

# The octant of $\theta_{23}$ and neutrino non-standard interactions degeneracy

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**NTN Workshop on Neutrino Non-Standard Interactions**  
**Washington University, St. Louis**

30.05.2019

# Outline

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- ▶ Current status of neutrino oscillation parameters
- ◆ Theoretical and analytical framework of neutrino non-standard interactions
- ◆ Impact of NSI on the resolution of octant of  $\theta_{23}$  taking DUNE as a case study
- ◆ Some other kind of degeneracies
- Conclusion

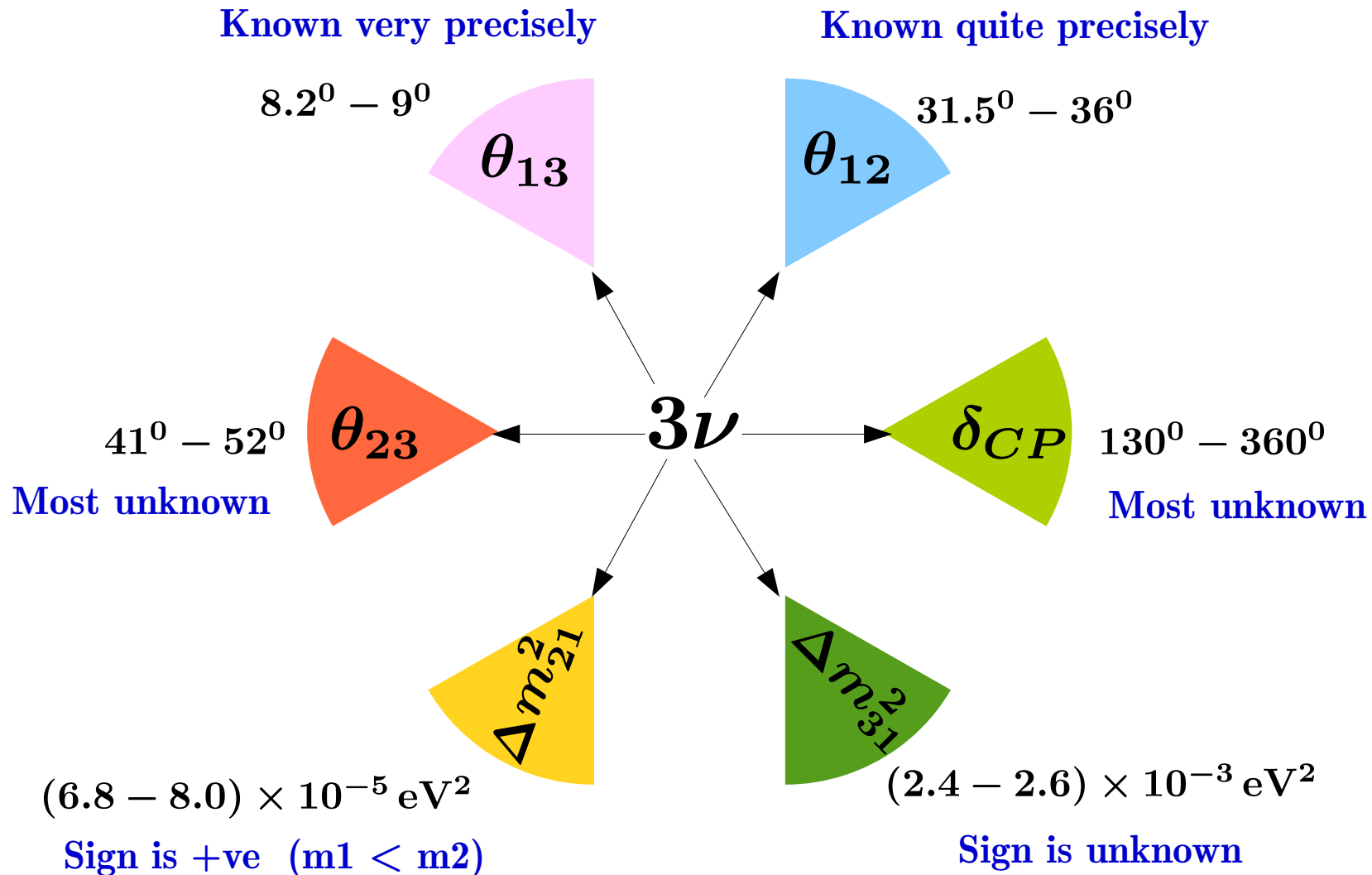
## 3ν Framework

1. Whether neutrino is Dirac or Majorana particle.
2. Absolute masses ( $m_1$ ,  $m_2$ , and  $m_3$ ) of neutrinos are unknown. We know the magnitude of mass squared differences ( $|\Delta m_{21}^2|$ ,  $|\Delta m_{31}^2|$ , or  $|\Delta m_{32}^2|$ ).
3. The sign of the solar mass splitting ( $|\Delta m_{21}^2|$ ) is known that is +ve that is  $m_2 > m_1$ . But the sign of the atmospheric mass splitting ( $|\Delta m_{31}^2|$ ) is unknown. This is known as mass hierarchy problem.  $m_1 < m_2 < m_3$ , called normal hierarchy, and  $m_3 < m_1 < m_2$  called inverted hierarchy.
4. The magnitude of 2-3 mixing angle ( $\theta_{23}$ ) is unknown. This is known as octant ambiguity.
5. No confirmation yet about the CP-violation in leptonic sector.

## New Physics ?

Presence of sterile neutrino, long-range forces, non-unitary nature of PMNS matrix, CPT violation, non-standard neutrino interactions, and many others.

## Current status of $3\nu$ parameters ( $3\sigma$ uncertainties)



For details please see arXiv: 1811.05487 by Esteban et al.

# NSI and its presence in the oscillation framework

The effective 4-Fermi flavor changing neutral current non-standard interactions can be written as

$$\mathcal{L}_{NSI} = \frac{G_F}{\sqrt{2}} \sum_{\alpha, \beta, f} \varepsilon_{\alpha\beta}^f [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\beta] [\bar{f} \gamma_\mu (1 \pm \gamma^5) f]$$

$$\alpha, \beta = e, \mu, \tau \text{ and } f = e, u, d$$

$$\varepsilon_{\alpha\beta} \equiv \sum_{f=e,u,d} \varepsilon_{\alpha\beta}^f \frac{N_f}{N_e}$$

$$\varepsilon_{\alpha\beta} \simeq \varepsilon_{\alpha\beta}^e + 3\varepsilon_{\alpha\beta}^u + 3\varepsilon_{\alpha\beta}^d \quad \longrightarrow \quad \text{Strength of NSI}$$

The time evolution Schrödinger equation for the neutrino flavor eigenstates in vacuum is given by

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \frac{1}{2E} \left[ U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger \right] \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}$$

Similarly, in matter this is given by

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \left[ \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} + V_{NC} & 0 & 0 \\ 0 & +V_{NC} & 0 \\ 0 & 0 & +V_{NC} \end{pmatrix} \right] \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e \quad \text{Charge current potential for neutrino}$$

$$V_{NC} = -\frac{G_F N_n}{\sqrt{2}} \quad \text{Neutral current potential for neutrino}$$

For antineutrino,  $V_{CC} \rightarrow -V_{CC}$  and  $V_{NC} \rightarrow -V_{NC}$

Now, the time evolution equation for the neutrino flavor eigenstates in presence of NSI is given by

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \left[ \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + V + V_{NSI} \right] \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}$$

Where,

$$V = \begin{pmatrix} V_{CC} + V_{NC} & 0 & 0 \\ 0 & +V_{NC} & 0 \\ 0 & 0 & +V_{NC} \end{pmatrix}, \quad V_{NSI} = V_{CC} \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix}$$

In our analysis we assume one NSI parameter at a time and we have explored the impact of two NSI parameters  $\varepsilon_{e\mu}$  &  $\varepsilon_{e\tau}$  respectively.

## Current constraints on neutral current NSI parameters

$$-0.006 < \varepsilon_{\mu\tau}^{dV} < 0.0054 \text{ (90\% C.L.)} \quad \text{ArXiv: 1609.03450}$$

Salvado et al.

