Detecting High-Frequency Gravitational Waves with Microwave Cavities

Jan Schütte-Engel

based on: *arxiv:2112.11465* (accepted in PRD)

in collaboration with A. Berlin, D. Blas, R. Tito D'Agnolo, S. A.R. Ellis, R. Harnik, Y. Kahn

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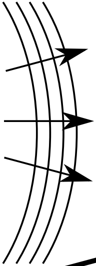
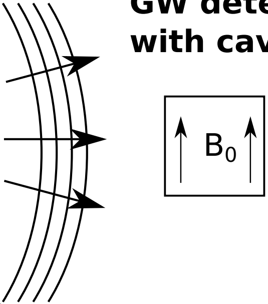
High Frequency GW Sources

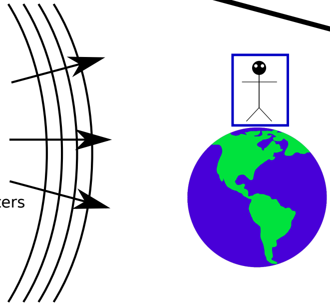


GW detection with cavities



Proper detector frame

- Sensitivity to direction and polariation
- Why using the proper detector frame matters



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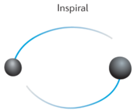

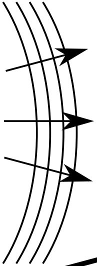

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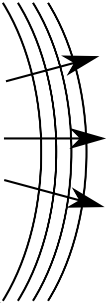

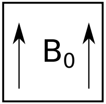

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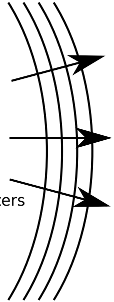
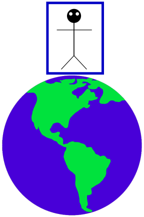
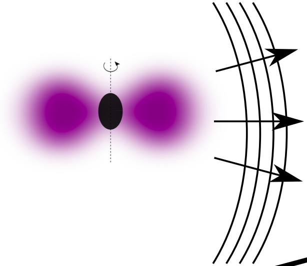




High Frequency GW Sources



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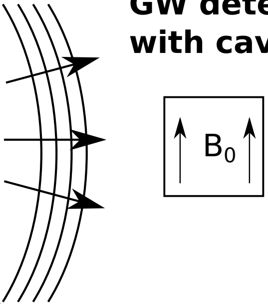

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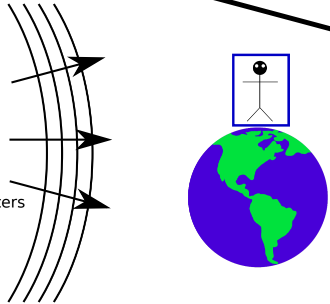
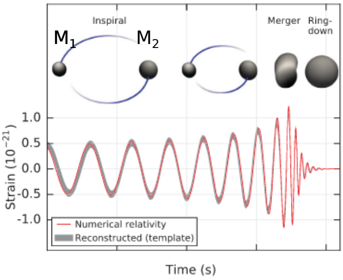


High Frequency GW Sources

Mergers of sub-solar mass objects



[1602.03837]

innermost stable circular orbit (ISCO)

$$r_{\rm ISCO} = 0.02\,{\rm m}\,\frac{M_b}{10^{-6}\,M_\odot}$$

$$f_{\rm ISCO} = 1.1\,{\rm GHz}\left(\frac{10^{-6}\,M_\odot}{M_b}\right)$$

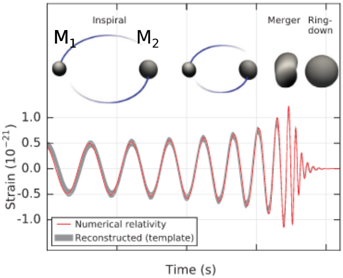
$$\omega_g \simeq 14\ {\rm GHz} \times \frac{10^{-6}M_\odot}{M_b}\left(\frac{r_{\rm ISCO}}{r_b}\right)^{\frac{3}{2}}$$

$M_1 = M_b = M_2$

$$\mathcal{N}_{\rm cyc} \geq Q \Rightarrow M_b \leq 10^{-11}\,M_\odot\left(\frac{10^5}{Q}\right)^{\frac{6}{5}}\left(\frac{1\,{\rm GHz}}{\omega_g}\right)$$

with $Q = \frac{f}{\Delta f}$.

Mergers of sub-solar mass objects



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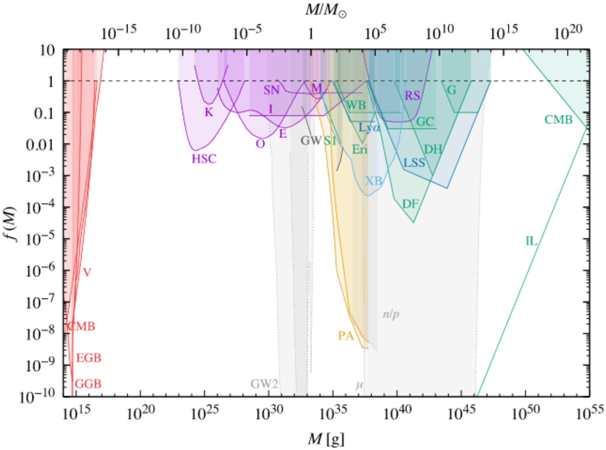
$$M_1 = M_b = M_2$$

Expected strain

$$h_0 \approx 10^{-29}\times\left(\frac{1\text{ pc}}{D}\right)\left(\frac{M_b}{10^{-11}\,M_\odot}\right)^{5/3}\left(\frac{\omega_g}{1\text{ GHz}}\right)^{2/3}$$

D is distance to Binary merger.

