Aspects of Neutrino Theory

K.S. Babu

Oklahoma State University

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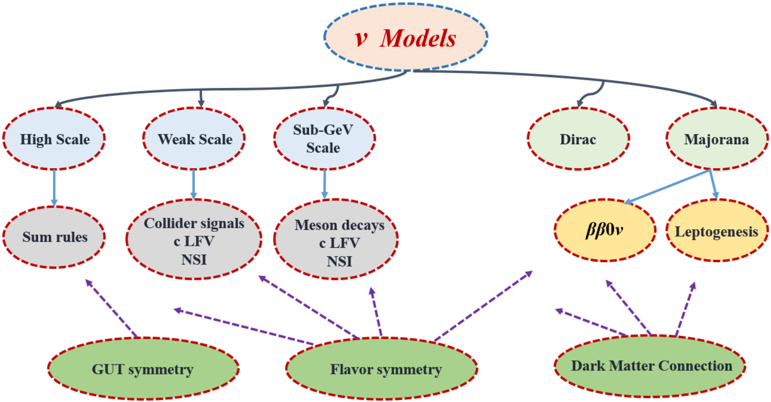
Current knowledge of 3-neutrino oscillations

NuFIT 5.0 (2020)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
	$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
	$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{\mathrm{CP}}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
	$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
	$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
	$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
	$\delta_{\mathrm{CP}}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

Roadmap for Neutrino Models



ν Models

High Scale

Weak Scale

Sub-GeV Scale

Dirac

Majorana

Sum rules

Collider signals
c LFV
NSI

Meson decays
c LFV
NSI

ββ0ν

Leptogenesis

GUT symmetry

Flavor symmetry

Dark Matter Connection

Effective Field Theory for neutrino masses

- Neutrino masses are zero in the Standard Model. Observed oscillations require new physics beyond Standard Model
- Neutrino masses and oscillations can be explained in terms of the celebrated Weinberg operator
- It is the leading operator in Standard Model EFT and arises at dimension-five, suppressed by one power of an inverse mass scale
- It violates lepton number by two units and generates neutrino masses:

$$\begin{aligned}\mathcal{O}_1 &= \frac{\kappa_{ab}}{2}(L_a^i L_b^j)H^k H^l \epsilon_{ik}\epsilon_{jl} \\ &= \frac{\kappa_{ab}}{2}(\nu_a H^0 - \ell_a H^+)(\nu_b H^0 - \ell_b H^+)\end{aligned}$$

$$\Rightarrow \quad (M_\nu)_{ab} = (\kappa)_{ab} v^2$$

- $\kappa^{-1} \sim (10^{14}\ \mathrm{GeV})$ can be inferred from data

Strong reasons to go beyond EFT

- EFT description cannot be the end goal, or else important phenomena would be missed
- What if neutrinos are Dirac particles? $\mathcal{O}_1$ is then the wrong description
- What if neutrino masses arose from $d = 7$ operators or $d = 9$ operators in a fundamental theory, and not through $\mathcal{O}_1$?
- Even when the scale of new physics is beyond reach of current experiments, opening the EFT operator can give new insights
- An example is baryon asymmetry generation via leptogensis
- Requires opening up the Weinberg operator. Baryon asymmetry originates from the decays of N^c, the mediator of the operator $\mathcal{O}_1$

Origin of neutrino mass: Seesaw mechanism

- Adding right-handed neutrino N^c which transforms as singlet under $SU(2)_L$,

$$\mathcal{L} = f_\nu \,(L \cdot H)\, N^c + \tfrac{1}{2} M_R N^c N^c$$

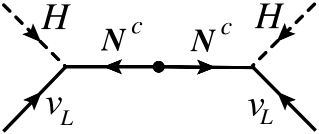
- Integrating out the N^c, $\Delta L = 2$ operator is induced:

$$\mathcal{L}_{\text{eff}} = -\frac{f_\nu^2}{2} \frac{(L \cdot H)\,(L \cdot H)}{M_R}$$

- Once H acquires VEV, neutrino mass is induced:

$$m_\nu \simeq f_\nu^2 \frac{v^2}{M_R}$$

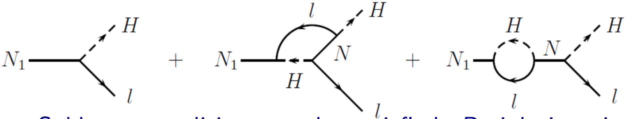
- For $f_\nu v \simeq 100$ GeV, $M_R \simeq 10^{14}$ GeV.



Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, Slansky (1980)
Mohapatra & Senjanovic (1980)

Baryogenesis via leptogenesis and type-I seesaw

- In the early history of the universe, a lepton asymmetry may be dynamically generated in the decay of N Fukugita, Yanagida (1986)
- N being a Majorana fermion can decay to $L + H$ as well as $\overline{L} + H^*$

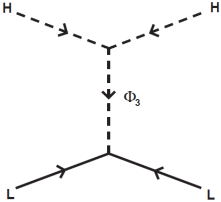


- Three Sakharov conditions can be satisfied: B violation via electroweak sphaleron, C and CP violation in Yukawa couplings of N, and out of equilibrium condition via expanding universe
- Lepton asymmetry in decay of N_1 (with $M_1 \ll M_{2,3}$):

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{1}{(f_\nu f_\nu^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[(f_\nu f_\nu^\dagger)^2_{i1}\right] \frac{M_1}{M_i}$$

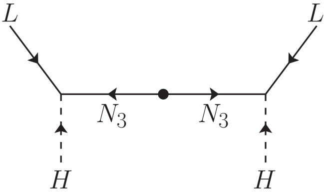
- $\varepsilon \sim 10^{-6}$ can explain observed baryon asymmetry of the universe
- Indirect tests in Majorana nature of ν and in CP violation in oscillations

Seesaw mechanism (cont.)



Type II seesaw: $\Phi_3 \sim (1, 3, 1)$

Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)



Type III seesaw: $N_3 \sim (1, 3, 0)$

Foot, Lew, He, & Joshi (1989)

Ma (1998)

- Φ_3 abd N_3 contain charged particles which can be looked for at LHC
- Eg: $\Phi^{++} \to \ell^+\ell^+$, $\Phi^{++} \to W^+W^+$ decays would establish lepton number violation

Dirac Neutrino Models

- Neutrinos may be Dirac particles without lepton number violation
- Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos
- Spin-flip transition rates (in stars, early universe) are suppressed by small neutrino mass:

$$\Gamma_{\rm spin-flip} \approx \left(\frac{m_\nu}{E}\right)^2 \Gamma_{\rm weak}$$

- Neutrinoless double beta decay discovery would establish neutrinos to be Majorana particles
- If neutrinos are Dirac, it would be nice to understand the smallness of their mass
- Models exist which explain the smallness of Dirac m_ν
- "Dirac leptogenesis" can explain baryon asymmetry

  Dick, Lindner, Ratz, Wright (2000)

Dirac Seesaw Models

- Dirac seesaw can be achieved in Mirror Models

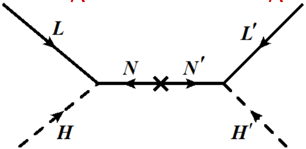
  Lee, Yang (1956); Foot, Volkas (1995); Berezhiani, Mohapatra (1995), Silagadze(1997)

- Mirror sector is a replica of Standard Model, with new particles transforming under mirror gauge symmetry:

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} ; \quad L' = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L ; \quad H' = \begin{pmatrix} H'^+ \\ H'^0 \end{pmatrix}$$

- Effective dimension-5 operator induces small Dirac mass:

$$\frac{(LH)(L'H')}{\Lambda} \Rightarrow m_\nu = \frac{vv'}{\Lambda}$$



- $B - L$ may be gauged to suppress Planck-induced Weinberg operator $(LLHH)/M_{\rm Pl}$ that would make neutrino pseudo-Dirac particle

Dirac Neutrinos from Left-Right Symmetry

- In left-right symmetric models with a “universal seesaw”, neutrinos are naturally Dirac particles
- Gauge symmetry is extended to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$
- These models are motivated on several grounds:
  - Provide understanding of Parity violation
  - Better understanding of smallness of Yukawa couplings
  - Requires right-handed neutrinos to exist
  - Provide a solution to the strong CP problem via Parity

Davidson, Wali (1987) – universal seesaw
Babu, He (1989) – Dirac neutrino
Babu, Mohapatra (1990) – solution to strong CP problem via parity
Babu, Dutta, Mohapatra (2018) – R_{D^*} solution
Craig, Garcia Garcia, Koszegi, McCune (2020) – flavor constraints
Babu, He, Su, Thapa (2022) – neutrino oscillations

Left-Right Symmetry

- Fermion transformation:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$

$$\Psi_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \qquad \Psi_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

- Vector-like fermions are introduced to realize seesaw for charged fermion masses:

$$P(3,1,1,4/3), \quad N(3,1,1,-2/3), \quad E(1,1,1,-2)\ .$$

- Higgs sector is very simple:

$$\chi_L\ (1,2,1,1) = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \qquad \chi_R\ (1,1,2,1) = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$$

- $\langle \chi_R^0 \rangle = \kappa_R$ breaks $SU(2)_R \times U(1)_X$ down to $U(1)_Y$, and $\langle \chi_L^0 \rangle = \kappa_L$ breaks the electroweak symmetry with $\kappa_R \gg \kappa_L$

Seesaw for charged fermions

- Yukaw interactions:

$$\mathcal{L} = y_u\ (\bar{Q}_L\tilde{\chi}_L + \bar{Q}_R\tilde{\chi}_R)P + y_d\ (\bar{Q}_L\chi_L + \bar{Q}_R\chi_R)N \\ + y_\ell\ (\bar{\Psi}_L\chi_L + \bar{\Psi}_R\chi_R)E + h.c.$$