[ArXiv: 1905.05203](#) (I. Esteban et al.)

$$-0.12 \lesssim \varepsilon_{e\mu} \lesssim 0.12 \text{ (90\% C.L.)}$$

$$-0.3 \lesssim \varepsilon_{e\tau} \lesssim 0.3 \text{ (90\% C.L.)}$$

$$-0.028 \lesssim \varepsilon_{\mu\tau} \lesssim 0.028 \text{ (90\% C.L.)}$$

$$-0.5 \lesssim \varepsilon_{ee} - \varepsilon_{\mu\mu} \lesssim 0.5 \text{ (90\% C.L.)}$$

$$-0.05 \lesssim \varepsilon_{\tau\tau} - \varepsilon_{\mu\mu} \lesssim 0.2 \text{ (90\% C.L.)}$$



# Impact of NSI on the octant resolution

The vacuum survival Probability  $\nu_\mu \rightarrow \nu_\mu$  in 3-flavor is given by

$$P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \alpha \Delta c_{12}^2 \sin^2 2\theta_{23} \sin 2\Delta - 4 s_{13}^2 s_{23}^2 \sin^2 \Delta$$

Insenstive to the resolution of octant as it gives rise to octant degeneracy

Where,  $\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$ ,  $\Delta = \frac{\Delta m_{31}^2 L}{4E}$

In a simplified case, we can write

$$P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \sin^2 \Delta$$

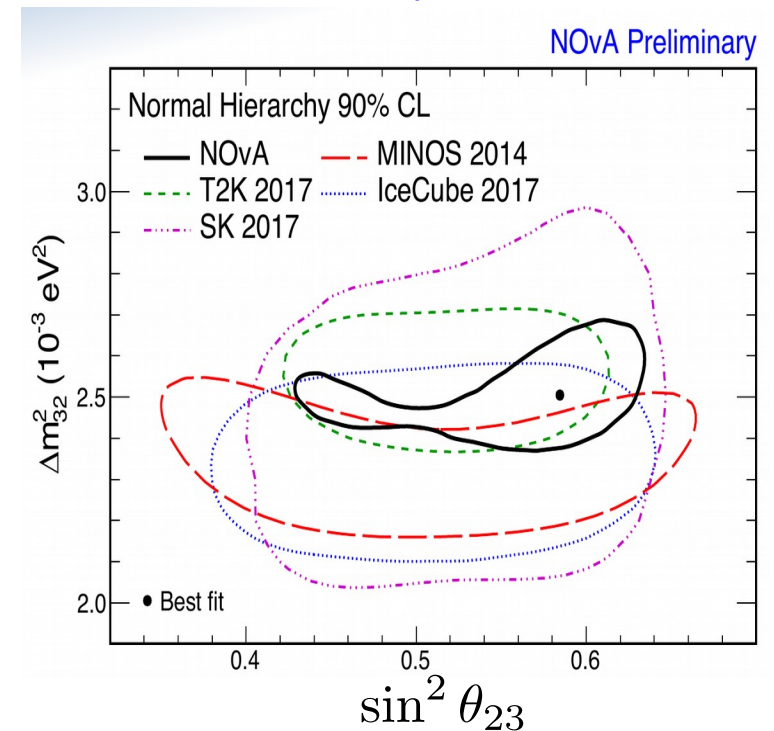
$$P_{\mu\mu}(\theta_{23}) = P_{\mu\mu}(\pi/2 - \theta_{23})$$

$\theta_{23} < 45^\circ$  Known as lower octant

$\theta_{23} > 45^\circ$  Known as higher octant

$\theta_{23} = 45^\circ$  Called maximal mixing

See the talk by Alex Himmel



There are two solutions,  
called Octant degeneracy

Our goal here is to see the capability of an experiment (say, DUNE) to distinguish between the two octants of  $\theta_{23}$  in presence of NSI.

The appearance probability  $\nu_\mu \rightarrow \nu_e$  is given by

$$P_{\mu e} \simeq 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \sin^2 \Delta \\ + 2 \sin \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} (\alpha \Delta) \sin \Delta \cos (\Delta \pm \delta_{13})$$

Sensitive to the resolution of octant degeneracy

$$P_{\mu e}(\theta_{23}) \neq P_{\mu e}(\pi/2 - \theta_{23})$$

Both appearance and survival channels play complementary role in resolving octant degeneracy.

We can rewrite  $\theta_{23}$  as,  $\theta_{23} = \pi/4 \pm \eta$

+ (-) corresponds to HO (LO).  $\eta$  is a deviation from maximality

In presence of NSI, the  $\nu_\mu \rightarrow \nu_e$  transition probability can be written approximately as,

$$P_{\mu e} \simeq P_0 + P_1 + P_2 .$$

NSI (e- $\mu$ ) sector

$$P_0 \simeq 4s_{13}^2 s_{23}^2 f^2$$

$$P_1 \simeq 8s_{13} s_{12} c_{12} s_{23} c_{23} \alpha f g \cos(\Delta + \delta)$$

$$P_2 \simeq 8s_{13} s_{23} v |\varepsilon_{e\mu}| [s_{23}^2 f^2 \cos(\delta + \phi_{e\mu}) + c_{23}^2 f g \cos(\Delta + \delta + \phi_{e\mu})]$$

NSI (e- $\tau$ ) sector

$$P_0 \simeq 4s_{13}^2 s_{23}^2 f^2$$

$$P_1 \simeq 8s_{13} s_{12} c_{12} s_{23} c_{23} \alpha f g \cos(\Delta + \delta)$$

$$P_2 \simeq 8s_{13} s_{23} v |\varepsilon_{e\tau}| [s_{23} c_{23} f^2 \cos(\delta + \phi_{e\tau}) - s_{23} c_{23} f g \cos(\Delta + \delta + \phi_{e\tau})]$$

$$f \equiv \frac{\sin[(1-v)\Delta]}{1-v}, \quad g \equiv \frac{\sin v\Delta}{v}, \quad |v| = \left| \frac{2V_{CC}E}{\Delta m_{31}^2} \right|$$

An experiment can be sensitive to the octant if, despite the freedom introduced by the unknown CP phases and other parameters, there is still a difference between the probabilities in the two octants, i.e.,

$$\Delta P \equiv P_{\mu e}^{\text{HO}}(\theta_{23}^{\text{HO}}, \delta^{\text{HO}}, \phi^{\text{HO}}) - P_{\mu e}^{\text{LO}}(\theta_{23}^{\text{LO}}, \delta^{\text{LO}}, \phi^{\text{LO}}) \neq 0$$

$$\Delta P = \Delta P_0 + \Delta P_1 + \Delta P_2$$

$$\Delta P_0 \simeq 8\eta s_{13}^2 f^2 \longrightarrow \text{+ve definite}$$

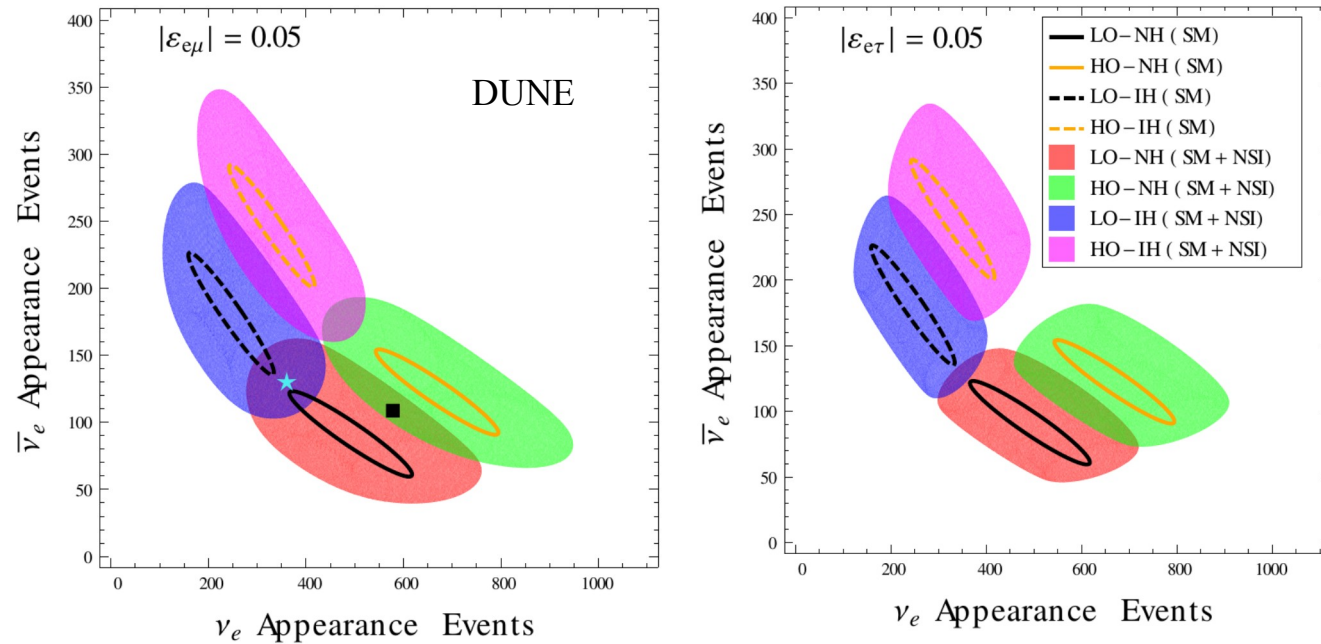
$$\Delta P_1 = A [\cos(\Delta + \delta^{\text{HO}}) - \cos(\Delta + \delta^{\text{LO}})] \longrightarrow \text{can be +ve or -ve}$$

$$\Delta P_2 = B [\cos(\delta^{\text{HO}} + \phi^{\text{HO}}) - \cos(\delta^{\text{LO}} + \phi^{\text{LO}})] \\ \pm C [\cos(\Delta + \delta^{\text{HO}} + \phi^{\text{HO}}) - \cos(\Delta + \delta^{\text{LO}} + \phi^{\text{LO}})] \longrightarrow \text{can be +ve or -ve}$$

$$\theta_{23} = \frac{\pi}{4} \pm \eta \quad \text{Here } \eta \text{ is the deviation from the maximality}$$