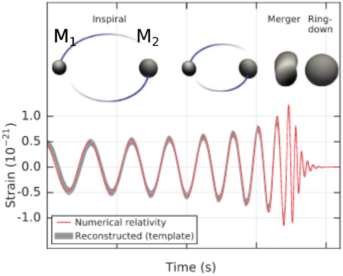
PBHs as dark matter: constraints



[Carr et al. 21]

Evaporation (red), lensing (magenta), dynamical effects (green), gravitational waves (black), accretion (light blue), CMB distortions (orange), large-scale structure (dark blue) and background effects (grey).

Mergers of sub-solar mass objects



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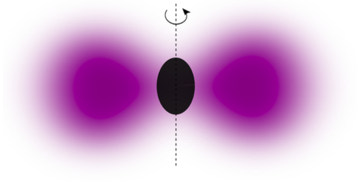
$$M_1 = M_b = M_2$$

If PBHs are 100% DM

$$D \approx 10^{-3}\,{\rm pc}\left(\frac{M_b}{10^{-11}\,M_\odot}\right)^{\frac{1}{3}} \Rightarrow \text{ best case scenario } \Rightarrow h_0 \approx 10^{-26}$$

Study taking into account merger rate: [2205.02153]

Boson clouds from PBH superradiance



[kipac.stanford.edu]

- Annihilation of bosons

$$m_a \approx \mu\text{eV} \times (10^{-4}\, M_\odot / M_{\text{PBH}})$$
$$\omega_g = 2m_a \approx \text{GHz} \times (m_a/\mu\text{eV})$$

- GW waveform is monochromatic and coherent over very long timescales

Expected strain

$$h_0 \approx 10^{-27} \left(\frac{\alpha/\ell}{0.5}\right) \left(\frac{\epsilon}{10^{-3}}\right) \times \left(\frac{10\ \text{kpc}}{D}\right) \left(\frac{M_{\text{PBH}}}{10^{-4}\, M_\odot}\right)$$

$\alpha = GM_{\text{PBH}} m_a$, ℓ is orbital quantum number, ϵ is fraction of PBH mass the axion cloud carries. [Arvanitaki et al.12]

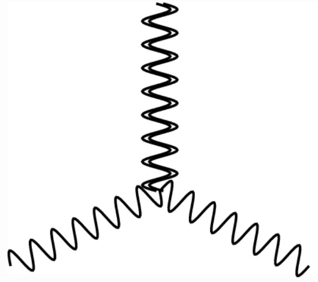
If $10^{-4}\, M_\odot$ PBHs are 1% of DM:

$$D \approx 1\,\text{pc} \quad \Rightarrow \quad \text{best case scenario} \ \Rightarrow h_0 \approx 10^{-23}$$

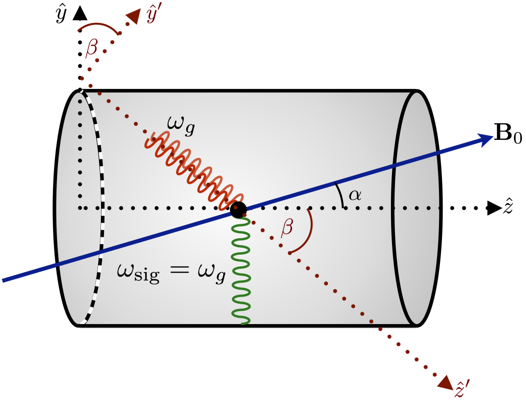
GW detection with cavities

$$\mathcal{L} \supset -\frac{1}{4}\eta^{\mu\alpha}\, h^{\nu\beta}\, F_{\mu\nu}F_{\alpha\beta}$$

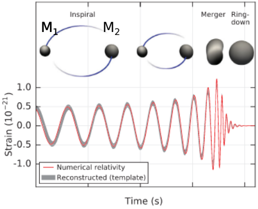
$$(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})$$



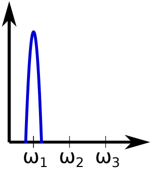
[Gertsenshtein 62]

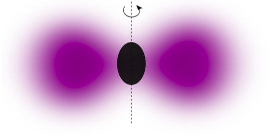

$\hat{y}$
$\hat{y}'$
β
ω_g
$\mathbf{B}_0$
α
$\hat{z}$
β
$\omega_{\text{sig}} = \omega_g$
$\hat{z}'$

Signal in the two benchmark scenarios

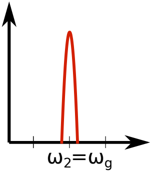


- Keep cavity fixed
- Sweep through all cavity modes





- Change cavity to scan different resonance frequencies
- GW frequency from superradiant cloud fixed



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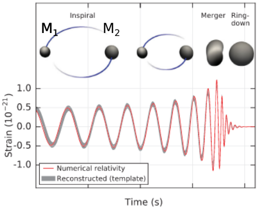

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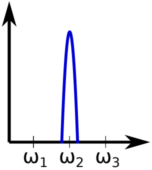

