New results for hadron structure in Holographic QCD

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PRD 102, 034011; PRD 101 034026

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Important Results of Ivan Schmidt

- PhD (1977) at Stanford U. with Richard Blankenbecler:
  Relativistic interactions between nuclei,
  structure function of nucleons and deuteron
  Blankenbecler, Sugar: PR 142, 1051 (1966)
  Linear integral equations for relativistic multichannel scattering
- Ivan started successful collaboration with Stan Brodsky
- Prediction of new bound state, so-called Nuclear-Bound Quarkonium

⋆ Brodsky, Schmidt, Teramond, PRL 64, 1011 (1990) (240 citations)
QCD van der Waals interaction due to multiple-gluon exchange provides a new kind of attractive nuclear force making the binding heavy quarkonia to nuclei

- First calculation of gluon PDFs in QCD

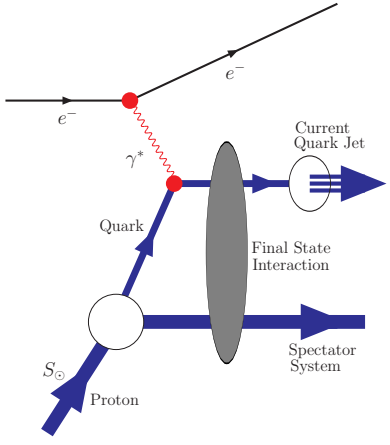
⋆ Brodsky, Schmidt, PLB 234, 144 (1990) (92 citations);
⋆ Brodsky, Burkardt, Schmidt, NPB 441, 197 (1995) (383 citations)

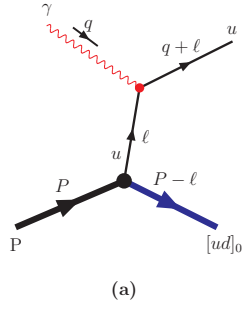
Important Results of Ivan Schmidt

- Discovery of new types of QCD asymmetries

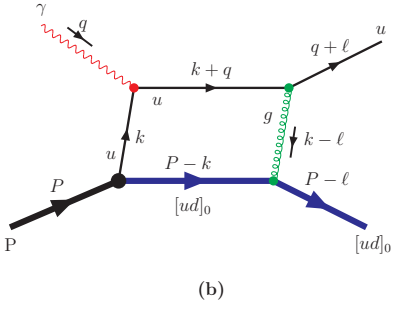
⋆ Brodsky, Hwang, Schmidt, PLB 530, 99 (2002) (894 citations)

Final state interactions in semiinclusive DIS (SIDIS) produce single spin asymmetries (SSA)





(a)



(b)

⋆ Brodsky, Hwang, Schmidt, NPB 642, 344 (2002) (417 citations)

SSA occurs due to Initial state interactions in Drell-Yan processes

- Light cone representation of the spin and orbital angular momentum of relativistic composite systems

⋆ Brodsky, Hwang, Ma, Schmidt, NPB 593, 311 (2001) (359 citations)

Plan

Introduction

Holographic QCD

Applications:

Parton Distributions in HQCD

Gluon and Quark TMDs

Sum Rules for Gluon and Quark TMDs

Numerical Resutls:

- Gluon TMDs
- Nucleon EM form factors
- Nucleon resonances: Roper (1440)
- Nuclei: Deuteron

QCD Compositeness and Quark Counting Rules

New QCD Asymmetries

Summary

Introduction

- Our Objective: Precise Description of Hadronic Reactions in Standard Model (SM) and New Physics Beyond SM
- Experimental Facilities: CERN, JLab, Fermilab, BNL, etc.
- SAPHIR is strongly involved in both experimental (ATLAS, NA64, SND, etc.) and theoretical studies
- Drell-Yan process $\mathbf{H_1} + \mathbf{H_2} \to \gamma(\mathbf{Z^0}, \mathbf{W^{\pm}}) \to \ell_1 + \ell_2$

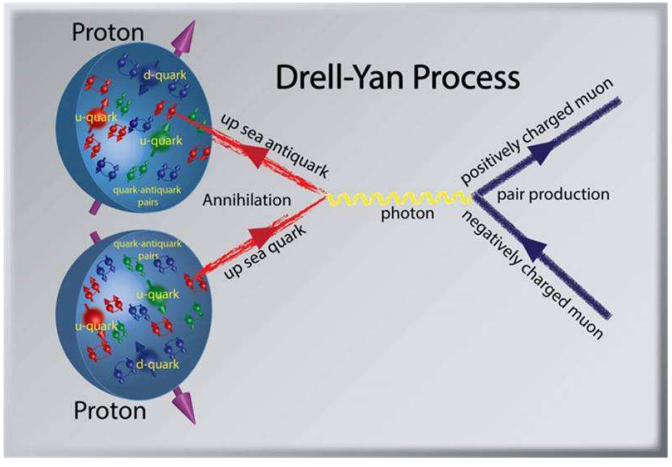


image courtesy of ANL (USA)

- Precise test and extraction of parameters of SM: Weinberg angle, Higgs couplings, angular distribution coefficients, etc.
- Precise determination of universal parton distribution functions (PDFs, TMDs, etc.)

Introduction

- Parton Distribution Functions (PDFs)
  1. Encode nonperturbative (long-distance) dynamics of partons inside a hadron
  2. Universal for each type of partons in specific hadron
  3. Fixed from global data analysis of observables
  4. Calculated in Lattice QCD, effective field approaches and quark models
  5. Big progress in Holographic QCD (see details in my talk)
  6. Included in observables (like cross sections) using factorization theorem

- QCD factorization formula for differential cross section of the DY process $\mathbf{pp} \to \ell\bar{\ell}$

$$\frac{\mathbf{d}\sigma_{\mathbf{pp}\to\ell\bar{\ell}}(\mathbf{P_1},\mathbf{P_2})}{\mathbf{d^4q\,d\Omega}} = \sum_{\mathbf{a,b}} \int_0^1 \mathbf{dx_1} \int_0^1 \mathbf{dx_2}\, \mathbf{f_{a/p}}(\mathbf{x_1})\, \mathbf{f_{b/p}}(\mathbf{x_2})\, \frac{\mathbf{d}\hat{\sigma}_{\mathbf{ab}\to\ell\bar{\ell}}(\mathbf{x_1P_1},\mathbf{x_2P_2})}{\mathbf{d^4q\,d\Omega}}$$

  where $\mathbf{f_{a/p}}(\mathbf{x})$ is the PDF of partons of sort $\mathbf{a}$ in the proton; $\mathbf{d}\hat{\sigma}_{\mathbf{ab}}/(\mathbf{d^4q\,d\Omega})$ is the corresponding partonic-level hard differential cross section, calculated in pQCD

- Transverse momentum distributions (TMDs) $\mathbf{f_{a/p}}(\mathbf{x},\mathbf{k}_\perp)$,
  Generalized parton distributions (GPDs), Fragmentation functions (FF), etc.