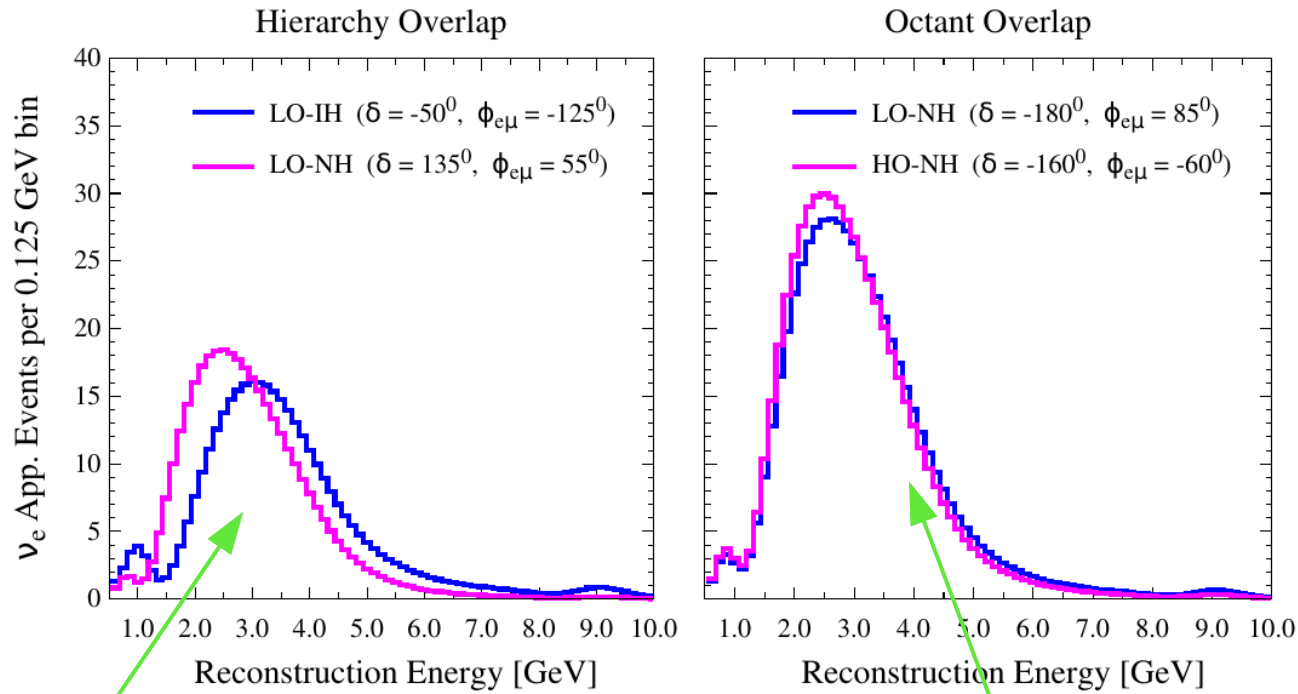
$$A = 4s_{13}s_{12}c_{12}\alpha fg, \quad B = 2\sqrt{2}v|\varepsilon|s_{13}f^2, \quad C = 2\sqrt{2}v|\varepsilon|s_{13}fg.$$

Bievents plot



$$\sin^2 \theta_{23}(\text{true}) = 0.42(0.58) \text{ as LO(HO)}$$

In presence of NSI, ellipses become blobs. Color blobs are the convolution of different combinations of  $\delta_{13}$  &  $\phi_{e\mu}$  ( $\delta_{13}$  &  $\phi_{e\tau}$ ) in the left (right) panel.

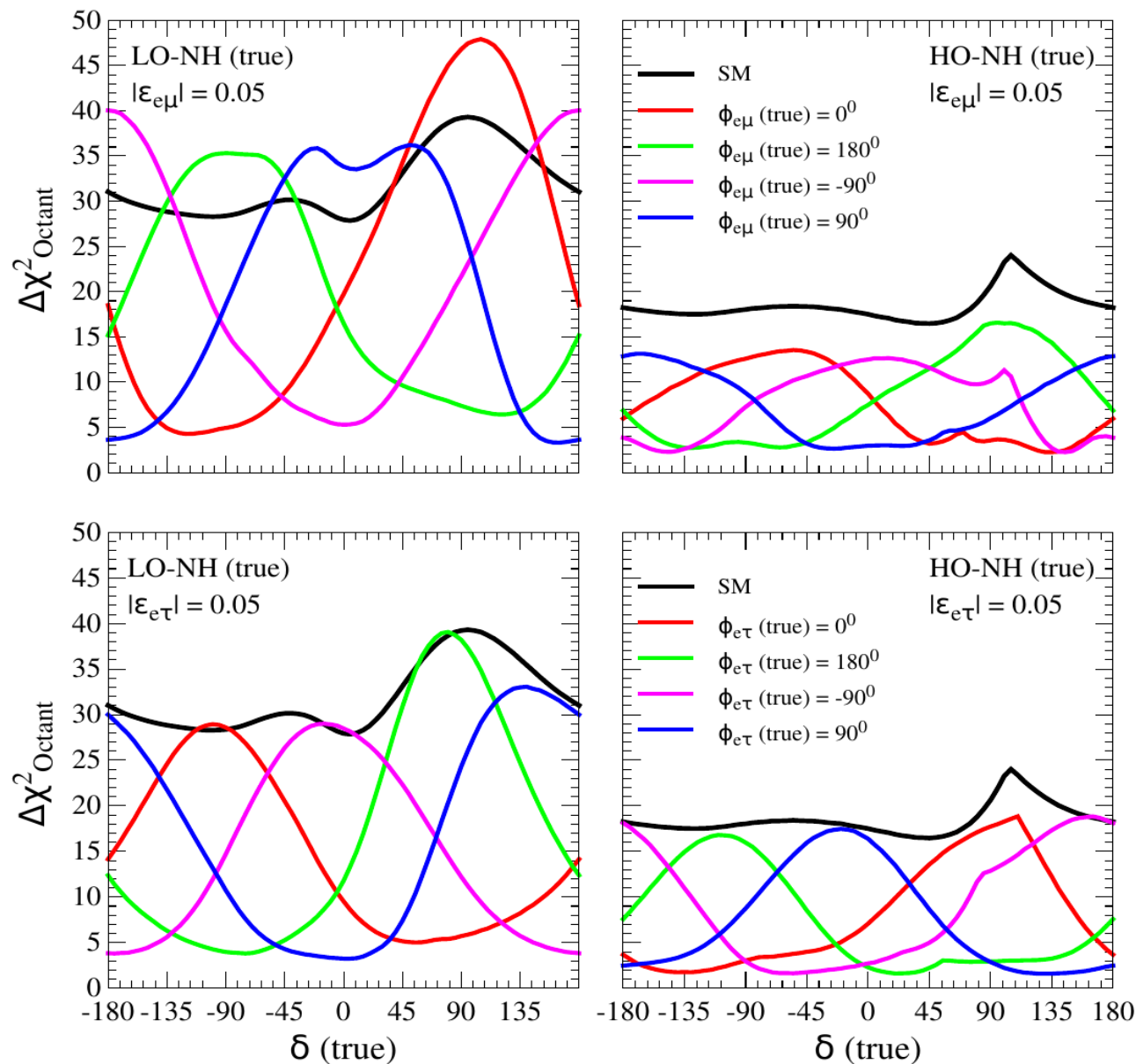


Spectral analysis helps to get good hierarchy sensitivity

Spectral analysis does not help much!

Phys.Lett. B762 (2016) 64-71 by Agarwalla, Chatterjee, and Palazzo

A good sensitivity to an octant means if an experiment excludes the wrong octant at certain confidence level, provided the true data is generated with the right octant.