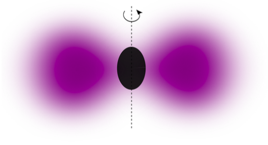

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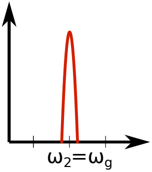

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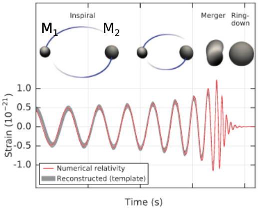

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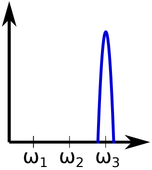

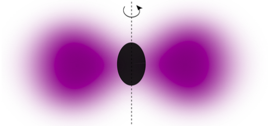

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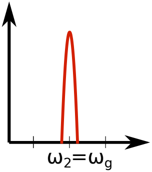

Maxwell equations on curved spacetime

$$\begin{aligned}\nabla \cdot \boldsymbol{E} &= \rho_{\rm eff} + \rho, \\ \nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} &= \boldsymbol{j}_{\rm eff} + \boldsymbol{j}\ ,\end{aligned}$$

$$j^{\mu}_{\rm eff} \equiv \partial_\nu \left(\frac{1}{2}\, h\, F^{\mu\nu} + h^{\nu}{}_{\alpha}\, F^{\alpha\mu} - h^{\mu}{}_{\alpha}\, F^{\alpha\nu} \right)$$

Maxwell equations on curved spacetime

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\begin{aligned}
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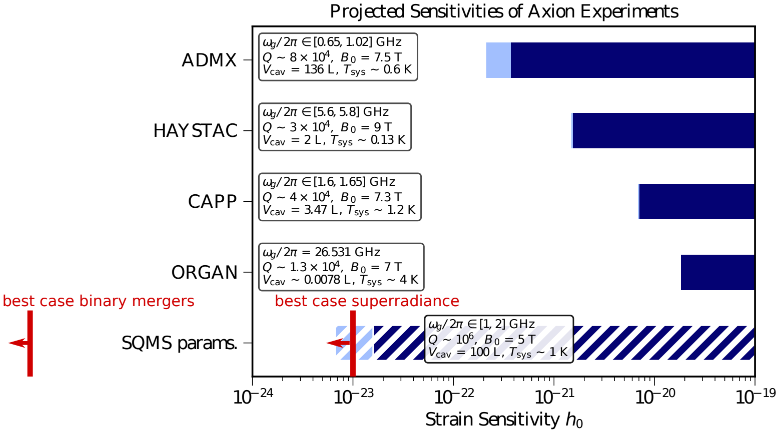
\end{aligned}
$$

$$
j^\mu_{\rm eff} \equiv \partial_\nu \left(\frac{1}{2} \, h \, F^{\mu\nu} + h^\nu{}_\alpha \, F^{\alpha\mu} - h^\mu{}_\alpha \, F^{\alpha\nu} \right)
$$

$$
h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ \text{GHz}}{\omega_g / 2\pi} \right)^{3/2} \left(\frac{0.1}{\eta_n} \right) \left(\frac{8\ \text{T}}{B_0} \right) \left(\frac{0.1\ \text{m}^3}{V_{\rm cav}} \right)^{5/6} \times
\times \left(\frac{10^5}{Q} \right)^{1/2} \left(\frac{T_{\rm sys}}{1\ \text{K}} \right)^{1/2} \left(\frac{\Delta\nu}{10\ \text{kHz}} \right)^{1/4} \left(\frac{1\ \text{min}}{t_{\rm int}} \right)^{1/4}
$$

$$
\eta_n \equiv \frac{\left| \int_{V_{\rm cav}} d^3\boldsymbol{x} \ \boldsymbol{E}_n^* \cdot \boldsymbol{j}_{\rm eff} \right|}{h_0 \, B_0 \omega_g^2 V_{\rm cav}^{\frac{5}{6}} \left(\int_{V_{\rm cav}} d^3\boldsymbol{x} \, |\boldsymbol{E}_n|^2 \right)^{\frac{1}{2}}}
$$

Sensitivity of existing axion experiments

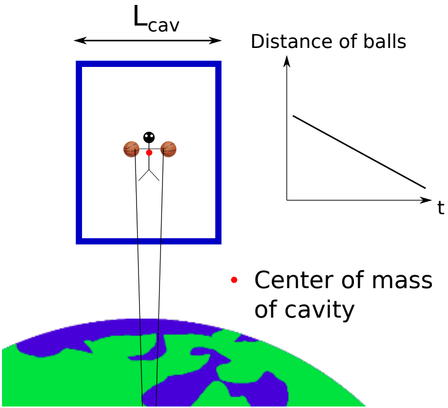


Existing axion experiments only need to reanalyze their data!

Proper detector frame

(Fermi Normal coordinates)

Proper detector frame

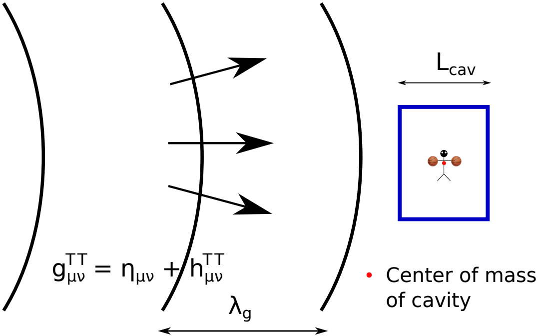


- Cavity freely falling towards Earth
- Coordinate system attached to the center of mass is proper detector frame
- Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\rm cav}}{K}\right)$$

K is scale on which the gravitational field varies.