Holographic QCD

- 1993 't Hooft Holographic Principle

  Information about string theory contained in some region of space can be represented as "Hologram" (theory which lives on the boundary of that region)

- 1997-1998 Maldacena, Polyakov, Witten et al AdS/CFT correspondence
  Duality of 4D conformal supersymmetric Yang-Mills and supersting theories

  - Correspondence: Matching partition functions gives relation between parameters of two theories

  - Symmetry arguments: Conformal group acting in boundary theory is isomorphic to the $SO(4,2)$ isometry group of AdS_5 space

- AdS/CFT $\rightarrow$ AdS/QCD upon breaking conformal invariance

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD:
  attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies:
  chiral symmetry, confinement, power scaling of hadron form factors

- Physical interpretation of extra 5th dimension as Scale

Holographic QCD

- AdS metric Poincaré form

  $ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$ R – AdS radius

- Manifestly scale-invariant $x \to \lambda x,\ z \to \lambda z.$

- z – extra dimensional (holographic) coordinate;
  $z = 0$ is UV boundary , $z = \infty$ is IR boundary

- Five Dimensions: L = Length, W = Width, H = Height, T = Time, S = Scale

- Action for scalar field

$$S_\Phi = \frac{1}{2}\int d^d x dz\, \sqrt{g}\, e^{-\varphi(z)}\left(\partial_M \Phi(x,z)\partial^M \Phi(x,z) - m^2\, \Phi^2(x,z)\right)$$

- Dilaton field $\varphi(z) = \kappa^2 z^2$, Determinant of metric tensor $g = |\det g_{MN}|$

- 5d mass $m^2 = \Delta(\Delta - 4)/R^2$, Conformal dimension Δ

Holographic QCD

- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum_n \phi_n(x)\,\Phi_n(z)$
- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons
- Bulk profiles $\Phi_n(z)$ dual to hadronic wave functions
- Use $-\partial_\mu\partial^\mu\phi_n(x) = M_n^2\phi_n(x)$ and Substitute $\Phi_n(z) = \left(\frac{R}{z}\right)^{1-d}\phi_n(z)$
- Identify $\Delta = \tau = N + L$

  $N = 2$ – number of partons in meson; L – orbital angular momentum;

  τ is twist = Canonical dimension - Sum of spins
- Schrödinger-type EOM

$$\Big[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + \kappa^4 z^2 - 2\kappa^2\Big]\phi_n(z) = M_n^2\phi_n(z)$$

- Solutions $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\;\kappa^{L+1}\;z^{L+1/2}\;e^{-\kappa^2 z^2/2}\;L_n^L(\kappa^2 z^2)$
- $M_{nL}^2 = 4\kappa^2\left(n + \frac{L}{2}\right)$
- Massless pion $M_\pi^2 = 0$ for $n = L = 0$

Holographic QCD

- Scattering problem for AdS field gives information about propagation of external field from $z \neq 0$ to the boundary $z = 0$

- Bulk-to-boundary propagator $\Phi_{\rm ext}(q,z)$ – Fourier-trasform of AdS field: $\Phi_{\rm ext}(q,z) = \int d^d x e^{-iqx}\, \Phi_{\rm ext}(x,z)$

- Vector field as example

  $$\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z V(q,z) \right) + q^2 \, \frac{e^{-\varphi(z)}}{z} V(q,z) = 0 \,.$$

  $$V(Q,z) \;=\; \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) \text{ with } Q^2 = -q^2$$

  Consistent with GI, fulfills UV and IR boundary conditions : $V(Q,0) = 1\,, \;\; V(Q,\infty) = 0$

- Hadron form factors

  $$F_\tau(Q^2) = \langle \phi_\tau | \hat{V}(Q) | \phi_\tau \rangle = \int\limits_0^\infty dz \, \phi_\tau^2(z) \, V(Q,z) = \frac{\Gamma(\tau)\,\Gamma(a+1)}{\Gamma(a+\tau)}$$

  is implemented by a nontrivial dependence of AdS fields on 5-th coordinate

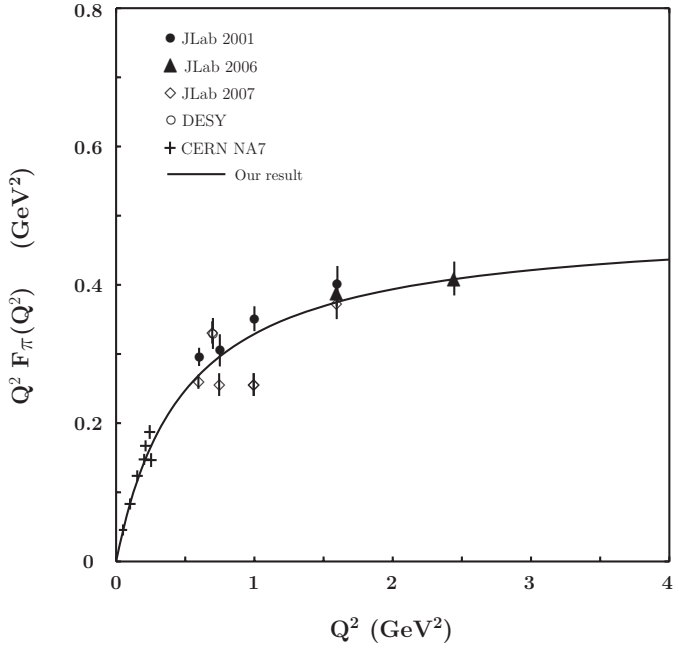
Holographic QCD

- Power scaling at large Q^2 $\quad F_\tau(Q^2) \sim 1/(Q^2)^{\tau-1}$

  Quark counting rules: Brodsky-Farrar-Matveev-Muradyan-Tavhelidze 1973

Pion	$1/Q^2$
Nucleon	$1/Q^4$ (Dirac), $\quad 1/Q^6$ (Pauli)
Deuteron	$1/Q^{10}$ (Charge)

- Pion form factor in normalization $Q^2 F_\pi(Q^2)$



Parton Distributions in HQCD

- Sum Rules relating Parton Distributions and Electromagnetic Form Factors
- Nucleon Electromagnetic Form Factors

$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p')[\gamma^\mu F_1^N(Q^2) + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu F_2^N(Q^2)]u(p),$$

F_1^N and F_2^N are Dirac and Pauli FF normalized to nucleon electric charge e_N and anomalous magnetic moment k_N: $F_1^N(0) = e_N$ and $F_2^N(0) = k_N$.

- SR relating EMFFs and GPDs: Ji (1996), Radyushkin (1997)

$$\begin{aligned}
F_1^p(Q^2) &= \int_0^1 dx \left(\frac{2}{3}H^u(x,Q^2) - \frac{1}{3}H^d(x,Q^2)\right), \\
F_1^n(Q^2) &= \int_0^1 dx \left(\frac{2}{3}H^d(x,Q^2) - \frac{1}{3}H^u(x,Q^2)\right), \\
F_2^p(Q^2) &= \int_0^1 dx \left(\frac{2}{3}E^u(x,Q^2) - \frac{1}{3}E^d(x,Q^2)\right), \\
F_2^n(Q^2) &= \int_0^1 dx \left(\frac{2}{3}E^d(x,Q^2) - \frac{1}{3}E^u(x,Q^2)\right),
\end{aligned}$$

Here $H^q(x,Q^2)$ and $E^q(x,Q^2)$ — nucleon charge and magnetization GPDs

Parton Distributions in HQCD

- In PRD 102, 034011 (2020) we showed that the GPDs are related to PDFs as

$$H_\tau(x,Q^2) = q_\tau(x)\,\exp\left(\frac{Q^2}{4\kappa^2}\,z_\tau(x)\right)$$

$$z_\tau(x) = \log\left(1-\Big[\int_x^1 dy\,q_\tau(y)\Big]^{1/(\tau-1)}\right)$$

- At $x\to 1$ we reproduce QCD relation $H_\tau(x,Q^2)\to q_\tau(x)$
- Pion $H_\pi(x,Q^2)=q_\pi(x)\sim(1-x)^2$
- Nucleon: $H^q_N(x,Q^2)=q_N(x)\sim(1-x)^3$, $E^q_N(x,Q^2)=e^q_N(x)\sim(1-x)^5$
- Large x scaling of PDFs is consistent with QCD counting rules
- Our idea is that the PDFs should be constrained from world data analysis, while HQCD gives clear prediction for GPDs in terms of PDFs
- PDFs obey the normalization condition $n_q=\int_0^1 dx\,q_\tau(x)$ and $k_q=\int_0^1 dx\,e_\tau(x)$ and where n_q is the number of quarks of flavor q in a hadron, e.g. for u-quark PDF in pion $n_u=1$, in proton $n_u=2$; k_q is anomalous magnetic moment of quark.