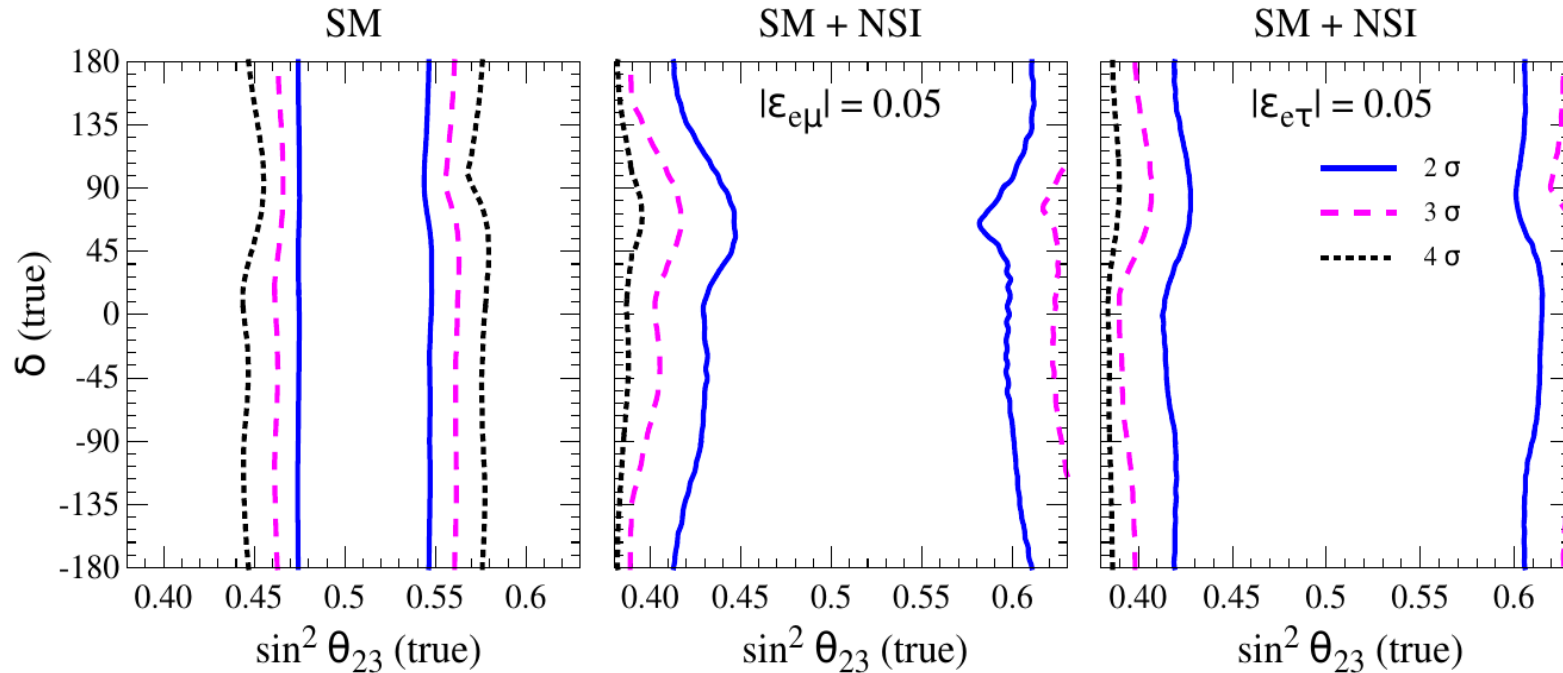


$\sin^2 \theta_{23}(\text{true}) = 0.42(0.58)$   
as LO (HO)

Sensitivity in presence of NSI goes  
down in compare to SM

$$\Delta\chi^2_{\text{Octant}} = \chi^2_{\text{test Octant}} - \chi^2_{\text{true Octant}}$$

## Octant sensitivity in the full parameter space of $[\sin^2 \theta_{23}, \delta]$ (true) plane

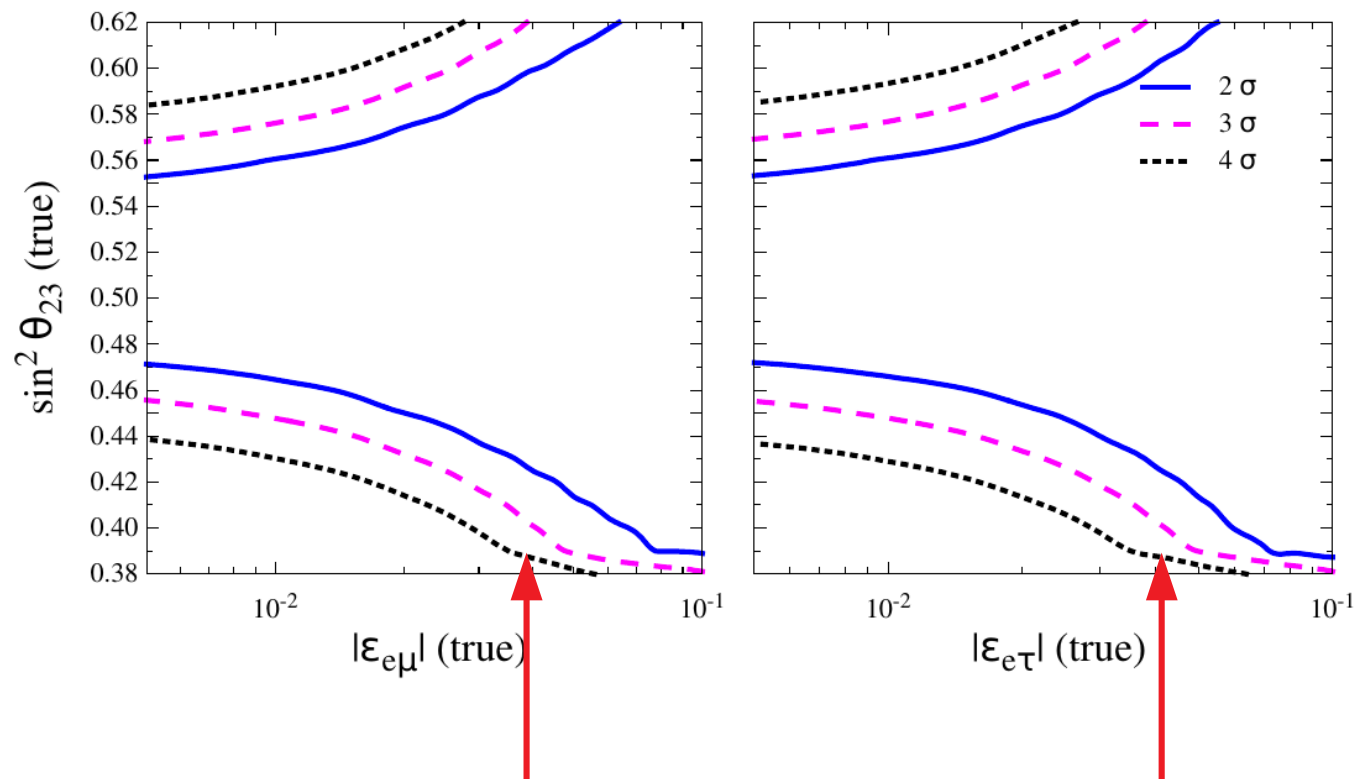


$$\Delta P_2 = B [\cos(\delta^{\text{HO}} + \phi^{\text{HO}}) - \cos(\delta^{\text{LO}} + \phi^{\text{LO}})] \pm C [\cos(\Delta + \delta^{\text{HO}} + \phi^{\text{HO}}) - \cos(\Delta + \delta^{\text{LO}} + \phi^{\text{LO}})]$$

extra degree of freedom

In SM+NSI, the sensitivity to the octant of  $\theta_{23}$  gets completely lost.