- Vector-like fermion masses:

$$\mathcal{L}_{\text{mass}} = M_{p^0}\ \bar{P}P + M_{N^0}\ \bar{N}N + M_{E^0}\ \bar{E}E$$

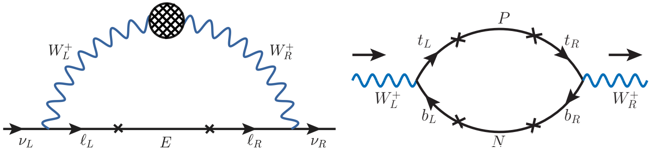
- Seesaw for charged fermion masses:

$$M_F = \begin{pmatrix} 0 & Y\kappa_L \\ Y^\dagger\kappa_R & M \end{pmatrix} \Rightarrow m_f = \frac{Y^2\kappa_L\kappa_R}{M}$$

- $\theta_{QCD} = 0$ due to Parity; ArgDet$(M_U M_D) = 0$; induced $\overline{\theta} = 0$ at one-loop; small and finite $\overline{\theta}$ arises at two-loop
- There is no seesaw for neutrinos, since there is no corresponding singlet fermion

Two-loop Dirac Neutrino Masses

- Higgs sector is very simple: $\chi_L(1,2,1,1/2) + \chi_R(1,1,2,1/2)$
- $W_L^+ - W_R^+$ mixing is absent at tree-level in the model
- $W_L^+ - W_R^+$ mixing induced at loop level, which in turn generates Dirac neutrino mass at two loop Babu, He (1989)



- Flavor structure of two loop diagram needs to be studied to check consistency
- Oscillation date fits well within the model regardless of Parity breaking scale Babu, He, Su, Thapa (2022)

Neutrino Fit in Two-loop Dirac Mass Model

Oscillation parameters	3σ **range** `NuFit5.1`	**Model prediction**			
		BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)
$\Delta m^2_{21} (10^{-5}\ \mathrm{eV}^2)$	6.82 - 8.04	7.42	7.32	7.35	7.30
$\Delta m^2_{23} (10^{-3}\ \mathrm{eV}^2)$(IH)	2.410 - 2.574	-	-	2.48	2.52
$\Delta m^2_{31} (10^{-3}\ \mathrm{eV}^2)$(NH)	2.43 - 2.593	2.49	2.46	-	-
$\sin^2 \theta_{12}$	0.269 - 0.343	0.324	0.315	0.303	0.321
$\sin^2 \theta_{23}$ (IH)	0.410 - 0.613	-	-	0.542	0.475
$\sin^2 \theta_{23}$ (NH)	0.408 - 0.603	0.491	0.452	-	-
$\sin^2 \theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0230	0.0234
$\sin^2 \theta_{13}$(NH)	0.02060 - 0.02435	0.0234	0.0223	-	-
δ_{CP} (IH)	192 - 361	-	-	271°	296°
δ_{CP} (NH)	105 - 405	199°	200°	-	-
m_{light} (10^{-3}) eV		0.66	0.17	0.078	4.95
M_{E_1}/M_{W_R}		917	321.3	639	3595
M_{E_2}/M_{W_R}		0.650	19.3	1.54	5.03
M_{E_3}/M_{W_R}		0.019	1.26	0.054	2.94

- Ten parameters to fit oscillation data
- Both normal ordering and inverted ordering allowed
- Dirac CP phase is unconstrained
- Left-right symmetry breaking scale is not constrained

Tests with $N_{\rm eff}$ in Cosmology

- Dirac neutrino models of this type will modify $N_{\rm eff}$ by about 0.14

$$\Delta N_{\rm eff} \simeq 0.027 \left(\frac{106.75}{g_\star \left(T_{\rm dec} \right)} \right)^{4/3} g_{\rm eff}$$

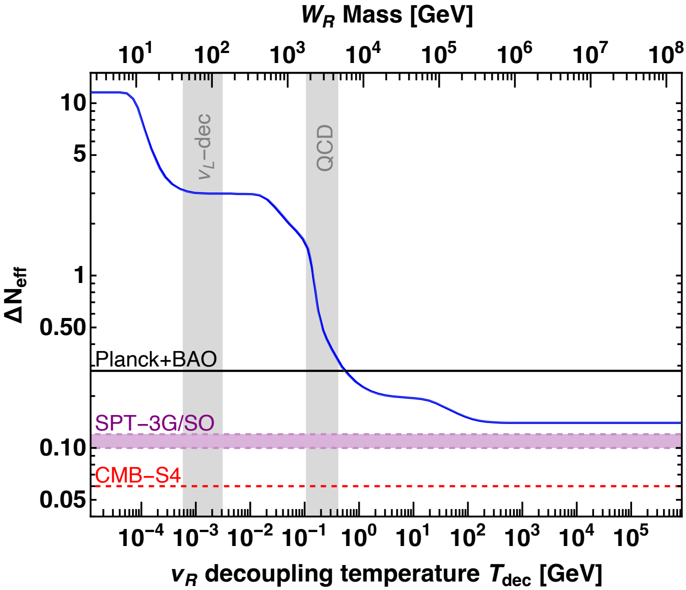
$$g_{\rm eff} = (7/8) \times (2) \times (3) = 21/4$$