Parton Distributions in HQCD

- TMDs $f_\tau(x, \mathbf{k}_\perp)$ are functions of x light-cone variable and transverse momentum of the parton (quark or gluon)

- TMDs and PDFs are related by integration over $\mathbf{k}_\perp$

$$q_\tau(x) = \int d^2\mathbf{k}_\perp \, f_\tau(x, \mathbf{k}_\perp)$$

- Drell-Yan-West formula (DYW) for the EM hadronic form factor with twist τ

$$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, \psi_\tau^\dagger(x, \mathbf{k}'_\perp) \, \psi_\tau(x, \mathbf{k}_\perp) \, ,$$

  where $\psi(x, \mathbf{k}_\perp)$ is the LFWF, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ is the transverse momentum of final hadron, $Q^2 = \mathbf{q}_\perp^2$.

- Our idea: to match expressions for the EM form factors obtained in HQCD and using DYW formula to derive the light-front wave functions (LFWFs), which serve as basic blocks for calculation of PDFs and TMDs

Parton Distributions in HQCD

- Predictions of HQCD and DYW formula for EM FF we get expression of the LFWFs $\psi^{\lambda_N}_{\lambda_q q}(x, \mathbf{k}_\perp)$ in terms of PDFs

$$\psi^{\pm}_{\pm q}(x, \mathbf{k}_\perp) = \varphi^{(1)}_q(x, \mathbf{k}_\perp)\,, \qquad \psi^{\pm}_{\mp q}(x, \mathbf{k}_\perp) = \mp \frac{k^1 \pm i k^2}{M_N}\, \varphi^{(2)}_q(x, \mathbf{k}_\perp)\,,$$

where

$$\begin{aligned} \varphi^{(1)}_q(x, \mathbf{k}_\perp) &= \frac{2\pi\sqrt{2}}{\kappa} \sqrt{q^+(x)\, D_q(x)}\, \exp\left[-\frac{\mathbf{k}^2_\perp}{2\kappa^2}\, D_q(x)\right], \\ \frac{1}{M_N}\, \varphi^{(2)}_q(x, \mathbf{k}_\perp) &= \frac{2\pi c_q \sqrt{2}}{\kappa^2} \sqrt{q^-(x)}\, D_q(x)\, \exp\left[-\frac{\mathbf{k}^2_\perp}{2\kappa^2}\, D_q(x)\right]. \end{aligned}$$

Here $c_u = 1$, $c_d = -1$, $\psi^{\lambda_N}_{\lambda_q q}(x, \mathbf{k}_\perp)$ are the LFWFs with specific helicities for the nucleon $\lambda_N = \pm$ and for the struck quark $\lambda_q = \pm$, where plus and minus correspond to $+\frac{1}{2}$ and $-\frac{1}{2}$,

- $q^{\pm}(x) = f^q_1(x) \pm g^q_1(x)$ combination of unpolarized and helicity PDFs
- $D_q(x)$ are arbitrary functions fixed from data on EM FF of nucleons

Parton Distributions in HQCD

- Simplest unpolarized TMD, where nucleon and active quark are unpolarized. is given by

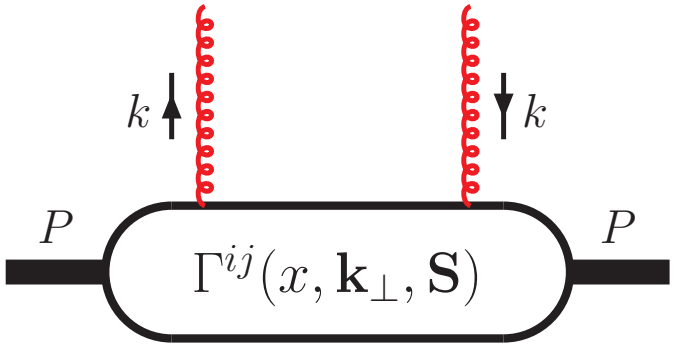
$$\begin{aligned} f_1^q(x,\mathbf{k}_\perp) &= \frac{1}{16\pi^3}\left[|\psi^+_{+q}(x,\mathbf{k}_\perp)|^2+|\psi^+_{-q}(x,\mathbf{k}_\perp)|^2\right] \\ &= \frac{1}{16\pi^3}\left[\left(\varphi_q^{(1)}(x,\mathbf{k}_\perp)\right)^2+\frac{\mathbf{k}_\perp^2}{M_N^2}\left(\varphi_q^{(2)}(x,\mathbf{k}_\perp)\right)^2\right] \end{aligned}$$

- Integration of TMD $f_1^q(x,\mathbf{k}_\perp)$ over $\mathbf{k}_\perp$ gives unpolarized PDF $f_1^q(x)$

$$\int d^2\mathbf{k}_\perp f_1^q(x,\mathbf{k}_\perp) = f_1^q(x)$$

Gluon and Quark TMDs

- Mulders and Rodrigues, PRD63, 094021 (2001):
  Expansion of gluon correlator $\Gamma^{ij}(x, \mathbf{k}_\perp, \mathbf{S})$ in QCD

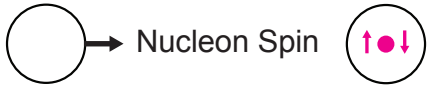


- i, j – gluon polarization indices; nucleon spin $S^\mu = (0, \mathbf{S})$ and $\mathbf{S} = (S_L, \mathbf{S}_T)$
- $S_L = \cos\theta$ and $\mathbf{S}_T = (\cos\phi \sin\theta, \sin\phi \sin\theta)$
- θ and ϕ – polar and azimuthal angles (orientation of the nucleon spin-vector $\mathbf{S}$)
- $k_\perp^\mu = (0, 0, \mathbf{k}_\perp) = \sqrt{\mathbf{k}_\perp^2}\,(0, 0, \cos\phi_k, \sin\phi_k)$,
  ϕ_k – azimuthal angle (orientation of parton $\mathbf{k}_\perp$ in the transverse plane).
- Basis matrices of the rotation $U(2)$ group

$$g_T^{ij} = -\delta^{ij}\,, \qquad \eta_T^{ij} = \tau_3^{ij}\,\cos 2\phi_k + \tau_1^{ij}\,\sin 2\phi_k\,,$$
$$\epsilon_T^{ij} = i\tau_2^{ij}\,, \qquad \xi_T^{ij} = -\tau_3^{ij}\,\sin 2\phi_k + \tau_1^{ij}\,\cos 2\phi_k$$

Gluon and Quark TMDs

Leading Gluon TMDPDFs

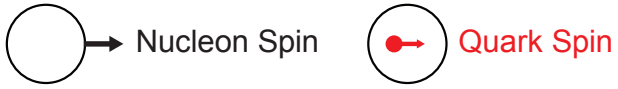


Nucleon Polarization	Gluon Operator Polarization: Un-Polarized	Helicity 0 antisymmetric	Helicity 2
U	f_1^g = Unpolarized		$h_1^{\perp g}$ = + Linearly Polarized
L		g_{1L}^g = − Helicity	$h_{1L}^{\perp g}$ = +
T	$f_{1T}^{\perp g}$ = −	$g_{1T}^{\perp g}$ = −	h_{1T}^g = + Transversity $h_{1T}^{\perp g}$ = +

Picture taken from R. Boussarie et al. “TMD Handbook,” arXiv:2304.03302 [hep-ph]

Gluon and Quark TMDs

Leading Quark TMDPDFs



Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity; $h_{1T}^\perp$ = Pretzelosity

Picture taken from R. Boussarie et al. “TMD Handbook,” arXiv:2304.03302 [hep-ph]