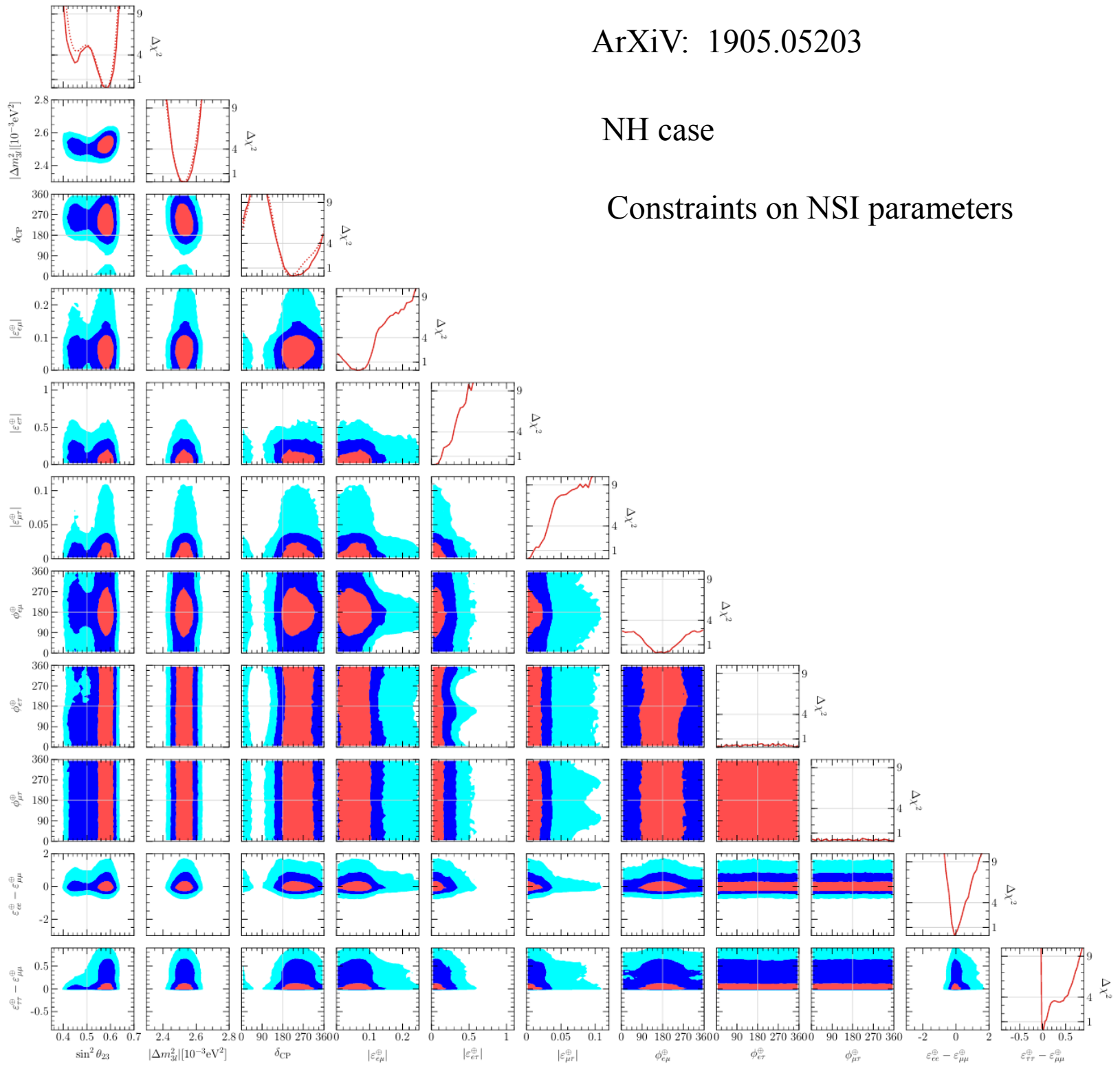


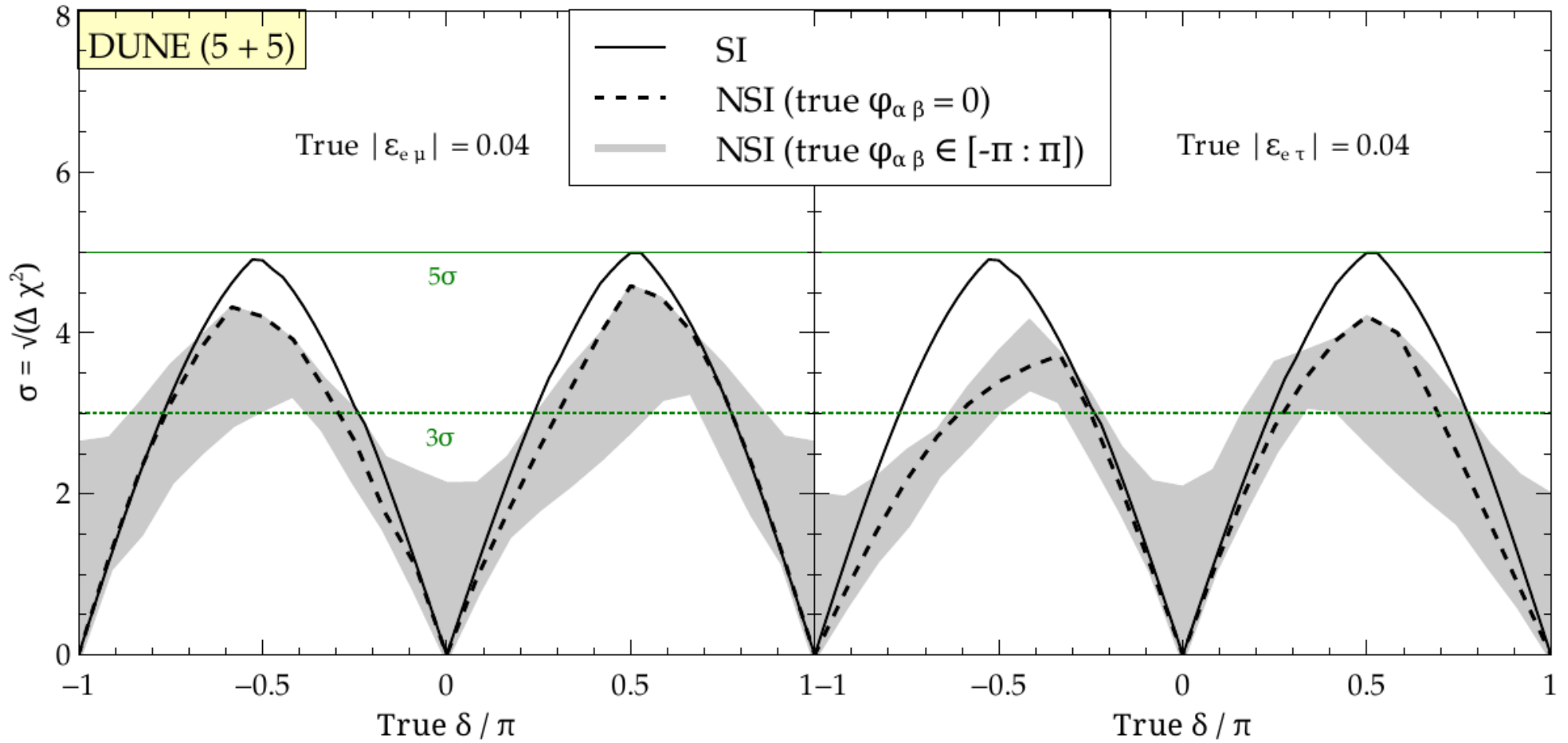


Even a small magnitude of NSI can spoil the sensitivity of distinguishing the two octants of  $\theta_{23}$