- Can be tested in CMB measurements: $N_{\rm eff} = 2.99 \pm 0.17$ (Planck+BAO)

$$G_F^2 \left(\frac{M_{W_L}}{M_{W_R}} \right)^4 T_{\rm dec}^5 \approx \sqrt{g^* (T_{\rm dec})} \, \frac{T_{\rm dec}^2}{M_{\rm Pl}}$$

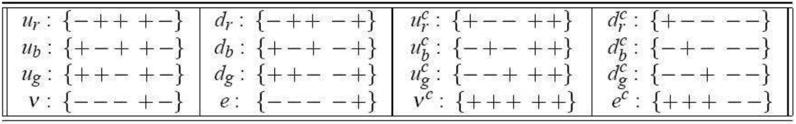
$$T_{\rm dec} \simeq 400 \text{ MeV} \left(\frac{g_* \left(T_{\rm dec} \right)}{70} \right)^{1/6} \left(\frac{M_{W_R}}{5 \text{ TeV}} \right)^{4/3}$$

- Present data sets a lower limit of 7 TeV on W_R mass


W_R Mass [GeV]
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
$\Delta N_{\rm eff}$
10
5
1
0.50
0.10
0.05
ν_L–dec
QCD
Planck+BAO
SPT–3G/SO
CMB–S4
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
ν_R decoupling temperature $T_{\rm dec}$ [GeV]

Unification of Forces & Matter in SO(10)

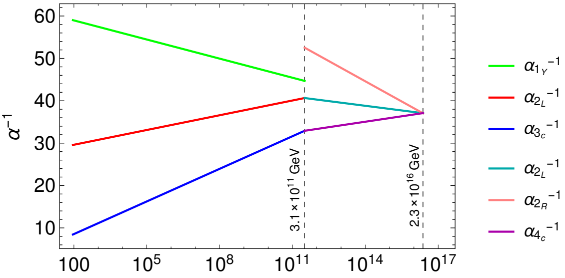
16 members of a family fit into a spinor of $SO(10)$



$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

First 3 spins refer to color, last two are weak spins

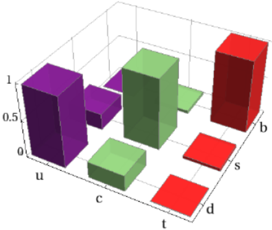
$$Y = \frac{1}{3}\Sigma(C) - \frac{1}{2}\Sigma(W)$$



Disparity in Quark & Lepton Mixings

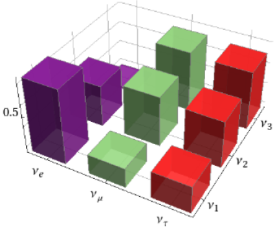
- **Quark Mixings**

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



- **Leptonic Mixings**

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



Yukawa Sector of Minimal $SO(10)$

$$16 \times 16 = 10_s + 120_a + 126_s$$

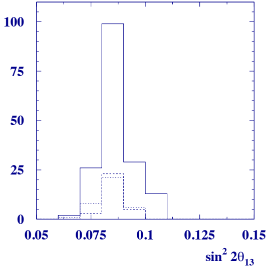
- At least two Higgs fields needed for family mixing
- Symmetric 10_H and $\overline{126}$ is the minimal model

$$W_{SO(10)}^{\text{Yukawa}} = 16^T \left(Y_{10}\, 10_H + Y_{126}\overline{126}_H \right) 16 \ .$$

$$\begin{aligned} M_U &= v_u^{10} Y_{10} + v_u^{126} Y_{126} \\ M_D &= v_d^{10} Y_{10} + v_d^{126} Y_{126} \\ M_E &= v_d^{10} Y_{10} - 3 v_d^{126} Y_{126} \\ M_{\nu_D} &= v_u^{10} Y_{10} - 3 v_u^{126} Y_{126} \\ M_R &= Y_{126} V_R \end{aligned}$$

Minimal Yukawa sector of SO(10)

- 12 parameters plus 7 phases to fit 18 observed quantities
- This setup fits all obsevables quite well
- Large neutrino mixings coexist with small quark mixings
- θ_{13} prediction turned out to be correct



Babu, Mohapatra (1993); Bajc, Senjanovic, Vissani (2001); (2003); Fukuyama, Okada (2002); Goh, Mohapatra, Ng (2003); Bajc, Melfo, Senjanovic, Vissani (2004); Bertolini, Malinsky, Schwetz (2006); Babu, Macesanu (2005); Dutta, Mimura, Mohapatra (2007); Aulakh et al (2004); Bajc, Dorsner, Nemevsek (2009); Joshipura, Patel (2011); Dueck, Rodejohann (2013); Ohlsson, Penrow (2019); Babu, Bajc, Saad (2018); Babu, Saad (2021)

