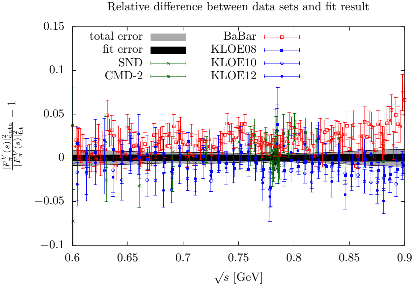
$$= 6931(40) \times 10^{-11}$$

- The “theory” prediction a_μ^{SM} is actually **based on experiments** (ISR, direct scan)
  ↪ propagation of experimental uncertainties
- Uncertainty estimate includes:
  - different methodologies for the combination of data sets Davier et al. 2019, Keshavarzi et al. 2020
  - conservative estimate of systematic errors from tensions in the data
  - cross checks from analyticity/unitarity constraints Colangelo et al. 2018, Ananthanarayan et al. 2018, Davier et al. 2019, MH et al. 2019
  - full NLO radiative corrections Campanario et al. 2019

Cross checks from analyticity and unitarity



BESIII 2009.05011



Colangelo, MH, Stoffer 2018

- For "simple" channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)
  $\hookrightarrow$ strong cross check on the data sets (covering about 80% of HVP)
- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

Cross checks from analyticity and unitarity

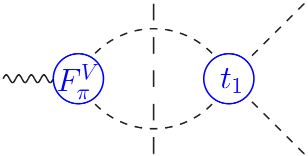
- In direct integration: local combination of data

  $\hookrightarrow$ **local scale factor** in case tensions arise

- $e^+e^- \to 2\pi$ determined by pion vector form factor F_π^V

- **Unitarity** for **pion vector form factor**

$$\text{Im}\, F_\pi^V(s) = \theta\big(s - 4M_\pi^2\big) F_\pi^V(s) e^{-i\delta_1(s)} \sin\delta_1(s)$$



$\hookrightarrow$ **final-state theorem**: phase of F_π^V equals $\pi\pi$ P-wave phase δ_1 Watson 1954

- Can derive a **global fit function** that depends on
  - Two values of δ_1 (elastic 2π intermediate states)
  - ω mass, width, and residue (3π intermediate states)
  - Conformal polynomial (4π intermediate states)

Breakdown of the HVP error

HVP from e^+e^- data

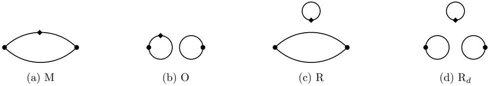
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error

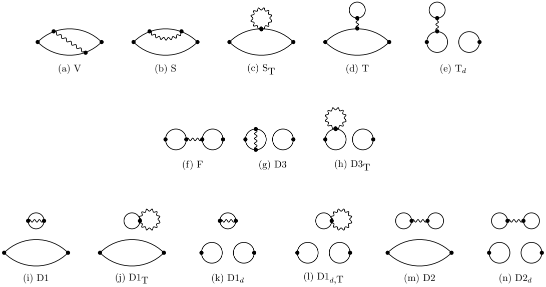
  $\hookrightarrow$ would get $4.2\sigma \to 4.8\sigma$ when ignoring additional systematic error
- There was broad consensus to adopt **conservative error estimates**

  $\hookrightarrow$ **merging procedure** in WP20 covers tensions in the data and different methodologies for the combination of data sets
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2

- **Strong isospin breaking** $\propto m_u - m_d$



- **QED effects** $\propto \alpha$

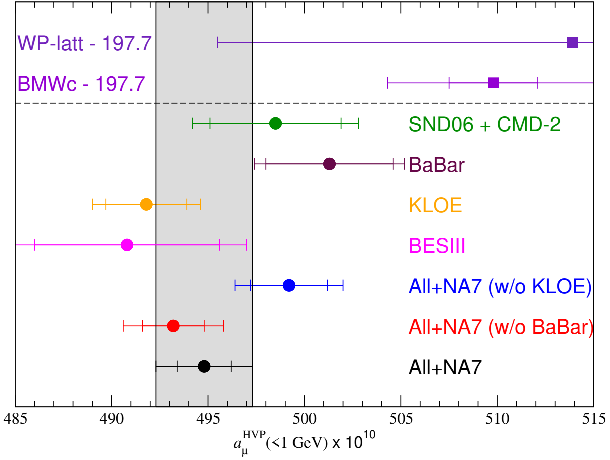


plots from Gülpers et al. 2018

- **Matches data-driven convention for leading-order HVP**

  ↪ diagram (f) F without additional gluons is subtracted

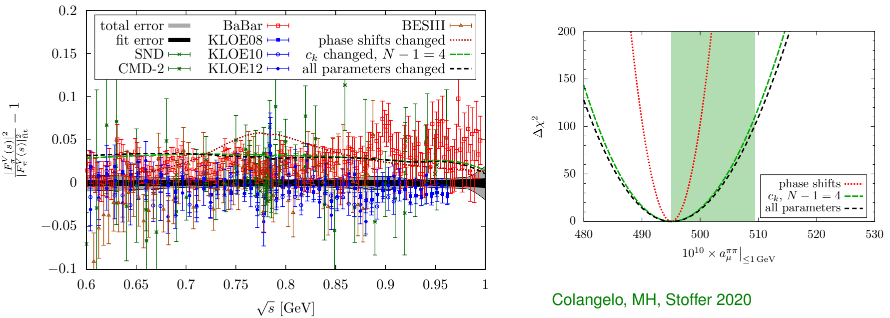
$\pi\pi$ contribution below 1 GeV



Assumption: suppose all changes occur in $\pi\pi$ channel below 1 GeV

$\hookrightarrow a_\mu^{\text{total}}[\text{WP20}] - a_\mu^{2\pi,<1\,\text{GeV}}[\text{WP20}] = 197.7 \times 10^{-10}$