Gluon and Quark TMDs

- We proposed expansion of the gluon correlation tensor in more compact form and separation of symmetric and antisymmetric correlation of transverse quantities nucleon spin $\mathbf{S}_T$ and $\mathbf{k}_\perp$:

$$\begin{aligned}\Gamma^{ij}(x,\mathbf{k}_\perp,\mathbf{S}) &= -g_T^{ij}\,F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) - i\epsilon_T^{ij}\,\mathbf{S}\,\mathbf{G}_1^g(x,\mathbf{k}_\perp^2)\\ &+ \eta_T^{ij}\,H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) + \xi_T^{ij}\,\mathbf{S}\,\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2)\end{aligned}$$

where

$$F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) = f_1^g(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) \qquad U-\text{polarized gluon}$$

$$\mathbf{G}_1^g(x,\mathbf{k}_\perp^2) = \left(g_{1L}^g(x,\mathbf{k}_\perp^2),\frac{\mathbf{k}_\perp}{M_N}g_{1T}^g(x,\mathbf{k}_\perp^2)\right) \qquad C-\text{polarized gluon}$$

$$H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) = h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,h_{1T}^{-g}(x,\mathbf{k}_\perp^2) \qquad L-\text{polarized gluon}$$

$$\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2) = \left(h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2),\frac{\mathbf{k}_\perp}{M_N}\,h_{1T}^{+g}(x,\mathbf{k}_\perp^2)\right) \qquad L-\text{polarized gluon}$$

and $\mathbf{S}=(S_L,\mathbf{S}_T)$

Gluon and Quark TMDs

- Following Mulders-Rodrigues expand the gluon tensor in the nucleon spin basis

$$\Gamma^{ii'}(x,\mathbf{k}_\perp,\mathbf{S}) = \sum_{\Lambda,\Lambda'} \rho_{\Lambda'\Lambda}(\mathbf{S})\,\Gamma^{ii'}_{\Lambda\Lambda'}(x,\mathbf{k}_\perp)\,, \qquad \rho(\mathbf{S}) = \frac{1}{2}\Big(\mathbf{1} + \mathbf{S}\,\sigma\Big)$$

- $4 \otimes 4$ matrix $\Gamma^{ii'}_{\Lambda\Lambda'}$ in the gluon-nucleon circular basis

$$\begin{pmatrix} F_1^+ & F_{1T}^+ & H_1^- & \Delta H_{1T} \\ \left(F_{1T}^+\right)^\dagger & F_1^- & H_{1T} & H_1^+ \\ \left(H_1^-\right)^\dagger & \left(H_{1T}\right)^\dagger & F_1^- & F_{1T}^- \\ \left(\Delta H_{1T}\right)^\dagger & \left(H_1^+\right)^\dagger & \left(F_{1T}^-\right)^\dagger & F_1^+ \end{pmatrix}$$

$$F_1^\pm = f_1^g \pm g_{1L}^g\,, \quad F_{1T}^\pm = \pm\frac{|\mathbf{k}_\perp|}{M_N}\,e^{-i\phi_k}\left[g_{1T}^g \pm i f_{1T}^{\perp g}\right],$$

$$H_1^\pm = -e^{-2i\phi_k}\left[h_1^{(1)\perp g} \pm i h_{1L}^{(1)\perp g}\right],$$

$$H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\,e^{-i\phi_k}\left[h_{1T}^{+g} + h_{1T}^{-g}\right], \quad \Delta H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\,e^{-3i\phi_k}\left[h_{1T}^{+g} - h_{1T}^{-g}\right]$$

Sum Rules for gluon TMDs

- We make the conjecture that the Sum Rules should exist as the consequence of the condition

$$\det\Big[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\Big] = 0$$

  signaling that the gluon TMDs are not independent and are related via SRs

- 3 SRs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$

- From $\det\Big[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\Big] = 0$ follows the condition

$$\Big[R_0 + 2R_1 + R_2\Big]\Big[R_0 - 2R_1 + R_2\Big] = 0$$

- R_0, R_1, R_2 are the combibations of TMDs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$

Sum Rules for gluon TMDs

- 3 Sum Rules

$$R_0 = \left[g_{1L}^g\right]^2 + \left[g_{1T}^{(1/2)g}\right]^2 + \left[h_1^{(1)\perp g}\right]^2 - \left[f_1^g\right]^2 = 0$$

$$R_1 = f_1^g\, h_{1T}^{(1/2)-g} + g_{1L}^g\, h_{1T}^{(1/2)+g} - g_{1T}^{(1/2)g}\, h_{1L}^{(1)\perp g} - h_1^{(1)\perp g}\, f_{1T}^{(1/2)\perp g} = 0$$

$$R_2 = \left[f_{1T}^{(1/2)\perp g}\right]^2 + \left[h_{1L}^{(1)\perp g}\right]^2 + \left[h_{1T}^{(1/2)+g}\right]^2 - \left[h_{1T}^{(1/2)-g}\right]^2 = 0$$

- 1st SR $R_0 = 0$ is exactly our SR involving four T-even TMDs
- 2nd SR $R_1 = 0$ couples T-even and T-odd TMDs
- 3rd SR $R_2 = 0$ involves only T-odd TMDs
- Large-x scaling: $f_1^g \sim g_{1L}^g \sim (1-x)^4, \quad g_{1T}^{(1/2)g} \sim h_1^{(1)\perp g} \sim (1-x)^5$

Sum Rules for quark TMDs

- Quark TMD Correlator

$$\begin{aligned}\Gamma^q(x,\mathbf{k}_\perp,\mathbf{S},\mathbf{S}_{T_q}) &= F_1^q(x,\mathbf{k}_\perp;\mathbf{S}) + \mathbf{S}\,\mathbf{G}_1^q(x,\mathbf{k}_\perp)\\ &+ H_{1;\mathrm{as}}^q(x,\mathbf{k}_\perp;\mathbf{S},\mathbf{S}_{T_q}) + \mathbf{S}\,\mathbf{H}_{1;\mathrm{s}}^q(x,\mathbf{k}_\perp;\mathbf{S}_{T_q})\end{aligned}$$

where

$$F_1^q = f_1^q + e^{\mathbf{S}_T\mathbf{n}_\perp} f_{1T}^{\perp(1/2)q} \qquad U-\text{polarized quark}$$

$$\mathbf{G}_1^q = \left(g_{1L}^q, \mathbf{n}_\perp g_{1T}^{\perp(1/2)q}\right) \qquad C-\text{polarized gluon}$$

$$H_{1;\mathrm{as}}^q = e^{\mathbf{S}_{T_q}\mathbf{n}_\perp}\left(h_1^{\perp(1/2)q} + e^{\mathbf{S}_T\mathbf{n}_\perp} h_{1T}^{\perp -q}\right) \qquad L-\text{polarized gluon}$$

$$\mathbf{H}_{1;\mathrm{s}}^q = \left(h_{1L}^{\perp(1/2)q}, \mathbf{n}_\perp h_{1T}^{\perp +q}\right) \qquad L-\text{polarized gluon}$$

$\mathbf{n}_\perp = \mathbf{k}_\perp/|\mathbf{k}_\perp|$

- Large-x scaling: $f_1^q \sim g_{1L}^q \sim h_{1T}^{\perp+q} \sim h_{1T}^{\perp-q} \sim (1-x)^4$, $g_{1T}^{(1/2)q} \sim h_{1L}^{(1)\perp q} \sim (1-x)^5$