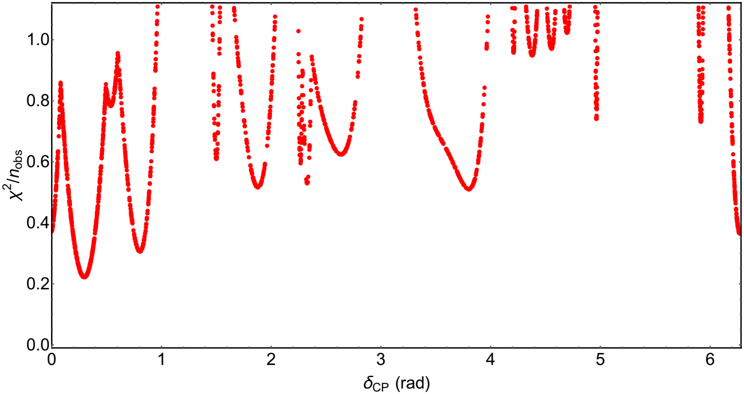
Best fit values for fermion masses and mixings

Observables	**SUSY**			**non-SUSY**		
(masses in GeV)	Input	Best Fit	Pull	Input	Best Fit	Pull
$m_u/10^{-3}$	0.502±0.155	0.515	0.08	0.442±0.149	0.462	0.13
m_c	0.245±0.007	0.246	0.14	0.238±0.007	0.239	0.18
m_t	90.28±0.89	90.26	-0.02	74.51±0.65	74.47	-0.05
$m_b/10^{-3}$	0.839±0.17	0.400	-2.61	1.14±0.22	0.542	-2.62
$m_s/10^{-3}$	16.62±0.90	16.53	-0.09	21.58±1.14	22.57	0.86
m_b	0.938±0.009	0.933	-0.55	0.994±0.009	0.995	0.19
$m_e/10^{-3}$	0.3440±0.0034	0.344	0.08	0.4707±0.0047	0.470	-0.03
$m_\mu/10^{-3}$	72.625±0.726	72.58	-0.05	99.365±0.993	99.12	-0.24
m_τ	1.2403±0.0124	1.247	0.57	1.6892±0.0168	1.688	-0.05
$\|V_{us}\|/10^{-2}$	22.54±0.07	22.54	0.02	22.54±0.06	22.54	0.06
$\|V_{cb}\|/10^{-2}$	3.93±0.06	3.908	-0.42	4.856±0.06	4.863	0.13
$\|V_{ub}\|/10^{-2}$	0.341±0.012	0.341	0.003	0.420±0.013	0.421	0.10
δ°_{CKM}	69.21±3.09	69.32	0.03	69.15±3.09	70.24	0.35
$\Delta m^2_{21}/10^{-5}(eV^2)$	8.982±0.25	8.972	-0.04	12.65±0.35	12.65	-0.01
$\Delta m^2_{31}/10^{-3}(eV^2)$	3.05±0.04	3.056	0.02	4.307±0.059	4.307	0.006
$\sin^2\theta_{12}$	0.318±0.016	0.314	-0.19	0.318±0.016	0.316	-0.07
$\sin^2\theta_{23}$	0.563±0.019	0.563	0.031	0.563±0.019	0.563	0.01
$\sin^2\theta_{13}$	0.0221±0.0006	0.0221	-0.003	0.0221±0.0006	0.0220	-0.16
δ°_{CP}	224.1±33.3	240.1	0.48	224.1±33.3	225.1	0.03
χ^2	-	-	7.98	-	-	7.96

Babu, Saad (2021)

Dirac CP phase

Multiple χ^2 minima make δ_{CP} prediction difficult



Babu, Bajc, Saad (2018)

Proton decay predictions

- Proton decay branching ratios determined by neutrino oscillation fits
- Mediated by superheavy gauge bosons
- Lifetime has large uncertainties, $\tau_p \approx (10^{32} - 10^{36})$ yrs.

Prediction of branching ratios

$$\Gamma(p \to \pi^0 e^+) \to 47\%$$
$$\Gamma(p \to \pi^0 \mu^+) \to 1\%$$
$$\Gamma(p \to \eta^0 e^+) \to 0.20\%$$
$$\Gamma(p \to \eta^0 \mu^+) \to 0.00\%$$
$$\Gamma(p \to K^0 e^+) \to 0.16\%$$
$$\Gamma(p \to K^0 \mu^+) \to 3.62\%$$
$$\Gamma(p \to \pi^+ \overline{\nu}) \to 48\%$$
$$\Gamma(p \to K^+ \overline{\nu}) \to 0.22\%$$

Nemesvek, Bajc, Dorsner (2009)
Babu, Khan (2015)

Radiative neutrino mass generation

- An alternative to seesaw is radiative neutrino mass generation, where neutrino mass is absent at tree level, but arises via quantum loop corrections
- The smallness of neutrino mass is explained by loop and chiral suppressions
- Loop diagrams may arise at 1-loop, 2-loop or 3-loop levels
- New physics scale typically near TeV and thus accessible to LHC
- Further tests in observable LFV processes and as nonstandard neutrino interaction (NSI) in oscillations

