Inflation from Dynamical Projective Connections

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Cosmological Inflation

Rapid expansion of early universe

Explains problems with Big Bang Model: Flatness, Horizon, Monopole, etc

Models of inflation involve scalar field, ϕ:

$$S = \frac{1}{2\kappa_0} \int d^4x \sqrt{|g|} f(\phi) R - \int d^4x \sqrt{|g|} [\frac{1}{2} \nabla_a \phi \nabla^a \phi - V(\phi)] \quad (1)$$

- $V(\phi)$, kinetic term, coupling to gravity choices guided by theoretical and observational constraints
- often no fundamental principle governing its dynamics

Our approach: Can we explain inflation as particular manifestation of tensor field arising from TW-gravity?

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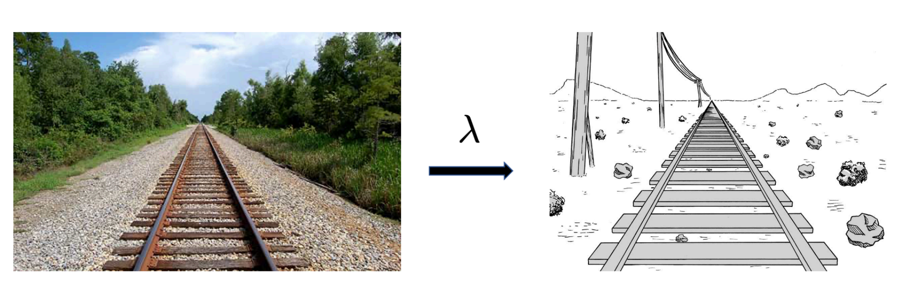
Projective Geometry



Projective Invariance ⇒ General Relativity

$$\mathcal{K}^a_{bcd} = R^a_{bcd} + \delta_{[c}{}^a \mathcal{P}_{d]b} - \delta^a{}_b \mathcal{P}_{[cd]}$$

TW - Gravity Action

$$S_{TW} = -\frac{1}{2\tilde{\kappa_0}}\int d\ell\, d^dx\sqrt{|G|}\mathcal{K}$$
$$-\tilde{J}_0 c\int d\ell\, d^dx\sqrt{|G|}\left[\mathcal{K}^2 - 4\mathcal{K}_{\alpha\beta}\mathcal{K}^{\alpha\beta} + \mathcal{K}^{\alpha}{}_{\beta\mu\nu}\mathcal{K}_{\alpha}{}^{\beta\mu\nu}\right] \quad (2)$$

Integrate $\ell = \frac{\lambda}{\lambda_0}$

$\downarrow$

$$S_{TW} = -\frac{1}{2\kappa_0}\int d^dx\sqrt{|g|}\left(R + (d-1)\mathcal{P} + 2\Lambda_0\right)$$
$$+ J_0 c\int d^dx\sqrt{|g|}\left[\lambda_0^2 K_{abc}K^{abc} - \mathcal{P}_{ab}\tilde{\mathcal{P}}_*^{ab}\right] - J_0 c S_{GB} \quad (3)$$

$$K_{abc} = \nabla_{[b}\mathcal{P}_{c]a}, \quad \tilde{\mathcal{P}}_*^{ab} \equiv (d-1)g^{ab}\tilde{\mathcal{P}} - 2(2d-3)\tilde{\mathcal{P}}^{ab} \quad (4)$$

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Coupling Constants

$d = 4, \quad c = \hbar = 1$ define dimensionless couplings:

$$\kappa_0 = n_\kappa M_p^{-2}, \quad \lambda_0 = n_\lambda M_p^{-1}, \quad J_0 = n_J \hbar = n_J$$

- n_κ = gravitational coupling
- n_λ = projective length scale
- n_J = angular momentum parameter

By definition: $n_\kappa, n_\lambda, n_J > 0$

$$\hat{n} = \frac{n_\lambda}{n_J n_\kappa} \tag{5}$$

Realizing Inflation

$$\mathcal{P}_{ab} = \frac{M_p}{n_\lambda}\phi g_{ab} \tag{6}$$

$$S = -\frac{M_p^2}{2n_\kappa}\int d^4x\sqrt{|g|}f(\phi)R + 12n_J\int d^4x\sqrt{|g|}\left[\tfrac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi)\right] \tag{7}$$