Sum Rules for quark TMDs

- 2 Sum Rules for quark TMDs
- T-even and T-odd TMDs decouple

$$R_0 = (F_+^q \mp H_+^q)^2 (F_-^q \pm H_-^q)^2 - (G_\pm^q)^2 = 0$$

$$R_2 = (f_{1T}^{\perp q} \pm h_{1T}^{\perp q})^2 = 0$$

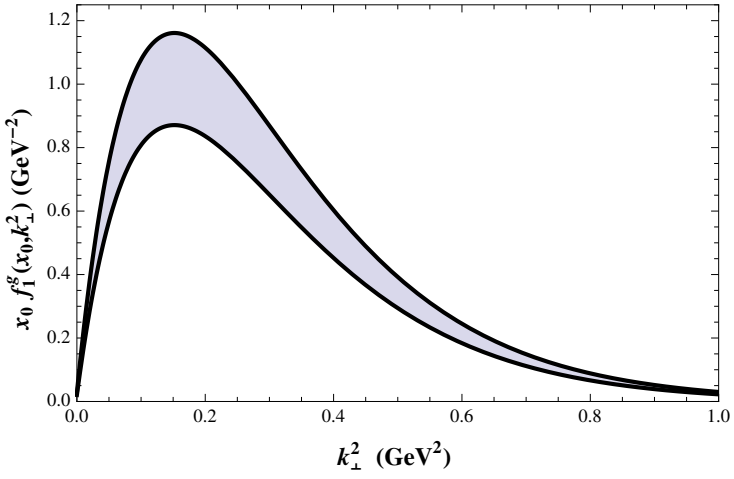
- 1st SR $R_0 = 0$ related T-even TMDs T-even TMDs

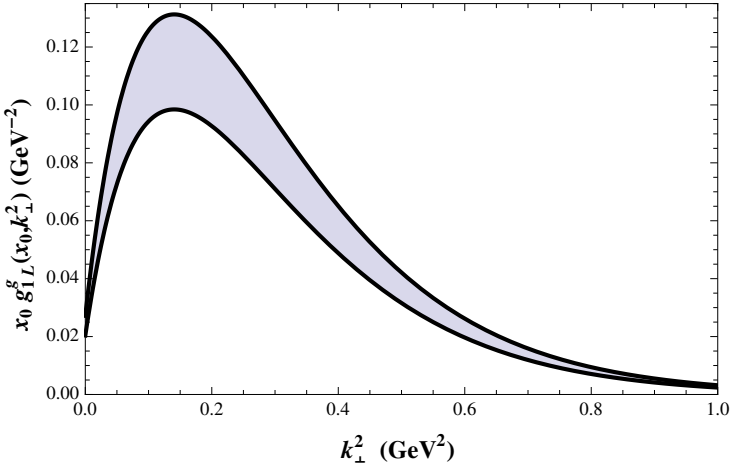
$$F_\pm^q = f_1^q \pm g_{1L}^q \,, \qquad H_\pm^q = h_{1T}^{\perp +q} \pm h_{1T}^{\perp -q} \,, \qquad G_\pm^q = g_{1T}^{(1/2)q} \pm h_{1L}^{(1/2)q}$$

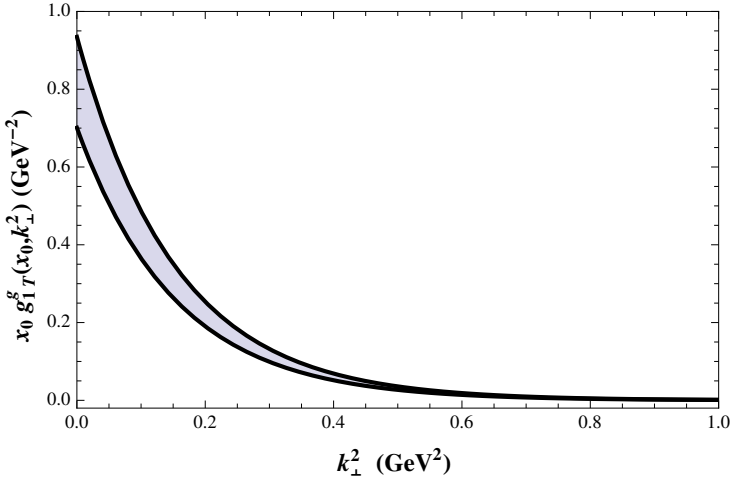
- 2nd SR $R_2 = 0$ relates T-odd TMDs (Sivers and Boer-Mulders functions)

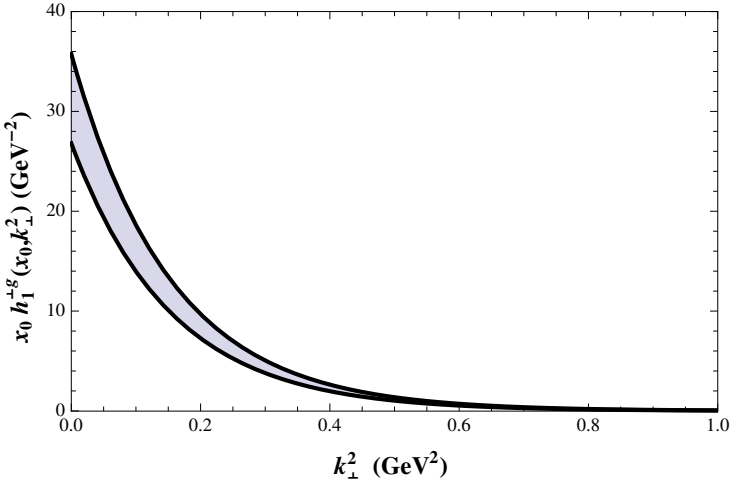
Gluon TMDs

- T-even $x\ \mathrm{TMD}(x, \mathbf{k}_\perp^2)$ at $x = 0.1$ and for $\kappa = 380 \pm 30$ MeV
- Very good agreement with Pavia group, Bacchetta et al, EPJC80, 733 (2020)