NH case

Constraints on NSI parameters

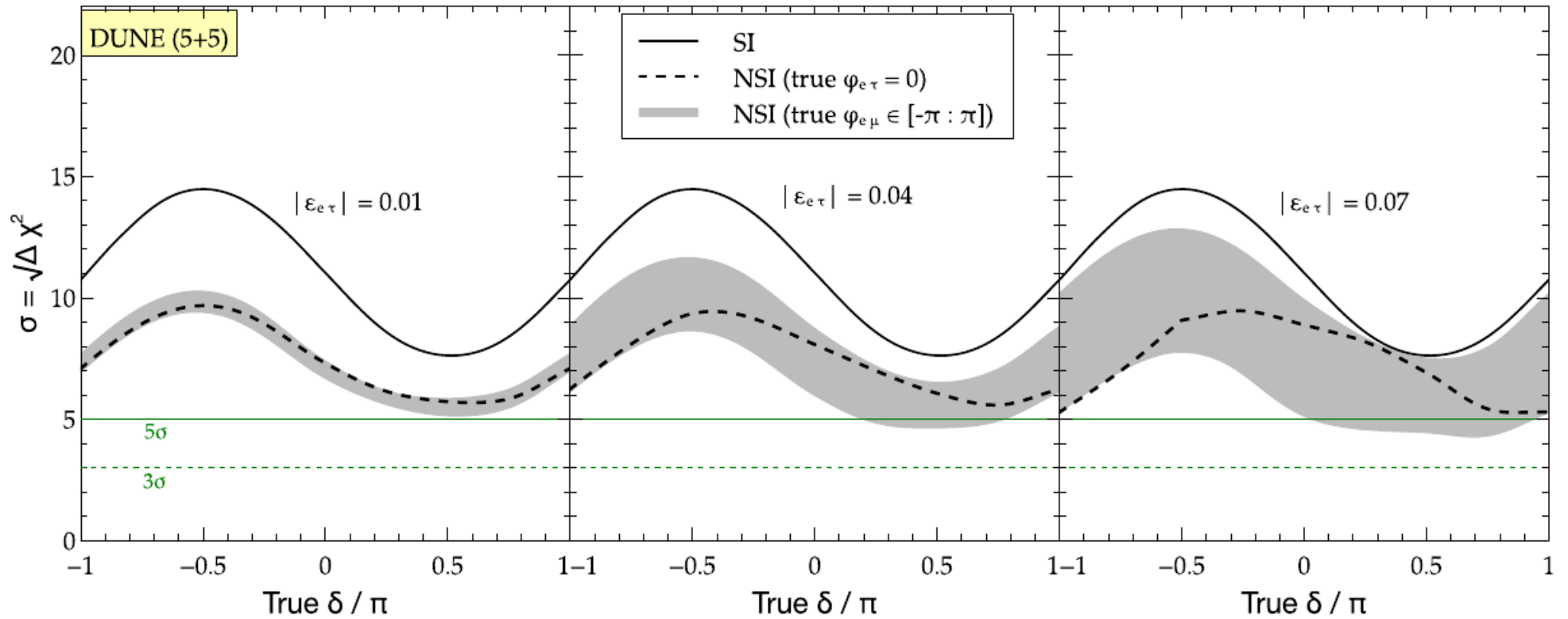




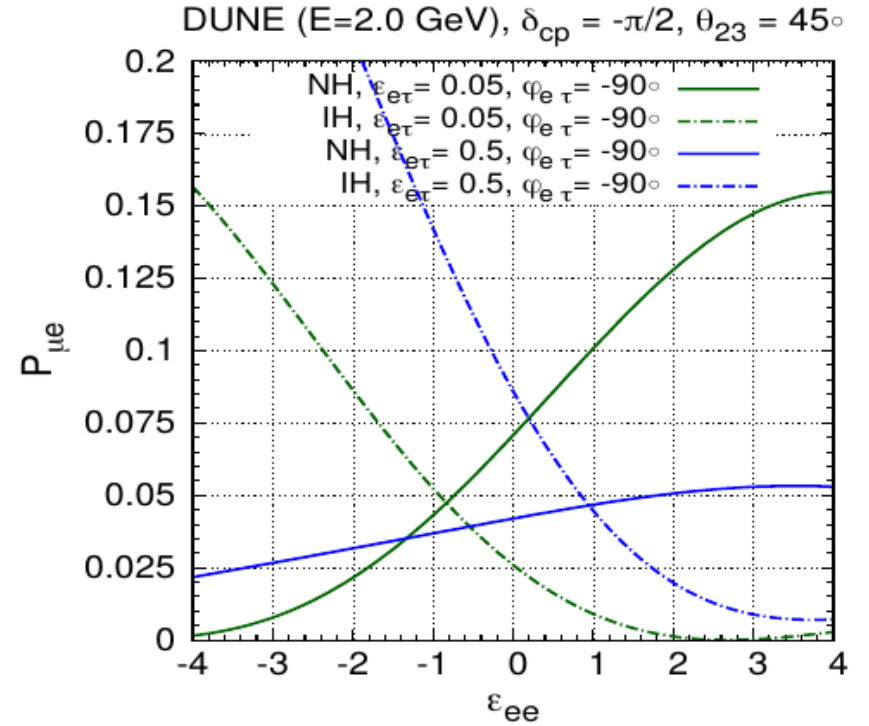
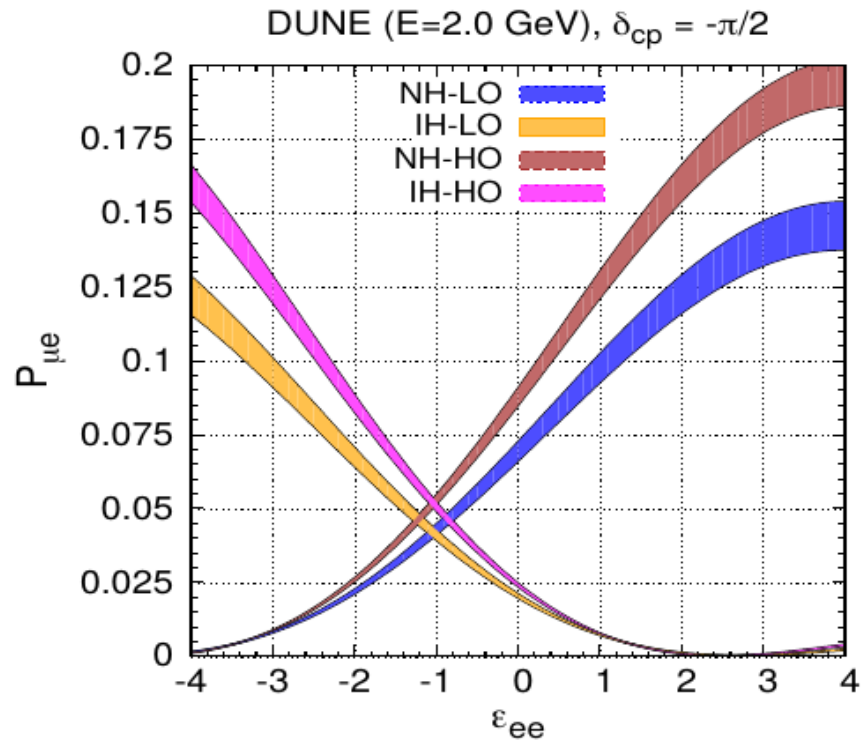
NSI can have drastic effect on CPV measurement

1510.08261 Masud, A Chatterjee, and Mehta

# Impact of NSI on MH determination

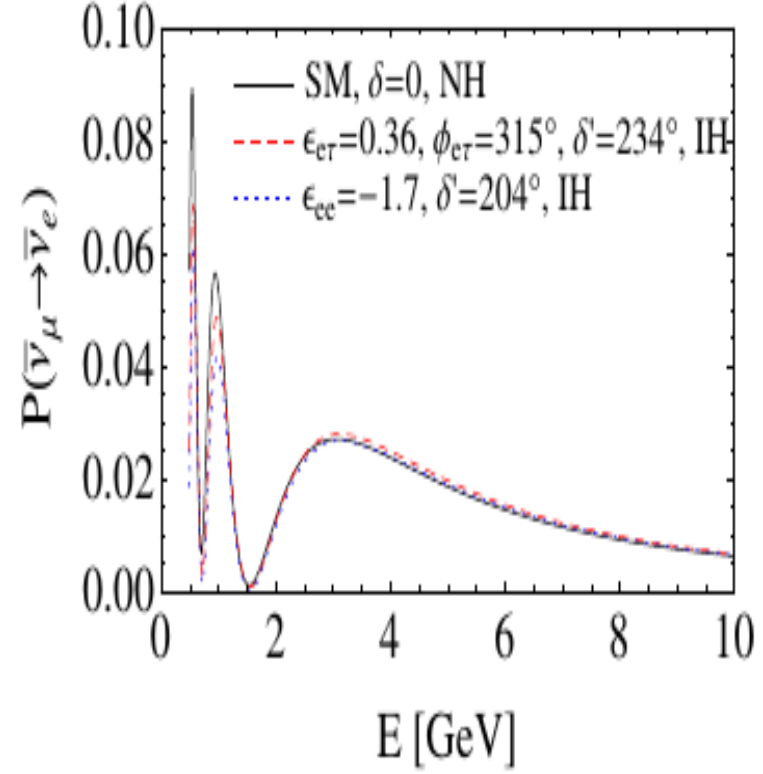
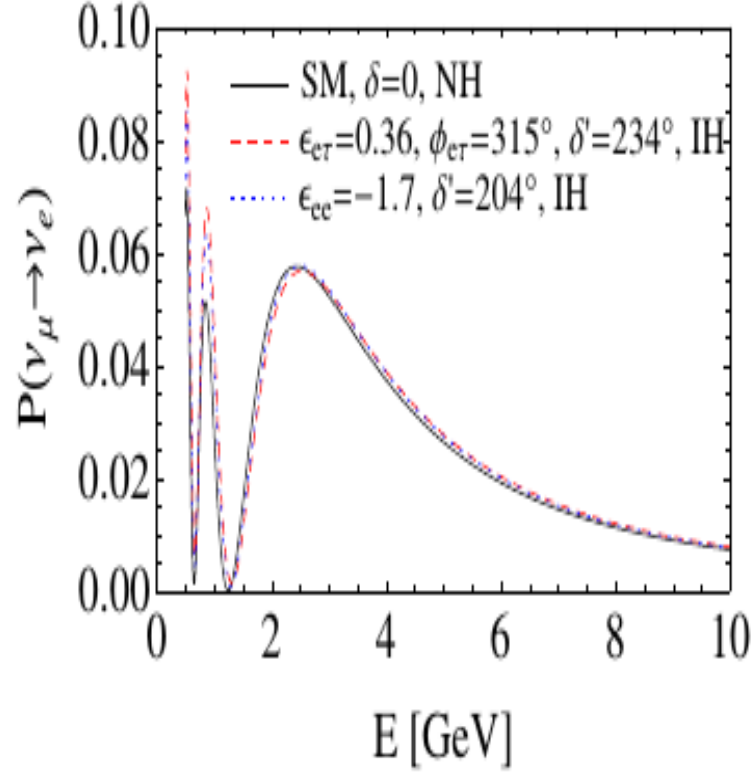