Proper detector frame



$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega_g(t-z)}$$

TT: transverse traceless

Metric in proper detector frame:

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\rm cav}}{\lambda_{\rm g}}\right)$$

$\lambda_{\rm g}$ wavelength of GW

Proper detector frame

Resonant excitation of cavity:

$$\lambda_g \simeq L_{cav}$$

$$g_{00} = \eta_{00} + h_{00} = -1 - 2\sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n,k_1,\cdots k_r} x^m x^n x^{k_1} \cdots x^{k_r}$$

(Similar eq. for g_{0i} and g_{ij})

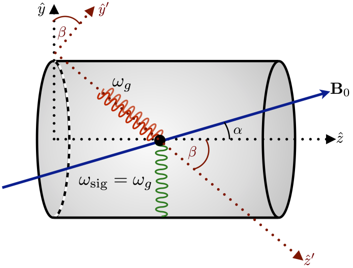
[Marzlin 94, Rakhmanov 14]

- Riemann tensor invariant under infinitesimal coordinate transformations. Evaluate therefore in TT-frame and plug in series expansion above.
- GW propagating in z-direction:

$$R_{ijkl} \sim e^{i\omega_g(t-z)}$$

$$h_{00} = -\omega_g^2\, h_{ab}^{TT}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

Why the frame matters

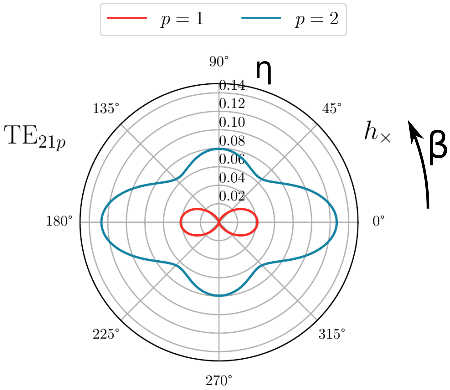


TT frame is not the right frame! Use proper detector frame.

$$\alpha = 0, \beta = 0$$

If we use TT metric $\boldsymbol{j}_{\rm eff} = 0$. NO SIGNAL!
However this is **WRONG**. Use proper detector metric.

Proper detector frame result ($\alpha = 0$)



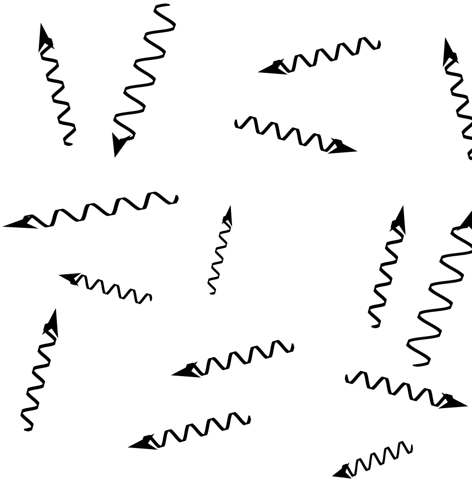
Conclusions

- PBH inspirals and PBH superradiance can generate GWs in GHz regime
- Difficult to probe PBH inspirals but PBH superradiance best case scenario can be probed
- Signal calculation in cavity: Use proper detector frame metric resummed to all orders
- Existing axion experiments only need to reanalyze data

Thank you for your attention

Backup

High Frequency GW Sources



Stochastic GWs: constraints

$$\Omega_{\rm GW}^{(0)} = \frac{1}{\rho_c^{(0)}} \frac{d\rho_{\rm GW}^{(0)}}{d\ln f}, \qquad \text{with} \quad \rho_c^{(0)} = \frac{3H_0^2}{8\pi G}$$

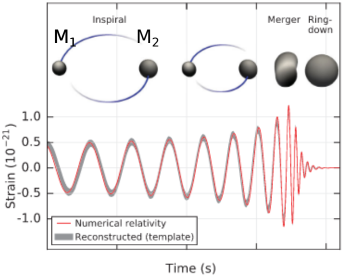
$$\frac{\rho_{\rm GW}^{(0)}}{\rho_c^{(0)}} = \int_0^\infty \frac{df}{f} \Omega_{\rm GW}^{(0)} \approx \Omega_{\rm GW}^{(0)}(f_{\rm max}^{(0)})$$

$$h^2 \frac{\rho_{\rm GW}^{(0)}}{\rho_c^{(0)}} \leq h^2 \frac{\Delta\rho_{\rm rad}^{(0)}}{\rho_c^{(0)}} = h^2 \Omega_\gamma \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} \Delta N_{\rm eff} = 5.6\times 10^{-6}\,\Delta N_{\rm eff}$$

$$h^2 \Omega_{\rm GW}^{(0)}(f_{\rm max}^{(0)}) \leq 1.68\times 10^{-6}\,\left(\frac{\Delta N_{\rm eff}}{0.3}\right)$$

$g_{*s}(T_0) = 3.90$, $g_{*s}(T) = 10.75$, $T_0 = 2.72\,\mathrm{K}$, $h^2\Omega_\gamma = 2.47\times 10^{-5}$

Mergers of sub-solar mass objects



[1602.03837]