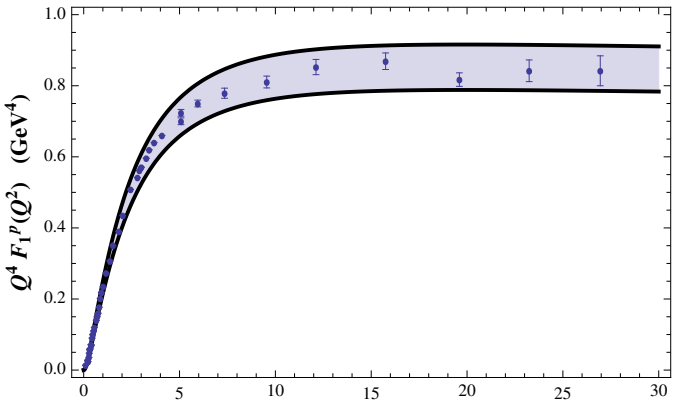


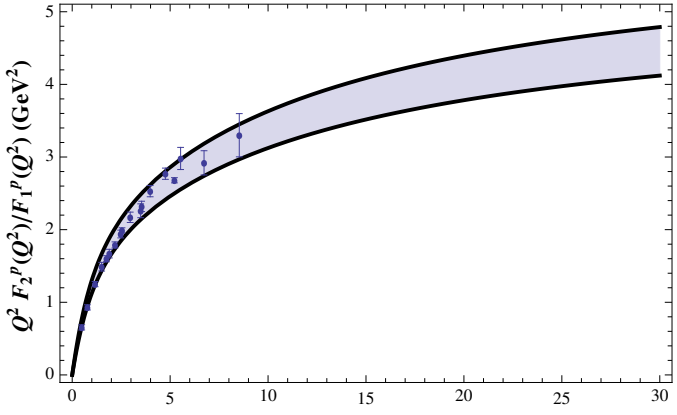


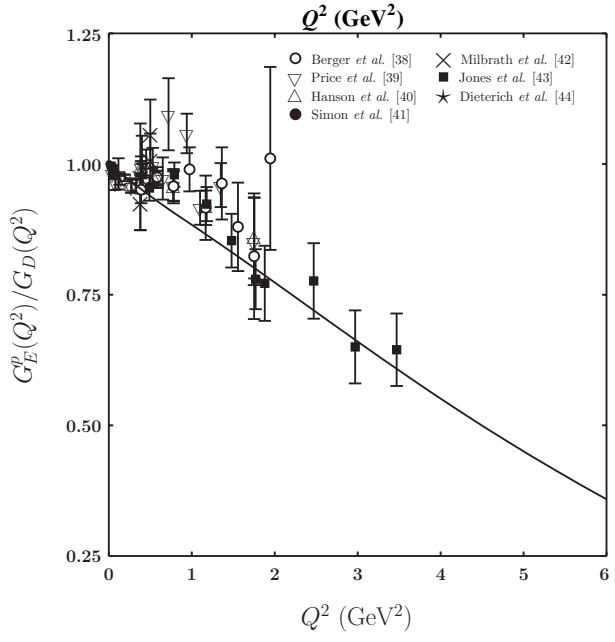


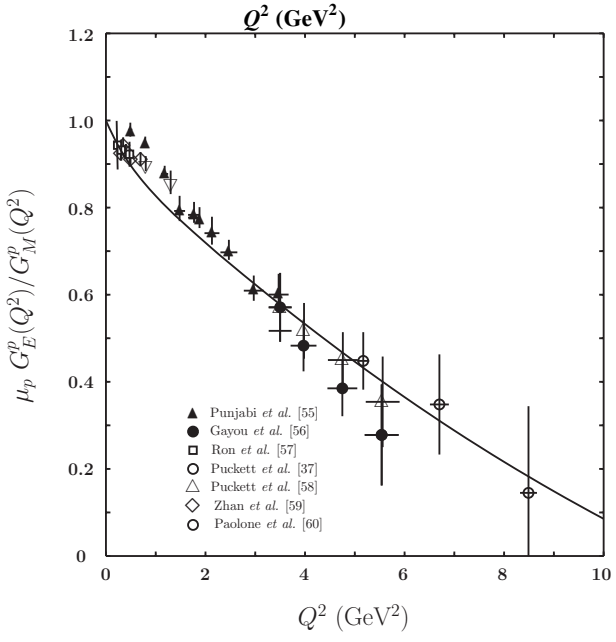
Nucleon electromagnetic form factors

- $Q^4 F_1^p(Q^2)$, ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$, ratio of Sachs FF



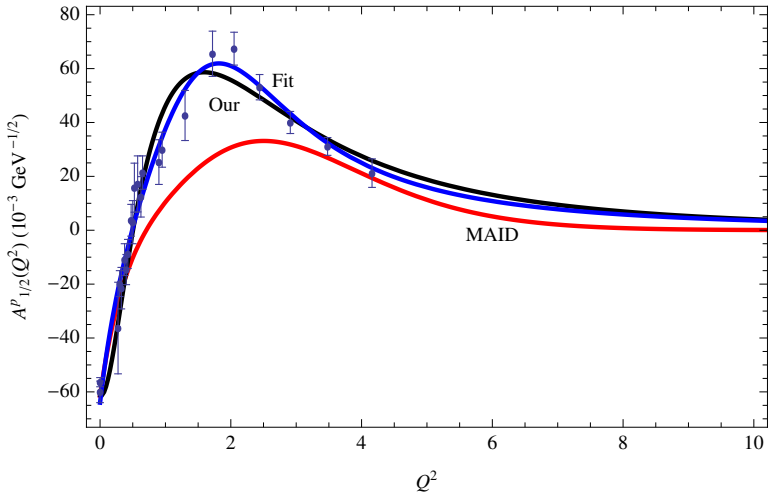




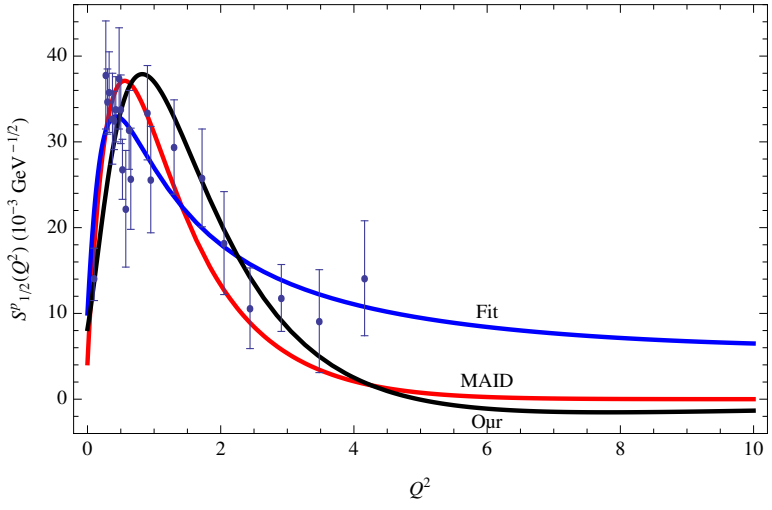


Nucleon resonances: Roper (1440)

Helicity-flip $\Delta\lambda = \pm 1$ amplitude $A^p_{1/2}(Q^2)$



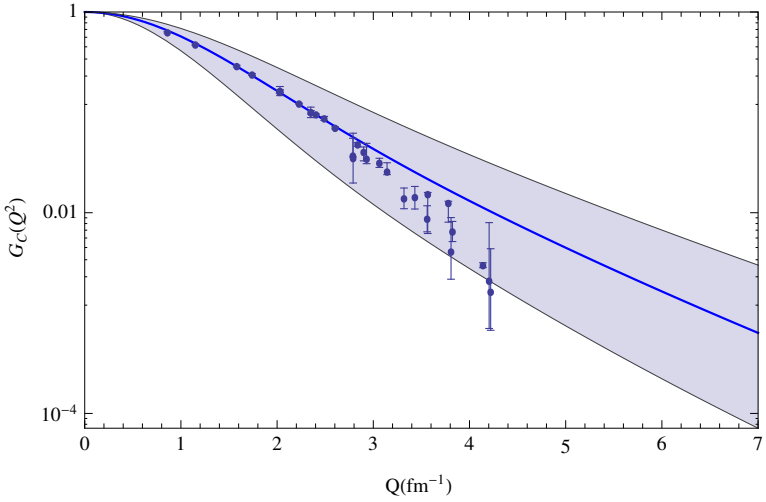
Helicity-nonflip $\Delta\lambda = 0$ amplitude $S^p_{1/2}(Q^2)$