Non-Minimally coupled Inflation with linear coupling (Brans-Dicke-like):

$$f(\phi) = 1 + \frac{8}{\hat{n}M_p}\phi \tag{8}$$

Predicted Inflaton Potential

$$V(\phi) = \frac{M_p^4}{32n_J^2n_\kappa^2}\left(f(\phi)^2 - 1\right) = \frac{M_p^2}{2n_\lambda^2}\left(4\phi^2 + M_p\hat{n}\phi\right) \tag{9}$$

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Canonical Scalar Field Action

Perform Conformal transformation from Jordan to Einstein frame:

$$g_{ab} = e^{-2\omega}\tilde{g}_{ab} \qquad \omega = \ln\sqrt{\frac{f(\phi)}{n_\kappa}} \tag{10}$$

$$h(\phi) = \begin{cases} \sqrt{6}M_p\left[\sqrt{1+\frac{\hat{n}n_\lambda}{8}f(\phi)} - \tanh^{-1}\sqrt{1+\frac{\hat{n}n_\lambda}{8}f(\phi)}\right] & -\frac{8}{\hat{n}n_\lambda} < f(\phi) < 0 \\ \sqrt{6}M_p\left[\sqrt{1+\frac{\hat{n}n_\lambda}{8}f(\phi)} - \coth^{-1}\sqrt{1+\frac{\hat{n}n_\lambda}{8}f(\phi)}\right] & f(\phi) > 0 \end{cases} \tag{11}$$

$$S = \int d^4x\sqrt{|\tilde{g}|}\left(-\frac{M_p^2}{2}\tilde{R} + \frac{1}{2}\tilde{\nabla}_a h\tilde{\nabla}^a h - \tilde{V}(\phi(h))\right) \tag{12}$$

$$\tilde{V}(\phi) = \frac{3M_p^4}{8n_J}\left(1 - \frac{1}{f(\phi)^2}\right) = \frac{6M_p^4\phi(M_p\hat{n} + 4\phi)}{n_J(M_p\hat{n} + 8\phi)^2} \tag{13}$$

Slow Roll Parameters

Accelerated expansion: $\ddot{a} > 0 \Rightarrow \epsilon < 1$, $\eta \ll 1$

$$\epsilon = \frac{M_p^2}{2}\left(\frac{\partial\tilde{V}/\partial h}{\tilde{V}}\right)^2, \quad \eta = M_p^2 \frac{\partial^2\tilde{V}/\partial h^2}{\tilde{V}} \tag{14}$$

$$C(\phi) \equiv \frac{12}{\hat{n}^2 f(\phi)^2}[8 + \hat{n} n_\lambda f(\phi)] = \frac{96}{\hat{n}^2 f(\phi)^2}\left[1 + \frac{\hat{n} n_\lambda}{8} + \frac{n_\lambda}{M_p}\phi\right] \tag{15}$$

$$\epsilon = \frac{M_p^2}{2C}\left(\frac{\partial\tilde{V}/\partial\phi}{\tilde{V}}\right)^2 \quad , \quad \eta = \frac{M_p^2}{\sqrt{C}\tilde{V}}\frac{\partial}{\partial\phi}\left[C^{-1/2}\frac{\partial\tilde{V}}{\partial\phi}\right] \tag{16}$$

Model Predictions:

$$\epsilon = \frac{32}{3(f^2-1)^2(8+\hat{n} n_\lambda f)} \quad , \quad \eta = -\frac{16}{3}\frac{32+5\hat{n} n_\lambda f}{(f^2-1)(8+\hat{n} n_\lambda f)^2} \tag{17}$$

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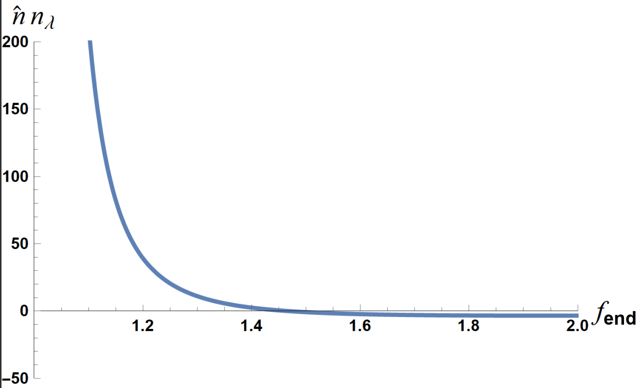
Observational Parameters