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$$f_{\text{ISCO}} = 1.1\,\text{GHz}\left(\frac{10^{-6}\,M_\odot}{M_b}\right)$$

$$\omega_g \simeq 14\text{ GHz} \times \frac{10^{-6}M_\odot}{M_b}\left(\frac{r_{\text{ISCO}}}{r_b}\right)^{\frac{3}{2}}$$

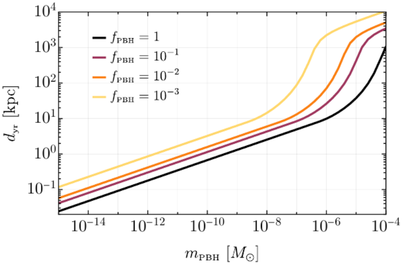
$$M_1 = M_b = M_2$$

- primordial black holes (PBHs) [Hawking 71]
- boson and fermion stars
  [Palenzuela et al. 07, Giudice et al. 16, Palenzuela et al. 17, Helfer et al. 18]
- gravitino stars [Narain et al. 06] and gravistars [Mazur et al. 04]
- dark matter blobs [Diamond et al. 21]

Sensitivity to stochastic GW background

- $S_{\rm sig}(\omega) = \frac{\omega_n}{Q}\frac{(\omega\omega_n)^2}{(\omega^2-\omega_n^2)^2+(\omega\omega_n/Q)^2}\left|\eta\right|^2 B_0^2 V_{\rm cav} S_h(\omega)$
- Thermal noise $S_{\rm noise}(\omega) = \frac{4\pi T(\omega\omega_n/Q)^2}{(\omega^2-\omega_n)^2+(\omega\omega_n/Q)^2}$
- Non-coherent signal appears as an additional noise source in the detector
- $\mathrm{SNR} = \frac{S_{\rm sig}}{S_{\rm noise}} = \frac{\omega_n Q}{4\pi T}\left|\eta\right|^2 B_0^2 V_{\rm cav} S_h(\omega_g)$
- $\Omega_{\rm GW}(\omega) = \frac{2}{3}\frac{\omega^3}{H_0^2} S_h(\omega)$
- $\Omega_{\rm GW} = 8 \times 10^{10} \times \left(\frac{(0.2)^2}{|\eta|^2}\right)\left(\frac{10\,\mathrm{T}}{B_0}\right)^2\left(\frac{\omega_n}{1\,\mathrm{GHz}}\right)^2\left(\frac{1\,\mathrm{m}^3}{V_{\rm cav}}\right)\left(\frac{10^{12}}{Q}\right)\left(\frac{T_{\rm sys}}{10\,\mathrm{mK}}\right)$
- Cosmologically produced GW backgrounds $\Omega_{\rm GW} < 10^{-6}$
- Without tricks the detection prospects are not great for stochastic GW backgrounds.

Alternative estimate for PBH case



$$d_{\rm yr} = 0.21 \text{ kpc} \left(\frac{m_{\rm PBH}}{10^{-11} M_\odot}\right)^{\frac{1}{3}}$$

[2205.02153]

$$h_0 = 10^{-31} \left(\frac{m_{\rm PBH}}{10^{-11}\, M_\odot}\right)^{\frac{4}{3}} \left(\frac{\omega_g}{1\,\text{GHz}}\right)^{\frac{2}{3}}$$

Detector sensitivity:

$$h_0 > 8.3 \times 10^{-20} \left(\frac{1\,\text{GHz}}{\omega_g/2\pi}\right)^{\frac{1}{2}} \left(\frac{0.1}{\eta}\right) \left(\frac{8\,\text{T}}{B_0}\right) \left(\frac{0.1\,\text{m}^3}{V_{\rm cav}}\right)^{\frac{5}{6}} \left(\frac{T_{\rm sys}}{1\,\text{K}}\right)^{\frac{1}{2}} \left(\frac{10^{-11}\, M_\odot}{m_{\rm PBH}}\right)^{\frac{6}{5}}$$

Mode decomposition

$$\nabla \times \nabla \times \boldsymbol{E} + \partial_t^2 \boldsymbol{E} = -\partial_t \boldsymbol{j}_{\text{eff}} - \partial_t \boldsymbol{j}$$

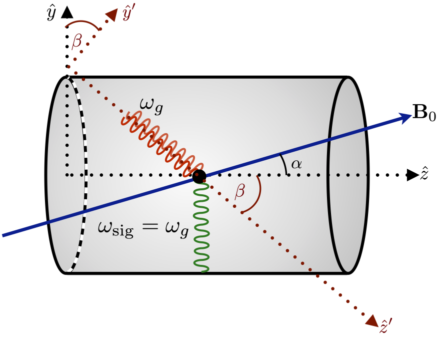
Cavity mode decomposition:

$$\boldsymbol{E}(\boldsymbol{x}, t) = \sum_n e_n(t)\, \boldsymbol{E}_n(\boldsymbol{x}) \ ,$$

$$P_{\text{sig}} = \frac{1}{2}\, Q\, \omega_g^3\, V_{\text{cav}}^{5/3}\, (\eta_n\, h_0\, B_0)^2$$

$$\eta_n \equiv \frac{\left| \int_{V_{\text{cav}}} d^3\boldsymbol{x}\ \boldsymbol{E}_n^* \cdot \boldsymbol{j}_{\text{eff}} \right|}{h_0\, B_0 \omega_g^2\, V_{\text{cav}}^{\frac{5}{6}} \left(\int_{V_{\text{cav}}} d^3\boldsymbol{x}\ |\boldsymbol{E}_n|^2 \right)^{\frac{1}{2}}}$$