Effective $\Delta L = 2$ Operators

$$
\begin{aligned}
\mathcal{O}_1 &= L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl} \\
\mathcal{O}_2 &= L^i L^j L^k e^c H^l \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_3 &= \{L^i L^j Q^k d^c H^l \epsilon_{ij}\epsilon_{kl}, \quad L^i L^j Q^k d^c H^l \epsilon_{ik}\epsilon_{jl}\} \\
\mathcal{O}_4 &= \{L^i L^j \bar{Q}_i \bar{u^c} H^k \epsilon_{jk}, \quad L^i L^j \bar{Q}_k \bar{u^c} H^k \epsilon_{ij}\} \\
\mathcal{O}_5 &= L^i L^j Q^k d^c H^l H^m \bar{H}_i \epsilon_{jl}\epsilon_{km} \\
\mathcal{O}_6 &= L^i L^j \bar{Q}_k \bar{u^c} H^l H^k \bar{H}_i \epsilon_{jl} \\
\mathcal{O}_7 &= L^i Q^j \bar{e^c} \bar{Q}_k H^k H^l H^m \epsilon_{il}\epsilon_{jm} \\
\mathcal{O}_8 &= L^i \bar{e^c} \bar{u^c} d^c H^j \epsilon_{ij} \\
\mathcal{O}_9 &= L^i L^j L^k e^c L^l e^c \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_1' &= L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl} H^{*m} H_m
\end{aligned}
$$

Babu & Leung (2001)

de Gouvea & Jenkins (2008)

Angel & Volkas (2012)

Cai, Herrero-Garcia, Schmidt, Vicente, Volkas (2017)

Lehman (2014) – all $d = 7$ operators

Li, Ren, Xiao, Yu, Zheng (2020); Liao, Ma (2020) – all $d = 9$ operators

Operator $\mathcal{O}_2$ and the Zee model

- Introduce a singly charged scalar and a second Higgs doublet to standard model:

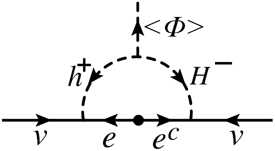
$$\mathcal{L} \quad = \quad f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + \mu H^a \Phi^b h^- \epsilon_{ab} + \text{h.c.}$$

$$\Downarrow$$

$$\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$$

Zee (1980)

- Neutrino mass arises at one-loop.



- A minimal version of this model in which only one Higgs doublet couples to a given fermion sector with a Z_2 symmetry yields: Wolfenstein (1980)

$$m_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}, \quad m_{ij} \simeq \frac{f_{ij}}{16\pi^2} \frac{(m_i^2 - m_j^2)}{\Lambda}$$

It requires $\theta_{12} \simeq \pi/4 \rightarrow$ ruled out by solar + KamLAND data.

Koide (2001); Frampton *et al.* (2002); He (2004)

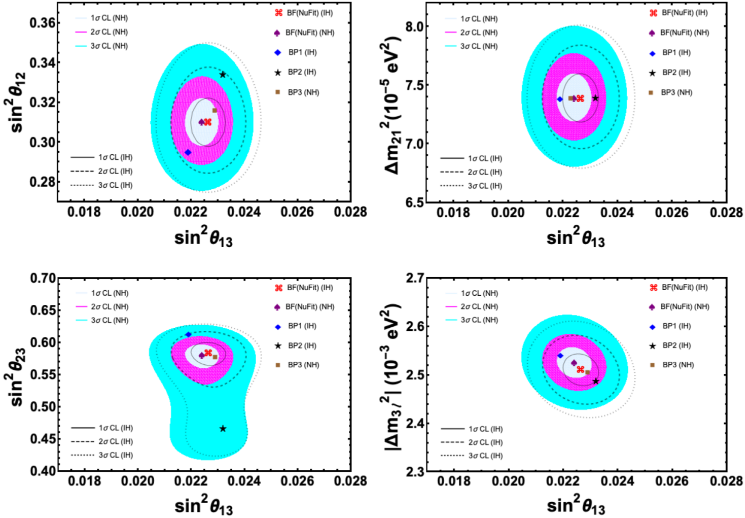
Neutrino oscillations in the Zee model

- Neutrino oscillation data can be fit to the Zee model consistently without the Z_2 symmetry
- Some benchmark points for Yukawa couplings of second doublet:

$$
\begin{aligned}
\text{BP I}: \ Y &= \begin{pmatrix} Y_{ee} & 0 & Y_{e\tau} \\ 0 & Y_{\mu\mu} & Y_{\mu\tau} \\ 0 & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix} \\
\text{BP II}: \ Y &= \begin{pmatrix} 0 & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & 0 & Y_{\mu\tau} \\ 0 & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix} \\
\text{BP III}: \ Y &= \begin{pmatrix} Y_{ee} & 0 & Y_{e\tau} \\ 0 & Y_{\mu\mu} & Y_{\mu\tau} \\ Y_{\tau e} & 0 & Y_{\tau\tau} \end{pmatrix}
\end{aligned}
$$