	Model Predictions	Constraints	
Scalar Spectral Index:			
	$n_s = 1 - 6\epsilon + 2\eta\|_{\phi_*}$	$n_s = 0.9649 \pm 0.0042$	(18)
Tensor to Scalar Ratio:			
	$r = 16\epsilon\|_{\phi_*}$	$r < 0.056$	(19)
Amplitude of Density Perturbations:			
	$A_s = \frac{1}{24\pi^2 M_p^4}\frac{\tilde{V}}{\epsilon}\|_{\phi_*}$	$2 \leq \ln 10^{10} A_s \leq 4$	(20)
Number of e-folds:			
	$N \simeq \frac{1}{M_p^2}\int_{h_{\text{end}}}^{h_*} \frac{\tilde{V}}{\partial\tilde{V}/\partial h} dh$	$N \in [50 - 70]$	(21)

Constraining f_{end} with $\epsilon|_{\phi_{end}} = 1$

$$\hat{n} n_\lambda = \frac{32}{3 f_{end}(f_{end}^2 - 1)^2} - \frac{8}{f_{end}}, \quad \hat{n} n_\lambda > 0 \tag{22}$$



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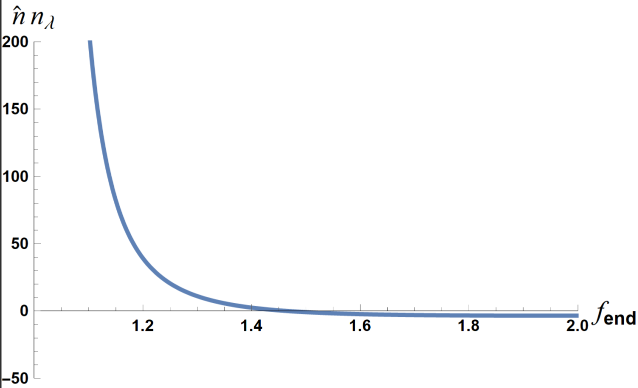

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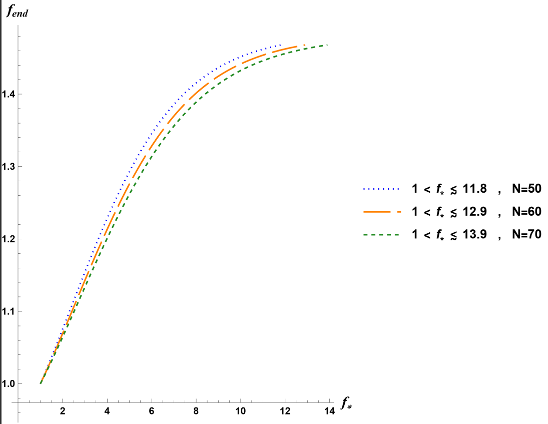
Constraining f_* to f_{end} with N efolds

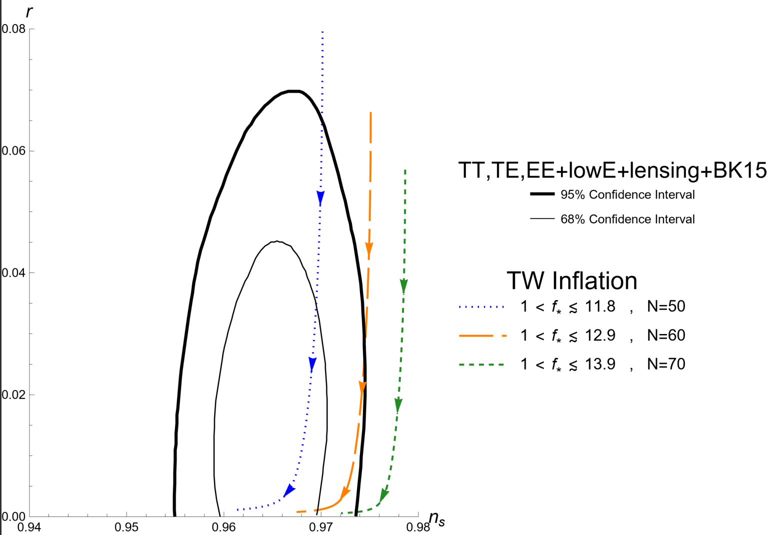
$$N \simeq \frac{1}{32}\left(\hat{n}n_\lambda(\alpha^3-1)f_{end}^3 + 12(\alpha^2-1)f_{end}^2 - 3\hat{n}n_\lambda(\alpha-1)f_{end} - 24\ln\alpha\right)$$

$$\alpha \equiv \frac{f(\phi_*)}{f(\phi_{end})} \quad , \quad f_{end} = f(\phi_{end}) \tag{23}$$