Changing the $\pi\pi$ cross section below 1 GeV

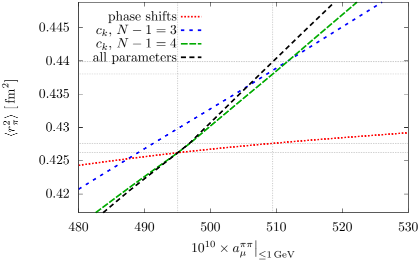


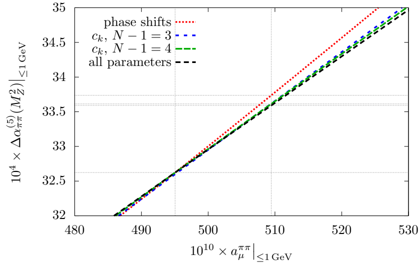
Colangelo, MH, Stoffer 2020

- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
  1. "Low-energy" scenario: **$\pi\pi$ phase shifts**
  2. "High-energy" scenario: **conformal polynomial**
  3. Combined scenario

$\hookrightarrow$ 2. and 3. lead to uniform shift, 1. concentrated in ρ region

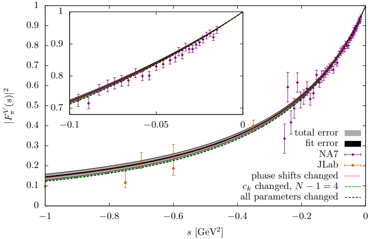
Correlations



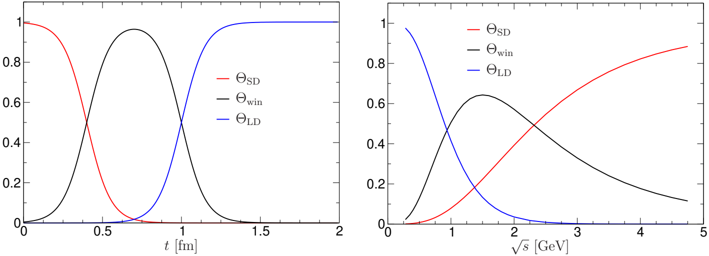


Correlations with other observables:

- **Pion charge radius $\langle r_\pi^2 \rangle$**
  - $\hookrightarrow$ significant change in scenarios 2. and 3.
  - $\hookrightarrow$ can be tested in lattice QCD
- **Hadronic running of α**
- **Space-like pion form factor**



Window quantities



- **Weight functions in Euclidean time** proposed by RBC/UKQCD 2018

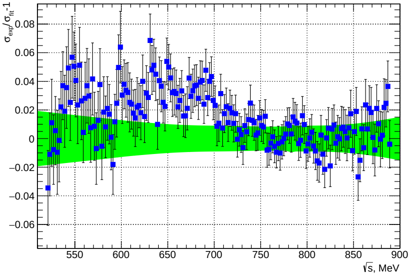
  $\hookrightarrow$ long-distance, intermediate, and short-distance window

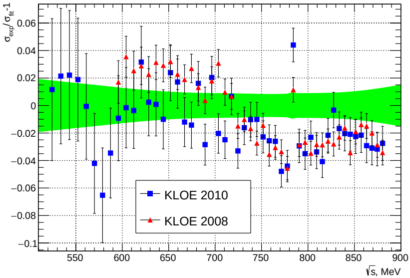
- For intermediate window a_μ^{int}[RBC/UKQCD] $= 231.9(1.5) \times 10^{-10}$ and a_μ^{int}[BMWc] $= 236.7(1.4) \times 10^{-10}$ **differ by 2.3σ**

- Difference between BMWc and e^+e^- in intermediate window is **3.7σ**, but $\pi\pi$ channel below 1 GeV split 69 : 28 : 3, relevant changes above 1 GeV?

- Detailed study of windows key tool for **comparison among lattice and with e^+e^-**

New data since WP20





BaBar vs. SND 20 2004.00263 KLOE vs. SND 20

- New data from SND experiment not yet included in WP20 number

  $\hookrightarrow$ lie between BaBar and KLOE

- **More data to come** from: CMD3, BESIII, BaBar, Belle II

- **MUonE project**: extract space-like HVP from μe scattering

Relation to global electroweak fit

Hadronic running of α

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ enters as input in **global electroweak fit**
  $\hookrightarrow$ integral weighted more strongly towards high energy
- Changes in $R_{\text{had}}(s)$ have to occur at low energies, $\lesssim 2\,\text{GeV}$ Crivellin et al. 2020, Keshavarzi et al. 2020, Malaescu et al. 2020
- This seems to happen for BMWc calculation (translated from the space-like), with only moderate increase of tensions in the electroweak fit ($\sim 1.8\sigma \to 2.4\sigma$)
  $\hookrightarrow$ need **large changes in low-energy cross section**

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

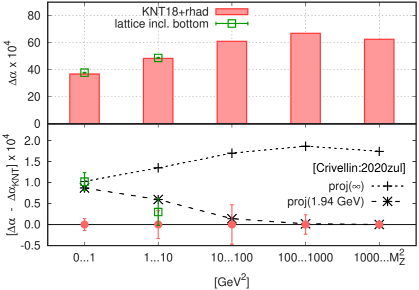
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020