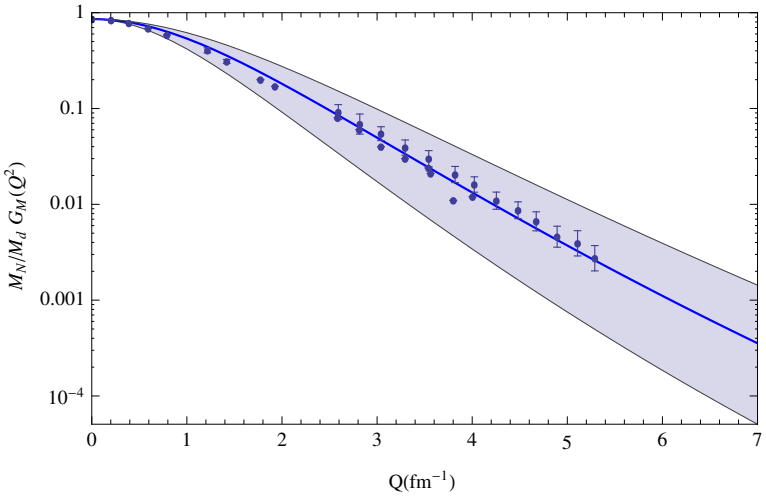


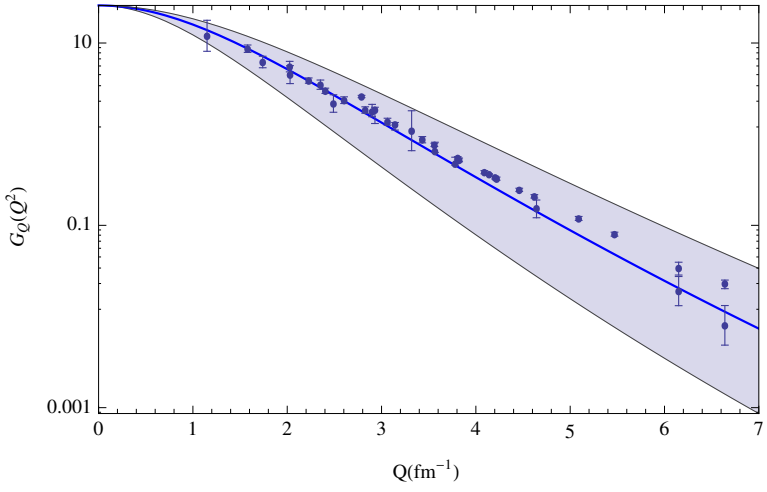
Data: Mokeev et al (CLAS Coll), PRC86 (2012) 035203

Deuteron

- Deuteron form factors

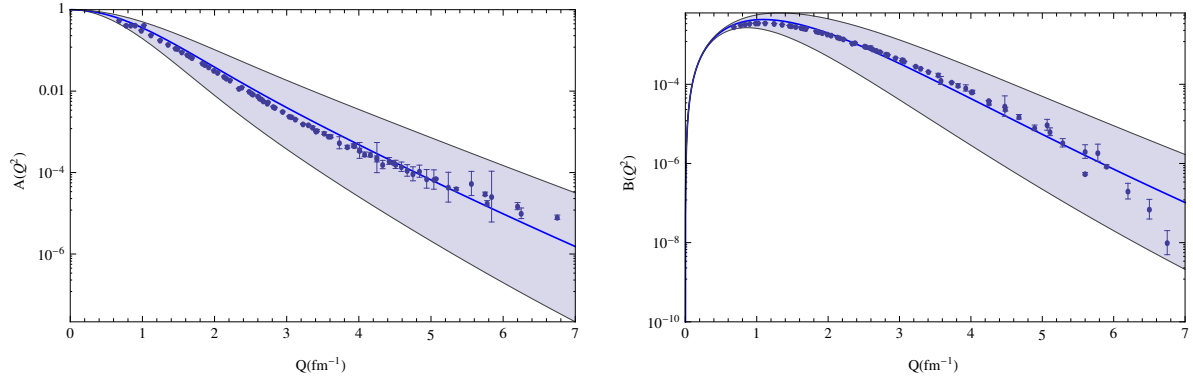






Deuteron

- Structure Functions



- Diff. cross section for the elastic e-D scattering (Rosenbluth formula)

$$
\begin{aligned}
d\sigma/d\Omega &\sim \left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}\right], \\
A(Q^2) &= G_C^2(Q^2) + \frac{2}{3}\tau_d G_M^2(Q^2) + \frac{8}{9}\tau_d^2 G_Q^2(Q^2), \\
B(Q^2) &= \frac{4}{3}\tau_d(1+\tau_d)G_M^2(Q^2), \qquad \tau_d = \frac{Q^2}{4M_d^2}
\end{aligned}
$$

QCD Compositeness and Quark Counting Rules

QCD compositeness vs. VMD (Vector Meson Dominance model)

Brodsky, Lebed, Lyubovitskij: Phys.Lett.B764 (2017) 174; Phys.Rev.D97 (2018) 034009

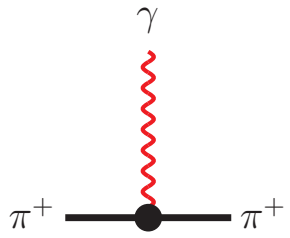
- Novel idea relevant for electrocouplings of baryon resonances
- QCD compositenens (vector mesons are bound states of quarks) leads to a nontrivial Q^2 dependence of vector meson - photon transition

$$\mathbf{V^0} = \rho^0, \omega, \phi, \mathbf{J}/\psi, \Upsilon$$

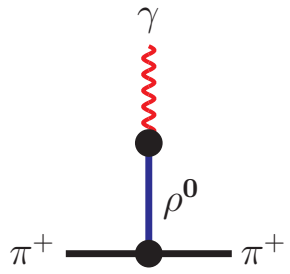
- Structure of matrix element $G_V(g^{\mu\nu}Q^2 - Q^\mu Q^\nu)$, where Q is Euclidean momentum of V^0 and γ
- In VMD G_V is constant
- $G_V(Q^2)$ must have $G_V(Q^2) \sim 1/\sqrt{Q^2}$ behavior at large Q^2

QCD Compositeness and Quark Counting Rules

- Consider the pion



(a) Direct



(b) ρ^0 exchange

- In VMD: contact diagram 1, vector meson diagram gives $-Q^2/(M_V^2+Q^2)$
- The sum is $M_V^2/(M_V^2+Q^2)$ scales as M_V^2/Q^2
- Direct diagram is 1, resonance is $-1+M_V^2/Q^2$
- In pQCD: direct diagram $1/Q^2$, vector meson diagram is subleading $1/Q^3$ because of falloff of the vector meson-photon form factor

QCD Compositeness and Quark Counting Rules

- New formula for electrocouplings of two hadrons with ajustable constituent content n_1 and n_2

$$F_{H_{n_1}H_{n_2}}(Q^2) = \frac{\Gamma(\frac{n_1+n_2}{2})\,\Gamma(\frac{n_1+n_2}{2}-1)}{\sqrt{\Gamma(n_1-1)\Gamma(n_2-1)}}\,\frac{\Gamma(a+1)}{\Gamma(a+1+\frac{n_1+n_2}{2}-1)}$$
$$\sim \frac{1}{a^{(n_1+n_2)/2-1}}\,,$$

where $a = Q^2/(4\kappa^2)$ and $\Gamma(n) = \int\limits_0^\infty dx x^{n-1}e^{-x}$ is gamma function.

For $n_1 = n_2 = n$ we get

$$F_{H_n} \sim \left(\frac{1}{Q^2}\right)^{n-1}$$

For $n_1 = n,\ \ n_2 = 0$ we get

$$F_{H_n} \sim \left(\frac{1}{Q^2}\right)^{(n-1)/2}$$

QCD Compositeness and Quark Counting Rules

- In particular, the scaling of the form factor corresponding to $\gamma^* \to Z_c^+ + \pi^-$ is

$$F_{Z_c^+\pi^-} \sim \frac{1}{Q^4}$$

in case of tetraquark structure of Z_c state, and

$$F_{Z_c^+\pi^-} \sim \frac{1}{Q^2}$$

in the case when Z_c^+ is a system of two tightly bound diquarks.
For $\gamma^* \to Z_c^+ + Z_c^-$,

$$F_{Z_c^+ Z_c^-} \sim \frac{1}{Q^6}$$

in case of a Z_c state with tetraquark structure, and

$$F_{Z_c^+ Z_c^-} \sim \frac{1}{Q^2}$$

in case when Z_c^+ is a system of two tightly bound diquarks

QCD Compositeness and Quark Counting Rules

- Off-shell behavior of Hadron FF
- EM FF is example of use of vector bulk-to-boundary propagator with $J = 1$
- Extension to arbitrary J in PRD 108, 054030 (2023)

$$\begin{aligned} S_J &= \frac{(-)^J}{2} \int d^4x dz\, e^{-B_J(z)}\, \partial_M V_{\mu_1 \cdots \mu_J}(x,z)\, \partial^M V^{\mu_1 \cdots \mu_J}(x,z)\,, \\ B_J(z) &= \varphi(z) - (2J-1)R/z \end{aligned}$$