Sensitivity estimate

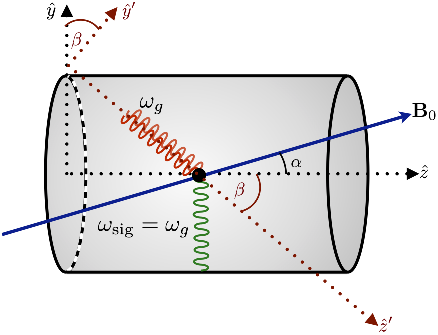


$$\mathrm{SNR} \simeq \frac{P_{\rm sig}}{T_{\rm sys}} \sqrt{\frac{t_{\rm int}}{\Delta\nu}}$$

$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ \mathrm{GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8\ \mathrm{T}}{B_0}\right) \left(\frac{0.1\ \mathrm{m}^3}{V_{\rm cav}}\right)^{5/6} \times \\ \times \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\rm sys}}{1\ \mathrm{K}}\right)^{1/2} \left(\frac{\Delta\nu}{10\ \mathrm{kHz}}\right)^{1/4} \left(\frac{1\ \mathrm{min}}{t_{\rm int}}\right)^{1/4}$$

Bandwidth $\Delta\nu = \frac{\omega_g}{2\pi Q}$.

Sensitivity estimate

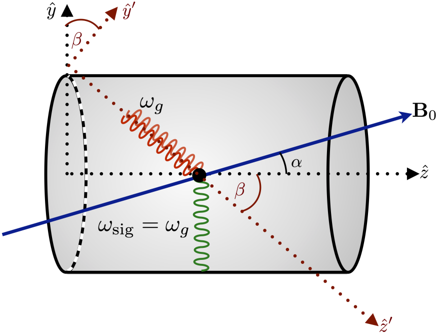


$$\mathrm{SNR} \simeq \frac{P_{\rm sig}}{T_{\rm sys}} \sqrt{\frac{t_{\rm int}}{\Delta\nu}}$$

$$h_0 \gtrsim 1 \times 10^{-23} \times \left(\frac{1\ \text{GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8\ \text{T}}{B_0}\right) \left(\frac{0.1\ \text{m}^3}{V_{\rm cav}}\right)^{5/6} \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\rm sys}}{1\ \text{K}}\right)^{1/2} \left(\frac{1\ \text{min}}{t_{\rm int}}\right)^{1/2}$$

Bandwidth $\Delta\nu = \frac{1}{t_{\rm int}}$

Sensitivity estimate



$$\mathrm{SNR} \simeq \frac{P_{\rm sig}}{T_{\rm sys}} \sqrt{\frac{t_{\rm int}}{\Delta\nu}}$$

$$h_0 \gtrsim 1 \times 10^{-23} \times \left(\frac{1\ \mathrm{GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8\ \mathrm{T}}{B_0}\right) \left(\frac{0.1\ \mathrm{m}^3}{V_{\rm cav}}\right)^{5/6} \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\rm sys}}{1\ \mathrm{K}}\right)^{1/2} \left(\frac{1\ \mathrm{min}}{t_{\rm int}}\right)^{1/2}$$

Bandwidth $\Delta\nu = \frac{1}{t_{\rm int}}$

-	Superradiance	Binary mergers
$t_{\rm int}$	1 min (single scan time)	10^{-4} s (no scan)
best case h_0	10^{-23}	10^{-26}

Coordinate transformations in linearized theory

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Coordinate transformation

$$x'^{\mu\nu} = x^{\mu} + \xi^{\mu}$$

$$g'_{\mu\nu}(x') = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} g_{\rho\sigma} \stackrel{!}{=} \eta_{\mu\nu} + h'_{\mu\nu}$$

with

$$h'_{\mu\nu}(x') = h_{\mu\nu}(x(x')) - (\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu})$$

In linearized theory we consider only the coordinate transformations that leave $h_{\mu\nu} \ll 1$. Therefore

$$\partial_{\mu}\xi_{\nu} \ll 1$$

Coordinate transformations in linearized theory

Example: First rank tensor that is separable in zeroth, first $\cdots$ order pieces

$$f_\mu = \left(f^{(0)} + f^{(1)} + \cdots\right)_\mu$$

Transformation:

$$f'_\mu(x') = \frac{\partial x^\rho}{\partial x'^\mu} f_\rho = \left(\delta^\rho_\mu + \frac{\partial \xi^\rho}{\partial x'^\mu}\right)\left(f^{(0)} + f^{(1)} + \cdots\right)_\rho = f^{(0)}_\mu + \frac{\partial \xi^\rho}{\partial x'^\mu} f^{(0)}_\rho + f^{(1)}_\mu + \cdots$$

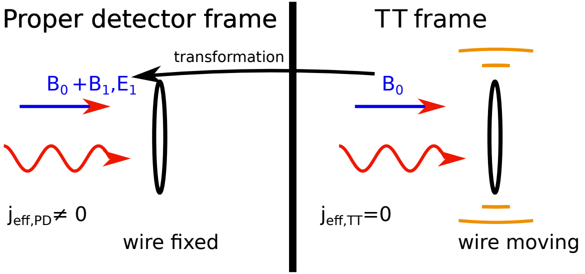
$$f'^{(1)} = \frac{\partial \xi^\rho}{\partial x'^\mu} f^{(0)}_\rho + f^{(1)}_\mu$$