ArXiv: 1606.05662    Masud, and Mehta



MH sensitivity can get seriously impacted

ArXiv: 1612.00784 Deepthi, Goswami, and Nath



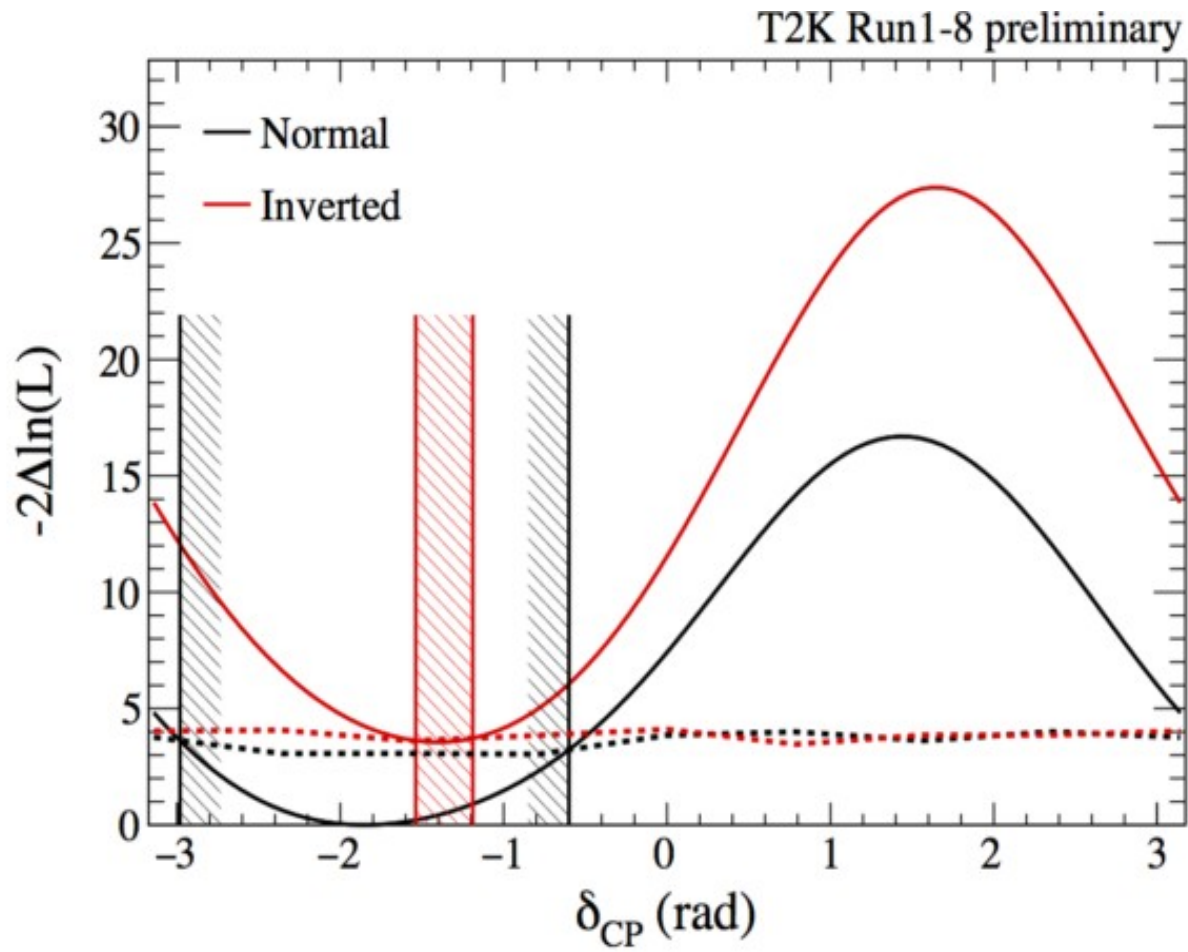
ArXiv: 1601.00927 Liao, Marfatia, and Whisnant

There are many more.....!

## Conclusion

- We have investigated the impact of NSI on the reconstruction of octant of  $\theta_{23}$  in the next generation LBL experiment DUNE.
- We have shown that in presence of NSI, a new interference term that enters into the  $\nu_{\mu} \rightarrow \nu_e$  transition probability can perfectly mimic a swap of the octant of  $\theta_{23}$  and as a result the sensitivity towards the resolution of octant of  $\theta_{23}$  may go to very low confidence level.
- ◆ This result has now become more important in the light of recent T2K and NOvA data which indicate towards the non maximal value of  $\theta_{23}$ .
- ◆ We hope that the analysis performed in these papers may give deep insight in exploring this new type of interactions.
- It remains to be seen whether any other kinds of experiments can lift or alleviate these kind of degeneracies.





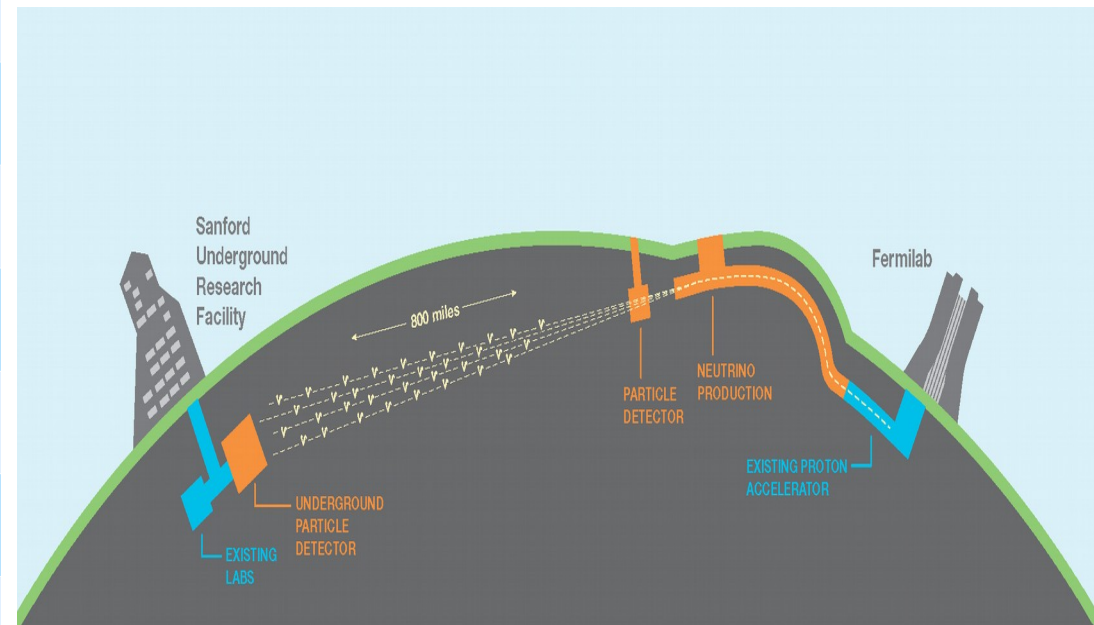
**T2K result of MH and CPV indication at 95% C.L.**