- Equation of motion

$$\partial_z \left(e^{-B_J(z)}\, \partial_z\, V_J(-q^2,z) \right) + e^{-B_J(z)}\, q^2\, V_J(-q^2,z) = 0\,,$$

$$a_J = a + J - 1\,, \qquad a = Q^2/(4\kappa^2)$$

- Analytical solution in terms of gamma $\Gamma(a)$ and Trikomi $U(a,b,z)$ functions

$$\begin{aligned} V_J(Q^2,z) &= (\kappa^2 z^2)^J\, \frac{\Gamma(a_J+1)}{\Gamma(J)}\, U(a_J+1, J+1, \kappa^2 z^2) \\ &= \frac{1}{\Gamma(J)} \int_0^\infty dy\, y^{J-1}\, e^{-y} \left(\frac{y}{y+\kappa^2 z^2} \right)^a . \end{aligned}$$

QCD Compositeness and Quark Counting Rules

- Charge conservation $V_J(0,z)=1$
  Ultraviolet boundary (local limit) $V_J(Q^2,0)=1$
  Confinement $V_1(Q^2,z)\to 0$ at $z\to\infty$
  Correct power scaling of Hadron FF $F(Q^2)\sim 1/Q^{2(\tau-1)}$ at large Q^2
- Massive bulk-to-boundary propagator of the weak bosons and Higgs

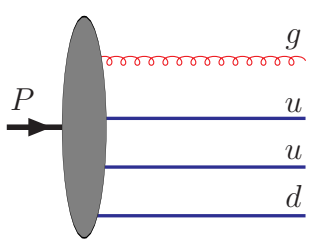
$$V(Q^2+M^2,z)=\int_0^\infty dy\, e^{-y}\left(\frac{y}{y+\kappa^2 z^2}\right)^{a(M^2)},$$

  where $a(M^2)=a+M^2/(4\kappa^2)=(Q^2+M^2)/(4\kappa^2)$ and $M=M_{W^\pm},M_{Z^0},M_H$.
- Coupling of off-shell meson $J\geq 1$ with two on-shell mesons reads

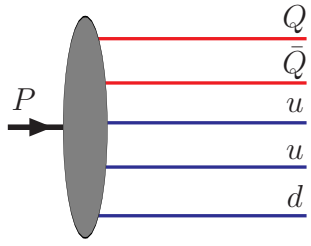
$$\begin{aligned}F_{M_{J,\tau}M_{\tau_1}M_{\tau_2}}(Q^2) &= \frac{\Gamma\left(\frac{\tau+\tau_1+\tau_2}{2}+J-2\right)}{\Gamma(J)\sqrt{\Gamma(\tau-1)\,\Gamma(\tau_1-1)\,\Gamma(\tau_2-1)}}\\ &\times \int_0^1 dx\, x^{a(M_M^2)+J-1}\,(1-x)^{\frac{\tau+\tau_1+\tau_2}{2}-3}\left(\frac{2}{3-x}\right)^{\frac{\tau+\tau_1+\tau_2}{2}+J-3}\end{aligned}$$

New QCD Asymmetries

- Intrinsic and extrinsic heavy quark distributions in nucleon Brodsky et al, PRD 23, 2745 (1980)

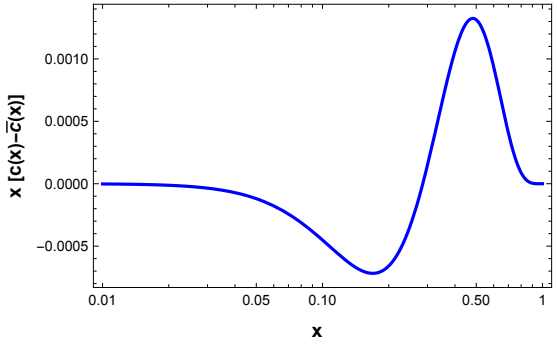


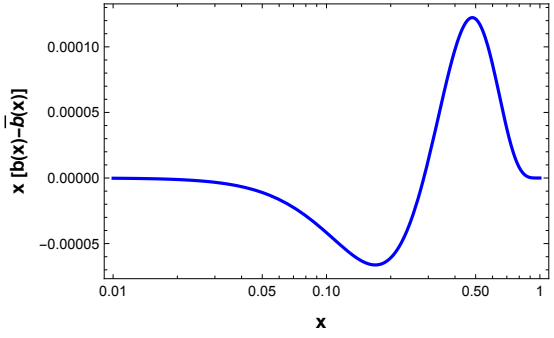
(a)



(b)

- Discovery of intrinsic heavy quark asymmetric distributions $Q_{\rm asym}(x) = Q(x) - \bar{Q}(x)$ in Holographic QCD

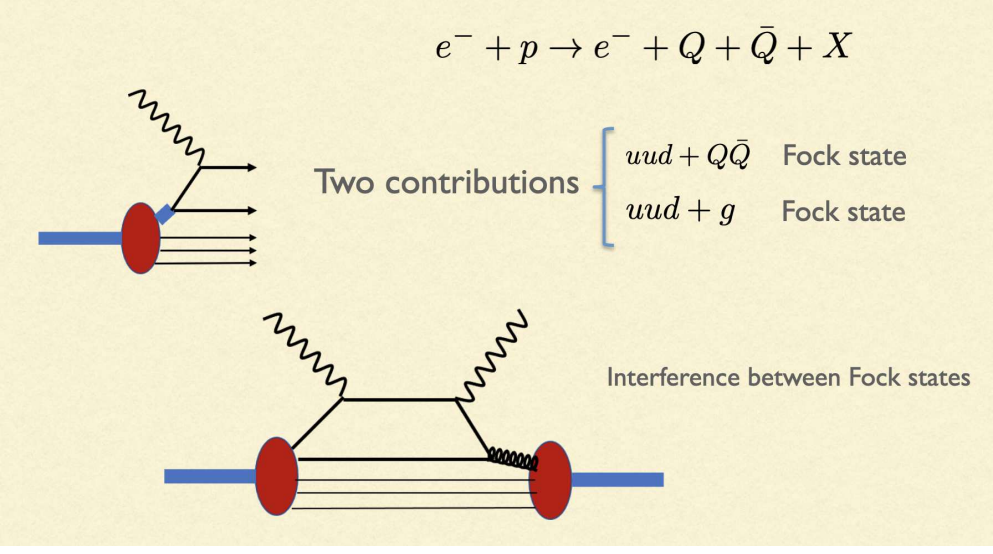




charm and bottom quark asymmetries

New QCD Asymmetries

- Fock state interference in heavy quark (HQ) electroproduction
- Asymmetry due to interference of intrinsic and extrinsic HQ in proton
- Similar effect was discovered before in case of QED
- Interference of the first-Born and second-Born amplitudes (Brodsky-Gillespie, Phys.Rev.173) leads to an ℓ vs. $\bar{\ell}$ asymmetry in the Bethe-Heitler cross section for lepton pairs produced on nuclei $\gamma Z \to \ell^+ \ell^- Z$ (DESY, 1967)
- Asymmetry in the $e^- + p \to e^- + Q + \bar{Q} + X$ reaction



Summary

- Novel approach to hadron structure – Holographic QCD works well
- Useful and accurate tool for analysis of data and predictions for planned experiments
- New Sum Rules involving gluon and quark TMDs
- New QCD asymmetries based on consideration of heavy quark conent in nucleon
- Next objectives which were discussed with Ivan and must be completed:
  - Using constraints derived for parton densitires in study QCD processes
  - Extension of HQCD on nuclei (using deuteron as the first example)
  - Extension of HQCD on fragmentation functions
  - Full analysis of QCD processes including hard and soft effects