Babu, Dev, Jana, Thapa (2019)

Neutrino fit in the Zee model



Babu, Dev, Jana, Thapa (2019)

Neutrino Non-Standard Interactions (NSI)

- Neutrino oscillation picture would change if there are non-standard interactions
- Modification of matter effects most important
- EFT for neutrino NSI:

$$\mathcal{L}_{\rm NSI}^{\rm NC} = -2\sqrt{2}G_F \sum_{f,X,\alpha,\beta} \varepsilon_{\alpha\beta}^{fX} \left(\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta\right) \left(\bar{f}\gamma_\mu P_X f\right) ,$$

$$\mathcal{L}_{\rm NSI}^{\rm CC} = -2\sqrt{2}G_F \sum_{f,f',X,\alpha,\beta} \varepsilon_{\alpha\beta}^{ff'X} \left(\bar{\nu}_\alpha \gamma^\mu P_L \ell_\beta\right) \left(\bar{f}'\gamma_\mu P_X f\right)$$

Wolfenstein (1978)

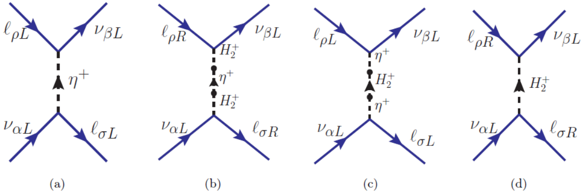
- Effective Hamiltonian for neutrino propagation in matter is now:

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \sqrt{2}G_F N_e(x) \begin{pmatrix} 1+\varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^\star & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^\star & \varepsilon_{\mu\tau}^\star & \varepsilon_{\tau\tau} \end{pmatrix}$$

- $\epsilon_{\alpha\beta}$ measure of NSI normalized to weak interaction strength

Neutrino NSI in the Zee model

- The two charged scalars of the Zee model mediate NSI



- The NSI parameters are given by:

$$\varepsilon_{\alpha\beta} = \frac{1}{4\sqrt{2}G_F} Y_{\alpha e} Y^*_{\beta e} \left(\frac{\sin^2\varphi}{m^2_{h^+}} + \frac{\cos^2\varphi}{m^2_{H^+}} \right)$$

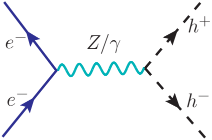
- Constrained by LHC and LEP direct limits; cLFV; precision electroweak tests; neutrino oscillation data; and theory

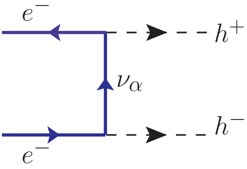
Babu, Dev, Jana, Thapa (2019)

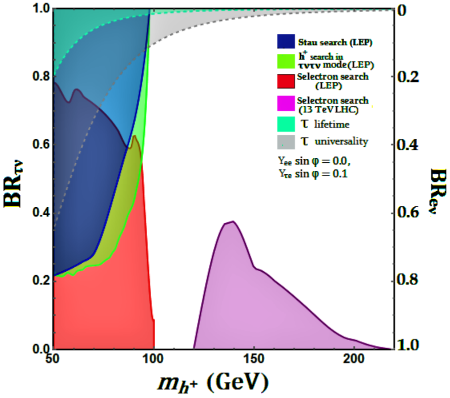
Constraints on Zee model parameters

- Electroweak T parameter sets limits on mixing $\sin\varphi$
- $\mu \to e + \gamma$ type processes limit products of couplings
- $\mu \to 3e$ type processes lead to further constraints
- τ lifetime and universality constraints
- Lepton universality in $W^{\pm}$ decays
- Theoretical constraint from avoiding charge breaking minima
- LEP direct search limits on charged scalars
- Constraints from LHC searches
- Higgs precision physics limits

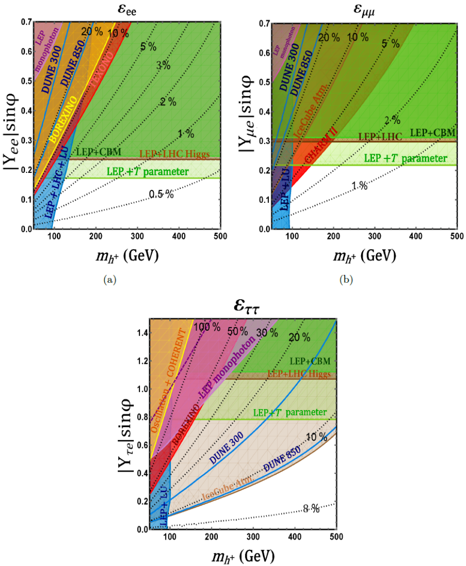
LEP and LHC constraints on Charged Scalar







Diagonal NSI in Zee model



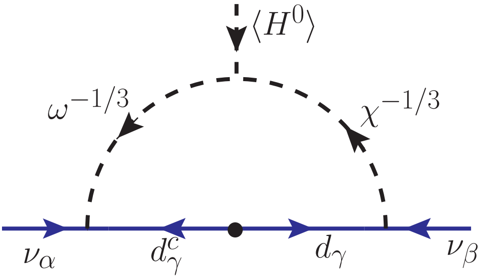
(a)