Constraining $f_\star$ with r vs. n_s Parameter space

Planck, Bicep2, Keck Array data



Values of $f_\star$, $f_{end}, \hat{n}n_\lambda$ constrained by observational values of r vs. n_s

		95% CL Boundary			68% CL Boundary		
		f_*	f_{end}	$\hat{n}n_\lambda$	f_*	f_{end}	$\hat{n}n_\lambda$
N	50	1.13	1.01	30300	1.62	1.05	1150
	60	1.70	1.05	1060	9.84	1.44	0.896
	70	11.1	1.45	0.625			

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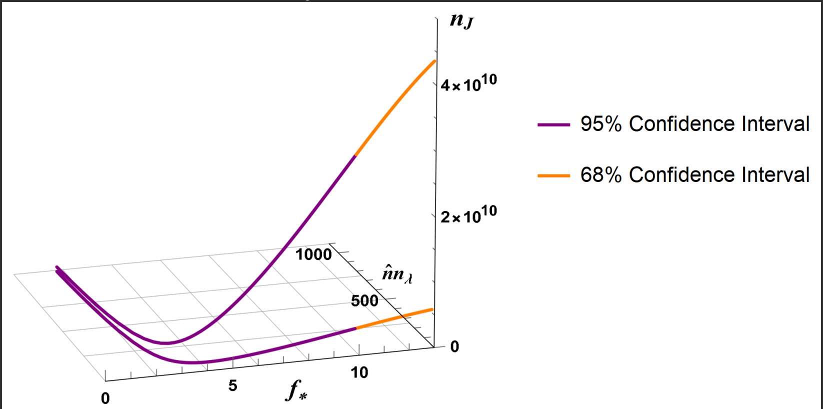
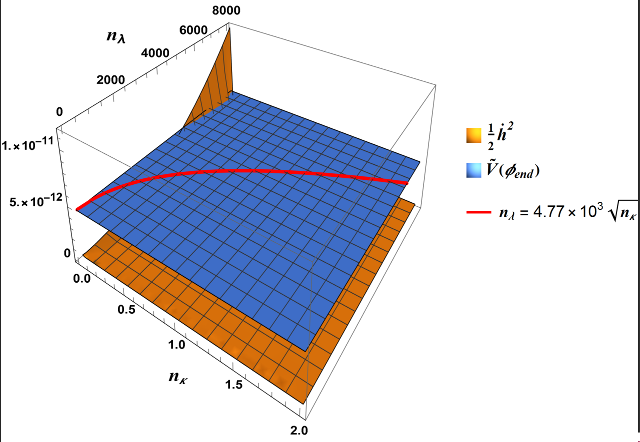
Constraining n_J with A_s

$N = 60$: $1.15 \times 10^8 \leq n_J \leq 4.37 \times 10^{10}$



Validating Slow Roll

Slow Roll: $\frac{1}{2}\dot{h}^2 \ll \tilde{V}$ for $N = 60$, $n_J = 4.37 \times 10^{10}$, $f_\star = 12.9$, $f_{end} = 1.47$



Parameter Space

Minimum-Maximum values of three free parameters

$$\begin{aligned} \text{Choosing:} \quad 0 < n_\kappa < 2 \\ 1.15 \times 10^8 < n_J < 4.37 \times 10^{10} \\ 0 < n_\lambda < 1.34 \times 10^6 \end{aligned} \tag{24}$$

Conclusion

- Thomas-Whitehead Gravity predicts a model of inflation when $P_{ab} \sim \phi g_{ab}$ consistent with data for range of parameters $(n_\kappa, n_\lambda, n_J)$

Future Work

$$P_{ab} \sim \phi g_{ab} + w_0 W_{ab}, \quad \text{with} \quad W_{ab}\, g^{ab} = 0 \tag{25}$$

Scalar field Ansatz for traceless W_{ab}:

$$W_a{}^b = u(t) \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{26}$$

- Can $u(t)$ be curvaton, responsible for reheating mechanism?
- Traceless tensor field W_{ab}: responsible for density perturbations of CMB without quantum fluctuations?
- We assumed connection is Levi-Civita: Investigate Inflation application in Palatini formalism of TW-gravity

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