This is what we mean when we say that a tensor is not invariant in linearized theory.
Example: Riemann tensor in linearized theory:

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\nu\partial_\rho h_{\mu\sigma} + \partial_\mu\partial_\sigma h_{\nu\rho} - \partial_\mu\partial_\rho h_{\nu\sigma} - \partial_\nu\partial_\sigma h_{\mu\rho}\right)$$

is invariant under infinitesimal coordinate transformations

Toy example



$$t_{TT} \simeq t - \frac{i}{4}\,\omega_g\,(x^2 - y^2)\,h_+\,e^{i\omega_g t} \;,\quad x_{TT} \simeq x - \frac{1}{2}\,x\,(1 - i\omega_g z)\,h_+\,e^{i\omega_g t}\;.$$

$$y_{TT} \simeq y + \frac{1}{2}\,y\,(1 - i\omega_g z)\,h_+\,e^{i\omega_g t} \;,\quad z_{TT} \simeq z - \frac{i}{4}\,\omega_g\,(x^2 - y^2)\,h_+\,e^{i\omega_g t}$$

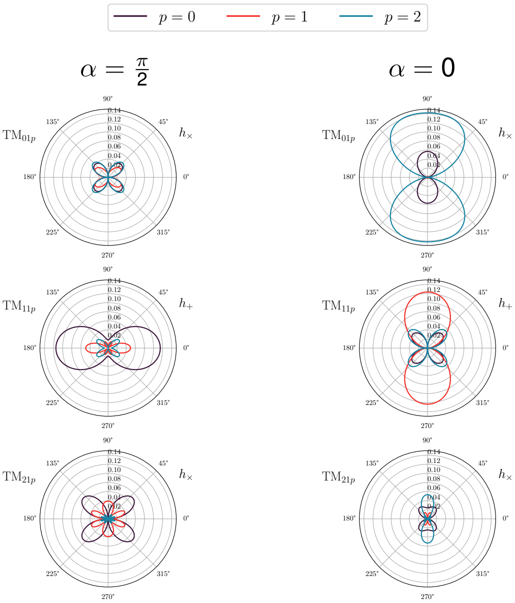
- Wire $U_\mu = (1,0,0,0)$
- Signal: E^1 induces current in wire

$$\boldsymbol{E} \simeq \frac{i}{2}\,B_0\,\omega_g\,h_+\,e^{i\omega_g t}\,(y,x,0),\, J_{\text{sig,PD}} = \sigma \boldsymbol{E}$$

- Wire moves $U_{TT,\mu} \neq (1,0,0,0)$
- Signal: moving wire in static B-field induces current in wire

$$J^i_{\text{sig,TT}} \simeq \frac{i}{2}\,\sigma\,B_0\,\omega_g\,h_+\,e^{i\omega_g t}\,(y,x,0)$$

Evaluation of coupling coefficient



- Directional sensitivity
- Sensitivity to polarization

PBHs production mechanism

General relation between PBH mass and production time in early universe:

$$M \sim 10^{-18}\, M_{\odot} \left(\frac{t}{10^{-23}\,\mathrm{s}} \right)$$

Production mechanisms:

- Primordial inhomogeneities
- Collapse in a matter-dominated era
- Collapse from inflationary fluctuations: Superhorizon density flucuations collapse at horizon reentry to PBHs. Gives PBHs in $10^{-15} - 10^{-11} M_{\odot}$ mass range.
- Collapse from scale-invariant fluctuations

GWs from merger of compact objects

$$
\begin{aligned}
h_{+}(t) &= \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{\rm g}}{c}\right)^{\frac{2}{3}}\frac{1+\cos^2\theta}{2}\cos\left(2\pi f_g t_{\rm ret}+2\phi\right) \\
h_{\times}(t) &= \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{\rm g}}{c}\right)^{\frac{2}{3}}\cos\theta\,\sin\left(2\pi f_g t_{\rm ret}+2\phi\right)
\end{aligned}
$$

[Maggiore]

$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$, θ emission angle, $t_{\rm ret} = t - \frac{r}{c}$.

Superradiance: Decay of bosons

$$f_g = 2\left(\frac{m_a}{10^{-9}\,\mathrm{eV}}\right) 10^6\,\mathrm{Hz}$$

$$h_0 \sim 10^{-27}\left(\frac{1\,\mathrm{GHz}}{f_g}\right)\left(\frac{M_{\mathrm{BH}}}{10^{-4}\,M_\odot}\right)^{\frac{1}{2}}\left(\frac{10\,\mathrm{kpc}}{D}\right)$$

Superradiance: Signal estimate with arxiv:2010.13157

Consider $\ell = m = 1$

$$f_g = 145\,\mathrm{kHz}\,\left(\frac{m_a}{3\times 10^{-10}\,\mathrm{eV}}\right) \overset{!}{=} 1\,\mathrm{GHz} \quad \Rightarrow m_a = 2\times 10^{-6}\,\mathrm{eV}$$

$$\alpha = 0.2\left(\frac{M_{\mathrm{BH}}}{M_\odot}\right)\left(\frac{m_a}{3\times 10^{-11}\,\mathrm{eV}}\right)$$