(b)

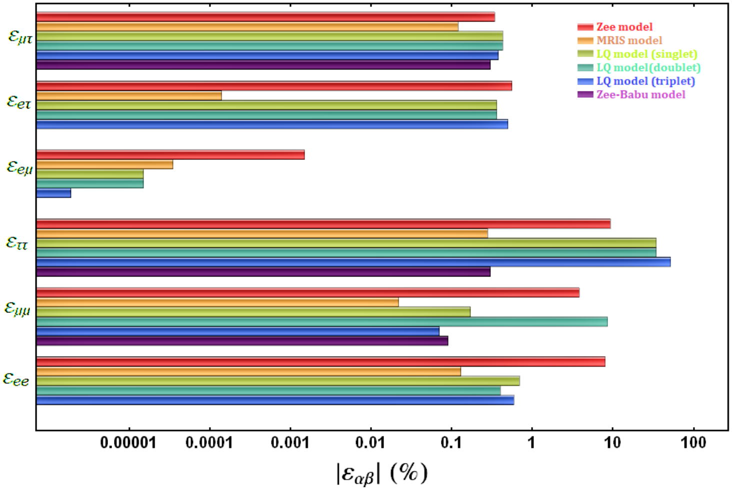
Babu, Dev, Jana, Thapa (2019)

Leptoquark models of radiative neutrino mass

- Charged lepton in Zee diagram may be replaced by quarks
- Charged scalars will then be replaced by Leptoquark scalars
- Several such models exist in literature
- More interest in context of B meson deacy anomalies



Summary of NSI in radiative models

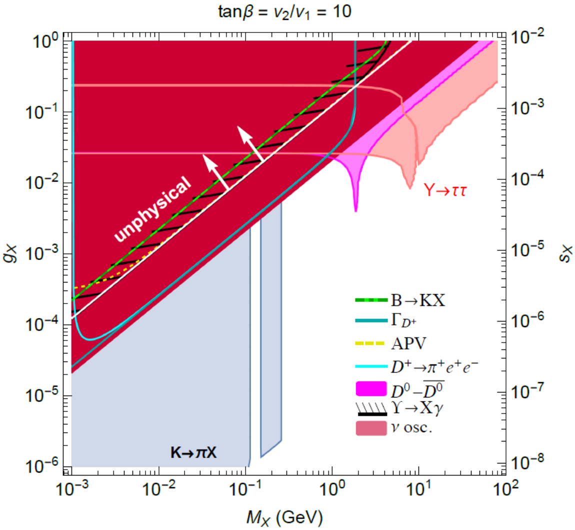


Babu, Dev, Jana, Thapa (2019)

Nutrino Mass Models with Light Mediators

- If the mediator generating $(\overline{\nu}_\alpha \gamma_\mu \nu_\beta)(\overline{f} \gamma^\mu f)$ interactions is light, the severe charged lepton flavor violation constraints may be evaded
- Gauging $(B - L)$ for the third family is an explicit example of this
  Babu, Friedland, Machado, Mocioiu (2017)
- The model has ν_R fields, a second Higgs doublet ϕ_2 and a singlet s, both with $(B - L)$ charge of $1/3$
- ϕ_2 generates quark mixings; charged leptons remain unmixed $\Rightarrow$ No flavor violation in charged leptons
- If mass of the new gauge boson X is of order 100 MeV, with the gauge coupling $g_X \sim 10^{-3}$ all constraints are satisfied
- This explicit model generates $\epsilon_{\tau\tau} \sim 0.5$
- ν_3^c is light and may serve as the sterile neutrino relevant for short baseline anomalies Babu, Friedland, Mocioiu, Machado (to appear)

$(B-L)_3$ Model Constraints



Babu, Friedland, Machado, Mocioiu (2017)

Other Models with large NSI

- Several models have been proposed to generate observable NSI
- Main challenge is to control charged lepton flavor violation and univesality constraints
- Some models use cancellations among $d = 6$ and $d = 8$ operators
  Gavela, Hernandez, Ota, Winter (2009)
- Light mediators help with satisfing such constraints Farzan, Shoemaker (2016); Farzan (2016); Denton, Farzan, Shoemaker (2018)
- Collider signals of these models have been studied, especailly for monojet signals Friedland, Graesser, Shoemaker (2012); Elahi, Martin (2019); Babu, Goncalves, Jana, Machado (2021)

Conclusions

- EFT description alone in neutrino sector is inadequate; we may miss important phenomena such as leptogenesis

- Grand Unification provides powerful tools to interconnect neutrino sector with quark sector

- Neutrino may very well be Dirac particles; interesting models of Dirac neutrino exist

- Various $d = 7$ and $d = 9$ lepton number violating EFT operators can lead to interesting neutrino mass models

- These models may be realized near the TeV scale, with potential signals for NSI, cFLV and direct detection at colliders