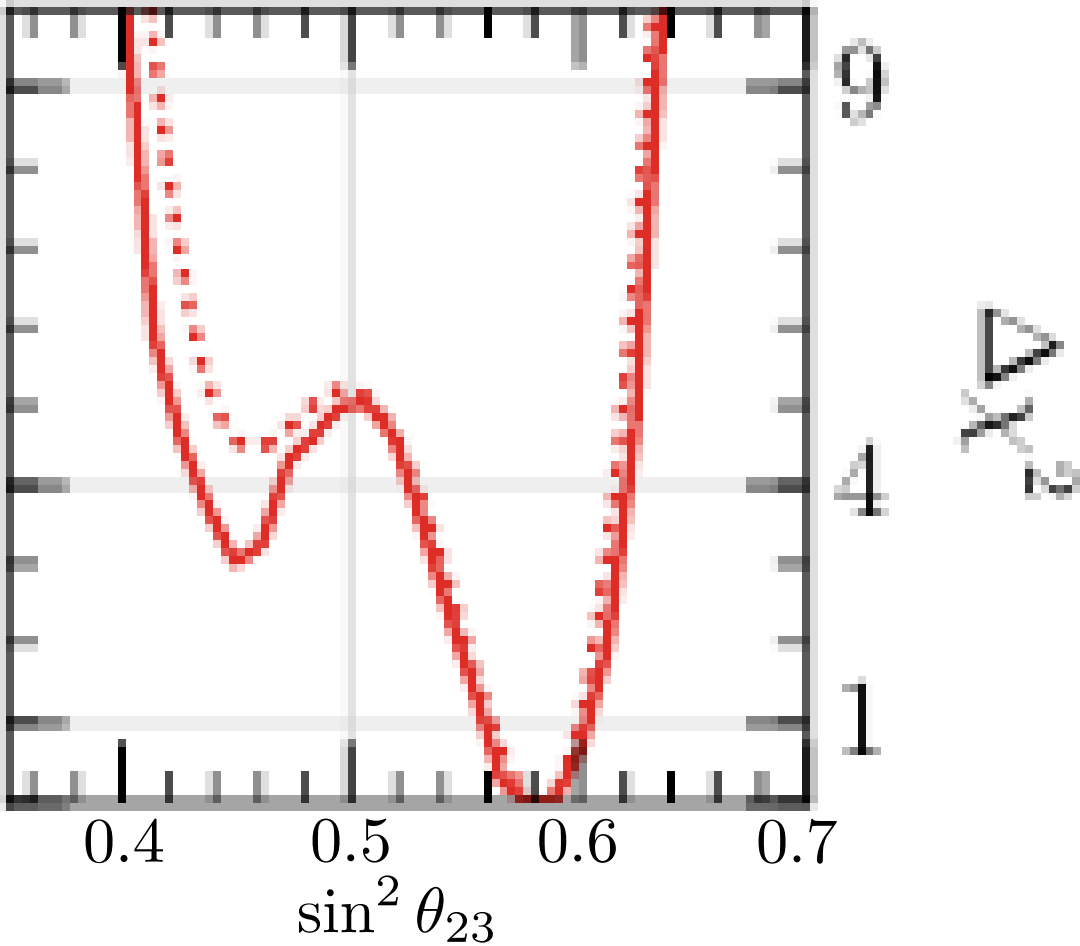
## Brief description of Long-Baseline Set-up

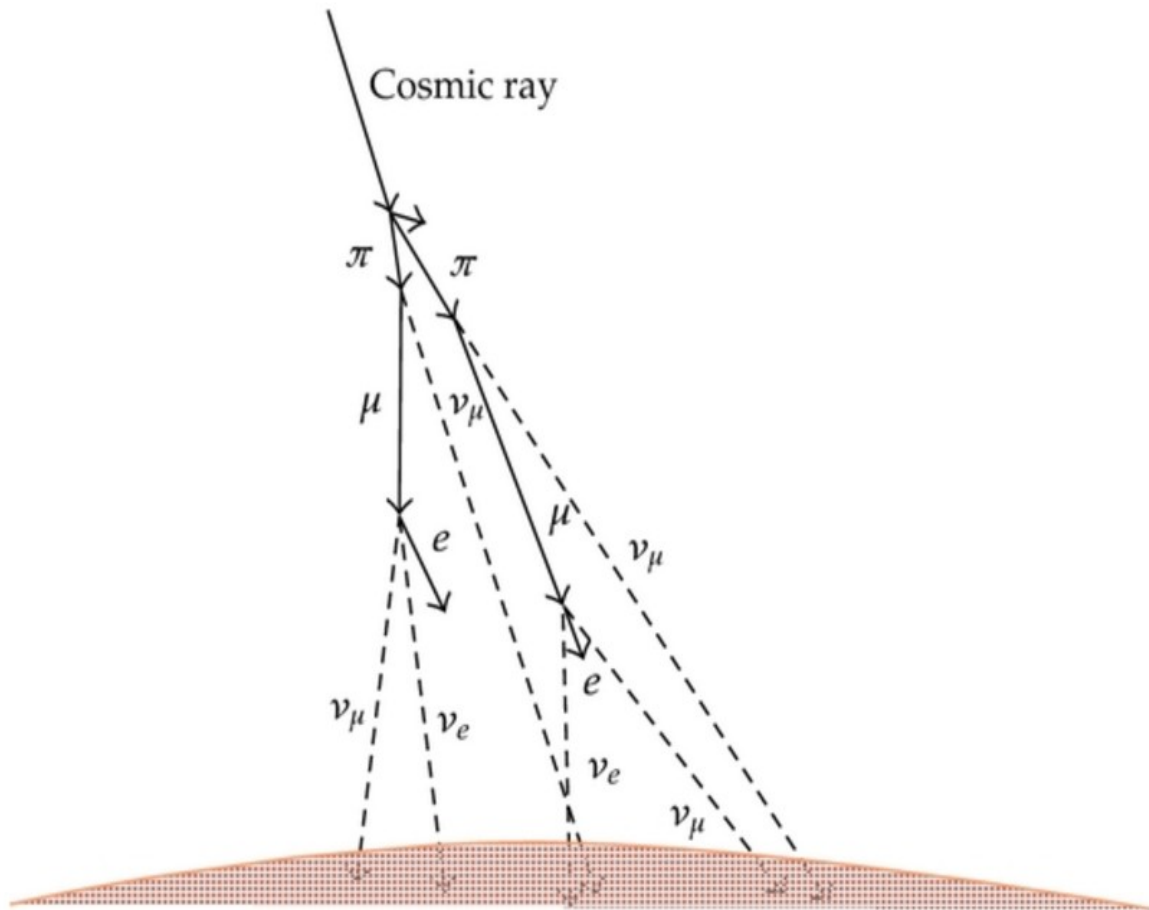
We have mainly considered the DUNE set-up to explore the impact of this new physics.

### DUNE (Fermilab to South Dakota)

Baseline	1300 KM
Detector mass	35 Kt
Run time	5 yrs + 5 yrs
Proton Energy	120 GeV
Beam Power	708 KW
Total POT / yr	$6 \times 10^{20}$
Signal app. error	5%
Signal disapp. error	5%
Background app. error	5%
Background disapp. error	5%







# Definition of $\chi^2$ function (implemented in GLoBES software)

$$\chi^2(\omega^{\text{true}}, \lambda^{\text{true}}; \omega^{\text{test}}, \lambda^{\text{test}}, \xi_s, \xi_b) = \min_{\{\xi_s, \xi_b\}} \left[ 2 \sum_{i=1}^n (\tilde{y}_i - x_i - x_i \ln \frac{\tilde{y}_i}{x_i}) + \frac{\xi_s^2}{\sigma_{\xi_s}^2} + \frac{\xi_b^2}{\sigma_{\xi_b}^2} \right]$$

$$\tilde{y}_i(\{\omega^{\text{test}}, \lambda^{\text{test}}\}, \{\xi_s, \xi_b\}) = N_i^{\text{pr}}(\{\omega^{\text{test}}, \lambda^{\text{test}}\})[1 + \pi^s \xi_s] + N_i^{\text{b}}(\{\omega^{\text{test}}, \lambda^{\text{test}}\})[1 + \pi^b \xi_b]$$

**Test events in ith reconstructed energy bin**

$\pi^s, \pi^b$  are systematic errors on signal and background events

$\xi_s, \xi_b$  are nuisance parameters, known with some accuracy  $\sigma_{\xi_s}$  and  $\sigma_{\xi_b}$  respectively

$x_i = N_i^{\text{obs}} + N_i^{\text{b}}$  True events in ith reconstructed energy bin

$$\chi_{\text{total}}^2 = \sum_{\text{channel}} \chi^2(\omega^{\text{true}}, \lambda^{\text{true}}; \omega^{\text{test}}, \lambda^{\text{test}}, \xi_s, \xi_b)$$

$$\chi_{\text{total}}^2 = \chi_{\nu_\mu \rightarrow \nu_e}^2 + \chi_{\nu_\mu \rightarrow \nu_\mu}^2 + \chi_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}^2 + \chi_{\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu}^2 \quad \text{For our case}$$

$$\Delta\chi_{\text{min}}^2 = \min_{\{\omega, \lambda\}} \left[ \sum_{\text{channel}} \chi^2(\omega, \lambda, \xi_s, \xi_b) \right] \quad \text{Final result}$$

Now, if we solve the above equation, we get the probability of oscillation from one flavor to another flavor with neutrino energy  $E$  and baseline  $L$ , given as

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re} (U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) + 2 \sum_{i>j} \text{Im} (U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin 2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right)$$

Where,  $\Delta m_{ij}^2 = m_i^2 - m_j^2$