Superradiance condition

$$\alpha \leq \frac{1}{2}\frac{a_*}{1+\sqrt{1-a_*^2}}$$

for $0 < a_* < 1$. We take $a_* = 1 \Rightarrow \alpha \leq \frac{1}{2}$.
Superradiance condition is fulfilled by

$$M_{\mathrm{BH}} \leq 3.75\times 10^{-5}\,M_\odot$$

Characteristic strain:

$$h_0 = 10^{-24}\left(\frac{\Delta a_*}{0.1}\right)\left(\frac{10\,\mathrm{kpc}}{D}\right)\left(\frac{M_{\mathrm{BH}}}{M_\odot}\right)\left(\frac{\alpha}{0.2}\right)^7 = 2.2\times 10^{-26}\,\left(\frac{\Delta a_*}{0.1}\right)\left(\frac{10\,\mathrm{kpc}}{D}\right.$$

Proper detector frame

$$
\begin{aligned}
g_{00} &= -1 - 2\sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n,k_1,\cdots k_r} x^m x^n x^{k_1} \cdots x^{k_r} \\
g_{0i} &= -2\sum_{r=0}^{\infty} \frac{r+2}{(r+3)!} R_{0nin,k_1,\cdots k_r} x^m x^n x^{k_1} \cdots x^{k_r} \\
g_{ij} &= \delta_{ij} - 2\sum_{r=0}^{\infty} \frac{r+1}{(r+3)!} R_{injn,k_1,\cdots k_r} x^m x^n x^{k_1} \cdots x^{k_r}
\end{aligned}
$$

Proper detector frame

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\begin{aligned}
h_{00} &= -\omega_g^2\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1-e^{-i\omega_g z}}{(\omega_g z)^2}\right] \\
h_{ij} &= \omega_g^2 \left[\left(\delta_{iz}\, h_{ja}^{\mathrm{TT}} + \delta_{jz}\, h_{ia}^{\mathrm{TT}}\right) z\, x^a - h_{ij}^{\mathrm{TT}}\, z^2 - \delta_{iz}\,\delta_{jz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b\right] \times \\
&\times \left[-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\,\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right] \\
h_{0i} &= -\omega_g^2 \left(h_{ia}^{\mathrm{TT}}\, z\, x^a - \delta_{iz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b\right) \left[-\frac{i}{2\,\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\,\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right]
\end{aligned}$$

Cross check: Long wavelength limit ($x^i \omega_g \ll 1$)

$$h_{00} = -\omega_g^2\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1-e^{-i\omega_g z}}{(\omega_g z)^2}\right]$$

Proper detector frame

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\begin{aligned}
h_{00} &= -\omega_g^2\, h^{\mathrm{TT}}_{ab}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right] \\
h_{ij} &= \omega_g^2 \left[\left(\delta_{iz}\, h^{\mathrm{TT}}_{ja} + \delta_{jz}\, h^{\mathrm{TT}}_{ia} \right) z\, x^a - h^{\mathrm{TT}}_{ij}\, z^2 - \delta_{iz}\, \delta_{jz}\, h^{\mathrm{TT}}_{ab}\, x^a\, x^b \right] \times \\
&\times \left[-\frac{1 + e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right] \\
h_{0i} &= -\omega_g^2 \left(h^{\mathrm{TT}}_{ia}\, z\, x^a - \delta_{iz}\, h^{\mathrm{TT}}_{ab}\, x^a\, x^b \right) \left[-\frac{i}{2\,\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]
\end{aligned}$$

Cross check: Long wavelength limit ($x^i \omega_g \ll 1$)

$$h_{00} = -\omega_g^2\, h^{\mathrm{TT}}_{ab}\, x^a\, x^b \left[\frac{1}{2} + \mathcal{O}(\omega_g z) \right] \ll h^{\mathrm{TT}}$$

More details on cavities

- Orthogonality and Eigenfunctions

$$
\begin{aligned}
\nabla^2 \boldsymbol{E}_n(\boldsymbol{x}) &= -\omega_n^2 \boldsymbol{E}_n(\boldsymbol{x}) \,, \\
\int_{V_{\text{cav}}} d^3\boldsymbol{x}\ \boldsymbol{E}_n(\boldsymbol{x}) \cdot \boldsymbol{E}_m^*(\boldsymbol{x}) &= \delta_{nm} \int_{V_{\text{cav}}} d^3\boldsymbol{x}\ \left|\boldsymbol{E}_n(\boldsymbol{x})\right|^2 \,,
\end{aligned}
$$

- Resonance frequencies

$$
\omega^2 = \frac{x_{nm}^2}{R^2} + \frac{\pi^2 p^2}{L^2}
$$

Scaling of coupling coefficient

Naive dimensionless coupling coefficient:

$$\eta_n'^2 = \frac{1}{h_0^2} \frac{\left| \int dV \boldsymbol{E}_n^*(\boldsymbol{x}) \cdot \boldsymbol{J}_{\text{eff}}(\boldsymbol{x}) \right|^2}{\int dV \left| \boldsymbol{E}_n(\boldsymbol{x}) \right|^2 B_0^2 \omega^2 V}$$

We see that this scales as p^2. Therefore we divide by an additional dimensionless factor $\left(\omega V^{1/3}\right)^2$:

$$\eta_n'^2 = \frac{1}{h_0^2} \frac{\left| \int dV \boldsymbol{E}_n^*(\boldsymbol{x}) \cdot \boldsymbol{J}_{\text{eff}}(\boldsymbol{x}) \right|^2}{\int dV \left| \boldsymbol{E}_n(\boldsymbol{x}) \right|^2 B_0^2 \omega^2 V \left(\omega V^{1/3}\right)^2}$$